Book Corrections for Fundamentals of Spacecraft Attitude Determination and Control

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This document provides corrections for the book: Markley, F.L., and Crassidis, J.L., *Fundamentals of Spacecraft Attitude Determination and Control*, Springer, New York, NY, 2014. Any other corrections are welcome via email to the authors.

Chapter 1

• On page 2 there should be a closed parenthesis after "1630" in Table 1.1.

Chapter 2

- On the sixth line of page 17 "conceptionally" should be "conceptually."
- On page 18, under Eq. (2.3), the sentence "Matrix addition and subtraction are both commutative, $A \pm B = B \pm A$, and associative, $(A \pm B) \pm C = A \pm (B \pm C)$ " should be replaced with "Matrix addition is commutative, A + B = B + A, and associative, (A+B)+C = A + (B+C). Matrix subtraction is not communicative or associative."
- On page 19, the definitions of upper and lower triangular matrices under Eq. (2.9) should read "An upper triangular matrix is a matrix in which all the entries below the main diagonal are zero, i.e. $a_{ij} = 0$ for i > j. A lower triangular has all zeros above the main diagonal, i.e. $a_{ij} = 0$ for i < j."
- On page 23, the sentence below Eq. (2.28) should read "In particular, if we have a scalar $f(\mathbf{x})$ in place of the vector function $\mathbf{y}(\mathbf{x})$, this reduces to the $1 \times n$ row vector..." Also, the next sentence should read "The transpose of this, an $n \times 1$ column vector..."
- On page 33, Eq. (2.68) should read

$$JD(Y, M, D, h, m, s) = 1,721,013.5 + 367 Y - INT \left\{ \frac{7}{4} \left[Y + INT \left(\frac{M+9}{12} \right) \right] \right\} + INT \left(\frac{275 M}{9} \right) + D + \frac{3,600 h + 60 m + s}{86,400^*}$$

where INT denotes the integer part and 86,400^{*} denotes using 86,401 seconds for days with a leap second.

• On page 62, in part a) of Problem 2.9, "GPRs" should be "GRPs."

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Chapter 3

- In the first paragraph of page 67 the translational momentum should be defined as $\mathbf{p} \equiv m\mathbf{v}$ not $\mathbf{p} \equiv m\dot{\mathbf{v}}$.
- In the second paragraph on page 89 (fourth line), the word "away" should read "a way."
- The inertia matrix in Exercise 3.6 on page 119 does not satisfy the triangle inequality. We recommend using the following inertia matrix:

$$J_B^c = \begin{bmatrix} 100 & 0 & 0\\ 0 & 75 & 0\\ 0 & 0 & 50 \end{bmatrix} \text{ kg-m}^2$$

Chapter 4

• On page 126, Eq. (4.3b) should read

$$v = v_0 + fs_2/s_3 = v_0 + f \tan \alpha$$

• On page 127, Eq. (4.17) should read

$$\mathbf{q}^{\text{true}} = \frac{1}{\sqrt{1 + \|(1/2)\boldsymbol{\theta}_T^{\text{aber}}\|^2}} \begin{bmatrix} -(1/2)\boldsymbol{\theta}_T^{\text{aber}} \\ 1 \end{bmatrix} \otimes \mathbf{q}^{\text{aber}}$$
$$= \frac{1}{\sqrt{1 + \|(1/2)\boldsymbol{\theta}_I^{\text{aber}}\|^2}} \mathbf{q}^{\text{aber}} \otimes \begin{bmatrix} -(1/2)\boldsymbol{\theta}_I^{\text{aber}} \\ 1 \end{bmatrix}$$
(1)

- On page 129, above Eq. (4.9) "finding N stars" should be changed to "finding exactly N stars." Also, below Eq. (4.9) and in the next paragraph "finding 4 stars" should be changed to "finding 4 or more stars" (3 places). Finally, below Eq. (4.9) and in the next paragraph "finding 5 stars" should be changed to "finding 5 or more stars" (3 places).
- On page 143, in the first sentence of Section 4.7.1, "rate-integrating gyro" should be replaced by "rate gyro."
- On page 159 the label "newpoly.eps" should be removed from Figure 4.10.
- On page 176 the subscript "min" in Eq. (4.127) should be text, not italic.

Chapter 5

• On page 200 the reference to Eq. (5.3) in the second line of the second (new) paragraph should be to Eq. (5.6).

