Wireless Networks
https://doi.org/10.1007/s11276-018-1699-y

Dynamic energy-efficient resource allocation in wireless powered communication network

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Abstract

In this paper, we investigate the dynamic energy-efficient resource allocation and analyze delay in newly emerging wireless powered communication network (WPCN). Considering the time-varying channel and stochastic data arrivals, we formulate the resource allocation (i.e., time allocation and power control) problem as a dynamic stochastic optimization model, which maximizes the system energy efficiency (EE) subject to both the data queue stability and the harvested energy availability, and simultaneously satisfies a certain quality of service (QoS) in terms of delay. With the aid of fractional programming, Lyapunov optimization theory and Lagrange method, we solve the problem and propose an dynamic energy-efficient resource allocation algorithm (DEERAA), which does not require any prior distribution knowledge of the channel state information (CSI) or stochastic data arrivals. We find that the performance of EE and delay can be adjusted by a system control parameter V. The effectiveness of the proposed algorithm is demonstrated by the mathematical analysis and simulation results.

Keywords WPCN · Energy efficiency · Delay · Dynamic resource allocation

1 Introduction

With the ability to prolong the life time of traditional networks, especially in energy-constrained scenarios, energy harvesting (EH) [1, 2] and wireless energy transfer (WET) [3] have attracted considerable attention. Since WET is more reliable than EH which is location-dependent or weather-dependent, WET has attracted increasingly interests [4].

There are two different lines of research in WET. One line focuses on simultaneous wireless information and power transfer [5–7], in which the signal received by the wireless devices are able to split into two parts, one for information decoding and the other one for energy harvesting. The other line pursues wireless powered communication network (WPCN) [8–10]. In WPCN, users first harvest energy in downlink WET stage and then utilize the energy to transmit information signal in uplink wireless information transmission (WIT) stage [11, 12], which means energy is consumed to combat the wireless channel attenuation for both the two stages. Recently, due to the explosive increasing of the number of mobile devices and mobile traffics, energy consumption is dramatically increasing, and thus energy efficiency (EE), measured in bits per joule, is gradually accepted as an important design criterion for future communication systems. For WPCN owing to its WET stage existing, which may consume plenty of energy to confront the wireless channel attenuation, it is especially important for WPCN to analyze EE [13, 14]. Thus, one challenge in the design of WPCN is that resource is needed to be allocated reasonably and dynamically for maximizing the system EE to face the time varying channel and stochastic data arrivals, especially when the prior distribution knowledge of CSI or data arrivals unavailable. Another critical challenge for WPCN is that queueing delay should be guaranteed accounting for the dramatical increasing of delay sensitive traffic. In this

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Published online: 24 May 2018

paper, the queueing delay is referred to as the time length that a packet waits in a queue until it can be served, and we refer the queueing delay to delay for convince. We always want to maximize EE and minimize delay simultaneously. However, the transmitter may always wait for a good channel state to transmit information for maximizing EE which may lead to undesirably high delay. Thus, how to allocate resource dynamically according to time-varying channel and stochastic data arrivals for maximizing EE and satisfying a certain of QoS requirement in terms of delay is an interesting but challenging subject in WPCN.

There have been extensive researches on optimizing EE in WPCN from different perspectives. The author in [15] developed an iterative resource allocation method to maximize EE in WPCN with time division mode switching system. EE is maximized by jointly considering the time allocation and energy beamforming in multi-user multi-cell WPCN with massive multi-input multi-output system [16]. By considering whether the CSI at relay is available, the authors in [17] studied a two-way amplify and forward relaying WPCN network to maximize EE. The weighted sum of the user EE is maximized without considering the QoS constraints and the tradeoff between the users with QoS constraints are investigated in WPCN with a time-division manner [18]. EE is maximized by taking into account the initial battery energy of each user via joint time allocation and power control in WPCN with time division multiplexing access (TDMA) mode [19].

It is obvious that resource is allocated for maximizing EE based on the static channel conditions in [15–19]. However, the channel conditions are time varying and data arrivals are stochastic in practical wireless networks, which means the resource should be allocated dynamically and thus the conclusions obtained in static state cannot be simply applied to practical systems. Besides, delay, an important metric to measure the QoS of traffics, is neglected in [15–19], which is impratical. Due to the fundamentally different system architectures compared to ours in [15–18] and different preconditions in [19], the conclusions and proposed method in [15–19] are not applicable to our scenario. The authors in [20] minimize the system energy consumption merely by allocated resource dynamically for studying the power-delay tradeoff for WPCN, which may affect the system performance, e.g. throughput, and decrease the user experience. Taking both the delay limited and delay tolerant transmission modes into account, the authors in [21] maximize the average throughput yet EE via dynamic resource allocation. As for WPCN, energy is consumed both in the WIT stage for data transmission and in the WET stage to combat the wireless channel attenuation. When optimizing the system throughput merely, the system always consume more energy to obtain a high throughput, which conflicts to the themes of using energy efficiently and reducing carbon emission for the next generation networks. Therefore, it is important to optimize EE in WPCN. EE and delay tradeoff is studied for time varying and interference-free wireless networks in [22] and for heterogeneous cloud radio access networks in [23]. Both the systems in [22, 23] are different compared to ours, and thus the algorithm obtained in [22, 23] cannot take directly effect in our interest scenario and model. To our best knowledge, by allocating resource dynamically to analyze EE and delay in WPCN has not yet been investigated in literatures.

After having studied the previous works, we notice the importance on analyzing EE and delay in a long-term perspective. In this paper, we consider a WPCN system, where wireless powered devices (WPDs) charge their energy buffers by harvesting energy from a power station (PS) in the downlink WET stage and then utilize the energy to transmit the information to the information receiving station (IRS) in the uplink WIT stage.The main contributions of this paper are as follows

- We formulate a stochastic optimization problem to analyze EE and delay, and address it with the aid of fractional programming, Lyapunov optimization theory and Lagrange method by allocating resource dynamically in a long-term perspective in WPCN.
- We find that the PS should transmit the energy with its maximum power to obtain the maximum EE when taking both circuit power consumption and transmission consumption of PS and WPDs into account.
- A dynamic energy-efficient resource allocation algorithm (DEERAA) is proposed to regulate time allocation and power control dynamically with prior distribution knowledge of CSI and data arrivals free in a long-term perspective, which makes DEERAA easy to apply to practical WPCN systems.
- We identify that both the EE and the time-averaged queueing delay increase at the speed of $\mathbf{O(V)}$, where V is a system control parameter which can balance the performace of EE and delay.

The remainder of this paper is organized as follows. In Sect. 2, we introduce our system model and formulate the stochastic optimization problem. We solve the problem with the help of three mathematical methods and describe our DEERAA in Sect. 3. Simulation results are presented in Sect. 4 and the paper is concluded in Sect. 5.

2 System model and preliminaries

2.1 System model

In this paper, we consider a WPCN system which consists of one PS identified by $k = 0$, K WPDs, identified by $k = 1, \cdots, K$, and one IRS. A system model is shown in Fig. 1. Each WPD is equipped with a single antenna, a data buffer and an energy buffer. Assuming all of the nodes in our system are operating in the TDMA mode over the same frequency band, and the WET and the WIT signals do not interfere with each other [24, 25]. Both PS and IRS have a fixed power supply while WPDs not. These K WPDs utilize the harvested energy to communicate with IRS.

We adopt a time frame structure, as showned in Fig. 2. We divide the time horizon into time-slots indexed by t and assume a normalized unit time-slot in the sequel. During $\tau_0(t)$, the PS transfers the RF signal to charge the energy buffers of WPDs. During $1 - \tau_0(t)$, the K WPDs send messages to the IRS, where user WPD k can only transmit the information to IRS during its allocated time fraction $\tau_k(t)$ in time-slot t.

2.2 Power consumption model

In this paper, we take both the over-the-air transmit energy consumption and the circuit energy consumption into account. And the total energy consumption in our model consists of two parts: the energy consumed during WET stage and WIT stage.

