Components of the bid-ask spread

Notation

Time sequence: $V_t \rightarrow M_t \rightarrow P_t$

V_t = the unobservable fundamental value of the stock in the absence of transaction costs. V_t is determined just prior to the posting of the bid and ask quotes at time t.

M_t = the quote midpoint just before a transaction.

P_t = the transaction price at time t.

Q_t = the buy-sell indicator variable:
+1 for a buyer-initiated trade,
–1 for a seller-initiated trade,
0 for a transaction at the quote midpoint.

Fundamental value:

(1) $V_t = V_{t-1} + \alpha(S/2)Q_{t-1} + \varepsilon_t$,

S = the traded spread,

α = the percentage of the spread attributable to adverse selection,

ε_t, = the serially uncorrelated public information shock.

The change in the fundamental value reflects the price information revealed by the last trade, $\alpha(S/2)Q_{t-1}$, and the public information innovation.

If there is no adverse selection,

$V_t = V_{t-1} + \varepsilon_t$.

If the adverse selection component is the only cost,

$V_t = V_{t-1} + (S/2)Q_{t-1} + \varepsilon_t$,

Quote midpoint:

(2) $M_t = V_t + \beta(S/2)\Sigma Q_i$,

β = the percentage of the spread attributable to inventory holding costs.

ΣQ_i = the cumulative inventory from the market open until time t-1.

The market maker lowers the quote midpoint if he has too much inventory to encourage customer buys and discourage customer sells, and vice versa.

Observations:

If $\beta = 0$, then $M_t = V_t$.

From (2), we have

(2a) $M_t = V_t + \beta(S/2)(Q_1 + Q_2 + Q_3 + Q_4 + \dots + Q_{t-1})$

(2b) $M_{t-1} = V_{t-1} + \beta(S/2)(Q_1 + Q_2 + Q_3 + Q_4 +.. + Q_{t-2})$

Subtracting (2b) from (2a), we have:

(3) $M_t - M_{t-1} = V_t - V_{t-1} + \beta(S/2)(Q_{t-1})$

Next note that from (1),

(4) $V_t - V_{t-1} = \alpha(S/2)Q_{t-1} + \varepsilon_t$

Substituting (4) into (3), we have:

$$M_t - M_{t-1} = \alpha(S/2)Q_{t-1} + \beta(S/2)(Q_{t-1}) + \varepsilon_t$$

$$= (\alpha + \beta)(S/2)Q_{t-1} + \varepsilon_t$$

(5) $\Delta M_t = (\alpha + \beta)(S/2)Q_{t-1} + \varepsilon_t$

Transaction price:

(6) $P_t = M_t + (S/2)Q_t + \eta_t$

<u>Observation:</u>

Transaction price can be different from quoted price (η_t is not zero).

From (6),

(6a) $P_{t-1} = M_{t-1} + (S/2)Q_{t-1} + \eta_{t-1}$

Subtracting (6a) from (6), we have:

$P_t - P_{t-1} = M_t - M_{t-1} + (S/2)(Q_t - Q_{t-1}) + \eta_t - \eta_{t-1}$

(7) $\Delta P_t = \Delta M_t + (S/2)(Q_t - Q_{t-1}) + \eta_t - \eta_{t-1}$

Finally, substituting (5) into (7), we obtain

$\Delta P_t = (\alpha + \beta)(S/2)Q_{t-1} + \varepsilon_t + (S/2)(Q_t - Q_{t-1}) + \eta_t - \eta_{t-1}$

or

$\Delta P_t = (\alpha + \beta)(S/2)Q_{t-1} + (S/2)(Q_t - Q_{t-1}) + \eta_t - \eta_{t-1} + \varepsilon_t$

or

(8) $\Delta P_t = \lambda(S/2)Q_{t-1} + (S/2)(Q_t - Q_{t-1}) + e_t$

$\Delta P_t = \lambda(S/2)Q_{t-1} + (S/2)\Delta Q_t + e_t$

[(8a) or $\Delta P_t = (\lambda - 1)(S/2)Q_{t-1} + (S/2)Q_t + e_t$]

where $e_t = \eta_t - \eta_{t-1} + \varepsilon_t$ and $\lambda = \alpha + \beta$

Observations:

ΔP_t, Q_{t-1}, and Q_t are all observable variables.

Using these variables, we can estimate λ and S.

