Book Corrections for
Optimal Estimation of Dynamic Systems

John L. Crassidis[*] and John L. Junkins[†]

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This document provides corrections for the book: Crassidis, J.L., and Junkins, J.L., *Optimal Estimation of Dynamics Systems*, Chapman & Hall/CRC, Boca Raton, FL, 2004. Any other corrections are welcome via email to the authors.

Chapter 1

- Underneath Eq. (1.104) it states that $(H^T\,W\,H)$ is a diagonal matrix. This is not true in general. The matrix will be diagonal when the sample points are symmetric about $T/2$ and integer number cycles exist in the period of consideration, T.

- In the sentence under eqn. (1.106) on page 37 the words "does affect" should be changed to "does not affect."

- On the top of page 40 it states "... so that $\mathbf{h}(t)\,\mathbf{h}^T(t)$ is a diagonal matrix, then the individual components of $\hat{\mathbf{x}}$ are simply given by the uncoupled equations." The matrix $\mathbf{h}(t)\,\mathbf{h}^T(t)$ is not diagonal, but rather $\int_0^T \mathbf{h}(t)\,\mathbf{h}^T(t)\;dt$ reduces down to a diagonal matrix with elements given by $\int_0^T [h_i(t)]^2\,dt$. Therefore, eqn. (1.222) is still correct.

- Equation (1.140) should be

$$z = f(x,\,y) = \sum_{p=0}^{M}\sum_{q=0}^{N} c_{pq}x^p y^q$$

- Equation (1.141) should be

$$\tilde{z}_{ij} = f(x_i,\,y_j) + v_{ij}$$

Chapter 2

- Under Eq. (2.129) $P \geq F^{-1}$ means that $P - F^{-1}$ is positive semi-definite as described in Appendix A.

[*]Professor, Department of Mechanical & Aerospace Engineering, University at Buffalo, State University of New York, Amherst, NY 14260-4400. E-mail: johnc@buffalo.edu.

[†]Royce E. Wisenbaker '39 Chair Professor of Aerospace Engineering, Department of Aerospace Engineering, Texas A&M University, College Station, TX 77843-3141. E-mail: junkins@tamu.edu.

- Equation (2.161) should be

$$\begin{aligned} E\left\{\left[\frac{\partial}{\partial \mathbf{x}}\ln f(\mathbf{x})\right]\left[\frac{\partial}{\partial \mathbf{x}}\ln f(\mathbf{x})\right]^T\right\} &= Q^{-1}E\left\{(\hat{\mathbf{x}}_a-\mathbf{x})(\hat{\mathbf{x}}_a-\mathbf{x})^T\right\}Q^{-1} \\ &= Q^{-1}E\left\{\mathbf{w}\,\mathbf{w}^T\right\}Q^{-1} = Q^{-1} \end{aligned}$$

- Equation (2.168) should be

$$c(\mathbf{x}^*|\mathbf{x}) = \frac{1}{2}(\mathbf{x}^*-\mathbf{x})^T S(\mathbf{x}^*-\mathbf{x})$$

  This also needs to be changed on page 110.

- After Eq. (2.245) it states that the assumption $\bar{s}_n > s_{n+1}$ must be valid. This condition is in fact required for the validity of the TLS solution itself in Eq. (2.242). This is shown as Theorem 4.1 in the paper Golub, G.H. and Van Loan, C.F., "An Analysis of the Total Least Squares Problem," *SIAM Journal on Numerical Analysis*, Vol. 17, No. 6, Dec. 1980, pp. 883-893.

Chapter 3

- Equation (3.9a) should be

$$y = b_n\frac{d^n x}{dt^n} + b_{n-1}\frac{d^{n-1}x}{dt^{n-1}} + \cdots + b_1\frac{dx}{dt} + b_0\,x$$

- Equation (3.46) should be

$$\Phi(t,t_0)\,\dot{\mathbf{g}}(t) + F(t)\,\Phi(t,t_0)\,\mathbf{g}(t) = F(t)\,\Phi(t,t_0)\,\mathbf{g}(t) + B(t)\,\mathbf{u}(t)$$

- On page 146 in the Lyapunov analysis for the example 3.6, we can only conclude stability, not asymptotic stability, based on the negative semi-definiteness of $\dot{V}(\mathbf{x})$. In this example, we have to go to the higher derivatives or invoke LaSalle's Invariance to conclude asymptotic stability.