Denote the time allocated to PS and WPD $1, 2, \ldots, K$ by $\tau_0(t), \tau_1(t), \tau_2(t), \ldots, \tau_K(t)$. Let $h_k(t)$ and $g_k(t)$ represent for the downlink and the uplink channel gains in time-slot t. During the WET stage, the PS broadcasts the RF signal with the power $P_0^{tr}(t)$. Then in time-slot t the energy harvested at WPD k is

$$W_k(t) = \eta_k \tau_0(t) P_0^{tr}(t) H_k(t), \quad \forall k \neq 0, \tag{1}$$

where $\eta_k \in (0, 1]$ represents energy conversion efficiency of WPD k and $H_k(t) \triangleq |h_k(t)|^2$. During the WET stage, the overall energy cost $W_{\mathrm{WET}}(t)$ is

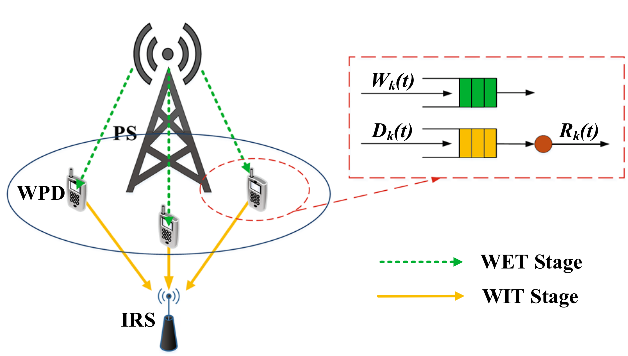

Fig. 1 System model

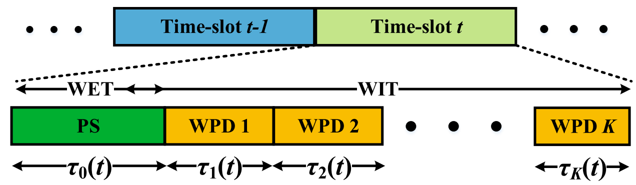

Fig. 2 Frame structure

$$W_{\mathrm{WET}}(t) = P_0^{tr}(t)\tau_0(t) - \sum_{k=1}^{K} W_k(t) + P_0^{ci}\tau_0(t), \tag{2}$$

where P_0^{ci} is the constant circuit power consumption of the PS. Note that $W_{\mathrm{WET}}(t)$ represents for the energy costs owing to energy buffers charging in WPDs. The energy consumption during the WIT stage is

$$W_{\mathrm{WIT}}(t) = \sum_{k=1}^{K} \left(P_k^{tr}(t) + P_k^{ci}\right)\tau_k(t), \quad \forall k \neq 0, \tag{3}$$

where $P_k^{tr}(t)$ and P_k^{ci} are the transmit power and the constant circuit consumption power at time-slot t for WPD k. And the dynamic energy queue for WPD k is

$$Q_k^W(t+1) = \left[Q_k^W(t) - \left(P_k^{tr}(t) + P_k^{ci}\right)\tau_k(t)\right]^+ + W_k(t), \quad \forall k \neq 0, \tag{4}$$

where $[x]^+$ denotes the maximum value between x and zero.

Above all, the total energy consumption of the system at time-slot t, denoted as $W_{\mathrm{A}}(t)$, is

$$W_{\mathrm{A}}(t) = W_{\mathrm{WET}}(t) + W_{\mathrm{WIT}}(t). \tag{5}$$

2.3 Energy efficiency

For WPD k, given $P_k^{tr}(t)$ and $\tau_k(t)$, the achievable data rate (in unit of bits/Hz/time-slot) at time-slot t is

$$R_k(t) = \tau_k(t)\log_2\left(1 + \frac{P_k^{tr}(t)G_k(t)}{B\sigma^2}\right), \tag{6}$$

where σ^2 is the noise power spectral density, B is the system bandwidth, and $G_k(t) = |g_k(t)|^2$. Then, the total throughput of the system at time-slot t is $R_A(t) = \sum_{k=1}^{K} R_k(t)$. And the dynamics of data queue $Q_k^D(t)$ is

$$Q_k^D(t+1) = \left[Q_k^D(t) - R_k(t)\right]^+ + D_k(t), \quad \forall k \neq 0, \tag{7}$$

where $D_k(t)$ is the arrival data rate at time-slot t for WPD k. In a practical system, the achievable transmission rate $R_k(t)$ and the arrival data rate $D_k(t)$ always have upper bounds, i.e., R_k^{max} and D_k^{max}[20]. According to Little's Theorem, for a given traffic arrival rate, the time-averaged queue delay is proportional to the time-averaged queue length [26]. Thus, we can depict the time-averaged queueing delay with data queue length.

Define the EE as the ratio of the long-term aggregate data delivered to the corresponding long-term total energy consumption in unit of bit/Joule/Hz. Then the EE denoted by ρ_{ee} of the whole system is

$$\begin{aligned}\rho_{ee} &= \lim_{t\to\infty} \frac{\sum_{v=1}^{t} \mathbb{E}\{R_A(\tau(v), \boldsymbol{P^{tr}}(v))\}}{\sum_{v=1}^{t} \mathbb{E}\{W_A(\tau(v), \boldsymbol{P^{tr}}(v))\}} \\ &= \frac{\lim_{t\to\infty} \frac{1}{t}\sum_{v=1}^{t} \mathbb{E}\{R_A(\tau(v), \boldsymbol{P^{tr}}(v))\}}{\lim_{t\to\infty} \frac{1}{t}\sum_{v=1}^{t} \mathbb{E}\{W_A(\tau(v), \boldsymbol{P^{tr}}(v))\}} = \frac{\overline{R}_A}{\overline{W}_A},\end{aligned} \tag{8}$$

where $\overline{W}_A$ and $\overline{R}_A$ represent the time-averaged energy consumption and the time-averaged transmission rate. And $\boldsymbol{P^{tr}}(v) = \{(P_k^{tr}(v)), k = 0, 1, 2, \ldots\}$, $\tau(v) = \{\tau_k(v), k = 0, 1, 2, \ldots\}$.

The definition of the EE in [27] is from an instantaneous perspective while we define EE in a time-averaged perspective, and the resource is allocated dynamically according to current CSI and data arricals which is more in line with the actual situation. Account for the different way of defining EE and preconditions on system models, the conclusions obtained in [27] cannot be applied to our system. In order to perform the resource allocation and study EE and delay simultaneously, we maximize the EE subject to both data queue stability and the harvested energy availability constraints. Hence, we formulate the problem as following

$$\begin{aligned}\mathcal{P}1:\ &\max_{\tau_k(t), P_k^{tr}(t)}\ \rho_{ee} = \frac{\overline{R}_A}{\overline{W}_A} \\ \text{s.t.}\quad & \text{C1}: \sum_{k=0}^{K} \tau_k(t) \le 1, \forall k \\ & \text{C2}: P_0^{tr} \le P_0^{max} \\ & \text{C3}: \left(P_k^{tr}(t) + P_k^{ci}\right)\tau_k(t) \le Q_k^{W}(t) + W_k(t), \forall k \ne 0 \\ & \text{C4}: \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} Q_k^{D}(t) < \infty.\end{aligned} \tag{9}$$

In (9), P_0^{max} is the maximum transmit power of PS. C1 means the sum of the allocated time to PS and WPDs is not larger than 1 due to the time-slot is normalized to be unit. C2 is the peak transmit power constraint of PS. C3 ensures that the energy consumed during WIT stage does not exceed the total available energy in the energy buffer for WPD k at the present moment. C4 ensures that the time-averaged data queue is stable and hence a finite average queueing delay.

It is obvious that problem $\mathcal{P}1$ is a stochastic optimization problem, which is non-convex. Such a problem can be solved by using techniques such as dynamic programming [28], while it may lead to the curse of dimensionality and is too difficult to solve. To avoid the defects, we propose a low computational complexity and practically operational algorithm, which is described detailedly in the next section.

3 Energy efficient resource allocation

In this section, we first change the form of the proposed stochastic optimization problem in Section II into a linear programming with fractional programming. Then, with the help of Lyapunov optimization theory we change the linear programming into a series deterministic optimization problems. Finally, we solve the ultima deterministic optimization problem which is convex via Lagrangian method and propose a general algorithm referred to as DEERAA to solve our problem.

