An analytical measure of market underreaction to earnings news#

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Abstract

Prior studies have provided a number of possible explanations for delayed market reactions to earnings announcements. However, there has been relatively little effort to predict the magnitude of the post-earnings announcement drift (PEAD). We show that the squared correlation coefficient ($\rho^2$) between order imbalance and earnings surprise determines the magnitude of market underreaction to earnings surprises and $PEAD = k \cdot \rho^2$, where $k$ is the information content of earnings. We discuss several testable implications of our analytical results, including a model-implied measure of information asymmetry that arises from the differential information processing ability of traders.

JEL Classification Code: G14

Key words: Strategic trading, Information asymmetry, Information precision, Liquidity demander, Liquidity provider, Order imbalance, Information content, Price impact
1. Introduction

Asset price reflects fundamental (intrinsic) value through the trading of informed investors. In this study, we develop an analytical model to examine trading and price reactions to corporate announcements when some traders have better abilities to process the information contained in these announcements than other traders. Although we describe our model in the context of corporate earnings announcements and analyze immediate and delayed price reactions to these announcements, our model is applicable to the analysis of trading/price reactions to other types of corporate announcements.

Our model differs from the strategic trading model of Kyle (1985) in that our model analyzes price reactions to corporate announcements that are subject to different interpretations between the liquidity demander and the liquidity supplier due to their differential information processing abilities, while Kyle’s model focuses on the market maker’s rational pricing in the presence of strategic informed traders. In our model, the liquidity demander uses both the public and private information that are extracted from public earnings announcements while the liquidity supplier uses only the public component of earnings information. In contrast, the strategic trading in Kyle (1985) is based only on private information. This model feature enables us to decompose the total information content of earnings into earning response coefficient (ERC) and post-earnings announcement drift (PEAD).

Prior studies have provided a number of possible explanations for delayed market reactions to earnings announcements.\textsuperscript{1} However, there has been relatively little effort to predict the magnitude of PEAD. Our study develops an analytical metric of market underreaction to earnings information and use it to predict the magnitude of PEAD. Information asymmetry

\textsuperscript{1} These explanations include omitted risk factors (Fama, 1991, 1998; Sadka, 2006), mispricing (Bernard and Thomas, 1989, 1990; Kang, Khurana, and Wang, 2017; Engelberg, Mclean, and Pontiff, 2018; Akbas, Jiang, and Koch, 2019), data mining (Fama, 1998), investor inattention (Hirshleifer et al., 2009, 2011; Mian and Sankaraguruswamy, 2012), and disposition effects (Shefrin and Statman, 1985; Odean, 1998; Frazzini, 2006; Barberis and Xiong, 2009).
plays an important role in trading and pricing. However, information asymmetry is difficult to measure because private information is generally unobservable. Our study contributes to the literature also by developing a model-implied measure of information asymmetry that arises from differential information processing abilities between the liquidity demander and supplier.2

A stream of research explains PEAD based on trading costs and information uncertainty. For example, the transaction cost hypothesis posits that trading costs deter informed traders from aggressively arbitraging pricing errors at the time of earnings announcements (Bhushan, 1994; Ke and Ramlingegowda, 2005; Sadka, 2006; Ng et al., 2008; Chordia et al., 2009). The information risk hypothesis explains that information risk or information uncertainty deters investors from reacting fully to the information in earnings announcements, creating initial underreaction and subsequent PEAD (Mendenhall, 2004; Zhang, 2006; Garfinkel and Sokobin, 2006; Francis et al., 2007, Zhang, Cai, and Keasey, 2013). Our paper extends this strand of research by providing an analytical model of PEAD and discussing the unique empirical implications of the model that have not been explored in the literature.3

To incorporate information asymmetry in an analytical framework, we employ a model in which some traders have better information processing skills than others, which gives rise to information asymmetry between traders. We invoke the assumption that some traders better utilize earnings news than others from the fact that (1) a typical earnings announcement

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2 Prior research suggests that earnings announcements increase information asymmetries among traders. Kim and Verrecchia (1994) maintain that earnings announcements provide information that allows some traders to make better judgments about a firm’s performance than others, increasing information asymmetries among traders during the earnings announcement period. Subsequent studies find evidence that is generally consistent with this prediction. [See, e.g., Lee, Mucklow, and Ready (1993), Coller and Yohn (1997), Yohn (1998), and Bhat and Jayaraman (2009).] Kaniel, Liu, Saar, and Tifman (2012) show that the private-information-based trading by individual investors around earnings announcements accounts for about half of the abnormal returns on and after earnings announcement dates. Recently, Back, Crotty, and Li (2018) find evidence that the extent of informed trading after earnings announcements is greater than that before earnings announcements. While Back, Crotty, and Li (2018) develop an information asymmetry based on availability of private information, our information asymmetry is based on the differential processing of public information (i.e., earnings news).

3 Although prior studies have examined the effect of information asymmetry and information risk on the market reaction to earnings announcements, there has been no formal analytical treatment of the issue. As a result, prior research (e.g., Zhang, Cai, and Keasey, 2013) does not make a clear distinction between fundamental information uncertainty and information asymmetry. Our study sheds further light on the issue using an analytical model.
contains a large amount of information and data that are subject to differential interpretations and (2) there are multiple sources of earnings forecasts with different information. The sheer size and diverse sources of information about earnings news are likely to result in processing-based information asymmetries between traders.

Our model has three types of traders: the liquidity provider, the liquidity demander, and the utilitarian trader. The liquidity provider furnishes liquidity (immediacy) to the other traders by offering a price quote based on partial earnings information and the total order received from both the liquidity demander and the utilitarian trader. The liquidity demander strategically submits (market) orders based on full earnings information and trades at the quote set by the liquidity provider. Finally, the utilitarian trader trades for non-informational reasons.

We define the information asymmetry as the ratio between the precision of the information used by the liquidity demander exclusively and the precision of the information used by both the liquidity demander and the liquidity provider. Hence, information asymmetry increases with the precision of the information used exclusively by the liquidity demander and decreases with the precision of the information used by both the liquidity demander and provider.

In our model, price impounds earnings information in two different ways. Price fully and instantly impounds the earnings information used by both the liquidity demander and the liquidity provider. This portion of earnings information is unrelated to either the liquidity demander’s order size or the total order imbalance because no market participants can generate a profit from this information. The portion of earnings information used only by the liquidity

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4 See, for example, Apple’s financial results for its fiscal 2018 third quarter ended on June 30, 2018 available at the following website: https://www.apple.com/newsroom/2018/07/apple-reports-third-quarter-results/. There were more than 30 analysts providing earnings forecasts for Apple in the third quarter of 2018.

