

Book Corrections for Optimal Estimation of Dynamic Systems, 2nd Edition

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This document provides corrections for the book: Crassidis, J.L., and Junkins, J.L., *Optimal Estimation of Dynamics Systems*, 2nd Edition, CRC Press, Boca Raton, FL, 2012. Any other corrections are welcome via email to the authors.

Chapter 1

- The upper limit for the index i in Equations (1.105) and (1.122) should be $2n + 1$ instead of n , so they should read, respectively, as

$$\hat{x}_i = \left[\sum_{j=1}^m h_i^2(t_j) \right]^{-1} \sum_{j=1}^m h_i(t_j) \tilde{y}_j, \quad i = 1, 2, \dots, 2n + 1$$

and

$$\hat{x}_i = \frac{\int_0^T y(t) h_i(t) dt}{\int_0^T [h_i(t)]^2 dt}, \quad i = 1, 2, \dots, 2n + 1$$

Also, the paragraph under Equation (1.106) should read “Orthogonality of the basis functions of Equation (1.106) means that the coefficients \hat{x}_i are computed independently as ratios of inner products in Equation (1.105), so adding additional terms to the series in Equation (1.102) does not require re-computation of the previously computed terms. This allows adaptation wherein additional terms can be added, so long as $n \leq (m - 1)/2$, until some convergence criterion is met.”

- The gradient of Equation (1.152) is actually evaluated at the current estimate. So, Equation (1.153) is given by

$$\nabla_{\hat{\mathbf{x}}} J|_{\mathbf{x}_c} = -H^T W [\tilde{\mathbf{y}} - \mathbf{f}(\mathbf{x}_c)] \equiv -H^T W \Delta \mathbf{y}_c$$

and Equation (1.154) is given by

$$H \equiv \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\mathbf{x}_c}$$

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Chapter 2

- On page 70 just before Eq. (2.62), we are making use of Eq. (2.58b) instead of Eq. (2.58a).
- On page 95, Eq. (2.175) should read

$$\left. \frac{\partial J_{\text{MR}}(\mathbf{x}^*)}{\partial \mathbf{x}^*} \right|_{\hat{\mathbf{x}}} = \mathbf{0} = S \int_{-\infty}^{\infty} (\hat{\mathbf{x}} - \mathbf{x}) p(\mathbf{x}|\tilde{\mathbf{y}}) d\mathbf{x}$$

- After Eq. (2.283) it states that the assumption $\bar{s}_n > s_{n+1}$ must be valid. This condition is in fact required for the validity of the TLS solution itself in Eq. (2.280). This is shown as Theorem 4.1 in the paper Golub, G.H. and Van Loan, C.F., “An Analysis of the Total Least Squares Problem,” *SIAM Journal on Numerical Analysis*, Vol. 17, No. 6, Dec. 1980, pp. 883-893.
- Exercise 2.11 should read:

Prove that the Cramér-Rao inequality given by Equation (2.100) achieves the equality if and only if

$$\frac{\partial}{\partial \mathbf{x}} \ln[p(\tilde{\mathbf{y}}|\mathbf{x})] = F(\mathbf{x})(\mathbf{x} - \hat{\mathbf{x}})$$

where $F(\mathbf{x})$ is the Fisher information matrix explicitly shown as a function of \mathbf{x} .

- Although exercise 2.12 is correct, another way to state the problem is as follows:
Suppose that an estimator of a non-random scalar x is biased, with bias denoted by $b(x)$, so that $E\{\hat{x}\} = x + b(x)$. Show that a lower bound on the variance of the estimate \hat{x} is given by

$$\text{var}(\hat{x}) \geq \left(1 + \frac{\partial b(x)}{\partial x}\right)^2 J^{-1}$$

where

$$J = E \left\{ \left[\frac{\partial}{\partial x} \ln[p(\tilde{\mathbf{y}}|x)] \right]^2 \right\}$$

Chapter 3

- On page 173, Eq. (3.188) should read

$$E \{ (x - \hat{x})^2 \} = \begin{cases} \infty & d < -1 \\ 0 & d > -1 \\ \frac{R}{\Delta t} & d = -1 \end{cases}$$

- On page 148, the innovations covariance in Eqs. (3.55b) and (3.56) should be $H_k P_k^- H_k^T + R_k$.