- Just under eqn. (3.216) the statement "Y is the side force due to rudder" should be changed to just "Y is the side force."

- Equation (3.222) should be

$$\begin{aligned} C_l &= C_{l_0} + C_{l_\beta}\beta + C_{l_{\delta_R}}\delta_R + C_{l_{\delta_A}}\delta_A + C_{l_p}\frac{\Delta\omega_1 b}{2\,v_{ss}} + C_{l_r}\frac{\Delta\omega_3 b}{2\,v_{ss}} \\ C_m &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_E}}\delta_E + C_{m_q}\frac{\Delta\omega_2\bar{c}}{2\,v_{ss}} \\ C_n &= C_{n_0} + C_{n_\beta}\beta + C_{n_{\delta_R}}\delta_R + C_{n_{\delta_A}}\delta_A + C_{n_p}\frac{\Delta\omega_1 b}{2\,v_{ss}} + C_{n_r}\frac{\Delta\omega_3 b}{2\,v_{ss}} \end{aligned}$$

  where $\Delta\omega_i$, $i = 1, 2, 3$, are the perturbed angular velocities, defined as the difference between the actual and steady-state values, and v_{ss} is the steady-state total velocity. This also needs to be changed on page 176.

Chapter 4

- The MATLAB code for Example 4.2 is incorrect. The boresight of the star camera sensor is along the body z-axis. Thus an identity quaternion would align the body z-axis with the inertial z-axis, which causes an incorrect motion compared to what is described in the example. The simulation has now been corrected in the MATLAB code to reflect what was stated in the text. Also, a magnitude of 6 is chosen for the stars. There are times when the number of available stars is less than 2. At these times a solution is not possible.

- The pitching torque coefficient in Example 4.4 should be

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_E}}\delta_E + C_{m_q}\frac{\Delta\omega_2\bar{c}}{2\,v_{ss}}$$

  Also, note that in this example the actual measurements are sampled at a rate that is ten times faster than shown in Figure 4.10. This is required to accurately determine the parameters.

- Reference [10] on page 240 is incorrect. It should be: Shuster, M.D., and Oh, S.D., "Three-Axis Attitude Determination from Vector Observations," *Journal of Guidance and Control*, Vol. 4, No. 1, Jan.-Feb. 1981, pp. 70-77.

Chapter 5

- Equations (5.77) and (5.78) should be

$$\begin{aligned}\tilde{\mathbf{z}}_k \equiv T_k^T\tilde{\mathbf{y}}_k &= T_k^T H_k\mathbf{x}_k + T_k^T\mathbf{v}_k\\ &\equiv \mathcal{H}_k\mathbf{x}_k + \boldsymbol{\upsilon}_k\end{aligned}$$

  where

$$\begin{aligned}\mathcal{H}_k &\equiv T_k^T H_k\\ \boldsymbol{\upsilon}_k &\equiv T_k^T\mathbf{v}_k\end{aligned}$$

  This also needs to be changed on page 322. The matrix T_k is still chosen to be the matrix whose columns are the eigenvectors of R_k, so the statements following eqn. (5.78) are still correct.

- Equation (5.79) should be given by

$$\begin{aligned}K_{i_k} &= \frac{P^+_{i-1_k}\mathcal{H}^T_{i_k}}{\mathcal{H}_{i_k}P^+_{i-1_k}\mathcal{H}^T_{i_k} + \mathcal{R}_{i_k}}, \quad P^+_{0_k} = P^-_k\\ P^+_{i_k} &= [I - K_{i_k}\mathcal{H}_{i_k}]P^+_{i-1_k}, \quad P^+_{0_k} = P^-_k\end{aligned}$$

  This also needs to be changed on page 322.