(9) $\Delta P_t = b_1 Q_{t-1} + b_2(Q_t - Q_{t-1}) + e_t$

where $b_1 = \lambda(S/2)$ and $b_2 = (S/2)$ and thus $\lambda = b_1/b_2$

$1-\lambda$ = an estimate of order processing costs

But cannot separately identify the adverse selection (α) and the inventory holding (β) components of the spread.

Roll (1984) - Q_t and Q_{t-1} are independent

Assume that $\lambda = 0$ (no adverse selection and inventory costs). Then from (8),

$\text{cov}(\Delta P_t, \Delta P_{t-1})$

$= \text{cov}[(S/2)(Q_t - Q_{t-1}), (S/2)(Q_{t-1} - Q_{t-2})]$

$= (S^2/4)\text{cov}[(Q_t - Q_{t-1}), (Q_{t-1} - Q_{t-2})]$

$= (S^2/4)[\text{cov}(Q_t, Q_{t-1}) - \text{cov}(Q_t, Q_{t-2})$

$\quad - \text{cov}(Q_{t-1}, Q_{t-1}) + \text{cov}(Q_{t-1}, Q_{t-2})]$

$= (S^2/4)[- \text{cov}(Q_{t-1}, Q_{t-1})] = (S^2/4)[- \text{var}(Q_{t-1})]$
[since Q_t and Q_{t-1} are independent]

$= -(S^2/4)[½(1-0)^2 + ½(-1-0)^2]$
[since $E(Q_{t-1}) = ½(1) + ½(-1) = 0$]

$= -(S^2/4)(1)$.

Hence $S = 2[- \text{cov}(\Delta P_t, \Delta P_{t-1})]^{½}$

Choi, Salandro, and Shastri (1988) - Q_t and Q_{t-1} are not independent (see below)

Suppose that the probability of a trade flow reversal [a trade at the bid (ask) is followed by a trade at the ask (bid)] is π and the probability of a trade flow continuation [a trade at the bid (ask) is followed by a trade at the bid (ask)] is 1- π. Then

$$\text{cov}(\Delta P_t, \Delta P_{t-1}) = -S^2\pi^2.$$

Hence $S = (1/\pi)[-\text{cov}(\Delta P_t, \Delta P_{t-1})]^{1/2}$.

Notice that if $\pi = 1/2$, then

$S = 2[-\text{cov}(\Delta P_t, \Delta P_{t-1})]^{1/2}$ – Roll (1984)

George, Kaul, and Nimalendran (1991)
– when $\pi = ½$ and $\beta = 0$.

From (8) $\Delta P_t = \lambda(S/2)Q_{t-1} + (S/2)\Delta Q_t + e_t$ and $\beta = 0$

$\Delta P_t = \alpha(S/2)Q_{t-1} + (S/2)\Delta Q_t + e_t$

$$\mathrm{cov}(\Delta P_t, \Delta P_{t-1}) = \mathrm{cov}[\alpha(S/2)Q_{t-1} + (S/2)(Q_t - Q_{t-1}), \alpha(S/2)Q_{t-2} + (S/2)(Q_{t-1} - Q_{t-2})]$$

$$= \mathrm{cov}[\alpha(S/2)Q_{t-1}, (S/2)Q_{t-1}] + \mathrm{cov}[(S/2)(-Q_{t-1}), (S/2)(Q_{t-1})]$$

$$= \alpha(S^2/4)[\mathrm{cov}(Q_{t-1}, Q_{t-1})] - (S^2/4)[\mathrm{cov}(Q_{t-1}, Q_{t-1})]$$

$$= -(1 - \alpha)(S^2/4)[\mathrm{cov}(Q_{t-1}, Q_{t-1})]$$

$$= -(1 - \alpha)(S^2/4)[\mathrm{var}(Q_{t-1})]$$

$$= -(1 - \alpha)(S^2/4)[½(1-0)^2 + ½(-1-0)^2]$$

$$= -(1 - \alpha)(S^2/4)$$

Hence, $\mathrm{cov}(\Delta P_t, \Delta P_{t-1}) = -(1 - \alpha)(S^2/4)$. [This is their equation (5) on p. 628 assuming $\mathrm{cov}(E_t, E_{t-1}) = 0$.]