5 An important necessary condition for our model is that the liquidity provider misses a piece of the pricing implications of announced earnings for some (rational or irrational) reasons and there is no alternative way for the liquidity provider to learn and incorporate the information he misses into price at the time of the earnings announcement except through his self-protection mechanism against the adverse selection problem he faces.
demander, however, gets into price through his strategic demand order. If the news is good (bad) the liquidity demander submits a buy (sell) order, and the liquidity provider raises (lowers) his price quote upon receiving the order. This portion of earnings news gets into price only partially because the liquidity provider cannot perfectly infer the earnings used exclusively by the liquidity demander from the total order flows received. When there is a higher degree of information asymmetry, a larger portion of earnings information gets into price through the liquidity demander’s order, and thus a smaller portion of earnings information gets into price at the time of announcement (i.e., a smaller ERC) and a larger portion gets into price during the post-earnings announcement period (i.e., a larger PEAD).

We define the degree of market underreaction to earnings announcements as the portion of the earnings news that the stock price fails to impound instantly. In our model, the underreaction increases with the information asymmetry but decreases with the information content of earnings. The information content of earnings in our model is the total stock price reaction to earnings surprises, and it increases with the precision of the earnings information released at the announcement relative to the precision of prior belief about earnings.

The liquidity demander’s strategic exploitation of information processing advantage leads to a positive correlation coefficient ($\rho$) between order imbalance and earnings surprise. As this information asymmetry increases, more earnings information gets into price through the liquidity demander’s order and thus $\rho$ increases. We show that $\rho^2$ is the degree of market underreaction and use it as our analytical measure of market underreaction. Since the underreaction is the direct cause of PEAD, $\rho^2$ is a predictor of PEAD, and the size of PEAD depends on both $\rho^2$ and the information content of earnings. Specifically, we show that

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6 Lee (1992) shows that quarterly earnings announcements with good (bad) news relative to the Value Line Investment Survey forecast trigger brief, but intense, buying (selling) in large trades. Bhattacharya (2001) and Battalio and Mendenhall (2005) show that small traders tend to trade in response to random-walk earnings forecast errors while large traders tend to trade in response to analysts' earnings forecast errors.
$PEAD = k \cdot \rho^2$, where $k$ is the information content of earnings.

To the best of our knowledge, this is the first paper that provides an analytical model of market underreaction to earnings announcements that results from strategic informed trading that arises from the processing-based information asymmetry.\(^7\) Another important contribution of our study is that we provide a unifying framework for analyzing the initial and subsequent market reaction to earnings announcements. A number of previous studies analyze the determinants of $ERC$ and a large literature tries to understand the causes and consequences of $PEAD$.\(^8\) However, there has been little attempt to understand the interrelatedness of $ERC$ and $PEAD$ within a unified analytical framework. The joint analysis of $ERC$ and $PEAD$ should prove useful because $ERC$ and $PEAD$ span the total information content of earnings. Our study provides such a framework.

We organize the rest of the paper as follows. Section 2 explains our model setup. Section 3 shows how the price impact of a trade is related to the information content of earnings and the information asymmetry. Sections 4 and 5 derive analytical expressions for $ERC$, $PEAD$, and $\rho$. Section 6 discusses some empirical implications of our analytical results. Finally, we provide a brief summary and concluding remarks in Section 7.

### 2. The model

This section introduces our analytical model in which we define information

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\(^7\) A common explanation for mispricing in the securities market is the limits of arbitrage. That is, arbitrage trading corrects mispricing only up to the point where the marginal benefit justifies the cost of arbitrage trading (e.g., Grossman and Stiglitz, 1980; Shleifer and Vishny, 1997). Therefore, it is the cost of arbitrage, not the intensity of arbitrage trading, that determines the magnitude of mispricing. This line of thought assumes that the market is perfectly competitive in the long run and the marginal arbitrageur is indifferent between trading and non-trading. However, the market does not always become perfectly competitive soon after the firm makes a public announcement. This is because the complete extraction of the value-relevant information in a public announcement requires efforts, skills, and time, and is costly for most of market participants (Hirshleifff and Teoh, 2003). If only a few investors were able to immediately and completely extract the pricing implication of the announced information, they would capitalize on it by strategically exercising their trading options. This strategic trading is exactly what generates the slow price adjustment and this mechanism gives rise to an association between order imbalance and $PEAD$ in our model.

\(^8\) See, e.g., Collins and Kothari (1989).
asymmetry and the information content of earnings. There are three types of traders in our model: the liquidity demander who fully utilizes earnings information at the time of earnings announcements, the liquidity provider who posts price quotes based on the partial earnings information, and the utilitarian trader who trades for non-informational reasons.

In the real market setting, both limit-order traders and market makers play the role of the liquidity provider in our model. They establish their price quotes based on the information available to them, and other traders buy and sell at their quotes. Because the role of market makers (limit-order traders) has declined (increased) significantly in recent years, we use the generic term ‘liquidity providers’ in our study.⁹ Traders who consume (or demand) liquidity by submitting market orders play the role of the liquidity demander in the model. They are aggressive traders who typically have informational advantages over other market participants (e.g., limit order traders and market makers) and strategically trade at prices set by the liquidity provider. Lastly, those traders who submit buy or sell orders for non-informational reasons play the role of the utilitarian trader in our model.

Consider a market with one risky asset (the firm) and a riskless bond. The final payoff from one unit of the risky asset is \( \bar{u} \), which is normally distributed with mean 0 and precision (inverse of variance) \( h \). There are three time periods. Period 1 is the pre-earnings-announcement period in which no traders possess any information other than the prior distribution of \( \bar{u} \). Period 2 is the earnings announcement period. The critical assumption in this model is that the earnings signal \( \bar{x} \), which arrives in period 2, consists of two noisy information signals, \( \bar{y} \) and \( \bar{z} \). Signal \( \bar{y} = \bar{u} + \bar{h} \) is a signal used by both the liquidity provider and the