• The matrix A^{true} in Problem 5.3 on page 225 should read

	0.352	0.864	0.360
$A^{\text{true}} =$	-0.864	0.152	0.480
	0.360	-0.480	0.800

That is, the (2,3) element is changed from 0.460 to 0.480 to ensure that A is orthogonal.

Chapter 6

- On page 243 in the last sentence of the paragraph below Eq. (6.25), $\delta \vartheta_k^-$ and $\Delta \xi^-$ should be $\delta \hat{\vartheta}$ and $\Delta \hat{\xi}$.
- The asterisk used for the unnormalized reset quaternion in Eqs. (6.27) and (6.28) and in Tables 6.1 and 6.3 conflicts with the notation for the conjugate quaternion, which is used throughout the book. To better differentiate between the two, replace Eqs. (6.27) and (6.28) with

$$\hat{\mathbf{q}}^{\mathrm{unnorm}} = \begin{bmatrix} \boldsymbol{\delta}\hat{\boldsymbol{\vartheta}}^{+\!/2} \\ 1 \end{bmatrix} \otimes \hat{\mathbf{q}}^{-} = \hat{\mathbf{q}}^{-} + \frac{1}{2}\Xi(\hat{\mathbf{q}}^{-})\boldsymbol{\delta}\hat{\boldsymbol{\vartheta}}^{+}$$

and

$$\hat{\mathbf{q}}^+ = rac{\hat{\mathbf{q}}^{\mathrm{unnorm}}}{\|\hat{\mathbf{q}}^{\mathrm{unnorm}}\|}$$

respectively. Also, replace both \mathbf{q}^* and $\hat{\mathbf{q}}^*$ by $\hat{\mathbf{q}}^{\text{unnorm}}$ in Tables 6.1 and 6.3.

• In the second line of Example 6.2 "a 90-min low-Earth orbit" should be changed to "a 91.5-min equatorial low-Earth orbit." The number of available stars changes slightly but the results are nearly identical. A plot of the available stars with a 91.5-min equatorial low-Earth orbit is shown in Figure 1.



Figure 1 Available Stars

• On page 264, just above Eq. (6.102a) the estimate should be defined as $E\{\mathbf{x}|\theta^{\text{out}}\}$. Thus, the sentence should read "Because η_u , η_v , and v_e have zero mean, the conditional expectation $\hat{\mathbf{x}} \equiv E\{\mathbf{x}|\theta^{\text{out}}\}$ and the state error vector $\Delta \mathbf{x} \equiv \mathbf{x} - \hat{\mathbf{x}}$ obey"

- On page 265 three lines from the bottom "Equation (6.107) are" should be "Equations (6.107) are."
- On page 266, in Eq. (6.108b) the 3-2 element of the matrix P^+ should be changed from $\zeta^{-1}S_e^2 S_u(\Delta t)^{-1}$ to $\zeta^{-1}S_e^2 S_u/\Delta t$ to make the notation consistent in the matrix.
- In Problem 6.7 on page 280, the sentence "Use the discrete-time covariance propagation in Eq. (E.132)..." should be replaced with "Use the discrete-time covariance propagation in Eq. (E.131)..."
- On page 284, the word "attitude" in Ref. 16 is misspelled.
- On page 284, the word "convergence" in Ref. 34 is misspelled.

Chapter 7

• In example 7.2, the initial the torque of the reaction wheels exceeds the capabilities of 0.01-1.0 Nm, as discussed on page 148. What would happen if one commands a reaction wheel to produce a torque above its maximum capability would depend on the electronics in the wheel drive. The most likely result is that the wheel will produce as much torque as it can. This can cause a problem if one wants the direction of the net torque vector in 3D space to be correct. In practice, the control algorithm is designed to check that all the wheel torque commands are at or below the maximum (with some safety factor). If any are above, then all the torque commands are scaled by the same factor to reduce the largest command to the maximum allowable in order preserve the correct direction of the torque vector in 3D space. Similar steps can be taken if a reaction wheel's angular momentum approaches its maximum values are not reached.