Thus $S = 2[-\mathrm{cov}(\Delta P_t, \Delta P_{t-1})/(1 - \alpha)]^{½}$

Glosten and Harris (1988) – Trade indicator model

(GH1: 1a) $V_t = V_{t-1} + \alpha(S/2)Q_t + e_t$,

V_t = the true price immediately after the trade at time t

S = the traded spread

α = the percentage of the spread attributable to adverse selection

e_t, = the serially uncorrelated public information shock.

(GH2: 1b) $P_t = V_t + (1 - \alpha)(S/2)Q_t$

From (GH2), $P_{t-1} = V_{t-1} + (1 - \alpha)(S/2)Q_{t-1}$

(GH3) $P_t - P_{t-1} = V_t - V_{t-1} + (1 - \alpha)(S/2)(Q_t - Q_{t-1})$

From (GH1),

(GH4) $V_t - V_{t-1} = \alpha(S/2)Q_t + e_t$

Substituting (GH4) into (GH3),

$$\text{(GH5)} \quad P_t - P_{t-1} = \alpha(S/2)Q_t + e_t + (1-\alpha)(S/2)(Q_t - Q_{t-1}) \quad \textbf{or}$$

$$\text{(GH6)} \quad \Delta P_t = \alpha(S/2)Q_t + (1-\alpha)(S/2)\Delta Q_t + e_t$$

From (GH5),

$$\Delta P_t = \alpha(S/2)Q_t + (S/2)(Q_t - Q_{t-1}) - \alpha(S/2)(Q_t - Q_{t-1}) + e_t$$

$$= (S/2)(Q_t - Q_{t-1}) + \alpha(S/2)Q_t - \alpha(S/2)(Q_t - Q_{t-1}) + e_t$$

$$= (S/2)(Q_t - Q_{t-1}) + \alpha(S/2)Q_{t-1} + e_t$$

Hence

$$\text{(GH7)} \quad \Delta P_t = (S/2)\,\Delta Q_t + \alpha(S/2)Q_{t-1} + e_t$$

Note that (GH7) is equal to (8) if $\beta = 0$.

$$\text{(8)} \quad \Delta P_t = \lambda(S/2)Q_{t-1} + (S/2)\Delta Q_t + e_t$$

Madhavan, Richardson, and Roomans (1996)

(MRR1) $\Delta P_t = (\phi + \theta)Q_t - \phi Q_{t-1} + e_t$,

where θ = the adverse selection component

ϕ = the order processing and inventory component

Upon rearrangement, we obtain

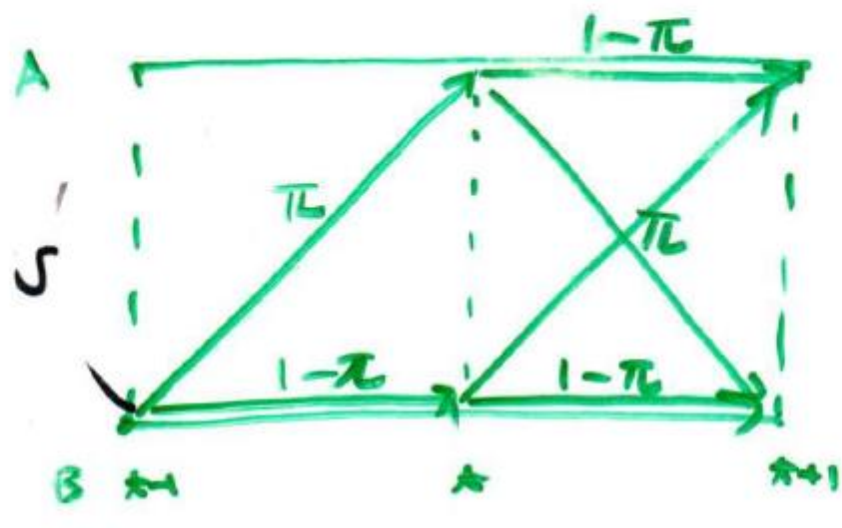
(MRR2) $\Delta P_t = \theta Q_t - \phi \Delta Q_{t-1} + e_t$

Note that (MRR2) is analogues to (GH6) above

(GH6) $\Delta P_t = \alpha(S/2)Q_t + (1 - \alpha)(S/2)\Delta Q_t + e_t$

Choi, Salandro, and Shastri (1988)

Suppose that the probability of a trade flow reversal [a trade at the bid (ask) is followed by a trade at the ask (bid)] is π and the probability of a trade flow continuation [a trade at the bid (ask) is followed by a trade at the bid (ask)] is $1-\pi$. And also suppose that at time t-1, there is an equal probability of a buy and sell.