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⁹ The NYSE implemented a new market model in 2008 that emphasizes speed, technology, and less human intermediation. The new model replaced specialists with designated market makers who have lesser obligations than their predecessors. NASDAQ amended its Rule 4613(c) in 2007 to eliminate the requirement that dealer quotes must be “reasonably related to the prevailing market.” Limit order traders play a critical role in the price discovery process on the NYSE and NASDAQ after the U.S. Securities and Exchange Commission introduced the limit order display rule in 1997 that requires market makers and specialists to reflect public limit orders in their quotes when the orders are better than their own quotes. This rule ensures that the general public competes directly with market makers and specialists in the price discovery process.
liquidity demander and \( \tilde{z} = \tilde{u} + \tilde{e} \) is a signal used only by the liquidity demander, where \( \tilde{y} \) and \( \tilde{e} \) are both normally and independently distributed with mean zero and precision \( m \) and \( s \), respectively. One may interpret \( \tilde{y} \) as the earnings signal that is used by both the liquidity provider and the liquidity demander and \( \tilde{z} \) as “soft” information contained in the earnings announcement that is used only by the liquidity demander. A typical earnings report comprises a large amount of information and multiple forecasts usually precede each earnings announcement. In the context of our model, \( \tilde{x} \) could be viewed as a composite measure of the earnings surprise implied by all the information contained in earnings reports and earnings forecasts. We define the earnings signal \( \tilde{x} \) as the precision-weighted average of \( \tilde{y} \) and \( \tilde{z} \):

\[
\tilde{x} = \frac{m}{m+s} \tilde{y} + \frac{s}{m+s} \tilde{z}.
\]

In period 3 the final payoff \( \tilde{u} \) is determined and consumption occurs. In period 2, the liquidity demander submits an order of size \( \tilde{d} \) to the liquidity provider based on his available information. The utilitarian trader also submits an order of size \( \tilde{\ell} \) to the liquidity provider based on his random needs. We assume that \( \tilde{\ell} \) is normally and independently distributed with mean 0 and variance \( L \). The two orders are batched together and the liquidity provider only observes the order imbalance \( \tilde{\omega} = \tilde{d} + \tilde{\ell} \). The liquidity provider conjectures that the liquidity demander’s order \( \tilde{d} \) is a linear function of the two information signals in the form of \( \tilde{d} = \beta \tilde{y} + \gamma \tilde{z} \) and sets the price as the expectation of \( \tilde{u} \) conditional on the available information at the time. That is, \( \tilde{P}_2 = E[\tilde{u}|\tilde{y}, \tilde{\omega}] \). The liquidity demander strategically decides his order based on a conjecture that the price chosen by the liquidity provider is a linear function of the information available to the liquidity provider at the time. That is, we can express the liquidity demander’s conjecture as \( \tilde{P}_2 = \alpha \tilde{y} + \lambda \tilde{\omega} \).

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10 Hong and Stein (1999, p. 2165) show that prices underreact to private information that gradually diffuses across population of bounded rational investors (i.e., news watchers and momentum traders). When applied their model to public news, they state that the conversion of public news into a judgement about value “requires some other, private, information (e.g., knowledge of the stochastic process generating earnings).”
In this model it is straightforward to show that there is a unique equilibrium in which:

\[ \tilde{P}_1 = 0, \]
\[ \alpha = \frac{m}{h+m}, \]
\[ \lambda = \frac{1}{2} \sqrt{\frac{s}{(h+m)(h+m+s)L}}, \]
\[ \beta = -\frac{ms}{2\lambda(h+m)(h+m+s)} = -\frac{m^2sL}{\sqrt{(h+m)(h+m+s)}}, \]

and

\[ \gamma = \frac{s}{2\lambda(h+m+s)} = \sqrt{\frac{(h+m)sL}{h+m+s}}. \]

Thus, the liquidity demander’s order size and the liquidity provider’s price are determined as

\[ \tilde{d} = \frac{s}{2\lambda(h+m+s)} \left[ \tilde{z} - \frac{m}{h+m} \tilde{y} \right] \]

and

\[ \tilde{P}_2 = \frac{m}{h+m} \tilde{y} + \lambda (\tilde{d} + \tilde{\eta}) \]
\[ = \frac{m(2h+2m+s)}{2(h+m)(h+m+s)} \tilde{y} + \frac{s}{2(h+m+s)} \tilde{z} + \frac{1}{2} \sqrt{\frac{s}{(h+m)(h+m+s)L}} \tilde{\eta}, \quad (1) \]

respectively. The liquidity demander’s order \( \tilde{d} \) is proportional to the difference between \( \tilde{z} \), which only the liquidity demander uses, and the liquidity provider’s expectation of \( \tilde{z} \), \( \frac{m}{h+m} \tilde{y} \), based on his information. As a result, we have

\[ \text{Cov}(\tilde{y}, \tilde{\omega}) = \text{Cov}(\tilde{y}, \tilde{d} + \tilde{\eta}) = \text{Cov}(\tilde{y}, \tilde{d}) \]
\[ \propto \text{Cov} \left( \tilde{y}, \tilde{z} - \frac{m}{h+m} \tilde{y} \right) = \text{Cov} \left( \tilde{u} + \tilde{\eta}, \frac{h\tilde{u} - m\tilde{y}}{h+m} \right) = 0. \quad (2) \]

That is, in equilibrium the liquidity demander’s order size and also the total order imbalance are not based on, and thus are not correlated with, the earnings signal used by both the liquidity demander and provider.

We define information asymmetry as the ratio between the precision \((s)\) of the
information (\( \frac{s}{m+s} \)) used by only the liquidity demander and the precision (\( m+s \)) of the liquidity demander’s total information (\( \tilde{y} \) and \( \tilde{z} \)), i.e., \( \theta \equiv \frac{s}{m+s} \). Information asymmetry increases with the precision (\( s \)) of the information exclusively used by the liquidity demander and decreases with the precision (\( m \)) of the information used by both the liquidity demander and provider.

Finally, consider regressing the final payoff, \( \tilde{u} \), on earnings surprise, \( \tilde{x} \). We define the information content of earnings (i.e., informativeness of \( \tilde{x} \)) as the regression coefficient on \( \tilde{x} \), i.e., \( k \equiv \frac{\text{Cov}[\tilde{u}, \tilde{x}]}{\text{Var}[\tilde{x}]} \). When the sum of precisions (\( m+s \)) of the information released at the time of earnings announcement is high, \( \tilde{x} \) is close to \( \tilde{u} \) and \( k \) is close to 1. On the other hand, when the precision is low, \( \tilde{x} \) is not informative and \( k \) is close to 0. In Appendix A we show that \( k \equiv \frac{\text{Cov}[\tilde{u}, \tilde{x}]}{\text{Var}[\tilde{x}]} = \frac{m+s}{h+m+s} \). Note that \( h \) is an inverse measure of the inherent (or fundamental) uncertainty in earnings, which is likely to be determined by the firm’s business and operating risks. If the firm’s business and operating risks were low (i.e., \( h \) is large), the information content of earnings would be low even if its precision (i.e., \( m+s \)) is high because there is little uncertainty in the final payoff to begin with. In contrast, if the firm’s business and operating risks were high (i.e., \( h \) is small), the information content of earnings would be high even if its precision is low.