- The model on page 269 should be

$$\mathbf{x}_{k+1} = \Phi\,\mathbf{x}_k + \Gamma\,\tilde{\omega}_k + \mathbf{w}_k$$
$$\tilde{y}_k = H\,\mathbf{x}_k + v_k$$

Also, in the second sentence below this model "$\tilde{\omega}$" should be replaced by "$\tilde{\omega}_k$".

- Choosing to minimize $\mathrm{Tr}[\dot{P}(t)]$ in eqn. (5.130) requires some more explanation. We wish to minimize the rate of increase of $P(t)$, which is $\dot{P}(t)$. Note that we cannot determine the definiteness of $\dot{P}(t)$ for general matrices of $F(t)$, $H(t)$ and $G(t)$, even though we assume that $R(t)$ is positive definite and that $Q(t)$ is at least positive semi-definite. Therefore, the trace of $\dot{P}(t)$ may be positive or negative at any given time. Also, the second derivative of eqn. (5.130) is $R(t)$, which is a positive definite matrix, leading to a minimization of $\mathrm{Tr}[\dot{P}(t)]$. Note that the time derivative of the trace of eqn. (5.126) is also equivalent to the trace of eqn. (5.129).

- It is important to note that Eqs. (5.139) and (5.140) are only valid for time-invariant systems. For time-varying systems, computing these quantities at each time step provides a good approximation if the sampling interval is "small" enough.

- Equation (5.177b) should be

$$X = -[W_{22} - P_0\,W_{12}]^{-1}[W_{21} - P_0\,W_{11}]$$

- The measurement in example 5.4 on page 280 should be represented by $\tilde{y}(t) = x(t) + v(t)$, not $y(t) = x(t) + v(t)$.

- Equation (5.181) should be

$$E\left\{\mathbf{w}(t)\,\mathbf{v}^T(\tau)\right\} = S(t)\,\delta(t-\tau)$$

- The sentence above eqn. (5.183) on page 282 that reads "... which has zero-mean and covariance, so" should be "... which has zero-mean and covariance given by"

- Equation (5.198) should read

$$\hat{\mathbf{x}}^+_{k_{i+1}} = \hat{\mathbf{x}}^-_k + K_{k_i}\left[\tilde{\mathbf{y}}_k - \mathbf{h}(\hat{\mathbf{x}}^+_{k_i}) - H_k(\hat{\mathbf{x}}^+_{k_i})\left(\hat{\mathbf{x}}^-_k - \hat{\mathbf{x}}^+_{k_i}\right)\right]$$
$$K_{k_i} = P^-_k H^T_k(\hat{\mathbf{x}}^+_{k_i})\left[H_k(\hat{\mathbf{x}}^+_{k_i})P^-_k H^T_k(\hat{\mathbf{x}}^+_{k_i}) + R_k\right]^{-1}$$
$$P^+_{k_{i+1}} = \left[I - K_{k_i}H_k(\hat{\mathbf{x}}^+_{k_i})\right]P^-_k$$
$$\hat{\mathbf{x}}^+_{k_0} = \hat{\mathbf{x}}^-_k$$

- Equation (5.205) should read

$$\begin{bmatrix}\hat{\boldsymbol{\alpha}}^+_k\\ \boldsymbol{\beta}_k\end{bmatrix} = \mathcal{Q}^T_k\begin{bmatrix}\hat{\boldsymbol{\alpha}}^-_k\\ \mathcal{V}\,\tilde{\mathbf{y}}_k\end{bmatrix}$$

Also note that the matrix $\mathcal{S}^+_k$ in eqn. (5.203) is given the first $n\times n$ rows and columns of the R matrix from the QR decomposition of $\tilde{\mathcal{S}}^+_k$.

- The sentence near the top of page 301 that reads "derived using the steady-state covariance equation in Table 5.2," should be "derived using the steady-state covariance equation in Table 5.5,"

- Equation (5.237) should be

$$\bar{\rho}_{k,j} = \frac{1}{\sqrt{m}} \sum_{i=1}^{M} \mathbf{e}_k^T(i) \left[\sum_{i=1}^{M} \mathbf{e}_k(i)\mathbf{e}_k^T(i) \sum_{i=1}^{M} \mathbf{e}_j(i)\mathbf{e}_j^T(i) \right]^{-1/2} \mathbf{e}_j(i)$$

  This should also be changed on page 327.