Chapter 4

- Equation (4.44) should read

$$\bar{\rho}_j = \frac{1}{\sqrt{m}} \sum_{k=1}^N \mathbf{e}_k^T \mathbf{e}_{k+j} \left[\sum_{k=1}^N \mathbf{e}_k^T \mathbf{e}_k \sum_{k=1}^N \mathbf{e}_{k+j}^T \mathbf{e}_{k+j} \right]^{-1/2}$$

- The term P_{xx}^+ is Eq. (4.54) should $P_{xx_k}^+$.
- Equation (4.110) on page 254 is incorrect. It should read:

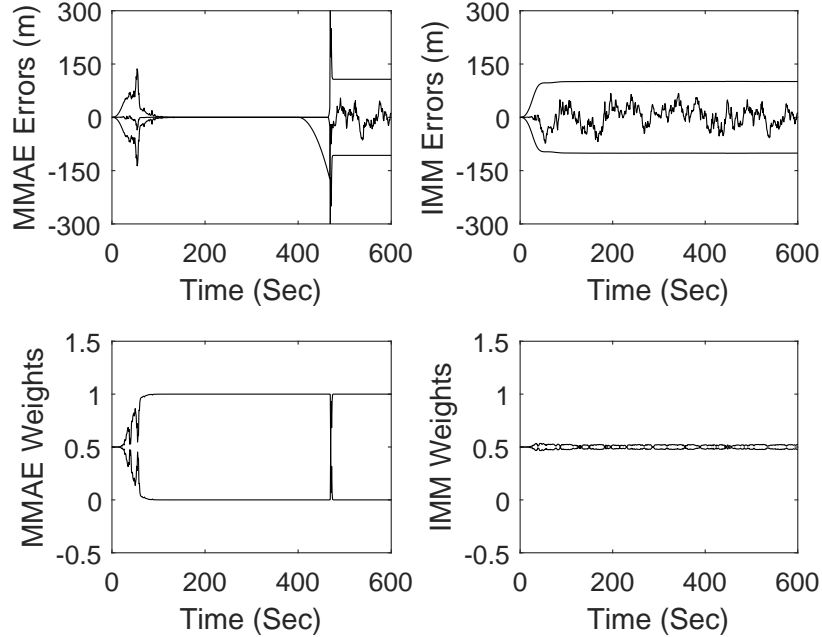
$$w_k^{(j)} = \bar{c}_{k-1}^{(j)} p(\tilde{\mathbf{y}}_k | \hat{\mathbf{x}}_k^{-(j)})$$

$$w_k^{(j)} \leftarrow \frac{w_k^{(j)}}{\sum_{j=1}^M w_k^{(j)}}$$

- The code has been updated for Example 4.6 to correct the above error as well as the following:

```
p_prop_imm2=phi*p_imm2*phi; --> p_prop_imm2=phi*p_imm2*phi';
```

Figure 4.9 is now replaced with the following:



- In Example 4.9 the transpose on $w(\tau)$ is not required in $E\{w(t)w^T(\tau)\} = q(t)\delta(t-\tau)$.
- In the second sentence after Eq. (4.153) it should read “then Equation (4.151) reduces...”

- On pages 283 and 284 Step 2 for Systematic Resampling and Stratified Resampling are incorrect. The corrected versions are:

2. Set $i = 1$. Perform the next steps for $j = 1, 2, \dots, N$. Execute a while loop:
 while $z^{(i)} < u^{(j)}$
 $i \leftarrow i + 1$
 end while

where \leftarrow denotes replacement; choose the resulting i after the while loop as the new index and replace $\mathbf{x}^{(j)}$ with $\mathbf{x}^{(i)}$.

- The MATLAB codes for Examples 4.11 and 4.12 are incorrect, so the Figures 4.14 and 4. 15 are no correct. Originally the posterior densities were plotted using “surf(xii,t,f)” but this is only correct for the last set of points, given by xii. This has been corrected by using “waterfall(xi,repmat(t,1,100),f)” for both examples. Note that the authors do not know how to plot the results correctly using the “surf” command. Please let the authors know if the reader knows how to plot the posterior densities using the “surf” command.