3.1 Equivalent problem transformation

While $\mathcal{P}1$ is a nonconvex optimization problem, we find it belongs to nonlinear fractional programming [29] and thus we utilize the fractional programming to change the problem form. Let $\mathfrak{S}$ ($\mathfrak{S} \ne \emptyset$) denote the set of feasible solutions of $\mathcal{P}1$ for simplicity. There must be an optimal time allocation $\tau^* = \{\tau_0^*, \tau_1^*, \ldots, \tau_K^*\}$ and power allocation $\boldsymbol{P^{tr*}} = \{P_0^{tr*}, P_1^{tr*}, \ldots, P_K^{tr*}\}$, while maximizing ρ_{ee} in (9) from a long-term perspective under the constraints of C1-C4, and then,

$$\rho_{ee}^* = \frac{\overline{R}_A(\tau^*, \boldsymbol{P^{tr*}})}{\overline{W}_A(\tau^*, \boldsymbol{P^{tr*}})} = \max_{(\tau,\boldsymbol{P})\in\mathfrak{S}} \frac{\overline{R}_A(\tau, \boldsymbol{P^{tr}})}{\overline{W}_A(\tau, \boldsymbol{P^{tr}})}. \tag{10}$$

Using a nonlinear programming method [4], we give the following theorem.

Theorem 1 *The maximum ρ_{ee}^* can be obtained if and only if*

$$\max \overline{R}_A(\tau, \boldsymbol{P^{tr}}) - \rho_{ee}^* \overline{W}_A(\tau, \boldsymbol{P^{tr}}) = 0. \tag{11}$$

Proof The proof is provided in "Proof of Theorem 1 in Appendix" section. □

Then we can change $\mathcal{P}1$ into $\mathcal{P}2$:

$$\begin{aligned}\mathcal{P}2:\ &\max_{\tau_k(t), P_k^{tr}(t)}\ \overline{R}_A(\tau, \boldsymbol{P^{tr}}) - \rho_{ee}^* \overline{W}_A(\tau, \boldsymbol{P^{tr}}) \\ &\text{s.t.}\ \text{C1}, \text{C2}, \text{C3}, \text{C4}.\end{aligned} \tag{12}$$

However, it is also difficult to solve $\mathcal{P}2$ because we cannot identify the value of ρ_{ee}^* which is the object we want to obtain. We propose $\rho_{ee}(t)$ for $t \in \{1, 2, 3, \ldots\}$ with $\rho_{ee}(0) = 0$ as

$$\rho_{ee}(t) = \frac{\sum_{v=1}^{t-1} \mathbb{E}\{R_A(\tau(v), \boldsymbol{P^{tr}}(v))\}}{\sum_{v=1}^{t-1} \mathbb{E}\{W_A(\tau(v), \boldsymbol{P^{tr}}(v))\}}. \tag{13}$$

Replace ρ_{ee}^* by $\rho_{ee}(t)$, then $\mathcal{P}2$ is casted as $\mathcal{P}3$:

$$\begin{aligned} \mathcal{P}3: \quad & \max_{\tau_k(t), P_k^{tr}(t)} \quad \overline{R}_A(\tau, \boldsymbol{P^{tr}}) - \rho_{ee}(t)\overline{W}_A(\tau, \boldsymbol{P^{tr}}) \\ & \text{s.t.} \quad \text{C1, C2, C3, C4.} \end{aligned} \tag{14}$$

From the definition of $\rho_{ee}(t)$, we can find that $\rho_{ee}(t)$ is depending on the past resource allocation decisions, which is a known parameter compared to ρ_{ee}^*. So, we can replace ρ_{ee}^* by $\rho_{ee}(t)$ to solve the $\mathcal{P}2$, which is an effective way to solve stochastic optimization problems with ratio objectives in renewal systems and has been widely used [30].

3.2 PS transmit power

The following *Theorem* 2 characters the operation of the PS to maximize EE.

Theorem 2 *The maximum EE can be obtained when PS transmits power to WPDs at its maximum power.*

Proof The proof is provided in "Proof of Theorem 2 in Appendix" section. □

Remark 1 *Theorem* 2 seems contradictory to intuition at first. However, in our WPCN system, both time and power are optimized and circuit energy consumption and transmission energy consumption are jointly considered. When PS transmits with its maximum allowed power, the allocated time for WET and the energy consumption are reduced simultaneously. Besides, it allows WPDs to have more time to improve the system throughput during WIT and thus maximizes EE. With the help of *Theorem* 2, we can change the formation of $\mathcal{P}3$ into $\mathcal{P}4$:

$$\begin{aligned} \mathcal{P}4: \quad & \max_{\tau_k(t), P_k^{tr}(t)} \quad \overline{R}_A(\tau, \boldsymbol{P^{tr}}) - \rho_{ee}(t)\overline{W}_A(\tau, \boldsymbol{P^{tr}}) \\ & \text{s.t.} \quad \text{C1, C3, C4,} \end{aligned} \tag{15}$$

where $\boldsymbol{P^{tr}} = \{P_0^{max}, P_1^{tr}, \cdots, P_K^{tr}\}$. For simplicity, we will use term $\overline{W}_A(t)$ to refer to $\overline{W}_A(\tau, \boldsymbol{P^{tr}})$, and $\overline{R}_A(t)$ to refer to $\overline{R}_A(\tau, \boldsymbol{P^{tr}})$ in the following discussion.

3.3 Lyapunov optimization theory

Observing $\mathcal{P}4$, we find it is a stochastic problem which couples with both instantaneous and time-averaged constraints. Lyapunov optimization theory is a method which considers the joint problem of stability queue and optimizing performance. Thus, $\mathcal{P}4$ can be solved with the aid of Lyapunov optimization theory. Define the Lyapunov function as

$$L(Z(t)) = \frac{1}{2}\sum_{k=1}^{K}\left(Q_k^D(t)\right)^2, \tag{16}$$

where $Z(t) = \{Q_k^D(t) : k = 1, 2, \ldots, K\}$. And then the one-step conditional Lyapunov drift is

$$\begin{aligned} \triangle\{Z(t)\} = \mathbb{E}\{L(Z(t+1)) - L(Z(t))|Z(t)\} \le C^M \\ - \sum_{k=1}^{K} E\{Q_k^D(t)R_k(t)|Z(t)\}. \end{aligned} \tag{17}$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator.

Proof The proof is provided in "Proof of (17) in Appendix" section. □

To solve $\mathcal{P}4$, we need to employ the Lyapunov drift-plus-penalty approach, as following

$$\begin{aligned} &\triangle\{Z(t)\} - VE\{R_A(t) - \rho_{ee}(t)W_A(t)|Z(t)\} \le C^M - \\ &E\left\{V\{R_A(t) - \rho_{ee}(t)W_A(t)\} + \sum_{k=1}^{K} Q_k^D(t)R_k(t)|Z(t)\right\}, \end{aligned} \tag{18}$$

where $V(V > 0)$ is an arbitrary system control parameter in our proposed algorithm, and it is regraded as the "importance weight" on how much we emphasize on EE or delay. By controlling V, we can balance the performance of EE and delay.

With the analysis above, minimize the righthand of (18) we can jointly optimize resource allocation with the queue being stable. Thus we can solve $\mathcal{P}4$ by solving the following optimization problem

$$\begin{aligned} \mathcal{P}5: \quad & \max_{\tau_k(t), P_k^{tr}(t)} \quad V\{R_A - \rho_{ee}(t)W_A(t)\} + \sum_{k=1}^{K} Q_k^D(t)R_k(t) \\ & \text{s.t.} \quad \text{C1, C3.} \end{aligned} \tag{19}$$

Hitherto, we transfer the original stochastic optimization problem $\mathcal{P}1$ into a static deterministic optimization problem $\mathcal{P}5$, which is nonconvex due to $\tau_k(t)$ and P_k^{tr} are coupled, while we can change the form of $\mathcal{P}5$ to solve it via Lagrange method in the following subsection.