The inertia matrix has now been changed to

$$J = \begin{bmatrix} 640 & -7.64 & -2.56 \\ -76.4 & 473 & -4 \\ -2.56 & -4 & 816 \end{bmatrix} \text{ kg-m}^2$$

Change "The gains are set to $k_p = 10$ and $k_d = 150$." to "The gains are set to $k_p = 1$ and $k_d = 15$." Change "...the norm of the momentum is $||J\omega(t_0)|| = 112.4586$ Nms" to "the norm of the momentum is $||J\omega(t_0)|| = 11.25$ Nms." Change "...pyramid configuration at the final time is 96.8044 Nms" to "...pyramid configuration at the final time is 96.8044 Nms" to "...pyramid configuration at the final time is 111.8695 Nms" to "NASA standard configuration at the final time is 111.8695 Nms" to "NASA standard configuration at the final time is 11.19 Nms." The new plots for Figure 7.2 in the book are given by Figure 2. The new plots for Figure 7.3 in the book are given by Figure 3. Note that Figure 7.2(a) is the same as before, however the other three have changed.

• Equation (7.28) should read

$$\dot{\mathbf{s}} = \delta J^{-1} \hat{J} \left\{ \frac{k}{2} \left[|\delta q_4| (\boldsymbol{\omega}_c - \boldsymbol{\omega}) - \operatorname{sign}(\delta q_4) \, \boldsymbol{\delta} \mathbf{q}_{1:3} \times (\boldsymbol{\omega} + \boldsymbol{\omega}_c) \right] + \dot{\boldsymbol{\omega}}_c \right\} \\ - J^{-1} \hat{J} G \, \bar{\mathbf{s}} + J^{-1} \mathbf{d}$$



Figure 2 Quaternion Errors and Wheel Momenta



Figure 3 Comparison Between Wheel Configurations

• On page 300 ω_c should be given by

$$\boldsymbol{\omega}_{c} = \begin{bmatrix} \dot{\phi}_{c} \sin \theta_{c} \sin \psi_{c} \\ \dot{\phi}_{c} \sin \theta_{c} \cos \psi_{c} \\ \dot{\psi}_{c} + \dot{\phi}_{c} \cos \theta_{c} \end{bmatrix} \frac{\mathrm{rad}}{\mathrm{sec}}$$

- On page 302 "Substituting Eqs. (7.1b) and (7.3.1) into Eq. (7.33), and after..." should read "Substituting Eqs. (7.1b) and (7.32) into Eq. (7.33), and after..."
- On page 328 the fourth line of the last paragraph of Section 7.7.3.2 should read "the rotation of the z-axis..."
- On page 342 the last page of Ref. [30] is 2566 not 2565.

Appendix A

- On page 346 the sentence above Eq. (A.6) should read "...then from Eq. (A.3b) we have."
- The third sentence on page 358 should read "Equation (A.50a) can be used to prove..."
- The last sentence on page 358 should read "Equation (A.50d) can be used to prove..."

Appendix C

• On page 387, the terms in Eq. (C.103) should read

$$\begin{aligned} \mathbf{a}_{J_2} &= -\frac{3}{2} J_2 \left(\frac{\mu}{r^2}\right) \left(\frac{R_{\oplus}}{r}\right)^2 \begin{bmatrix} \left(1-5\left(\frac{z}{r}\right)^2\right) \frac{x}{r} \\ \left(1-5\left(\frac{z}{r}\right)^2\right) \frac{y}{r} \\ \left(3-5\left(\frac{z}{r}\right)^2\right) \frac{z}{r} \end{bmatrix} \\ \mathbf{a}_{J_3} &= -\frac{1}{2} J_3 \left(\frac{\mu}{r^2}\right) \left(\frac{R_{\oplus}}{r}\right)^3 \begin{bmatrix} 5\left(3\left(\frac{z}{r}\right)-7\left(\frac{z}{r}\right)^3\right) \frac{x}{r} \\ 5\left(3\left(\frac{z}{r}\right)-7\left(\frac{z}{r}\right)^3\right) \frac{y}{r} \\ -\left(3-30\left(\frac{z}{r}\right)^2+35\left(\frac{z}{r}\right)^4\right) \end{bmatrix} \\ \mathbf{a}_{J_4} &= \frac{5}{8} J_4 \left(\frac{\mu}{r^2}\right) \left(\frac{R_{\oplus}}{r}\right)^4 \begin{bmatrix} 3\left(1-14\left(\frac{z}{r}\right)^2+21\left(\frac{z}{r}\right)^4\right) \frac{x}{r} \\ 3\left(1-14\left(\frac{z}{r}\right)^2+21\left(\frac{z}{r}\right)^4\right) \frac{y}{r} \\ \left(15-70\left(\frac{z}{r}\right)^2+63\left(\frac{z}{r}\right)^4\right) \frac{z}{r} \end{bmatrix} \\ \mathbf{a}_{J_5} &= \frac{3}{8} J_5 \left(\frac{\mu}{r^2}\right) \left(\frac{R_{\oplus}}{r}\right)^5 \begin{bmatrix} 7\left(5\left(\frac{z}{r}\right)-30\left(\frac{z}{r}\right)^3+33\left(\frac{z}{r}\right)^5\right) \frac{x}{r} \\ 7\left(5\left(\frac{z}{r}\right)-30\left(\frac{z}{r}\right)^3+33\left(\frac{z}{r}\right)^5\right) \frac{y}{r} \\ -\left(5-105\left(\frac{z}{r}\right)^2+315\left(\frac{z}{r}\right)^4-231\left(\frac{z}{r}\right)^6\right) \end{bmatrix} \\ \mathbf{a}_{J_6} &= -\frac{7}{16} J_6 \left(\frac{\mu}{r^2}\right) \left(\frac{R_{\oplus}}{r}\right)^6 \begin{bmatrix} \left(5-135\left(\frac{z}{r}\right)^2+495\left(\frac{z}{r}\right)^4-429\left(\frac{z}{r}\right)^6\right) \frac{x}{r} \\ \left(35-315\left(\frac{z}{r}\right)^2+693\left(\frac{z}{r}\right)^4-429\left(\frac{z}{r}\right)^6\right) \frac{x}{r} \end{bmatrix} \end{aligned}$$