Chapter 5

- Table 5.5 on page 350 should be given by Table 1 shown below.

Chapter 6

- The MATLAB code for Example 6.1 is incorrect. The boresight of the star camera sensor is along the body z -axis. Thus an identity quaternion would align the body z -axis with the inertial z -axis, which causes an incorrect motion compared to what is described in the example. It is assumed that the Earth-pointing spacecraft is in an equatorial 350 km circular orbit, which is equivalent to a 91.5 minute orbital period. The spacecraft’s z -axis is pointed in the nadir direction, the y -axis is pointed in the negative orbit momentum’s vector, and the x -axis is pointed in the orbit velocity direction. The true angular velocity is given by $\boldsymbol{\omega}(t) = [0 \quad -1.11445 \times 10^{-3} \quad 0]^T$ rad/sec. First rotate +90 degrees about x -body axis. Then rotate 180 degrees about the new x -body axis, which correctly places the boresight in the anti-nadir direction (i.e. the radial direction). The initial quaternion is then given by $q_0 = \frac{\sqrt{2}}{2}[0 \quad 1 \quad 1 \quad 0]^T$. Also, a magnitude of 6 is chosen for the stars. There are times when the number of available stars is less than 2. At these times a solution is not possible.
- The equation for h_2 in Eq. (6.72c) should be given by $h_2 = h_1 - h_3$.
- The attitude matrix in Exercise 6.17 should be

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

Table 1 Continuous-Discrete Nonlinear RTS Smoother

Model	$\frac{d}{d\tau} \mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) + G(t) \mathbf{w}(t), \mathbf{w}(t) \sim N(\mathbf{0}, Q(t))$ $\tilde{\mathbf{y}}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \mathbf{v}_k \sim N(\mathbf{0}, R_k)$
Forward Initialize Forward Gain Forward Update Forward Propagation	$\hat{\mathbf{x}}_f(t_0) = \hat{\mathbf{x}}_{f0}$ $P_{f0} = E \left\{ \tilde{\mathbf{x}}_f(t_0) \tilde{\mathbf{x}}_f^T(t_0) \right\}$ $K_{fk} = P_{fk}^- H_k^T (\hat{\mathbf{x}}_{fk}^-) [H_k (\hat{\mathbf{x}}_{fk}^-) P_{fk}^- H_k^T (\hat{\mathbf{x}}_{fk}^-) + R_k]^{-1}$ $H_k(\hat{\mathbf{x}}_{fk}^-) \equiv \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right _{\hat{\mathbf{x}}_{fk}^-}$ $\hat{\mathbf{x}}_{fk}^+ = \hat{\mathbf{x}}_{fk}^- + K_{fk} [\tilde{\mathbf{y}}_k - \mathbf{h}(\hat{\mathbf{x}}_{fk}^-)]$ $P_{fk}^+ = [I - K_{fk} H_k(\hat{\mathbf{x}}_{fk}^-)] P_{fk}^-$ $\frac{d}{dt} \hat{\mathbf{x}}_f(t) = \mathbf{f}(\hat{\mathbf{x}}_f(t), \mathbf{u}(t), t)$ $\frac{d}{dt} P_f(t) = F(\hat{\mathbf{x}}_f(t), t) P_f(t) + P_f(t) F^T(\hat{\mathbf{x}}_f(t), t) + G(t) Q(t) G^T(t)$ $F(\hat{\mathbf{x}}_f(t), t) \equiv \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right _{\hat{\mathbf{x}}_f(t)}$
Gain Smoother Covariance Smoother Estimate	$K(t) \equiv G(t) Q(t) G^T(t) P_f^{-1}(t)$ $\frac{d}{d\tau} P(t) = -[F(\hat{\mathbf{x}}_f(t), t) + K(t)] P(t) - P(t) [F(\hat{\mathbf{x}}_f(t), t) + K(t)]^T + G(t) Q(t) G^T(t), \quad P(T) = P_f(T)$ $\frac{d}{d\tau} \hat{\mathbf{x}}(t) = -[F(\hat{\mathbf{x}}_f(t), t) + K(t)] [\hat{\mathbf{x}}(t) - \hat{\mathbf{x}}_f(t)] - \mathbf{f}(\hat{\mathbf{x}}_f(t), \mathbf{u}(t), t), \quad \hat{\mathbf{x}}(T) = \hat{\mathbf{x}}_f(T)$