3.4 Lagrangian method

Using the property of the perspective function and letting $Y_k(t) \triangleq \eta_k P_0^{max} H_k(t), \forall k \neq 0$, we obtain the equivalent problem $\mathcal{P}6$, which is a convex function. Note that $e_k(t) \triangleq \tau_k(t)P_k^{tr}, \forall k \neq 0$.

$$\mathcal{P}6: \max_{\tau_k(t),P_k^{tr}(t)} \sum_{k=1}^{K}\left(Q_k^D(t)+V\right)\tau_k(t)\log_2\left(1+\frac{e_k(t)\gamma_k(t)}{\tau_k(t)}\right)$$
$$-V\rho_{ee}(t)\sum_{k=1}^{K}\left(e_k(t)+P_k^{ci}\tau_k(t)\right)$$
$$-V\rho_{ee}(t)\left(P_0^{max}(t)+P_0^{ci}-\sum_{k=1}^{K}Y_k(t)\right)\tau_0(t)$$
$$\text{s.t.}\ \begin{cases}\sum_{k=0}^{K}\tau_k(t)\le 1,\\ e_k(t)+P_k^{ci}\tau_k(t)\le Q_k^E(t)+Y_k(t)\tau_0(t), \forall k\neq 0.\end{cases} \tag{20}$$

The Lagrangian function of $\mathcal{P}6$ is written as:

$$\mathcal{L}(\tau_0(t),e_k(t),\tau_k(t),\zeta,\mu)=\sum_{k=1}^{K}\left(Q_k^D(t)+V\right)\tau_k(t)$$
$$\log_2\left(1+\frac{e_k(t)\gamma_k(t)}{\tau_k(t)}\right)$$
$$-V\rho_{ee}(t)\sum_{k=1}^{K}\left(e_k(t)+p_k^{ci}\tau_k(t)\right)-\mu\left(\sum_{k=0}^{K}\tau_k(t)-1\right)$$
$$-V\rho_{ee}(t)\left(P_0^{max}(t)+P_0^{ci}-\sum_{k=1}^{K}Y_k(t)\right)\tau_0(t)$$
$$-\sum_{k=1}^{K}\zeta_k\left(e_k(t)+P_k^{ci}\tau_k(t)-Q_k^E(t)-Y_k(t)\tau_0(t)\right), \tag{21}$$

where $\gamma_k(t)(t)=\frac{G_k(t)}{B\sigma^2}$ and $\forall k\neq 0$. In (21), ζ and μ are the non-negative Lagrange multipliers. Then we turn to maximize $\mathcal{L}(\tau_0(t),e_k(t),\tau_k(t),\zeta,\mu)$ to solve the problem. By taking the partial derivative of $\mathcal{L}$ with respect to $\tau_0(t),e_k(t)$, respectively, then we obtain

$$\frac{\partial\mathcal{L}}{\partial\tau_0(t)}=\sum_{k=1}^{K}(\zeta_k+V\rho_{ee}(t))Y_k(t)-V\rho_{ee}(t)(P_0^{max}+p_0^{ci})-\mu, \tag{22}$$

$$\frac{\partial\mathcal{L}}{\partial e_k(t)}=\frac{\left(Q_k^D(t)+V\right)\tau_k(t)\gamma_k(t)}{(\tau_k(t)+e_k(t)\gamma_k(t))\ln2}-V\rho_{ee}(t)-\zeta_k, \tag{23}$$

and the complementary slackness conditions are given by

$$\zeta_k\left(e_k(t)+P_k^{ci}\tau_k(t)-Q_k^E(t)-Y_k(t)\tau_0(t)\right)=0, \tag{24}$$

$$\mu\left(1-\sum_{k=0}^{K}\tau_k(t)\right)=0. \tag{25}$$

Let $f_0(\zeta)\triangleq\frac{\partial\mathcal{L}}{\partial\tau_0(t)}$. From (22), we know that $\mathcal{L}$ is a linear function of $\tau_0(t)$. Since $\tau_0(t)\ge 0$, to make sure that Lagrangian function $\mathcal{L}$ is bounded above, we have the following equation

$$\tau_0^*(t)\begin{cases}\in[0,1), & \text{if } f_0(\zeta)=0,\\ =0, & \text{if } f_0(\zeta)=0.\end{cases} \tag{26}$$

Setting $\frac{\partial\mathcal{L}}{\partial e_k(t)}=0$, we find the relation between $E_k(t)$ and $\tau_k(t),\forall k\neq 0$

$$P_k^{tr}(t)=\frac{e_k(t)}{\tau_k(t)}=\left[\frac{Q_k^D(t)+V}{(V\rho_{ee}(t)+\zeta_k)\ln2}-\frac{1}{\gamma_k(t)}\right]^+. \tag{27}$$

Substituting (27) into (21), and taking the partial derivative of $\mathcal{L}$ with $\tau_k(t)$, we obtain

$$f(\gamma_k(t),\zeta_k)\triangleq\frac{\partial\mathcal{L}}{\partial\tau_k(t)}=(V\rho_{ee}(t)+\zeta_k\ln2)$$
$$H\log_2\left(1+\left[H-\frac{1}{\gamma_k(t)}\right]^+\gamma_k(t)\right)$$
$$-(V\rho_{ee}(t)+\zeta_k)\left(\left[H-\frac{1}{\gamma_k(t)}\right]^++P_k^{ci}\right)-\mu, \tag{28}$$

where $H\triangleq\frac{Q_k^D(t)+V}{(V\rho_{ee}(t)+\zeta_k)\ln2}$. Therefore, we obtain the value of $\tau_k^*(t)$

$$\tau_k^*(t)\begin{cases}=\dfrac{Q_k^E(t)+Y_k(t)\tau_0^*(t)}{p_k^{tr*}+p_k^{ci}}, & \text{if } \gamma_k(t)>x_k^*(t),\\ \in[0,\dfrac{Q_k^E(t)+Y_k(t)\tau_0^*(t)}{p_k^{tr*}+p_k^{ci}}], & \text{if } \gamma_k(t)>x_k^*(t),\\ =0, & \text{if } \gamma_k(t)<x_k^*(t),\end{cases} \tag{29}$$

where $x_k^*(t)$ is the solution of

$$(Q_k^D(t)+V)\log_2\left(\frac{(Q_k^D(t)+V)x_k(t)}{V\rho_{ee}(t)\ln2}\right)+\frac{V\rho_{ee}(t)}{x_k(t)}$$
$$-\frac{Q_k^D(t)+V}{\ln2}-V\rho_{ee}(t)P_k^{ci}-\mu=0. \tag{30}$$

Moreover, $\tau_0^*(t)$ and $\tau_k^*(t)$ should satisfy

$$\tau_0^*(t)+\sum_{k=1}^{K}\tau_k^*(t)\begin{cases}=1, & \mu>0,\\ \le 1, & \mu=0.\end{cases} \tag{31}$$

The proof of (29) and (31) are described as following.

Proof Since $\tau_k(t)\ge 0$, using a similar analysis as for $\tau_0(t)$, the optimal solution $\tau_k(t)$ must satisfy

$$\frac{\partial\mathcal{L}}{\partial\tau_k(t)}=f(\gamma_k(t),\zeta_k)\begin{cases}<0, & \tau_k(t)=0,\\ =0, & \tau_k(t)\ge 0.\end{cases} \tag{32}$$

Lemma 1 *If $\frac{Q_k^D(t)+V}{(V\rho_{ee}(t)+\zeta_k)\ln2}\ge\frac{1}{\gamma_k(t)}$, then we find that $f(\gamma_k(t),\zeta_k)$ is an increasing function of $\gamma_k(t)$ and a decreasing function of ζ_k.*

Lemma 1 is easily proved by taking the derivative of $f(\gamma_k(t),\zeta_k)$ with respect to $\gamma_k(t)$ and ζ_k, respectively. The proof is thus omitted due to the space limitation. Based on *Lemma* 1, we know when $\zeta_k=0$ with ignoring $\gamma_k(t)$, $f(\gamma_k(t),\zeta_k)$ attains its maximum value $f(\gamma_k(t),0)$.