3. Price impact of a trade

Kyle (1985) shows that the price impact (\( \lambda \)) of a trade depends on both the variance (\( h^{-1} \)) of the liquidation value of the risky asset and the variance (\( L \)) of the quantity traded by noisy traders. To shed further light on the determinants of the price impact of a trade in a market where the precision of information differs between the liquidity demander and the liquidity provider, we obtain the following partial derivatives with respect to

\[
\lambda = \frac{1}{2} \sqrt{\frac{s}{(h+m)(h+m+s)L}}.
\]
\[
\frac{\partial \lambda}{\partial s} = \frac{1}{4} \left( \frac{s}{(h+m)(h+m+s)L} \right)^{-\frac{1}{2}} \left( \frac{h^2L^2+2hmL+m^2L}{h^4L^2+2h^3L^2m+h^3L^2s+L^2h^2m^2+L^2h^2ms} \right) > 0,
\]

\[
\frac{\partial \lambda}{\partial m} = \frac{1}{4} \left( \frac{s}{(h+m)(h+m+s)L} \right)^{-\frac{1}{2}} \left( \frac{-2shL-2smL-s^2L}{h^4L^2+2h^3L^2m+h^3L^2s+L^2h^2m^2+L^2h^2ms} \right) < 0,
\]

\[
\frac{\partial \lambda}{\partial h} = \frac{1}{4} \left( \frac{s}{(h+m)(h+m+s)L} \right)^{-\frac{1}{2}} \left( \frac{-2shL-2smL-s^2L}{h^4L^2+2h^3L^2m+h^3L^2s+L^2h^2m^2+L^2h^2ms} \right) < 0, \text{ and}
\]

\[
\frac{\partial \lambda}{\partial L} = \frac{1}{4} \left( \frac{s}{(h+m)(h+m+s)L} \right)^{-\frac{1}{2}} \left( \frac{-sh^2-2shm-s^2h-sm^2-s^2m}{h^4L^2+2h^3L^2m+h^3L^2s+L^2h^2m^2+L^2h^2ms} \right) < 0.
\]

Not surprisingly, the price impact of a trade increases with the precision \((s)\) of the information used exclusively by the liquidity demander, but decreases with the precision \((m)\) of the information used by both the liquidity demander and the liquidity provider. Similar to Kyle (1985), the price impact of a trade decreases with both the precision \((h)\) (inverse of variance) of the final payoff from the risky asset and the variance \((L)\) of the utilitarian trader’s order size.

From \(k \theta = \frac{s}{h+m+s} \cdot \frac{h}{h+m} = \frac{1-k}{1-k \theta} \) and \(k = \frac{\text{Cov}[ar{U}, \bar{X}]}{\text{Var}[\bar{X}]} = \frac{1}{h \cdot \text{Var}[\bar{X}]} \), we can further show that

\[
\lambda = \frac{1}{2} \sqrt{k \theta \left( \frac{1-k}{1-k \theta} \right) \frac{1}{hL}}.
\]

Hence, our analytical model provides a new insight that the price impact of a trade depends on both the information content of earnings \((k)\) and the information asymmetry \((\theta)\) between the liquidity demander and the liquidity provider, in addition to its previously recognized determinants (i.e., \(h\) and \(L\)).

4. ERC and PEAD as functions of information content and information asymmetry

In our model we introduce strategic informed trading as a new property of market reaction to an earnings announcement independent from the information content of earnings. Thus, the two factors, information asymmetry and information content, determine the impact of the earnings announcement on share price and PEAD. Consider regressing the price change in the earnings announcement period, \(\tilde{P}_2 - \tilde{P}_1\), on earnings surprise, \(\tilde{X}\). We derive in Appendix
A the following result for the earnings response coefficient (ERC):

\[
ERC[\bar{x}] = \frac{\text{Cov}[\bar{P}_2, \bar{x}]}{\text{Var}[\bar{x}]} = \frac{2m(h + m + s) + hs}{2(h + m)(h + m + s)} = k \cdot \frac{2-(1+k)\theta}{2-2k\theta}.
\]

(3)

In the last expression above, we use two ratio parameters, information content \((k)\) and information asymmetry \((\theta)\).\(^{11}\) If there is no information asymmetry between the liquidity provider and the liquidity demander, i.e., if \(\theta = 0\), ERC is equal to the information content of earnings, \(k\). If there is information asymmetry between the liquidity provider and the liquidity demander, i.e., if \(\theta > 0\), ERC is smaller than \(k\). In the extreme case of \(\theta = 1\), i.e., if the entire earnings information is asymmetrically utilized, ERC is equal to \(\frac{k}{2}\).

From equation (3) we have two comparative static results: \(\frac{\partial ERC}{\partial k} > 0\) and \(\frac{\partial ERC}{\partial \theta} < 0\). That is, ERC increases with the information content of earnings and decreases with information asymmetry. The following proposition summarizes these results:

**Proposition 1:** \(ERC \equiv \frac{\text{Cov}[\bar{P}_2, \bar{x}]}{\text{Var}[\bar{x}]} = k \cdot \frac{2-(1+k)\theta}{2-2k\theta}\) is increasing in \(k\) and decreasing in \(\theta\), where \(k\) is the information content of earnings, and \(\theta\) is the information asymmetry between the liquidity demander and the liquidity provider.

From the definition of \(k\) (i.e., \(k = \frac{m+s}{h+m+s}\)), the information content \((k)\) of earnings announcements increases with information risk (inverse of \(h\)). The positive relation between ERC and \(k\) is consistent with the information content hypothesis of Zhang, Cai, and Keasey (2013) that information risk reduces the informativeness of price, raises the relative importance of earnings announcements in the price discovery process, and thus increases the initial market reaction per unit of earnings surprise. Our study provides further insight that the information

\(^{11}\) The range of ERC is between 0 and 1 in our one period model. In a model where an earnings number represents a permanent stream, the ERC would be multiplied by \(1+1/r\), where \(r\) is the discount rate.
asymmetry between the liquidity demander and the liquidity provider is another determinant of the initial market reaction.