- On page 304 the residual is defined as $\mathbf{e}_k = \tilde{\mathbf{y}}_k - H_k\mathbf{x}_k$, which should be $\mathbf{e}_k = \tilde{\mathbf{y}}_k - H_k\hat{\mathbf{x}}_k$.

- Equation (5.245) should be

$$C_i = \begin{cases} H\,E\left\{\tilde{\mathbf{x}}_k^-\tilde{\mathbf{x}}_{k-i}^{-T}\right\}\,H^T - H\,E\left\{\tilde{\mathbf{x}}_k^-\mathbf{v}_{k-i}^T\right\} & i > 0 \\ H\,P\,H^T + R & i = 0 \end{cases}$$

  Also, eqn. (5.249b) should be

$$E\left\{\tilde{\mathbf{x}}_k^-\mathbf{v}_{k-i}^T\right\} = [\Phi\,(I - K\,H)]^{i-1}\Phi\,K\,R$$

  To prove eqn. (5.250) use the following identity:

$$[\Phi\,(I - K\,H)]^i = [\Phi\,(I - K\,H)]^{i-1}\Phi\,(I - K\,H)$$

  which gives for $i > 0$

$$\begin{aligned} C_i &= H\,[\Phi\,(I - K\,H)]^{i-1}\,\Phi\,[(I - K\,H)\,P\,H^T - K\,R] \\ &= H\,[\Phi\,(I - K\,H)]^{i-1}\,\Phi\,[P\,H^T - K\,H\,P\,H^T - K\,R] \\ &= H\,[\Phi\,(I - K\,H)]^{i-1}\,\Phi\,[P\,H^T - K\,C_0] \end{aligned}$$

- The covariance of Eq. (5.271) is derived from $E\left\{(\tilde{\tilde{\mathbf{x}}} - \boldsymbol{\mu}_{\tilde{\mathbf{x}}})(\tilde{\tilde{\mathbf{x}}} - \boldsymbol{\mu}_{\tilde{\mathbf{x}}})^T\right\} = V_{\tilde{\mathbf{x}}}$. The quantity $P_{\tilde{\mathbf{x}}}$ is actually the mean square error of the estimate. So the sentence above Eq. (5.272) should read "The mean square error of $\tilde{\tilde{\mathbf{x}}}$ can be shown to be given by"

- Equation (5.275b) should be

$$\dot{\boldsymbol{\mu}}_{\mathbf{x}} = F\,\boldsymbol{\mu}_{\mathbf{x}} + B\,\mathbf{u}$$

  This should also be changed on page 328.

- Equation (5.287) should be

$$\boldsymbol{\chi}_{k+1}^x(i) = \mathbf{f}(\boldsymbol{\chi}_k^x(i),\, \boldsymbol{\chi}_k^w(i),\, \mathbf{u}_k,\, k)$$

  This should also be changed on page 327.

- Exercise 5.18 shows an incorrect result for the measurement model. It should be

$$\tilde{\mathbf{y}}_k = (H_k + J_k\Phi_{k-1}^{-1})\mathbf{x}_k - J_k\Phi_{k-1}^{-1}\Gamma_{k-1}\mathbf{u}_{k-1} + (\mathbf{v}_k - J_k\Phi_{k-1}^{-1}\Upsilon_{k-1}\mathbf{w}_{k-1})$$

- The first equation in exercise 5.33 is missing an equal sign:

$$\begin{bmatrix} \dot{\lambda}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} -1/\beta & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda(t) \\ \theta(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(t)$$

- Reference [4] on page 339 is incorrect. It should be: Stengel, R.F., *Optimal Control and Estimation*, Dover Publications, New York, NY, 1994.