Moreover, when $\gamma_k(t) = \frac{(V\rho_{ee}(t)+\zeta_k)\ln 2}{Q_k^D(t)+V}$, $f(\gamma_k(t),0) = -VP_k^{ci} -\mu < 0$ holds, and when $\gamma_k(t) \longrightarrow +\infty$, $f(\gamma_k(t),0) \longrightarrow +\infty$ holds. It is obvious that there exists a $x_k^*(t)$ such that $f(x_k^*(t),0) = 0$, i.e., $x_k^*(t)$ is the solution of (30). Now, we analyze the following three cases:

1. For $\gamma_k(t) < x_k^*(t)$, we obtain $f(\gamma_k(t),\zeta_k) \leq f(\gamma_k(t),0) < f(x_k^*(t),0) = 0$. And according to (32), we can obtain $\tau_k(t) = 0$.
2. For $\gamma_k(t) > x_k^*(t)$, it is obvious that there always exists a $\zeta_k > 0$ satisfying $f(\gamma_k(t),\zeta_k) = 0 < f(\gamma_k(t),0)$. However, there may exist $\zeta_k > 0$ such that $f(\gamma_k(t),\zeta_k) < 0$. Then according to (32), we obtain $\tau_k(t) = 0$, and according to (32) we get $e_k(t) + P_k^{ci}\tau_k(t) = 0 < Q_k^E(t) + Y_k(t)\tau_0^*(t)$, which contradicts (24) and this is not the optimal solution. Nevertheless, for users $\gamma_k(t)$ larger than $x_k^*(t)$, $\zeta_k > 0$ implies that they use up all the energy in their energy buffers. Thus, from (24) we obtain $\tau_k(t) = \frac{Q_k^E(t)+Y_k(t)\tau_0^*(t)}{p_k^{tr}+p_k^{ci}}$. Correspondingly, as $\tau_k(t) > 0$, the value of ζ_k is obtained from $f(\gamma_k(t),\zeta_k) = 0$.
3. For $\gamma_k(t) = x_k^*(t)$, if $\zeta_k > 0$, then $f(\gamma_k(t),\zeta_k) < f(\gamma_k(t),0) = f(x_k^*(t),0) = 0$ and $\tau_k(t) = 0$, which contradicts (24). Therefore, $\zeta_k = 0$ follows from (24), and we obtain $\tau_k(t) \in \left[0, \frac{Q_k^E(t)+Y_k(t)\tau_0^*(t)}{p_k^{tr}+p_k^{ci}}\right]$

This completes the proof of (29). As the Lagrangian function $\mathcal{L}$ is a linear function of $\tau_0^*(t)$ and $\tau_k^*(t)$, the maximum value of $\mathcal{L}$ can always be achieved at the vertices of the region created by (26) and (29). Moreover, $\tau_0^*(t)$ and $\tau_k^*(t)$ should satisfy the complementary slackness conditions (25). Therefore, if $\mu > 0$, the time constraint should be strictly met with equality, otherwise, we obtain an associated inequality for limiting the range of time variables $\tau_0(t)$ and $\tau_k(t)$. Then we finish the proof of (31). □

3.5 Dynamic energy-efficient resource allocation algorithm

Based on the analysis in last subsections, we propose a dynamic energy-efficient resource allocation algorithm, referred to as DEERAA in the next page. Steps 2–12 in the proposed DEERAA algorithm are executed once within each time-slot and taking the dynamic CSI and data queues into account. With prior distribution knowledge of CSI and data arrivals free, and thus our algorithm is an online algorithm and can be easily implemented in practical WPCN systems. In steps 2–10, we adopt the Lagrange method to solve $\mathcal{P}6$.

The computational complexity of DEERAA algorithm can be analyzed as follows. The complexity of lines 6–8 in Algorithm 1 is linear in the number of users, K. Furthermore, the complexity of the Dinkelbach method [29] for updating $\rho_{ee}(t)$, and the bisection method [31] for updating μ are both independent of K. Hence the complexity of DEERAA algorithm is $O(K)$ within each time-slot of the network states.

Algorithm 1 DEERAA

1: **Input**: $P_0^{max}, \{P_k^{ci} : k = 0,1,...,K\}, K, \sigma^2, B, V, \epsilon$;

2: Observe $Q_k^D(t), Q_k^E(t), \forall k \neq 0$ and $h_k(t), g_k(t), \forall k \neq 0$.

3: Initialize $\tilde{\mu}$

4: Set Lagrange multipliers $\mu_{max} = \tilde{\mu}, \mu_{min} = 0$

5: **while** $\mu_{max} - \mu_{min} \geq \epsilon$

6: $\mu = \frac{1}{2}(\mu_{max} + \mu_{min})$; Compute $x_k^*(t)$ from (30);

7: **If** $\gamma_k(t) > x_k^*(t)$ Compute $\zeta_k(t)$ from (28) and (32); **else** $\zeta_k(t) = 0$; **end**

8: Compute $P_k^{tr}(t), \tau_0(t)$ and $\tau_k(t)$ from (27), (26) and (29), respectively;

9: **If** there exist $\tau_0(t)$ and $\tau_k(t)$ satisfying (31) then, **break**;

10: **else if** $\tau_0(t) + \sum_{k=1}^K \tau_k(t) > 1$ $\mu_{min} = \mu$; **else** $\mu_{max} = \mu$; **end**

11: **end while**

12: Update $Q_k^D(t), Q_k^W(t)$ and $\rho_{ee}(t)$ according to (7), (4) and (13) based on $\tau_0(t), \tau_k(t)$ and $P_k^{tr}(t)$, respectively.

13: Repeat step 2-12 in each time-slot.

3.6 Performance analysis

In the above subsections, we change the original problem $\mathcal{P}1$ into $\mathcal{P}6$ and solve $\mathcal{P}6$ to investigate EE and delay in WPCN. However, $\mathcal{P}6$ is not equivalent to $\mathcal{P}1$ due to the property of Lyapunov function and hence we need to analyze the performance of our proposed algorithm. In order to evaluate the performance of the proposed algorithm and analyze the performance of EE and delay for the considered WPCN, we introduce a Lemma first and make some assumptions.

Lemma 2 *Suppose that λ is strictly interior to the capacity region Ψ, and that $\lambda + \varepsilon$ is still in Ψ for a positive ε. Suppose that $\mathcal{P}1$ is feasible, i.e., given average arrival data rate $\lambda_k, \forall k \neq 0$, that there exists at least a power and time allocation solution to satisfy all of the constraints in $\mathcal{P}1$, and that is, given average arrival data rate $\lambda_k + \varepsilon, \forall k \neq 0$, $\mathcal{P}1$ is also feasible. Then, for any $\delta > 0$, there exists an i.i.d. algorithm*[1] *$\Theta^{\omega} = \{\tau_k(t), P_k^{tr}, \quad \forall k\}$ that satisfies*

$$E\{R_A^{\omega}(t)\} \geq E\{W_A^{\omega}(t)\}(\rho_{ee}^{opt} - \delta), \tag{33}$$

$$E\{R_k^{\omega}(t)|Z(t)\} = E\{R_k^{\omega}(t)\} \geq \lambda_k + \varepsilon, \tag{34}$$

where $\tau(t)$ represents $\{\tau_0(t), \tau_1(t), ..., \tau_K(t)\}$, and $\mathbf{P^{tr}(t)}$ represents $\{P_0^{tr}(t), P_1^{tr}(t), ..., P_K^{tr}(t)\}$. And $W_A^{\omega}(t)$, $R_A^{\omega}(t)$ and $R_k^{\omega}(t)$ are the resulting values under Θ^{ω}.

Proof The similar proof of *Lemma* 2 can be found in [22, 32]. □

Noting that the transmit power of PS is bounded by P_0^{max} in reality and the circuit power consumption is constant, hence we assume that the time-averaged power consumption is bounded by P_{min} and P_{max}, and thus the overall energy consumption has a lower bound $W_{A_{min}}$ and an upper bound $W_{A_{max}}$ for a long time. Similarly, we have an upper bound of overall transmit rate $R_{A_{max}}$. By exploiting *Lemma* 2 and the assumption above, then we have the following two performance bounds

$$\rho_{ee}^{opt} \geq \rho_{ee} \geq \rho_{ee}^{opt} - \frac{c^M / W_{A_{min}}}{V}, \tag{35}$$

$$\overline{Q} \leq \frac{c^M + V\{R_{A_{max}} + \rho_{ee}^{opt}(W_{A_{max}} - W_{A_{min}})\}}{\varepsilon}, \tag{36}$$

where ρ_{ee} denotes for the solution obtained by our proposed algorithm and ρ_{ee}^{opt} represents for the optimal value obtained by solving $\mathcal{P}1$. $\overline{Q}$ is the time-averaged data queue length.

Proof The correlative proof of (35) and (36) can be found in "Proof of (17) in Appendix" section. □

Remark 2 The gap between ρ_{ee} and ρ_{ee}^{opt} is reveled by (35). As V increases , ρ_{ee} is arbitrarily close to the optimal value ρ_{ee}^{opt}. That is, with a sufficiently large V, the resource allocation scheme obtained via DEERAA is asymptotically optimal in terms of solving the original problem $\mathcal{P}1$. The boundary of $\overline{Q}$ is given by (36), and we find that the time-averaged data queue is stable [22]. From (35) and (36), we find that both ρ_{ee} and $\overline{Q}$ are increasing functions with respect to V, and thus we can balance the performance of EE and delay by adjusting V. If we pay more attention on EE, we can choose a larger V which leads to a larger delay at the same time. And if we pay more attention on delay, we can choose a smaller V and thus smaller EE.