Now consider regressing the price change during the post-earnings announcement period, \(\bar{u} - \bar{P}_2\), on earnings surprise, \(\bar{x}\). We derive (in Appendix B) the following result for the post-earnings announcement drift (PEAD):

\[
PEAD \equiv \frac{\text{cov}[\bar{u} - \bar{P}_2, \bar{x}]}{\text{var}[\bar{x}]} = k - ERC = k \cdot \frac{(1-k)\theta}{2(1-k\theta)}
\] (4)

From equation (4), when the liquidity demander does not have an information advantage (i.e., \(\theta = 0\)), PEAD is zero. In this case, ERC reflects the entire information content of earnings (i.e., \(ERC = k\)) and thus there is no residual price adjustment after the earnings announcement period (i.e., \(PEAD = 0\)). When all earnings information gets into price through the liquidity demander’s order (i.e., \(\theta = 1\) and \(ERC = \frac{k}{2}\)), PEAD is equal to \(\frac{k}{2}\). It is easy to show that \(\frac{\partial PEAD}{\partial \theta} > 0\). The partial derivative of PEAD with respect to \(k\), however, is not monotonic because \(k\) differs from ERC when \(\theta\) is positive. We find that \(\frac{\partial PEAD}{\partial \theta} > 0\) if \(k(2-k\theta) < 1\), but \(\frac{\partial PEAD}{\partial \theta} < 0\) if \(k(2-k\theta) > 1\). The following proposition summarizes these results:

**Proposition 2:** PEAD \(\equiv \frac{\text{cov}[\bar{u} - \bar{P}_2, \bar{x}]}{\text{var}[\bar{x}]} = k \cdot \left[\theta \cdot \frac{1-k}{2(1-k\theta)}\right]\) is increasing in \(\theta\), where \(k\) is the information content of earnings and \(\theta\) is information asymmetry between the liquidity demander and the liquidity provider. PEAD is increasing in \(k\) if \(k(2-k\theta) < 1\), but decreasing in \(k\) if \(k(2-k\theta) > 1\).

It is useful to characterize the degree of market underreaction (DMU) to earnings announcement as the proportion of the information content of earnings that stock price fails to impound instantly. From equation (4), it is straightforward to show that this proportion is DMU \(\frac{\theta}{2} \cdot \frac{1-k}{(1-k\theta)}\). Note from equation (3) that \(ERC = k \cdot \left[\theta \cdot \frac{1-k}{(1-k\theta)}\right] = k \cdot (1 - DMU)\). Hence, \(1 - \left(\frac{\theta}{2} \cdot \frac{1-k}{(1-k\theta)}\right) = 1 - DMU\) represents the proportion of the information content of earnings
that is incorporated into stock price at the time of earnings announcement.

A simple comparative statics analysis shows that the degree of underreaction \((\text{DMU})\) is increasing in \(\theta\) but decreasing in \(k\). Figure 1a and Figure 1b show the relation between the degree of underreaction and \(k\) at different levels of \(\theta\) and the relation between the degree of underreaction and \(\theta\) at different levels of \(k\), respectively.\(^{12}\) The positive relation between the degree of underreaction and \(\theta\) shown in Figure 1b is generally steeper than the negative relation between the degree of underreaction and \(k\) for most values of \(k\) shown in Figure 1a. These results suggest that variation in the degree of underreaction is more likely driven by variation in information asymmetry than by variation in information content. The following corollary summarizes these results.

**Corollary 1:** The degree of market underreaction to earnings announcement, \(\frac{\theta}{2} \cdot \frac{1-k}{(1-k\theta)}\), is increasing in \(\theta\) but decreasing in \(k\), where \(\theta\) is the information asymmetry between the liquidity demander and the liquidity provider and \(k\) is the information content of earnings.

5. **Correlation (\(\rho\)) between order imbalance and earnings surprise and \(\text{PEAD}\)**

Now, consider the correlation coefficient between order imbalance and earnings surprise (\(\rho\)).\(^{13}\) We show in equation (2) that the correlation between order imbalance (\(\tilde{o}\)) and the part of earnings information (\(\tilde{y}\)) that both the liquidity demander and the liquidity provider use is zero. In contrast, we expect a positive correlation between order imbalance and the earnings information (\(\tilde{z}\)) used only by the liquidity demander because the intensity and direction of the information-based trading (and thus \(\tilde{o}\)) are a function of \(\tilde{z}\). Because \(\text{Corr} (\tilde{o}, \tilde{y}) = 0\), we expect the correlation between order imbalance (\(\tilde{o}\)) and the entire earnings

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\(^{12}\) The maximum possible value for the degree of underreaction is 0.5, and this is the case when \(\theta\) is equal to 1.

\(^{13}\) Chordia, Roll, and Subrahmanyam (2002) and Chordia and Subrahmanyam (2004) analyze the relation between order imbalance and stock returns.
information (i.e., $\bar{x} = (1 - \theta)\bar{y} + \theta \bar{z}$) to capture the extent to which a subset of market participants (i.e., the liquidity demander in our model) exploit the information processing advantage and the resulting information asymmetry among traders.

We show in Appendix A that the correlation coefficient between $\bar{x}$ and $\bar{\omega}$ can be expressed as:

$$\rho \equiv \text{Corr}[\bar{x}, \bar{\omega}] = \frac{\text{Cov}[\bar{x}, \bar{\omega}]}{\sqrt{\text{Var}[\bar{x}] \text{Var}[\bar{\omega}]}} = \frac{\theta s}{\sqrt{(h+s)(m+s)}} = \frac{\theta - k \theta}{2(1-k\theta)} \quad (5)$$

Note that equation (5) can be rewritten as follows:

$$\rho^2 = \frac{\theta(1-k)}{2(1-k\theta)} \quad (6)$$

and from equations (3) and (4) we have $\text{ERC} = k \cdot (1 - \rho^2)$ and $\text{PEAD} = k \cdot \rho^2$. That is, the degree of market underreaction is simply $\rho^2$.