- On page 390, in the ninth line of Section C.3.4 "SRP only contributes as times..." should be replaced with "SRP only contributes at times..."
- On page 397 two lines under Eq. (C.129), it should say "greater than 90°," not 95.68°.

Appendix D

• On page 412, the ΔT_c term in Eq. (D.18) should read

$$\Delta T_c = A + Bh_p + Ch_p^2 + Dh_p^3$$

• Also, the B term in Eq. (D.20) should read

$$B = B_{13}y + B_{14}yT + B_{15}yT^2 + B_{16}yT^3 + B_{17}yT^4 + B_{18}yT^5$$

Appendix E

• Equation (E.69) on page 446 should read

$$s = \left(\frac{d}{dt} + \lambda\right)^{n-1} \Delta x$$

Also, the sliding surface for n = 3 should be $s = \Delta \ddot{x} + 2\lambda \Delta \dot{x} + \lambda^2 \Delta x$. Note that Eq. (E.69) is a notational simplification for the weighted sum average used to generate s, which is discussed in [25].

Equations (E.70)-(E.72) and the associated wording should be changed as follows. As an example consider the following second-order system:

$$\ddot{x} = f(x, \dot{x}) + u$$

where an assumed model $\bar{f}(x, \dot{x})$ will be used to develop the control law. Taking the derivative of s in Eq. (E.65) and substituting $\ddot{x} = f(x, \dot{x}) + u$ gives

$$\dot{s} = \ddot{x} - \ddot{x}_c + \lambda \,\Delta \dot{x}$$
$$= f(x, \,\dot{x}) + u - \ddot{x}_c + \lambda \,\Delta \dot{x}$$

Using the condition $\dot{s} = 0$ from Eq. (E.67) with the assumed model $\bar{f}(x, \dot{x})$ leads to the following control law:

$$u_e = -\bar{f}(x, \dot{x}) + \ddot{x}_c - \lambda \,\Delta \dot{x}$$

• In Example E.3 on page 448, \dot{x}_2 should be

$$\dot{x}_2 = -(k_1/m)x_1 - (k_2/m)x_1^3 - (c/m)x_2|x_2| + u/m$$

- On the first line of page 459 "Nonlinear Least squares implies that an fairly..." should be replaced with "Nonlinear Least squares implies that a fairly..."
- Equation (E.113) should read

$$\hat{\mathbf{p}} = \frac{\left(1 - \|\mathbf{p}_c\|^2\right)\Delta\mathbf{x} + \left(1 - \|\Delta\mathbf{x}\|^2\right)\mathbf{p}_c - 2\Delta\mathbf{x}\times\mathbf{p}_c}{1 + \|\mathbf{p}_c\|^2\|\Delta\mathbf{x}\|^2 - 2\Delta\mathbf{x}\cdot\mathbf{p}_c}$$

• On line two of page 465 "This the preferred form..." should be replaced with "This is the preferred form..."