4 Simulation results

In this section, simulations are provided to validate our theoretical results and analyze EE and delay. We set four WPDs randomly and uniformly distributed on the right hand side of PS with the reference distance of 2 m and the maximum service distance is 15 m. The IRS is located 150 m far away from the PS. We take both the pathloss and small-scale Rayleigh fading into consideration. The simulation parameters are shown in Table 1. For simplicity, we assume the energy conversion efficiency are equal for all WPDs, i.e., $\eta_k = \eta$, and the circuit consumption power $P_k^{ci} = P^{ci}, k \neq 0$ keeps constant. Similarly, we set $\lambda_k = \lambda, \forall k \neq 0$.

Figure 3 displays the resulting EE against control parameter V. As expected, ρ_{ee} increases at the speed of $\mathbf{O(V)}$ as V increases for any given λ. Besides, with the energy conversion efficiency η increases, the energy consumption during WET is decreasing and thus lead to the EE increases. In general, with the data arrival rate λ increases, much more transmit power is needed to guarantee the stability of the data queues. However, an increase in transmit power does not result in a proportional increase in transmit rate due to the diminishing slope of logarithmic rate-power function. Thus we can find the EE is decreasing with λ increasing in Fig. 3. λ increasing in Fig. 3.

The time-averaged queue length $\overline{Q}$ against control parameter V is shown in Fig. 4. As expected, $\overline{Q}$ increases at the speed of $\mathbf{O(V)}$ as V increases. And $\overline{Q}$ is larger with a larger λ, which is following from Little's theorem that the time-averaged queue length is proportional to the arrival data rate. Besides, it is obvious that with a larger energy

[1] Define an i.i.d algorithm as one that, at the beginning of each slot $t \in \{0, 1, 2, \ldots\}$, choose a policy $\mathbf{P(t)}$ by independently and probabilistically selecting $\mathbf{P} \in \mathbf{A}_{\tau(t)}$ according to some distribution that is the same for all slots t. Let $\boldsymbol{P}^{\omega}(t)$, $t \in \{0, 1, 2, \ldots\}$ represent such an i.i.d algorithm, then $W_A(t)$ and $R_A(t)$ are also i.i.d over slots. $A_{\tau(t)}$ denotes the set of all feasible power allocation options under $\tau(t)$.

Table 1 List of Simulation Parameters

Path loss exponent ϱ	3
System bandwidth B	1 MHz
Power spectral density of noise σ^2	− 174 dBm/Hz
Maximum transmit power of the PS P_0^{max}	10 W
Data buffer size B^D	1000 bit/Hz
Energy buffer size B^W	10 J

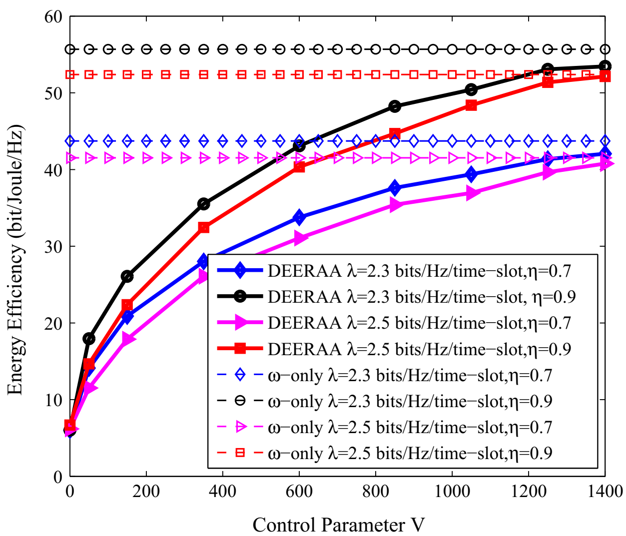

Fig. 3 EE against V with different λ and η under: $P_0^{ci} = 0.05$W and $P^{ci} = 0.001$W

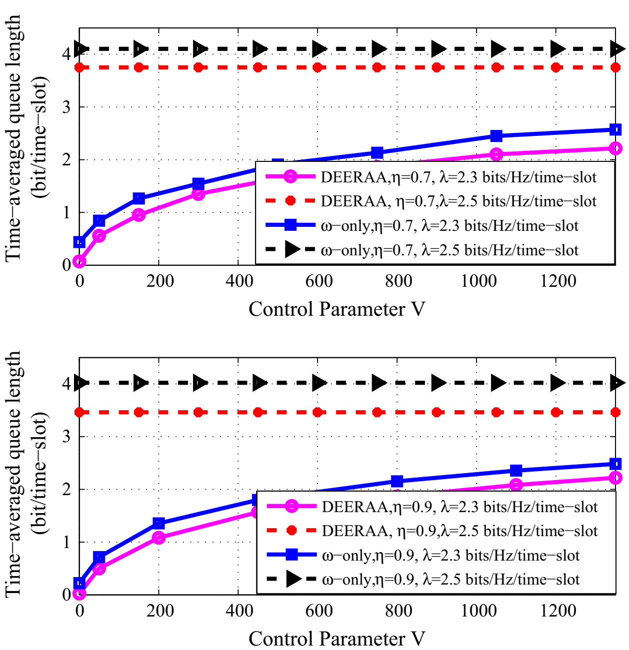

Fig. 4 Time-averaged queue length against V with different λ and η under: $P_0^{ci} = 0.05$W and $P^{ci} = 0.001$ W

conversion efficiency η, $\overline{Q}$ is smaller which is account for energy for data transmission is increasing as η increasing and thus leading to larger transmit rates and smaller time-averaged queue length. Jointly considering Figs. 3 and 4, it is obvious that the EE and time-averaged delay performance can be adjusted by controlling V, which is verified in Fig. 5. Figure 5 displays the EE against delay with different λ for the considered WPCN. It is shown that with larger V, the time-averaged delay and EE are increasing. If we pay attention to EE, we can select a large V and thus lead to a large delay, and if we pay more attention to delay, we can select a smaller V. Besides, we also can find that data arrival rates affect the EE and time-averaged delay performance. With larger λ, EE is decreasing and delay is increasing.

In Fig. 6, we show how the circuit power consumption (i.e. P_0^{ci} and P_k^{ci}) affects the EE and time-averaged delay. We observe that when the circuit power consumption increases, the EE decreases. This is owing to the fact that, the total energy is constant, the increasing of circuit power consumption leads to the decreasing of transmit power, and thus the transmission rates decrease, which decreases EE. And it is obvious that with arrival data rate increases, the EE is decreasing and delay is increasing, which is consistent with the analysis before.

As shown in Fig. 3, with V increasing, our proposed algorithm DEERAA's EE is gradually approaching to the ω-only algorithm's EE which is optimal and it is correspondence with the formula (35). As shown in Fig. 4, with V increasing, our proposed algorithm DEERAA's time-averaged queue length is gradually becoming constant, while ω-only algorithm's time-averaged queue length is independent with V and larger. Comparing with ω-only algorithm under different arrival rate in Fig. 7, it is obvious

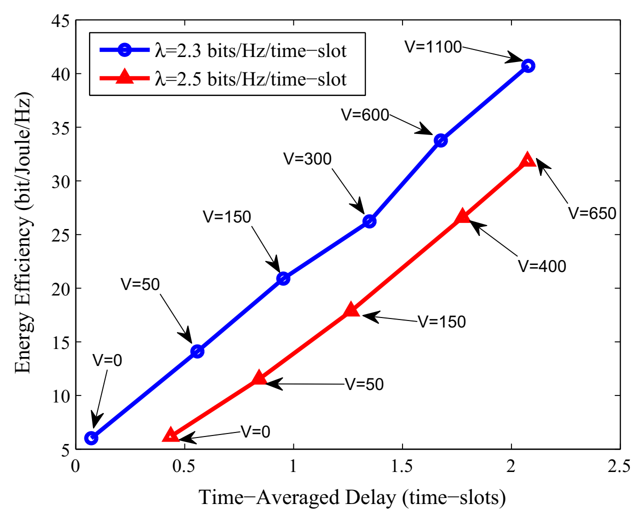

Fig. 5 EE against time-averaged delay with different λ under: $\eta = 0.7$, $P_0^{ci} = 0.05$ W and $P^{ci} = 0.001$ W