P_{t-1}	P_t	P_{t+1}	Probability (PROB)	ΔP_t	ΔP_{t+1}
B	A	A	$(1/2)\pi(1-\pi)$	S	0
B	A	B	$(1/2)\pi\pi$	S	-S
B	B	A	$(1/2)(1-\pi)\pi$	0	S
B	B	B	$(1/2)(1-\pi)(1-\pi)$	0	0
A	A	A	$(1/2)(1-\pi)(1-\pi)$	0	0
A	A	B	$(1/2)(1-\pi)\pi$	0	-S
A	B	A	$(1/2)\pi\pi$	-S	S
A	B	B	$(1/2)\pi(1-\pi)$	-S	0

$$\mathrm{cov}(\Delta P_t, \Delta P_{t+1}) = \Sigma PROB[\{\Delta P_t - E(\Delta P_t)\}\{\Delta P_{t-1} - E(\Delta P_{t+1})\}]$$

$$E(\Delta P_t) = \Sigma PROB\ \Delta P_t$$

$$= (1/2)\pi(1-\pi)(S) + (1/2)\pi\pi(S) + (1/2)\pi\pi(-S) + (1/2)\pi(1-\pi)(-S) = 0.$$

$$E(\Delta P_{t+1}) = \Sigma PROB\ \Delta P_{t+1}$$

$$= (1/2)\pi\pi(-S) + (1/2)(1-\pi)\pi(S) + (1/2)(1-\pi)\pi(-S) + (1/2)\pi\pi(S) = 0.$$

Hence, $cov(\Delta P_t, \Delta P_{t+1}) = \Sigma PROB(\Delta P_t)(\Delta P_{t+1})$

$= (1/2)\pi\pi(S)(-S) + (1/2)\pi\pi(-S)(S) = -\pi^2 S^2$

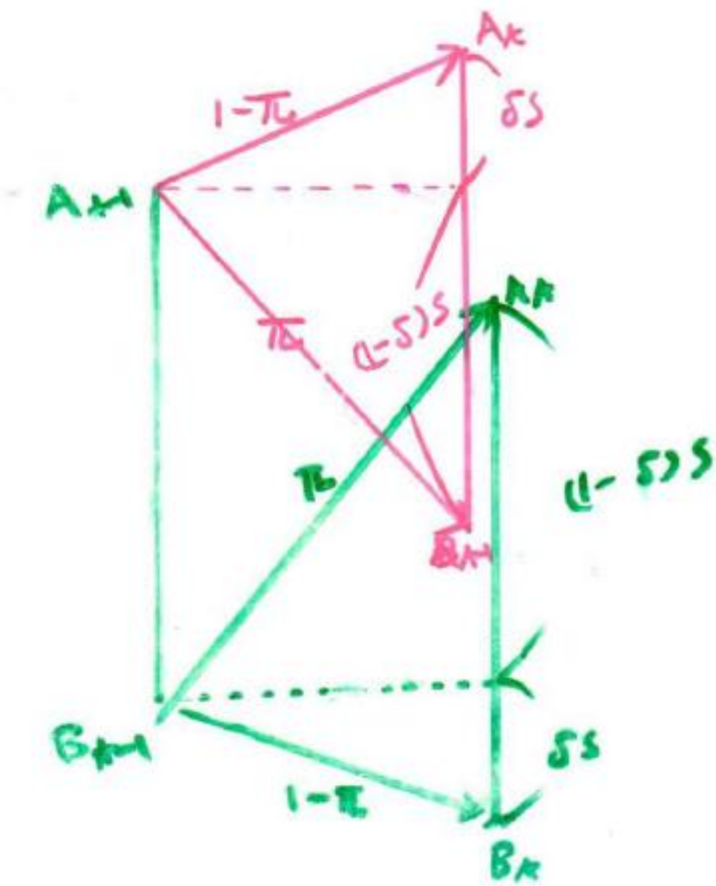
If $\pi = ½$ (Roll, 1984), then

$cov(\Delta P_t, \Delta P_{t+1}) = -(1/4)S^2$

Stoll (1989)

Suppose that the probability of a trade flow reversal [a trade at the bid (ask) is followed by a trade at the ask (bid)] is π and the probability of a trade flow continuation [a trade at the bid (ask) is followed by a trade at the bid (ask)] is $1- \pi$. And also suppose that at time zero, there is an equal probability of a buy and sell.