Because $\rho^2$ is the coefficient of determination ($R^2$) in the regression of order imbalance on earnings surprise, the degree of underreaction is equal to the proportion of the variation in order imbalance that is due to the variation in earnings surprise. To see the economic intuition behind this result, note first that underreaction would occur only if there were informed trading because the liquidity provider fails to fully incorporate all information into price. If there were no informed trading, there would be no underreaction, and order imbalance would also be unrelated to earnings surprise because the uninformed traders’ order sizes ($\beta \bar{y}$ and $\bar{\ell}$) are unrelated to earnings surprise. Hence, no underreaction would be observed when there is zero correlation between order imbalance and earnings surprise. As informed trading increases, the degree of underreaction and the correlation between order imbalance and earnings surprise would increase simultaneously. These considerations suggest why the degree of underreaction increases with the proportion of the variation in order imbalance that is due to the variation in earnings surprise.

The following proposition summarizes the above results:
**Proposition 3:** \( ERC = k \cdot (1 - \rho^2), \) \( PEAD = k \cdot \rho^2, \) and \( \rho^2 = \frac{\theta (1-k)}{2(1-k \theta)}, \) where \( \rho^2 \) is the squared correlation coefficient between order imbalance and earnings surprise, \( k \) is the information content of earnings, and \( \theta \) is the information asymmetry between the liquidity demander and the liquidity provider.

Although there is a large body of literature analyzing the effect of earnings announcements on share prices, prior research does not explicitly recognize that the post-earnings announcement drift depends not only on the information content of earnings (\( k \)) but also on \( \rho^2 \), which is largely determined by the information asymmetry (\( \theta \)) between traders.\(^{14}\)

For instance, Zhang, Cai, and Keasey (2013) examine the effects of information risk and transaction costs on \( ERC \) and \( PEAD \). They show that the earnings announcements of firms with higher levels of information risk (i.e., lower \( h \) in our model) convey more information and thus result in higher \( ERC \) and that higher transaction costs decrease \( ERC \) and increase \( PEAD \). Our study differs from theirs in that earnings announcements invoke information asymmetry in our study whereas earnings announcements provide new information that reduces information risk in their study. Proposition 3 shows how to disentangle the effects of these two properties (i.e., information content and information asymmetry) of earnings announcements on the post-earnings announcement drift.

Finally, from \( \lambda = \frac{1}{2} \sqrt{\frac{k \theta (1-k)}{\lambda - k \theta} \frac{1}{h L}} \) and Proposition 3, we obtain the following result:

**Proposition 4:** \( PEAD = 4h \cdot L \cdot \lambda^2 \), where \( h \) is the precision (inverse of variance) of the final payoff from the risky asset, \( L \) is the variance of the utilitarian trader’s order size, and \( \lambda \) is the price impact of a trade.

Proposition 4 shows that fundamental uncertainty (inverse of \( h \)) has both a direct effect

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\(^{14}\) We showed earlier in Figure 1a and Figure 1b that although \( \rho \) is a function of both information content (\( k \)) and information asymmetry (\( \theta \)), \( \rho \) is largely determined by information asymmetry.
and an indirect effect operating through the price impact of a trade ($\lambda$) on $PEAD$. It is easy to show that $\frac{\partial PEAD}{\partial h} > 0$ when $h^2 < m(m + s)$ and $\frac{\partial PEAD}{\partial h} \leq 0$ when $h^2 \geq m(m + s)$. Hence, a higher precision of the final payoff results in a higher (lower) $PEAD$ only when the precision of the final payoff is relatively low (high). In other words, a higher information risk (i.e., lower $h$ in the model) results in a higher (lower) $PEAD$ only when information risk is relatively low (high), indicating that the total effect of information risk or fundamental uncertainty on $PEAD$ is positive only for firms with low fundamental uncertainty.

Zhang, Cai, and Keasey (2013) examine the effects of information risk and transaction costs on $PEAD$. Specifically, they suggest that information risk induces a higher level of transaction costs, leading to a lower initial market reaction to the earnings surprise. They call this the transaction cost hypothesis. They also argue that the transaction cost effect on the initial market reaction will be corrected through the price movement driven by the gradual incorporation of the earnings surprise information by market participants, predicting that transaction costs will be positively correlated with $PEAD$. In contrast, our model predicts that $PEAD$ increases with transaction costs (i.e., the price impact of a trade $\lambda$) and the price impact of a trade is determined by information risk ($h$), the information content of earnings announcements ($k$), the information asymmetry ($\theta$), and the variance of the utilitarian trader’s order size ($L$).

In general, prior research neither makes a clear distinction between fundamental uncertainty and information asymmetry nor recognizes the direct and indirect effects of information risk on $PEAD$. Proposition 4 provides a theoretical clarification for conditions under which fundamental uncertainty (inverse of $h$) and price impact/trading cost ($\lambda$) may deter investors from reacting fully to the information in earnings announcements, creating initial underreaction and subsequent $PEAD$ (see, e.g., Mendenhall, 2004; Zhang, 2006; Garfinkel and Sokobin, 2006; Francis et al., 2007, Zhang, Cai, and Keasey, 2013).
6. Empirical implications

In this section we discuss several empirically testable implications of our analytical results that have not been explored in prior research. Our measure of information asymmetry ($\theta$) is not directly observable because it is a function of two unobservable variables $m$ and $s$, where $m$ denotes the precision of the information used by both the liquidity demander and provider and $s$ denotes the precision of the information exclusively used by the liquidity demander. However, Proposition 3 enables us to estimate $\theta$ by expressing $\theta$ as a function of two empirically measurable variables:

$$\hat{\theta} = \frac{2\hat{\rho}^2}{1 - k + 2k\hat{\rho}^2},$$

where $\hat{\theta}$ is the model-implied information asymmetry measure, $\hat{\rho}$ is the estimate of the correlation coefficient between order imbalance and earnings surprise, $\hat{k}$ is the coefficient on $\hat{x}$ in the regression of the final payoff ($\hat{u}$) on earnings surprise ($\hat{x}$).

If $\theta$ measures the liquidity demander’s information advantage, then we expect $\hat{\theta}$ to be associated with the firm/stock attributes that prior research has used to proxy for the information asymmetry around corporate events, such as the price impact of a trade, the adverse selection component of the spread, the effective bid-ask spread, the probability of informed trading ($PIN$), the divergence of investors’ opinions, firm size, the number of analysts, and the number of institutional investors.\(^{15}\) Note that our estimate of information asymmetry is based on the difference in the processing ability of public information (i.e., earnings news) while the measure of information asymmetry in Back, Crotty, and Li (2018) is based on the differential accessibility to private information.