Chapter 7

- The MATLAB code for Example 7.1 is incorrect. The boresight of the star camera sensor is along the body z -axis. Thus an identity quaternion would align the body z -axis with the inertial z -axis, which causes an incorrect motion compared to what is described in the example. It is assumed that the Earth-pointing spacecraft is in an equatorial 350 km circular orbit, which is equivalent to a 91.5 minute orbital period. The spacecraft's z -axis is pointed in the nadir direction, the y -axis is pointed in the negative orbit momentum's vector, and the x -axis is pointed in the orbit velocity direction. The true angular velocity is given by $\boldsymbol{\omega}(t) = [0 \quad -1.11445 \times 10^{-3} \quad 0]^T$ rad/sec. First rotate +90 degrees about x -body axis. Then rotate 180 degrees about the new x -body axis, which correctly places the boresight in the anti-nadir direction

(i.e. the radial direction). The initial quaternion is then given by $q_0 = \frac{\sqrt{2}}{2}[0 \ 1 \ 1 \ 0]^T$. Also, a magnitude of 6 is chosen for the stars. There are times when the number of available stars is less than 2. The extended Kalman filter still provides an update even when only 1 star is available.

- Equation (7.60) should read

$$\sqrt{p_{\theta\theta}^-} = \sqrt{p_{\theta\theta}^+} \equiv \sigma_c = \Delta t^{1/4} \sigma_n^{1/2} \left(\sigma_v^2 + 2\sigma_u \sigma_n \Delta t^{1/2} \right)^{1/4} \quad (1)$$

The σ_v term in the original $2\sigma_u \sigma_n \Delta t^{1/2}$ should be σ_n .

- Equation (7.83a) should read

$$Z_{11} = \frac{v_D}{R_\phi + h}, \quad Z_{12} = -\frac{2v_E \tan \phi}{R_\lambda + h} - 2\omega_e \sin \phi, \quad Z_{13} = \frac{v_N}{R_\phi + h}$$

The $2\omega_e \sin \phi$ term in Z_{12} should be subtracted not added, and Z_{13} was originally labeled as Z_{12} . The MATLAB code for Example 7.2 has been corrected.

Appendix A

- The derivation of Equation (A.111) is not correct. The goal is to determine the initial condition $\mathbf{x}(t_0)$, so replace t_0 with t , and replace t with t_f in Equation (A.109)

$$W_o(t, t_f) \equiv \int_t^{t_f} \Phi^T(\tau, t) H^T(\tau) H(\tau) \Phi(\tau, t) d\tau$$

Note that the notation for $W_o(t)$ has changed here, and that the integrations in Equations (A.107) to (A.109) should be done from t_0 and t_f . The time derivative of $\Phi(\tau, t) = \Phi^{-1}(t, \tau)$ will be needed. Take the time derivative of $V V^{-1} = I$ for some matrix V :

$$V \dot{V}^{-1} + \dot{V} V^{-1} = 0 \quad \rightarrow \quad \dot{V}^{-1} = -V^{-1} \dot{V} V^{-1}$$

Letting $V \equiv \Phi(t, \tau)$ and noting $V^{-1} = \Phi(\tau, t)$ leads to

$$\begin{aligned} \dot{\Phi}(\tau, t) &= -\Phi(\tau, t) \dot{\Phi}(t, \tau) \Phi(\tau, t) \\ &= -\Phi(\tau, t) F(t) \Phi(t, \tau) \Phi(\tau, t) \\ &= -\Phi(\tau, t) F(t) \end{aligned}$$

where the following identities were used

$$\begin{aligned} \dot{\Phi}(t, \tau) &= F(t) \Phi(t, \tau) \\ \Phi(t, \tau) \Phi(\tau, t) &= \Phi(t, t) = I \end{aligned}$$