Chapter 6

- Equation (6.171) should be

$$\begin{aligned} \hat{\mathbf{x}}_{k+1|k+1+N} &= \Phi_k \hat{\mathbf{x}}_{k|k+N} + \Gamma_k \mathbf{u}_k \\ &+ \Upsilon_k Q_k \Upsilon_k^T \Phi_k^{-T} (P_{fk}^+)^{-1} [\hat{\mathbf{x}}_{k|k+N} - \hat{\mathbf{x}}_{fk}^+] \\ &+ \mathcal{B}_{k+1+N} K_{fk+1+N} \{\tilde{\mathbf{y}}_{k+1+N} \\ &- H_{k+1+N} [\Phi_{k+N} \hat{\mathbf{x}}_{fk+N}^+ + \Gamma_{k+N} \mathbf{u}_{k+N}]\} \end{aligned}$$

  This also needs to be changed in Table 6.8 and on page 403 under "Fixed-Lag Smoother (Discrete-Time)."

- The second sentence after Eq. (6.185) should read "Also, we treat $\mathbf{w}_k$ as a deterministic input."

Chapter 7

- The initialization for the bias vector in Table 7.1 on page 424 should be $\hat{\boldsymbol{\beta}}(t_0) = \hat{\boldsymbol{\beta}}_0$.

- The MATLAB code for Example 7.2 is incorrect. The boresight of the star camera sensor is along the body z-axis. Thus an identity quaternion would align the body z-axis with the inertial z-axis, which causes an incorrect motion compared to what is described in the example. The simulation has now been corrected in the MATLAB code to reflect what was stated in the text. Also, a magnitude of 6 is chosen for the stars. There are times when the number of available stars is less than 2. The extended Kalman filter still provides an update even when only 1 star is available.

- To be consistent with the notation used throughout the text, Eqs. (7.57) and (7.58) should be

$$P_{k+1}^- = \Phi_k P_k^+ \Phi_k^T + \Upsilon_k Q_k \Upsilon_k^T$$

  where Υ_k is given by

$$\Upsilon_k = \begin{bmatrix} -I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} \end{bmatrix}$$

- Equation (7.60) is incorrect. With G_k given by eqn. (7.58), Q_k should be

$$Q_k = \begin{bmatrix} \left(\sigma_v^2 \Delta t + \frac{1}{3}\sigma_u^2 \Delta t^3\right) I_{3\times3} & \left(\frac{1}{2}\sigma_u^2 \Delta t^2\right) I_{3\times3} \\ \left(\frac{1}{2}\sigma_u^2 \Delta t^2\right) I_{3\times3} & (\sigma_u^2 \Delta t)\, I_{3\times3} \end{bmatrix}$$

Note that the product $G_k Q_k G_k^T$ with this corrected version for Q_k gives eqn. (7.60) as it appears in the book.

- It should be noted that corrected version of eqn. (7.60) is only an approximation, since the coupling effects of the cross-product matrix in eqn. (7.37a) have not been considered. Equation (7.60) is exact when $F(\hat{\mathbf{x}}(t), t)$ is given by

$$F(\hat{\mathbf{x}}(t), t) = \begin{bmatrix} 0_{3\times 3} & -I_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} \end{bmatrix}$$

  The approximation is valid if the sampling rate is below Nyquist's limit. For example, with a safety of 10 we require $||\hat{\boldsymbol{\omega}}(t)||\,\Delta t < \pi/10$.

- Equation (7.95b) should be

$$p_{\dot{r}\dot{r}}^{+} = \left(\frac{\sigma_n}{\Delta t}\right)^2 \left[\xi - S_q^2\left(\frac{1}{\xi} + \frac{1}{2}\right)\right]$$

- The stability conditions in eqn. (7.116) are valid even if α and β are chosen independently. If q is tuned to determine α and β, then from eqns. (7.92) and (7.107) the asymptotic limits are given by $\alpha = 1$ and $\beta = 3 - \sqrt{3} = 1.2679$, which are shown in Figure 7.9. These limits are within the upper bounds given in eqn. (7.116). So the filter will remain stable as long as $q > 0$. Note that choosing $q = 0$ gives $\alpha = \beta = 0$, which yields poles at +1. This leads to an unstable filter, which seems to contradict the stability result of §5.3.2 that $q \geq 0$. However, we must remember that the α-β filter uses a *constant* gain. The time-varying gain approaches zero when $q = 0$, but only in a asymptotic sense not in a strict sense (i.e. the time-varying gain never actually reaches zero).