In addition, suppose dealers change the position of the spread relative to the true price in order to induce public transactions that would even out the inventory position: bid and ask prices are lowered after a dealer purchase in order to induce dealer sales and inhibit additional dealer purchases and bid and ask prices are raised after a dealer sale in order to induce dealer purchases and inhibit additional dealer sales. The size of a price reversal (conditional on a reversal) is given by $(1-\delta)S$ and the size of a price continuation (conditional on a continuation) is δS.



P_{t-1}	P_t	P_{t+1}	Probability (PROB)	ΔP_t	ΔP_{t+1}
B	A	A	$(1/2)\pi(1-\pi)$	$(1-\delta)S$	δS
B	A	B	$(1/2)\pi\pi$	$(1-\delta)S$	$-(1-\delta)S$
B	B	A	$(1/2)(1-\pi)\pi$	$-\delta S$	$(1-\delta)S$
B	B	B	$(1/2)(1-\pi)(1-\pi)$	$-\delta S$	$-\delta S$
A	A	A	$(1/2)(1-\pi)(1-\pi)$	δS	δS
A	A	B	$(1/2)(1-\pi)\pi$	δS	$-(1-\delta)S$
A	B	A	$(1/2)\pi\pi$	$-(1-\delta)S$	$(1-\delta)S$
A	B	B	$(1/2)\pi(1-\pi)$	$-(1-\delta)S$	$-\delta S$

$$\text{cov}(\Delta P_t, \Delta P_{t+1}) = \Sigma\text{PROB}[\{\Delta P_t - E(\Delta P_t)\}\{\Delta P_{t+1} - E(\Delta P_{t+1})\}]$$

$$E(\Delta P_t) = \Sigma\text{PROB}\ \Delta P_t$$

$$= (1/2)E(\Delta P_t/B_{t-1}) + (1/2)E(\Delta P_t/A_{t-1})$$

$$= (1/2)(\pi - \delta)S + (1/2)(-1)(\pi - \delta)S = 0.$$

Similarly, $E(\Delta P_{t+1}) = \Sigma\text{PROB}\ \Delta P_{t+1} = 0$

Hence, $\text{cov}(\Delta P_t, \Delta P_{t+1}) = \Sigma \text{PROB}(\Delta P_t)(\Delta P_{t+1})$

$= (1/2)\pi(1-\pi)(1-\delta)S\delta S$
$+ (1/2)\pi\pi(1-\delta)S(-1)(1-\delta)S$
$+ (1/2)(1-\pi)\pi(-\delta S)(1-\delta)S$
$+ (1/2)(1-\pi)(1-\pi)(-\delta S)(-\delta S)$
$+ (1/2)(1-\pi)(1-\pi)\delta S\delta S$
$+ (1/2)(1-\pi)\pi\delta S(-1)(1-\delta)S$
$+ (1/2)\pi\pi(-1)(1-\delta)S(1-\delta)S$
$+ (1/2)\pi(1-\pi)(-1)(1-\delta)S(-1)\delta S$

$= S^2[(1-\pi)^2\delta^2 - \pi 2 (1 - \delta)^2]$

$= S^2[\delta^2(1-2\pi) - \pi^2(1 - 2\delta)]$

Hence $\text{cov}(\Delta P_t, \Delta P_{t+1}) = S^2[\delta^2(1-2\pi) - \pi^2(1 - 2\delta)]$

If $\delta = 0$, then

$\text{cov}(\Delta P_t, \Delta P_{t+1}) = -S^2\pi^2$

If $\delta = 0$ and $\pi = ½$ (Roll, 1984), then $\text{cov}(\Delta P_t, \Delta P_{t-1})$
$= -(1/4)S^2$

Similarly, it can be shown that $\text{cov}(\Delta Q_t, \Delta Q_{t+1}) =$
$S^2\delta^2(1-2\pi)$

From

$$\mathrm{cov}(\Delta P_t, \Delta P_{t+1}) = S^2[\delta^2(1- 2\pi) - \pi^2(1 - 2\delta)]$$

$$\mathrm{cov}(\Delta Q_t, \Delta Q_{t+1}) = S^2\delta^2(1- 2\pi)$$

Estimate

$$\mathrm{cov}(\Delta P_t, \Delta P_{t+1}) = a_0 + a_1 S^2 + u$$

$$\mathrm{cov}(\Delta Q_t, \Delta Q_{t+1}) = b_0 + b_1 S^2 + v$$

Then calculate δ and π by solving the following simultaneous equations:

$$a_1 = \delta^2(1- 2\pi) - \pi^2(1 - 2\delta)$$

$$b_1 = \delta^2(1- 2\pi)$$

Realized Spread:

Realized spread = the expected revenue on two transactions (i.e., a buy and sell)

$= E(\Delta P_t/B_{t-1}) - E(\Delta P_t/A_{t-1})$

$= \pi(1-\delta)S + (1-\pi)(-\delta S) - [(-1)\pi(1-\delta)S + (1-\pi)(\delta S)]$

$= 2(\pi - \delta)S.$

<u>Observation</u>:

With only order processing costs ($\pi = ½$ and $\delta = 0$), the realized spread is the same as the quoted spread, i.e., the realized spread = $2(½ - 0)S = S$.

With only adverse selection costs ($\pi = ½$ and $\delta = ½$), the realized spread is zero, i.e., the realized spread = $2(½ - ½)S = 0$.

When the quoted spread is determined by inventory costs, $\pi > ½$ and $\delta = ½$, so that the realized spread is positive but less than the quoted spread, i.e., the realized spread = $2(\pi - ½)S > 0$.

Spread Components:

Realized spread = Order processing costs
+ Inventory costs

Adverse selection costs = Quoted spread
– Realized spread

$$= S - 2(\pi - \delta)S = [1 - 2(\pi - \delta)]S$$

Inventory costs = Realized spread in the absence of order processing costs ($\delta = ½$)

$$= 2(\pi - ½)S$$

Order processing costs = Realized spread
– Inventory costs

$$= 2(\pi - \delta)S - 2(\pi - ½)S = 2S(\pi - \delta - \pi + ½)$$

$$= 2S(- \delta + ½) = S(- 2\delta + 1)$$

<u>Observation</u>: If $\delta = 0$ (i.e., no inventory effect and no adverse selection), Order processing costs = S.

Serial correlation in trade flows

Suppose that the probability of a trade flow reversal [a trade at the bid (ask) is followed by a trade at the ask (bid)] is π and the probability of a trade flow continuation [a trade at the bid (ask) is followed by a trade at the bid (ask)] is $1-\pi$. Then

(10) $E(Q_{t-1}/Q_{t-2}) = (1 - 2\pi)Q_{t-2}$

[since $P(Q_{t-1} = Q_{t-2}) = (1-\pi)$ and $P(Q_{t-1} = -Q_{t-2}) = \pi$.]

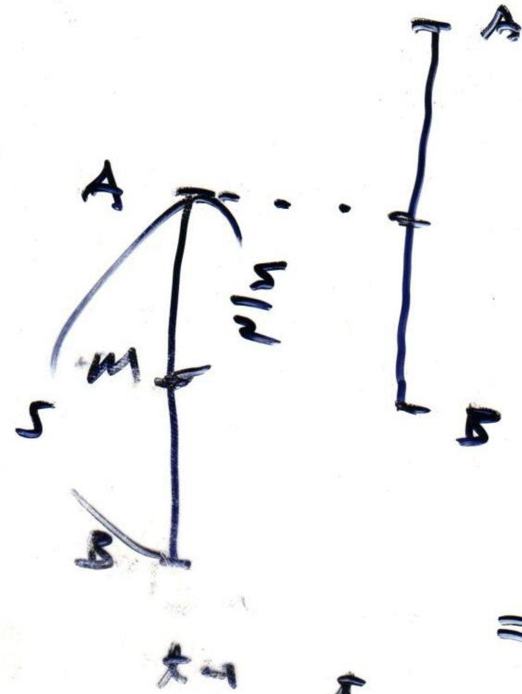
From (4), we have $V_t - V_{t-1} = \alpha(S/2)Q_{t-1} + \varepsilon_t$

Now, note that Q_{t-1} is not complete unpredictable because of (10). Because the market knows (10), (4) must be modified to account for the predictable information contained in the trade at time t-2.