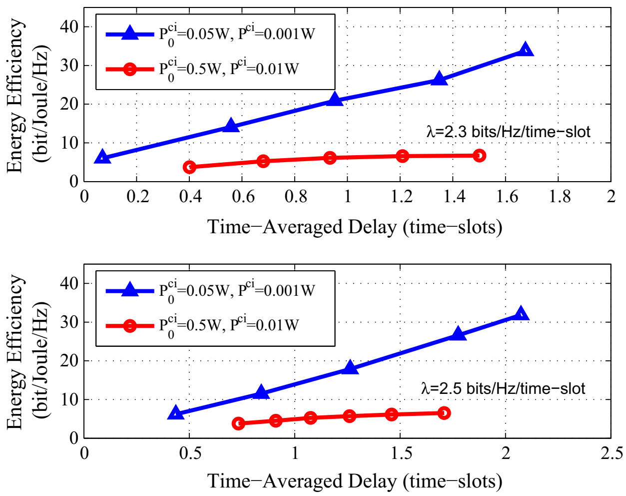

Fig. 6 EE against time-averaged delay under: $\eta = 0.7$

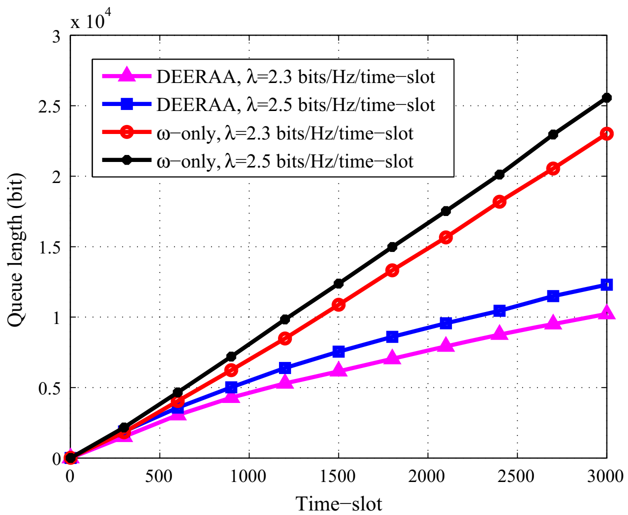

Fig. 7 Queue length against time-slot with different λ under: $V = 1000$, $P_0^{ci} = 0.05$ W, $P^{ci} = 0.001$ W and $\eta = 0.7$

that DEERAA's queue length would keep stability as time-slot increasing while ω-only algorithm doesn't. This is due to the reason that DEERAA takes queue length into consideration, while ω-only algorithm only optimizes EE, and the system may backlog data for waiting a good channel state and accumulating enough power to transmit data in order to achieve the optimal EE. When the arrival data rate is large, there would be a long backlog queue and immerse queue delay, which is uncontrolled and impractical. While by controlling V, in our proposed algorithm with prior distribution knowledge of data arrivals free, the performance between EE and delay can be adjusted.

In Fig. 8, the EE of DEERAA and Optima which optimizes the power consumption without considering system throughput in [20] is investigated. As expected, our proposed algorithm DEERAA outperforms the Optima.

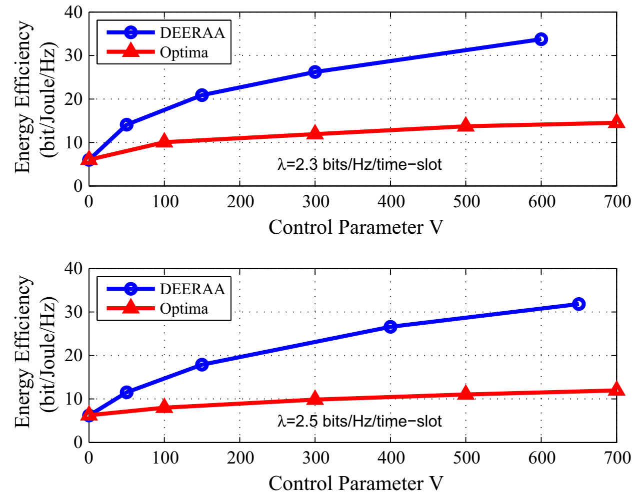

Fig. 8 EE against control parameter V in DEERAA and Optima with different λ under: $P_0^{ci} = 0.05$ W and $P^{ci} = 0.001$ W

Merely optimizing power consumption would ignore the throughput and leads to poor user experience and lower EE. While taking both throughput and power consumption into account, our proposed algorithm satisfies the users' requirements on throughput and makes full use of energy, which is environment friendly and coincidence with the theme of green communication. And thus, our DEERAA is more practical than Optima when they are applied into real system.

In Fig. 9, we investigate the effect of η on the performance of EE and time-averaged delay of EEDRAA and compare it with a Baseline algorithm which allocates time among WPDs equally, i.e., the Baseline only optimizes time allocation between PS and WPDs and power control among PS and WPDs. As expected, our proposed algorithm

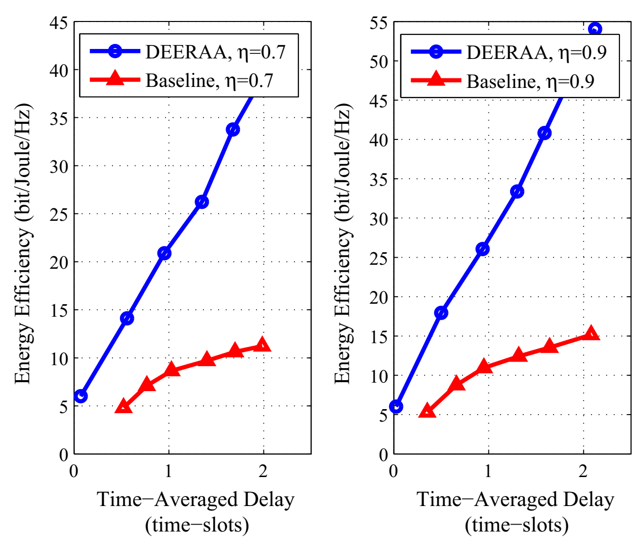

Fig. 9 EE against time-averaged delay in DEERAA and Baseline algorithm under: $\lambda = 2.5$, $P_0^{ci} = 0.05$ W and $P^{ci} = 0.001$ W

EEDRAA outperforms the Baseline, which accounts for EEDRAA adaptively allocates time among WPDs according to dynamics channel and data queue length. Besides, EE and time-averaged delay performance is improved with energy conversion efficiency increasing for both the EEDRAA and the Baseline, which is in accordance with the analysis before. And thus our proposed analytical framework can be readily extended to WPCNs with fixed time-allocation mechanism.

5 Conclusions

In this paper, the dynamic energy-efficient resource allocation for the newly emerging WPCN was investigated. We formulated a stochastic optimization problem subject to both data queue stability and the harvested energy availability to address the dynamic resource allocation (i.e, time allocation and power control) problem. With the aid of fractional programming, Lyapunov optimization theory and Lagrangian method, we solved the problem and analyze the EE and delay performance. Correspondingly, we proposed a dynamic energy-efficient resource allocation algorithm (DEERAA), which does not require any prior distribution knowledge of CSI and data arrivals. We have further found that the EE and delay performance can be adjusted with V, which is a system control parameter. Simulation results had confirmed the adaptiveness of the DEERAA and the theoretical analysis on resource allocation.

