MULTI-INDEX

MODELS

Fall 2000

1. Assumptions that residuals are uncorrelated from a single-index model turns out to be inaccurate. This assumption can be improved by assuming additional indexes.

2. Once again multi-index models have developed a life of their own beyond their original purpose of estimating covariances.

Consider two index model:

$$r_{it} = a_i + b_{i1} I_{1t} + b_{12} I_{2t} + e_{it}$$

Where:

(1). $E\left(I_{1t} I_{2t}\right) = 0$

(2). $E\left(e_{it}\right) = 0$

(3). $E\left(e_{it} I_{1t} \text{ or } I_{2t}\right) = 0$

(4). $E\left(e_{it} e_{jt}\right) = 0$

(5). $a_i, b_{i1} \text{ and } b_{i2}$ **are constant.**

only real assumption is $E\left(e_{it} e_{jt}\right) = 0$

Mean return:

$$E\left(r_{it}\right)=a_i+b_{i1}\bar{I}_1+b_{i2}\bar{I}_2$$

$$\mathrm{Var}\left(r_{it}\right)=b_{i1}^2 s_1^2+b_{i2}^2 s_2^2+s_{ei}^2$$

in other words:

$$s_{ei}^2=b_{i2}^2 s_2^2+s_{ei}^2$$

$$\mathrm{cov}\left(r_i r_j\right)=b_{i1}b_{j1}s_{I1}^2+b_{i2}b_{j2}s_{I2}^2$$

Note:

1. **if add** $\mathrm{E}\left(\mathrm{e}_{\mathrm{i}}\mathrm{e}_{\mathrm{j}}\right)$ **get historical.**

2. **assuming** $\mathrm{E}\left(e_{\mathrm{i}}e_{\mathrm{j}}\right)=\mathrm{b}_{\mathrm{i2}}\mathrm{b}_{\mathrm{j2}}s_{\mathrm{I2}}^{2}+0$

Generalizing to more than two indexes is straight forward.

<u>Types of indexes</u>:

(1). Statistically derived

(2). Portfolios of securities

A.	**S&P, H-L, B-M**	**Fama French**
B.	**S&P, H-L, B-M, Bonds**	**E&G**

(3). Economic Factors

- **A. ΔIP, surprise in inflation**
- **B. Surprise GNP**

(4). Market and Industry

- **A. S&P**
- **B. Industry factors**

In bond area:

$$R_{it} = \overline{R}_i + OAS - D_i \frac{\Delta r}{1+r} - V_i dV_i + C_i(\text{spread}) + e_i$$

Issues:

(1). **Real influences**

(2). **Parsimonious**

Uses:

(1). Covariance structure

(2). Selection of exposure

(3). Return attribution

(4). Portfolio evaluation