SIMPLE RULES

Fall 2000

<u>Basic Ideas</u>:

1. **Covariances will be estimated via model.**

2. **With any standard model can develop easy selection rule.**

<u>Uses</u>:

1. **Makes clear why security enters.**

2. **Useful in sensitivity analysis.**

3. **Easy calculation.**

Example:

Assume $r_{jk} = \overline{r}$ **constant correlation.**

First order condition

$$\overline{R}_i - R_F = Z_i s_i^2 + \sum_{\substack{j=1 \\ j \neq 1}}^{N} Z_j s_{ij}$$

if $s_{ij} = \overline{r} s_i s_j$

in what follows $r = \overline{r}$

$$\overline{R}_i - R_F = Z_i s_i^2 + \sum_{\substack{j=1 \\ j \neq 1}}^{N} Z_j r s_i s_j$$

add and subtract $Z_i r s_i s_i$

$$\bar{R}_i - R_F = Z_i s_i^2 (1 - r) + r s_i \sum_{j=1}^{N} Z_j s_j$$

Note $\sum_{j=1}^{N} Z_j s_j$ **is a constant.**

solving for Z_i

$$Z_i = \frac{\bar{R}_i - R_F}{(1-r)s_i^2} - \frac{r s_i}{(1-r)s_i^2} \sum_{j=1}^{N} Z_j s_j$$

$$Z_i = \frac{1}{(1-r)s_i}\left[\frac{\bar{R}_i - R_F}{s_i} - \frac{r}{(1-r)} \sum_{j=1}^{N} Z_j s_j\right]$$

let $C^* = \frac{r}{1-r} \sum_{j=1}^{N} Z_j s_j$

Then

$$Z_i = \frac{1}{(1-r)s_i}\left[\frac{\bar{R}_i - R_F}{s_i} - C^*\right]$$

It can be shown that

$$C^* = \frac{r}{1-r+Nr}\sum_{j=1}^{N}\frac{\bar{R}_j - R_F}{s_j}$$

Note C^* is a constant. If short sales are not allowed, summation goes to number in set not N.

$$\frac{r}{1-r+Nr} = \frac{1}{\frac{1}{r}+(N-1)} \cong \frac{1}{N}$$

$$\therefore C^* \cong \frac{1}{N}\sum_{j=1}^{N}\frac{\overline{R}_j - R_F}{s_j}$$

If single-index model:

$$Z_i = \frac{b_i}{s^2_{e_i}}\left[\frac{\overline{R}_i - R_F}{b_i} - C^*\right]$$

where

$$C^* = s^2_m \sum_{j=1}^{N} Z_j b_j$$

$$C^{*}=\frac{s_{m}^{2}\sum_{j=1}^{N}\frac{\left(\overline{R}_{j}-R_{F}\right)b_{j}}{s_{e_{j}}^{2}}}{1+s_{m}^{2}\sum_{j=1}^{N}\frac{b_{j}^{2}}{s_{e_{j}}^{2}}}$$

If short sales are not allowed, summation goes to numbers that are held in positive proportion, not N.