CHAPTER 6: CAPITAL ALLOCATION TO RISKY ASSETS

**PROBLEM SETS**

4. a. The expected cash flow is: (0.5 × $70,000) + (0.5 × 200,000) = $135,000.

With a risk premium of 8% over the risk-free rate of 6%, the required rate of return is 14%. Therefore, the present value of the portfolio is:

$135,000/1.14 = $118,421

b. If the portfolio is purchased for $118,421 and provides an expected cash inflow of $135,000, then the expected rate of return [*E*(*r*)] is as follows:

$118,421 × [1 + *E*(*r*)] = $135,000

Therefore, *E*(*r*) =14%. The portfolio price is set to equate the expected rate of return with the required rate of return.

c. If the risk premium over T-bills is now 12%, then the required return is:

6% + 12% = 18%

The present value of the portfolio is now:

$135,000/1.18 = $114,407

d. For a given expected cash flow, portfolios that command greater risk premiums must sell at lower prices. The extra discount from expected value is a penalty for risk.

6. Points on the curve are derived by solving for *E*(*r*) in the following equation:

*U* = 0.05 = *E*(*r*) – 0.5*A*σ2 = *E*(*r*) – 1.5σ2

The values of *E*(*r*), given the values of σ2, are therefore:

|  |  |  |
| --- | --- | --- |
| σ | σ 2 | *E*(*r*) |
| 0.00 | 0.0000 | 0.05000 |
| 0.05 | 0.0025 | 0.05375 |
| 0.10 | 0.0100 | 0.06500 |
| 0.15 | 0.0225 | 0.08375 |
| 0.20 | 0.0400 | 0.11000 |
| 0.25 | 0.0625 | 0.14375 |

The bold line in the graph on the next page (labeled Q6, for Question 6) depicts the indifference curve.

7. Repeating the analysis in Problem 6, utility is now:

*U* = *E*(*r*) – 0.5*A*σ2 = *E*(*r*) – 2.0σ2 = 0.05

The equal-utility combinations of expected return and standard deviation are presented in the table below. The indifference curve is the upward sloping line in the graph on the next page, labeled Q7 (for Question 7).

|  |  |  |
| --- | --- | --- |
| σ | σ 2 | *E*(*r*) |
| 0.00 | 0.0000 | 0.0500 |
| 0.05 | 0.0025 | 0.0550 |
| 0.10 | 0.0100 | 0.0700 |
| 0.15 | 0.0225 | 0.0950 |
| 0.20 | 0.0400 | 0.1300 |
| 0.25 | 0.0625 | 0.1750 |

The indifference curve in Problem 7 differs from that in Problem 6 in slope. When *A* increases from 3 to 4, the increased risk aversion results in a greater slope for the indifference curve since more expected return is needed in order to compensate for additional σ.



8. The coefficient of risk aversion for a risk neutral investor is zero. Therefore, the corresponding utility is equal to the portfolio’s expected return. The corresponding indifference curve in the expected return-standard deviation plane is a horizontal line, labeled Q8 in the graph above (see Problem 6).

9. A risk lover, rather than penalizing portfolio utility to account for risk, derives greater utility as variance increases. This amounts to a negative coefficient of risk aversion. The corresponding indifference curve is downward sloping in the graph above (see Problem 6), and is labeled Q9.

10. The portfolio expected return and variance are computed as follows:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| (1)  *W*Bills | (2)  *r*Bills | (3)  *W*Index | (4)  rIndex | *r*Portfolio  (1)×(2)+(3)×(4) | σPortfolio  (3) × 20% | σ 2 Portfolio |
| 0.0 | 5% | 1.0 | 13.0% | 13.0% = 0.130 | 20% = 0.20 | 0.0400 |
| 0.2 | 5 | 0.8 | 13.0 | 11.4% = 0.114 | 16% = 0.16 | 0.0256 |
| 0.4 | 5 | 0.6 | 13.0 | 9.8% = 0.098 | 12% = 0.12 | 0.0144 |
| 0.6 | 5 | 0.4 | 13.0 | 8.2% = 0.082 | 8% = 0.08 | 0.0064 |
| 0.8 | 5 | 0.2 | 13.0 | 6.6% = 0.066 | 4% = 0.04 | 0.0016 |
| 1.0 | 5 | 0.0 | 13.0 | 5.0% = 0.050 | 0% = 0.00 | 0.0000 |