From Corollary 1 and Proposition 3, the degree of underreaction ($\rho^2$) is positively

associated with $\theta$ and negatively associated with $k$. Although we do not expect that the observed degree of underreaction is exactly equal to our estimate of $\rho^2$ because underreaction could be driven by many other factors that are not considered in our analytical model, we expect the observed degree of underreaction to be positively related to our estimate of $\rho^2$. These considerations suggest that the observed degree of underreaction to earnings announcement is expected to be positively related to information asymmetry ($\theta$), negatively related to the information content of earnings ($k$), and positively related to the square of the correlation between order imbalance and earnings surprise ($\rho^2$).

According to Proposition 2, we expect $PEAD$ to be positively related to various empirical measures of information asymmetry ($\theta$) suggested in the literature. For instance, we conjecture that $PEAD$ is larger for firms with more intangible assets because our processing-based information asymmetry ($\theta$) would be increasing in information opacity. Prior studies show that information disclosure of intangible intensive firms are more opaque. While new economy firms make heavy investments in intangibles, they do not recognize these internally-generated intangibles (e.g., brand names, R&D assets, advertising assets, etc.) on the balance sheet due to the conservative tradition in accounting. The pricing implication of earnings surprise is likely to vary across firms with different levels of intangible assets due to the earnings quality difference (Dechow, Ge, and Schrand, 2010). Earnings of intangible intensive firms may be a poor indicator of how much value was created during a period due to the difficulty in figuring out the future earnings implications of current earnings surprises (Srivastava 2014). To the extent that the poor earnings quality results in higher information asymmetries between the liquidity demander and provider, firms with larger intangible assets are likely to exhibit smaller $ERC$ and larger $PEAD$.

Chung, Elder, and Kim (2010) find a strong negative cross-sectional relation between the quality of corporate governance and various empirical measures of information asymmetry,
such as PIN and the price impact of a trade. Hence, we expect that PEAD decreases with the quality of corporate governance. Similarly, we expect PEAD to vary with measures of firms’ informational environment such as firm size, analyst following, and institutional ownership. According to Proposition 3, the positive relation between PEAD and $\rho^2$ is steeper for firms with a higher information content of earnings ($k$). Beaver, McNichols, and Wang (2018) show that market reactions on earnings announcement dates are greater for larger firms and firms with higher analyst coverage. Hence, we predict that the relation between PEAD and $\rho^2$ is steeper for these firms. Finally, to the extent that the information content of earnings is lower for firms with larger intangible assets, we predict that the relation between PEAD and $\rho^2$ is flatter for these firms.

7. Summary and concluding remarks

Numerous studies have provided empirical evidence regarding the effects of trading costs, information uncertainty, and information asymmetry on the speed of price adjustment to corporate earnings announcements. However, there is surprising lack of analytical (theoretical) investigation of the issue. As a result, the exact mechanisms (or channels) through which these variables exert an impact on the speed of price adjustment have not been well understood. Notably, most prior studies do not make a clear distinction between information content, information risk (uncertainty), and information asymmetry in the empirical analysis of initial and subsequent market reaction to earnings announcements. In this study, we develop an analytical framework that enables us to disentangle the effects of the price impact of a trade, fundamental uncertainty, the information content of earnings announcements, and the information asymmetry between traders on the post-earnings announcement drift. Our metric

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of market underreaction ($\rho^2$) makes a methodological contribution to the literature by enabling researchers to estimate the magnitude of market underreaction at the time of earnings announcements and prior to $PEAD$.

Prior research implicitly assumes that information risk induces a higher level of transaction costs, leading to a lower initial market reaction to the earnings surprise and a larger post-earnings announcement drift. We show that information risk has both a direct effect and an indirect effect operating through the price impact of a trade on the post-earnings announcement drift and that a higher information risk results in a higher post-earnings announcement drift only for firms with relatively low fundamental uncertainty. We also show that the post-earnings announcement drift increases with the price impact of a trade ($\lambda$), where the latter increases with the precision ($s$) of the information used only by the liquidity demander and decreases with the precision ($m$) of the information used by both the liquidity demander and the liquidity provider.

This study shows that the post-earnings announcement drift increases with the squared correlation coefficient between order imbalance and earnings surprise. Consistent with this analytical prediction, Chung, Kim, Lim, and Yang (2018) show that $PEAD$ is positively associated with $\rho^2$ after controlling for various firm characteristics (e.g., firm size, analyst coverage, institutional holding, the bid-ask spread, earnings persistence) using quarterly earnings announcement data. A fruitful area for future research may be empirical tests of other implications of the model discussed in Section 6.

Although we describe our analytical model in the context of earnings announcements, it may be used for the study of trading/price reactions to other types of corporate announcements. For example, our model can provide predictions/explanations of immediate and delayed price reactions to corporate dividend, stock repurchase, capital and R&D expenditure, or new equity issue announcements based on order imbalances during the
announcement period and surprise (unexpected) components of these announcements. This may be another fruitful area for future research.
References


Figure 1a. Relation between the degree of underreaction and $k$ at different levels of $\theta$.

Figure 1b. Relation between the degree of underreaction and $\theta$ at different levels of $k$. 
Appendix A
Derivation of $k$, ERC, and $\rho$ equations
From:

$$\tilde{P}_2 = \left[ \frac{m}{h+m} + \frac{s}{2(h+m+s)} \cdot \frac{h}{h+m} \right] \cdot \tilde{u} + \left[ 1 - \frac{s}{2(h+m+s)} \right] \cdot \frac{m}{h+m} \cdot \tilde{\eta}$$

$$+ \frac{s}{2(h+m+s)} \cdot \tilde{\epsilon} + \frac{1}{2} \sqrt{\frac{s}{(h+m)(h+m+s)}} \tilde{\rho},$$

$$\tilde{\omega} = \sqrt{\frac{(h+m)sl}{h+m+s}} \cdot \left[ \frac{h}{h+m} \cdot \tilde{u} - \frac{m}{h+m} \cdot \tilde{\eta} + \tilde{\epsilon} \right] + \tilde{\rho},$$

and $\tilde{x} = \tilde{u} + \frac{m}{m+s} \tilde{\eta} + \frac{s}{m+s} \tilde{\epsilon}$, it is straightforward to show that:

$$\text{Cov}[\tilde{\omega}, \tilde{x}] = \frac{1}{h} \cdot \frac{(h+m)sl}{(h+m)(m+s)} = \sqrt{\frac{(h+m)sl}{(h+m)(m+s)^2}},$$

$$\text{Var}[\tilde{\omega}] = 2L,$$

$$\text{Var}[\tilde{x}] = \frac{1}{h} + \frac{m}{(m+s)^2} + \frac{s}{(m+s)^2} = \frac{h+m+s}{h(m+s)},$$