(11) $V_t - V_{t-1} = \alpha(S/2)[Q_{t-1} - E(Q_{t-1}/Q_{t-2})] + \varepsilon_t$

Substituting (10) into (11), we obtain;

(12) $V_t - V_{t-1} = \alpha(S/2)[Q_{t-1} - (1 - 2\pi)Q_{t-2}] + \varepsilon_t$
(note that if $\pi = 0.5$, (12) becomes (4)).



$$P(Q_{t-1} = Q_{t-2}) = 1 - \pi$$

$$P(Q_{t-1} = -Q_{t-2}) = \pi$$

$$E(Q_{t-1} / Q_{t-2}) = Q_{t-2} \cdot (1-\pi) - Q_{t-2} \cdot \pi$$

$$\cancel{= Q_{t-2}(1-2\pi)}$$

$$= Q_{t-2}(1 - 2\pi)$$

From (3), $M_t - M_{t-1} = V_t - V_{t-1} + \beta(S/2)(Q_{t-1})$ and (12),

$$M_t - M_{t-1} = \alpha(S/2)[Q_{t-1} - (1-2\pi)Q_{t-2}] + \varepsilon_t + \beta(S/2)(Q_{t-1})$$

$$= \alpha(S/2)[Q_{t-1} - (1 - 2\pi)Q_{t-2}] + \beta(S/2)(Q_{t-1}) + \varepsilon_t$$

$$= (\alpha + \beta)(S/2)Q_{t-1} - \alpha(S/2)(1 - 2\pi)Q_{t-2} + \varepsilon_t$$

Hence

$$(13)\quad \Delta M_t = (\alpha + \beta)(S/2)Q_{t-1} - \alpha(S/2)(1 - 2\pi)Q_{t-2} + \varepsilon_t$$

Note from (7), $\Delta P_t = \Delta M_t + (S/2)(Q_t - Q_{t-1}) + \eta_t - \eta_{t-1}$.

Substituting (13) into (7), we obtain:

$$\Delta P_t = (\alpha + \beta)(S/2)Q_{t-1} - \alpha(S/2)(1 - 2\pi)Q_{t-2} + \varepsilon_t + (S/2)(Q_t - Q_{t-1}) + \eta_t - \eta_{t-1}$$

$$= (S/2)Q_t + (\alpha + \beta - 1)(S/2)Q_{t-1} - \alpha(S/2)(1 - 2\pi)Q_{t-2} + \varepsilon_t + \eta_t - \eta_{t-1}$$

Hence,

$$(14\text{: HS } (25))\qquad \Delta P_t = (S/2)Q_t + (\alpha + \beta - 1)(S/2)Q_{t-1} - \alpha(S/2)(1 - 2\pi)Q_{t-2} + e_t,$$

where $e_t = \eta_t - \eta_{t-1} + \varepsilon_t$

Note that:

(10) $E(Q_{t-1}/Q_{t-2}) = (1 - 2\pi)Q_{t-2}$ and

(14) $\Delta P_t = (S/2)Q_t + (\alpha + \beta - 1)(S/2)Q_{t-1}$
$- \alpha(S/2)(1 - 2\pi)Q_{t-2} + e_{t,}$

Hence, we can estimate parameter values using the following regression models:

$E(Q_{t-1}/Q_{t-2}) = b_1 Q_{t-2} + e_{1t}$

$\Delta P_t = b_2 Q_t + b_3 Q_{t-1} + b_4 Q_{t-2} + e_{2t,}$

where $b_1 = (1 - 2\pi)$, $b_2 = (S/2)$, $b_3 = (\alpha + \beta - 1)(S/2)$,
and $b_4 = - \alpha(S/2)(1 - 2\pi)$.

Here we have four unknowns (π, S, α, and β) and four equations, hence solvable!

Alternatively, we can estimate the component of the spread directly using (13). Replacing the traded spread with the quoted spread, we can rewrite (13) as:

(15) $\Delta M_t = (\alpha + \beta)(S_{t-1}/2)Q_{t-1} - \alpha(S_{t-1}/2)(1-2\pi)Q_{t-2} + \varepsilon_t$

Now using the two-equations regression model:

$E(Q_{t-1}/Q_{t-2}) = b_1 Q_{t-2} + e_{1t}$

$\Delta M_t = b_2 (S_{t-1}/2)Q_{t-1} - b_3(S_{t-1}/2)Q_{t-2} + e_{2t,}$

where $b_1 = (1 - 2\pi)$, $b_2 = (\alpha + \beta)$, and $b_3 = \alpha(1 - 2\pi)$.

We now have three unknowns (π, α, and β) and three equations.