Then the derivative of the observability Gramian is given by

$$\begin{aligned}\dot{W}_o(t, t_f) = & -\Phi^T(t, t) H^T(t) H(t) \Phi(t, t) \\ & - F^T(t) \underbrace{\int_t^{t_f} \Phi^T(\tau, t) H^T(\tau) H(\tau) \Phi(\tau, t) d\tau}_{W_o(t, t_f)} \\ & - \underbrace{\int_t^{t_f} \Phi^T(\tau, t) H^T(\tau) H(\tau) \Phi(\tau, t) d\tau}_{W_o(t, t_f)} F(t)\end{aligned}$$

Thus,

$$\boxed{\dot{W}_o(t, t_f) = -F^T(t) W_o(t, t_f) - W_o(t, t_f) F(t) - H^T(t) H(t)}$$

which is integrated backwards with $W_o(t_f, t_f) = 0$.

The controllability Gramian can be derived in a similar fashion but is integrated forward in time.

Appendix B

- On the top of page 662 the sentence “Matrix addition and subtraction are both commutative, $A \pm B = B \pm A$, and associative, $(A \pm B) \pm C = A \pm (B \pm C)$ ” should be replaced with “Matrix addition is commutative, $A + B = B + A$, and associative, $(A + B) + C = A + (B + C)$. Matrix subtraction is not commutative or associative.”

Appendix C

- Above Eq. (C.32) it should read “Two processes, $\{\mathbf{x}(t_k)\}$ and $\{\mathbf{y}(t_k)\}$, are uncorrelated if $E \{\mathbf{x}(t_i) \mathbf{y}^T(t_j)\} = E \{\mathbf{x}(t_i)\} E \{\mathbf{y}^T(t_j)\}$ for all t_i and t_j .”
- In the second sentence after Eq. (C.83b) “as $t \rightarrow 0$ ” should read “as $\Delta t \rightarrow 0$.”

Appendix D

- Below Eq. (D.16) it should read “Equations (D.16) provide three equations...”
- The necessary conditions in Example D.2 are given by

$$\begin{aligned}\frac{\partial \phi}{\partial y} &= -\frac{1}{2} - 18\lambda(y - 4) = 0 \\ \frac{\partial \phi}{\partial z} &= -\frac{1}{3} - 8\lambda(z - 5) = 0 \\ \psi(\mathbf{x}) &= 9(y - 4)^2 + 4(z - 5)^2 - 36 = 0\end{aligned}$$

The equation for $\partial \phi / \partial z$ is incorrect in the book.

- On page 715, it should read:

Therefore, the stationary points are given by

$$\begin{aligned} y^* &= 4 - \frac{1}{36\lambda} = 4 \pm \sqrt{2} \\ z^* &= 5 - \frac{1}{24\lambda} = 5 \pm \frac{3}{2}\sqrt{2} \\ \lambda^* &= \pm \frac{1}{36\sqrt{2}} \end{aligned}$$

The sufficient condition of Equation (D.19) for this problem is given by

$$\nabla_{\mathbf{x}}^2 \phi = - \begin{bmatrix} 18\lambda^* & 0 \\ 0 & 8\lambda^* \end{bmatrix}$$

Also, Equation (D.21) gives

$$Q \equiv q = -8\lambda^* \left[\frac{8}{18} \frac{(z^* - 5)^2}{(y^* - 4)^2} + 1 \right]$$

Clearly, if $\lambda^* = -1/(36\sqrt{2})$, then the stationary point given by $y^* = 4 + \sqrt{2}$ and $z^* = 5 + (3/2)\sqrt{2}$ is a local minimum with $\phi = (7/3) - \sqrt{2}$. Likewise, if $\lambda^* = +1/(36\sqrt{2})$, then the stationary point given by $y^* = 4 - \sqrt{2}$ and $z^* = 5 - (3/2)\sqrt{2}$ is a local maximum with $\phi = (7/3) + \sqrt{2}$.