Objective Functions

I. Classical Markowitz

Trade-off is not explicitly made.
II. Trade-off explicit:

A. Use utility functions

Problem is specifying utility.
B. Specify risk tolerance.

By tradition, divide variance by risk tolerance.

Mean Return

- risk penalty = \( \frac{\text{variance}}{\text{risk tolerance}} \)

risk adjusted expected return

Example:

\( \bar{r} = 12 \)
\( \sigma = 15 \)

Tolerance = 50

Risk adjusted expected return: \( 12 - \frac{225}{50} = 7 \frac{1}{2} \)

Same issue is how tolerance specified but maybe easier to work with investor to determine range.
III. Safety first criteria (emphasis is on avoidance of risk).

A. Roy's Criteria:

Minimize Prob \( R_p < R_L \)

B. Katoka's Criteria

Maximize \( R_L \)

Subject to: Prob \( R_p < R_L \) \( \leq \alpha \)
C. Telser's Criteria

\[ \text{Max } \overline{R}_P \]

Subject to: \( \text{Prob } \left( \overline{R}_P \leq \overline{R}_L \right) \leq \alpha \)
Analysis of criteria:

The following analysis assumes normal returns.

A. Consider Roy's criteria:

\[
\text{Min Prob}\left( R_P < R_L \right)
\]
Thus, want to maximize:

\[
\frac{\overline{R}_p - R_L}{\sigma_p}
\]

\(R_L\) serves as role of \(R_F\).
$R_L$ serves as role of $R_F$.

B. Katoka's criteria

Maximize $R_L$

Subject to:

\[
\text{Prob} \left( R_P < R_L \right) \leq \alpha
\]

\[
R_L \leq \overline{R}_P - K \sigma_P
\]

Where $K$ is set to match above constraint - example $1.65$. 
Expression of straight line

Note if riskless lending and borrowing get funny results.
Consider Telser's criteria:

\[
\text{max } \bar{R}_P \\
\text{Subj to } \text{Prob } \left[ R_P \leq R_L \right] \leq \alpha
\]

Constraint is:

\[
R_L \leq \bar{R}_P - K \sigma_P
\]