and

$$\text{Cov}[\tilde{P}_2, \tilde{x}] = \frac{m}{h+m} + \frac{s}{2(h+m+s)} \cdot \frac{h}{h+m} \cdot \frac{1}{h} + \left[ 1 - \frac{s}{2(h+m+s)} \right] \cdot \frac{m}{h+m} \cdot \frac{1}{m+s}$$

$$\cdot \frac{1}{m} + \frac{s}{2(h+m+s)} \cdot \frac{1}{s} \cdot \frac{1}{m+s},$$

$$= \frac{m}{(h+m)h} + \frac{2(h+m+s)(h+s)}{s} + \frac{(h+m)(m+s)}{2(h+m+s)(h+m)(m+s)} - \frac{2m(h+m+s)}{2(h+m)(m+s)}$$

$$= \frac{m(m+s) + mh}{h(h+m)(m+s)} + \frac{s(m+s)}{2(h+m+s)(h+m)(m+s)} = \frac{2m(h+m+s) + hs}{2(h+m)(m+s)}$$

From the above equations we get:

$$k \equiv \frac{\text{Cov}[\tilde{u}, \tilde{x}]}{\text{Var}[\tilde{x}]} = \frac{1}{h} \cdot \frac{h(m+s)}{h+m+s} = \frac{m+s}{h+m+s}$$

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\[ \rho \equiv \text{Corr}[\bar{\omega}, \bar{x}] = \frac{Cov[\bar{\omega}, \bar{x}]}{\sqrt{\text{Var}[\bar{\omega}]} \sqrt{\text{Var}[\bar{x}]}}, \]
\[ = \frac{(h + m + s)sL}{\sqrt{(h + m)(m + s)^2}} \cdot \frac{1}{2L} \cdot \frac{h(m + s)}{h + m + s} = \frac{hs}{2(h + m)(m + s)} \]

\[ \text{ERC} = \frac{Cov[\bar{p}_2, \bar{x}]}{\text{Var}[\bar{x}]} = \frac{2m(h + m + s) + hs}{2h(h + m)(m + s)} \cdot \frac{h(m + s)}{h + m + s} = \frac{2m(h + m + s) + hs}{2(h + m)(h + m + s)} \]
\[ = \frac{1}{2} \left( \frac{m(h + m + s)}{(h + m)(h + m + s)} + \frac{m(h + m + s) + hs}{(h + m)(h + m + s)} \right) = \frac{1}{2} \left( \frac{m + s}{h + m + s} + \frac{m}{h + m} \right) \]

To express ERC and \( \rho \) as functions of \( k \) (information content) and \( \theta \) (information asymmetry), we use the following identities:

\[ k \equiv \frac{m + s}{h + m + s}, \quad \theta \equiv \frac{s}{m + s}. \]

From the above two definitions we get:

\[ k(1 - \theta) = \frac{m}{h + m + s}, \quad 1 - k\theta = \frac{h + m}{h + m + s}, \text{ and thus, } \frac{k(1 - \theta)}{1 - k\theta} = \frac{m}{h + m}. \]

Now \( \rho \) and \( \text{ERC} \) can be rewritten as:

\[ \rho = \frac{hs}{\sqrt{2(h + m)(m + s)}} = \sqrt{\frac{\theta}{2} \left( \frac{1 - k}{1 - k\theta} \right)}, \]
\[ \text{ERC} = \frac{1}{2} \left( \frac{m + s}{h + m + s} + \frac{m}{h + m} \right) = \frac{1}{2} \left( k + \frac{k(1 - \theta)}{1 - k\theta} \right) = k \cdot \frac{2 - (1 + k)}{2 - 2k\theta}. \]
Appendix B
Derivation of PEAD

From

\[
\tilde{u} - \tilde{p}_2 = \left[ 1 - \frac{m}{h + m} - \frac{s}{2(h + m + s)} \cdot \frac{h}{h + m} \right] \cdot \tilde{u} + \left[ \frac{s}{s(h + m + s)} - 1 \right] \cdot \frac{m}{h + m} \cdot \tilde{\eta} - \frac{s}{2(h + m + s)} \cdot \tilde{\varepsilon} - \frac{1}{2} \sqrt{\frac{s}{(h + m)(h + m + s)}} \cdot \tilde{p},
\]

and \( \tilde{x} = \tilde{u} + \frac{m}{m + s} \tilde{\eta} + \frac{s}{m + s} \tilde{\varepsilon} \), we have

\[
\begin{align*}
\text{Cov}[\tilde{u} - \tilde{p}_2, \tilde{x}] & = \left[ 1 - \frac{m}{h + m} - \frac{s}{2(h + m + s)} \cdot \frac{h}{h + m} \right] \cdot \frac{1}{h + m} + \left[ \frac{s}{s(h + m + s)} - 1 \right] \cdot \frac{1}{m} - \frac{1}{2} \cdot \frac{s}{m + s} - \frac{1}{s} \\
& = \left[ 1 - \frac{s}{2(h + m + s)} \cdot \frac{1}{(h + m)(m + s)} - \frac{m}{s} \right] - \frac{1}{2} \cdot \frac{s}{2(h + m + s)(m + s)} \\
& = \frac{2s(h + m + s) - s^2 - s(h + m)}{2(h + m + s)(h + m)(m + s)} - \frac{2s(h + m + s) - s(h + m + s)}{2(h + m + s)(h + m)(m + s)} = \frac{s}{2(h + m)(m + s)} \\

\text{PEAD} & \equiv \frac{\text{Cov}[\tilde{u} - \tilde{p}_2, \tilde{x}]}{\text{Var}[\tilde{x}]} = \frac{s}{2(h + m)(m + s)} \cdot \frac{h(m + s)}{h + m + s} = \frac{sh}{2(h + m)(h + m + s)}
\end{align*}
\]

Since \( k\theta = \frac{s}{h + m + s} \) and \( 1 - \frac{k(1 - \theta)}{1 - k\theta} = 1 - \frac{m}{h + m} = \frac{h}{h + m} \), it follows that

\[
\text{PEAD} = \frac{\text{Cov}[\tilde{u} - \tilde{p}_2, \tilde{x}]}{\text{Var}[\tilde{x}]} = \frac{1}{2} k\theta \cdot \left[ 1 - \frac{k(1 - \theta)}{1 - k\theta} \right] = k \cdot \frac{\theta}{2(1 - k\theta)}. \]