- The sentence below eqn. (7.120) on page 444 that reads "Note that the lower left 2×2 sub-matrix ..." should be "Note that the lower right 2×2 sub-matrix ..."

- The third to last sentence in the first paragraph under §7.4.3 on page 447 should be "The state vector, $\mathbf{x}$, consists of v_1, v_3, ω_2, θ, C_{D_0}, C_{L_0}, and C_{m_0}."

- Equations (7.128), (7.130) and (7.134) should be

$$\begin{aligned} \frac{\partial D}{\partial v_1} &= C_D \rho\, v_1 S - \frac{1}{2}\rho\, C_{D_\alpha} v_3\, S \\ \frac{\partial D}{\partial v_3} &= C_D \rho\, v_3 S + \frac{1}{2}\rho\, C_{D_\alpha} v_1\, S \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial v_1} &= C_L \rho\, v_1 S - \frac{1}{2}\rho\, C_{L_\alpha} v_3\, S \\ \frac{\partial L}{\partial v_3} &= C_L \rho\, v_3 S + \frac{1}{2}\rho\, C_{L_\alpha} v_1\, S \end{aligned}$$

$$\frac{\partial \dot{\omega}_2}{\partial v_1} = \frac{\rho\, S\, \bar{c}}{J_{22}} \left[\left(C_{m_0} + C_{m_\alpha} \alpha + C_{m_{\delta_E}} \delta_E + C_{m_q} \frac{\Delta\omega_2 \bar{c}}{2\, v_{ss}} \right) v_1 - \frac{1}{2} C_{m_\alpha} v_3 \right]$$

$$\frac{\partial \dot{\omega}_2}{\partial v_3} = \frac{\rho\, S\, \bar{c}}{J_{22}} \left[\left(C_{m_0} + C_{m_\alpha} \alpha + C_{m_{\delta_E}} \delta_E + C_{m_q} \frac{\Delta\omega_2 \bar{c}}{2\, v_{ss}} \right) v_3 + \frac{1}{2} C_{M_\alpha} v_1 \right]$$

$$\frac{\partial \dot{\omega}_2}{\partial \omega_2} = \frac{1}{4 J_{22} v_{ss}} \rho\, S\, ||\mathbf{v}||^2 \bar{c}^2 C_{m_q}$$

- Exercise 7.33 involves a 6-state filter, not an 8-state filter.

Chapter 8

- Reference [13] on page 531 is incorrect. It should be: Stengel, R.F., *Optimal Control and Estimation*, Dover Publications, New York, NY, 1994.

Appendix A

- The definition of Unitary matrix on page 542 should be "inverse is equal to its conjugate transpose" not Hermitian transpose.
- Equation (A.72) should be

$$\nabla_{\mathbf{x}}^2 f \equiv \frac{\partial^2 f}{\partial \mathbf{x}\, \partial \mathbf{x}^T} = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1 \partial x_1} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n \partial x_n} \end{bmatrix}$$

Appendix B

- Equation (B.30) should be

$$\mathbf{x} \sim N(\boldsymbol{\mu}, R)$$

Appendix C

- The vector $\mathbf{x}$ in example C.2 should be $\mathbf{x} \equiv \begin{bmatrix} y & z \end{bmatrix}^T$.
- The correct necessary conditions in example C.2 are given by

$$\frac{\partial \phi}{\partial y} = -\frac{1}{2} - 18\lambda(y-4) = 0$$

$$\frac{\partial \phi}{\partial z} = -\frac{1}{3} - 8\lambda(z-5) = 0$$

The stationary points are still correct as stated.

- In the sentence just before eqn. (C.24) on page 577, change "differential derivative" to "directional derivative."

Appendix D

- No Corrections