11. Computing utility from *U* = *E*(*r*) – 0.5 × *A*σ2 = *E*(*r*) – σ2, we arrive at the values in the column labeled *U*(*A* = 2) in the following table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *W*Bills | *W*Index | *r*Portfolio | σPortfolio | σ2Portfolio | *U*(*A* = 2) | *U*(*A* = 3) |
| 0.0 | 1.0 | 0.130 | 0.20 | 0.0400 | 0.0900 | .0700 |
| 0.2 | 0.8 | 0.114 | 0.16 | 0.0256 | 0.0884 | .0756 |
| 0.4 | 0.6 | 0.098 | 0.12 | 0.0144 | 0.0836 | .0764 |
| 0.6 | 0.4 | 0.082 | 0.08 | 0.0064 | 0.0756 | .0724 |
| 0.8 | 0.2 | 0.066 | 0.04 | 0.0016 | 0.0644 | .0636 |
| 1.0 | 0.0 | 0.050 | 0.00 | 0.0000 | 0.0500 | .0500 |

The column labeled *U*(*A* = 2) implies that investors with *A* = 2 prefer a portfolio that is invested 100% in the market index to any of the other portfolios in the table.

12. The column labeled *U*(*A* = 3) in the table above is computed from:

*U* = *E*(*r*) – 0.5*A*σ2 = *E*(*r*) – 1.5σ2

The more risk averse investors prefer the portfolio that is invested 40% in the market, rather than the 100% market weight preferred by investors with *A* = 2.

13. Expected return = (0.7 × 18%) + (0.3 × 8%) = 15%

Standard deviation = 0.7 × 28% = 19.6%

|  |  |  |  |
| --- | --- | --- | --- |
| 14. | Investment proportions: |  | 30.0% in T-bills |
|  |  | 0.7 × 25% = | 17.5% in Stock A |
|  |  | 0.7 × 32% = | 22.4% in Stock B |
|  |  | 0.7 × 43% = | 30.1% in Stock C |

15. Your reward-to-volatility ratio: ****

Client's reward-to-volatility ratio: ****

16.



17. a. *E*(*rC*) = *rf* + *y* × [*E*(*rP*) – *rf*] = 8 + *y* × (18 − 8)

If the expected return for the portfolio is 16%, then:

16% = 8% + 10% × *y* ⇒

Therefore, in order to have a portfolio with expected rate of return equal to 16%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b.

|  |  |  |
| --- | --- | --- |
| Client’s investment proportions: | | 20.0% in T-bills |
|  | 0.8 × 25% = | 20.0% in Stock A |
|  | 0.8 × 32% = | 25.6% in Stock B |
|  | 0.8 × 43% = | 34.4% in Stock C |

c. σ*C* = 0.8 × σ*P* = 0.8 × 28% = 22.4%

18. a. σ*C* = *y* × 28%

If your client prefers a standard deviation of at most 18%, then:

*y* = 18/28 = 0.6429 = 64.29% invested in the risky portfolio.

b. 

19. a. *y*\*

Therefore, the client’s optimal proportions are: 36.44% invested in the risky portfolio and 63.56% invested in T-bills.

1. *E*(*rC*) = 0.08 + 0.10 × *y*\* = 0.08 + (0.3644 × 0.1) = 0.1164 or 11.644%

σ*C* = 0.3644 × 28 = 10.203%

20. a. If the period 1927–2018 is assumed to be representative of future expected performance, then we use the following data to compute the fraction allocated to equity: *A* = 4, *E*(*rM*) − *rf* = 8.34%, σ*M* = 20.36% (we use the standard deviation of the risk premium from Table 6.7). Then *y*\* is given by:



That is, 50.30% of the portfolio should be allocated to equity and 49.70% should be allocated to T-bills.

1. If the period 1973–1995 is assumed to be representative of future expected performance, then we use the following data to compute the fraction allocated to equity: *A* = 4, *E*(*rM*) − *rf* = 6.11%, σ*M* = 18.34% and *y*\* is given by:

**

Therefore, 45.41% of the complete portfolio should be allocated to equity and 54.59% should be allocated to T-bills.

c. In part (b), the market risk premium is expected to be lower than in part (a) and market risk is higher. Therefore, the reward-to-volatility *ratio* is expected to be lower in part (b), which explains the greater proportion invested in T-bills.

21. a. *E*(*rC*) = 8% = 5% + *y* × (11% – 5%) ⇒ 

b. σ*C* = *y* × σ*P* = 0.50 × 15% = 7.5%

c. The first client is more risk averse, preferring investments that have less risk as evidenced by the lower standard deviation.