Appendix

Proof of Theorem 1

Theorem 1 is proved by separately proving the necessary condition and sufficient condition of it. First, we prove the necessary condition. Let τ^* and $\boldsymbol{P^{tr*}}$ be the solution of (9) and according to the formulation (10), we arrive in

$$\rho_{ee}^* = \frac{\overline{R}_A(\tau^*, \boldsymbol{P^{tr*}})}{\overline{W}_A(\tau^*, \boldsymbol{P^{tr*}})} \geq \frac{\overline{R}_A(\tau, \boldsymbol{P^{tr}})}{\overline{W}_A(\tau, \boldsymbol{P^{tr}})}, \forall \tau, \boldsymbol{P^{tr}} \in \mathfrak{S}. \tag{37}$$

Rewrite the formulation above, i.e. (37), then we obtain

$$\overline{R}_A(\tau, \boldsymbol{P^{tr}}) - \rho_{ee}^* \overline{W}_A(\tau, \boldsymbol{P^{tr}}) \leq 0, \forall \tau, \boldsymbol{P^{tr}} \in \mathfrak{S} \tag{38}$$

$$\overline{R}_A(\tau^*, \boldsymbol{P^{tr*}}) - \rho_{ee}^* \overline{W}_A(\tau^*, \boldsymbol{P^{tr*}}) = 0, \forall \tau, \boldsymbol{P^{tr}} \in \mathfrak{S}. \tag{39}$$

According to (38), we have $max\{\overline{R}_A(\tau, \boldsymbol{P^{tr}}) - \rho_{ee}^* \overline{W}_A(\tau, P^{tr}), \forall \tau | \boldsymbol{P^{tr}} \in \mathfrak{S} = 0$. Combining with (38), we have the maximum value is taken when $\{\tau, \boldsymbol{P^{tr}}\} = \{\tau^*, \boldsymbol{P^{tr*}}\}$. Thus finished the proof of necessary condition. To prove the sufficient conditon, let $\{\{\tau^*, \boldsymbol{P^{tr*}}\}$ be a solution of (11), i.e. $\overline{R}_A(\tau^*, \boldsymbol{P^{tr*}}) - \rho_{ee}^* \overline{W}_A(\tau^*, \boldsymbol{P^{tr*}}) = 0$, and then

$$\begin{aligned} \overline{R}_A(\tau, \boldsymbol{P^{tr}}) - \rho_{ee}^* \overline{W}_A(\tau, \boldsymbol{P^{tr}}) &\leq \overline{R}_A(\tau^*, \boldsymbol{P^{tr*}}) \\ &- \rho_{ee}^* \overline{W}_A(\tau^*, \boldsymbol{P^{tr*}}) = 0, \forall \tau, \boldsymbol{P^{tr}} \in \mathfrak{S}. \end{aligned} \tag{40}$$

Hence

$$\overline{R}_A(\tau, \boldsymbol{P^{tr}}) - \rho_{ee}^* \overline{W}_A(\tau, \boldsymbol{P^{tr}}) \leq 0, \forall \tau, \boldsymbol{P^{tr}} \in \mathfrak{S}, \tag{41}$$

$$\overline{R}_A(\tau^*, \boldsymbol{P^{tr*}}) - \rho_{ee}^* \overline{W}_A(\tau^*, \boldsymbol{P^{tr*}}) = 0, \forall \tau, \boldsymbol{P^{tr}} \in \mathfrak{S}. \tag{42}$$

From (41)we have $\rho_{ee}^* \geq \frac{\overline{R}_A(\tau, \boldsymbol{P^{tr}})}{\overline{W}_A(\tau, \boldsymbol{P^{tr}})}, \forall \tau, \boldsymbol{P^{tr}} \in \mathfrak{S}$, that is ρ_{ee}^* is the maximum of (10). From (42) we have $\frac{\overline{R}_A(\tau^*, \boldsymbol{P^{tr*}})}{\overline{W}_A(\tau^*, \boldsymbol{P^{tr*}})} = \rho_{ee}^*$, that is $\{\{\tau^*, \boldsymbol{P^{tr*}}\}$ is a solution of (10). Thus finished the proof of Theorem 1. □

Proof of Theorem 2

Theorem 2 with is proved with the aid of contradiction. We propose two solutions to $\mathcal{P}3$. The first one is supposing the optimal solution to problem $\mathcal{P}3$ in time-slot t is $\Phi^* = \{P_0^{tr*}(t), P_k^{tr*}(t), \tau_0^*(t), \tau_k^*(t)\}, \forall k \neq 0$, and supposing the PS transmit power satisfies $P_0^{tr*}(t) < P_0^{max}(t)$. Under such a assumption we attain the optimal solution Φ^* is denoted as ρ^*. The other one solution is $\Phi' = \{P_0^{tr\prime}(t), P_k^{tr\prime}(t), \tau_0'(t), \tau_k'(t)\}, \forall k \neq 0$, and suppose the PS transmit power satisfies $P_0^{tr\prime}(t) = P_0^{max}(t)$. The corresponding optimal value is ρ'. Furthermore, we assume the difference only exists in the WET stage, i.e., the condition in WIT stage between two solutions is same. Then we have $\forall k \neq 0, P_0^{tr*}(t)\tau_0^*(t) = P_0^{tr\prime}(t)\tau_0'(t), \tau_k^*(t) = \tau_k'(t), P_k^{tr*}(t) = P_k^{tr\prime}(t), W_k^*(v) = W_k'(v), R_k^*(v) = R_k'(v)$. Now, we can compare the value ρ^* and ρ', which are given by Eqs. (43) and (44), at the top of next page to find the relationship between them. According to the assumption that $P_0^{tr\prime}(t) = P_0^{max}(t) > P_0^{tr*}(t)$, $P_0^{tr*}(t)\tau_0^*(t) = P_0^{tr\prime}(t)\tau_0'(t)$, then we obtain $\tau_0'(t) < \tau_0^*(t)$. Comparing the two equations, i.e.,Eqs. (43) and (44), we can obtain $\rho^* < \rho'$ which contradicts the assumption that ρ^* is the optimal solution. Thus *Theorem* 2 is proved. □

$$\rho' = \frac{\sum_{v=1}^{t}\left(\sum_{k=1}^{K} R_k'(v)\right)}{\sum_{v=1}^{t-1}\left(P_0^{tr\prime}(v)\tau_0'(v) + P_0^{ci}\tau_0'(v) - \sum_{k=1}^{K} W_k'(v) + \sum_{k=1}^{K}\left(P_k^{tr\prime}(v) + P_k^{ci\prime}(v)\right)\tau_k'(v)\right)}. \quad (43)$$

$$\rho^* = \frac{\sum_{v=1}^{t}\left(\sum_{k=1}^{K} R_k^*(v)\right)}{\sum_{v=1}^{t-1}\left(P_0^{tr*}(v)\tau_0^*(v) + P_0^{ci}\tau_0^*(v) - \sum_{k=1}^{K} W_k^*(v) + \sum_{k=1}^{K}\left(P_k^{tr*}(v) + P_k^{ci*}(v)\right)\tau_k^*(v)\right)}. \quad (44)$$

Proof of (17)

Squaring both sides of Eq. (7) results in

$$\begin{aligned}\left(Q_k^D(t+1)\right)^2 \le (Q_k^D(t))^2 + (R_k(t))^2 + (D_k(t))^2 \\ - 2Q_k^D(t)(R_k(t) - D_k(t)).\end{aligned} \quad (45)$$

Sum overall $k(k = 1, \ldots, K)$, take the conditional expectation $\mathbb{E}\{\cdot|Z(t)\}$, and recall $R_k(t) \le R_k^{max}$, $D_k(t) \le D_k^{max}$, then

$$\begin{aligned}\varDelta\{Z(t)\} &\le \frac{1}{2}\sum_{k=1}^{K}\mathbb{E}\{(D_k(t))^2 + (R_k(t))^2|Z(t)\} \\ &\quad - \sum_{k=1}^{K}\mathbb{E}\{Q_k^D(t)((R_k(t) - D_k(t))|Z(t))\} \\ &\le c^M - \sum_{k=1}^{K}\mathbb{E}\{Q_k^D(t)(R_k(t)|Z(t))\} \\ &\quad + \sum_{k=1}^{K}\mathbb{E}\{Q_k^D(t)(D_k(t)|Z(t))\},\end{aligned} \quad (46)$$

where $c^M = \frac{1}{2}\sum_{k=1}^{K}\left((D_k^{max}(t))^2 + (R_k^{max}(t))^2\right)$. Define $C^M = c^M + \sum_{k=1}^{K}\mathbb{E}\{Q_k^D(t)(D_k(t)|Z(t))\}$, and then we obtain (17). This finished the proof. □

Proof of (35) and (36)

Adopt an i.i.d. algorithm and we can transform (18) into (47)

$$\begin{aligned}&\triangle\{Z(t)\} - VE\{R_A(t) - \rho_{ee}(t)W_A(t)|Z(t)\} \\ &\quad \le C^M - E\{V\{R_A^\omega(t) - \rho_{ee}(t)W_A^\omega(t)\}|Z(t)\} \\ &\quad - E\left\{\sum_{k=1}^{K} Q_k^D(t)R_k^\omega(t)|Z(t)\right\}\end{aligned} \quad (47)$$

Plugging (33) and (34) into (47) and taking a limit as $\varepsilon \to 0$ yield

$$\begin{aligned}&\triangle\{Z(t)\} - VE\{R_A(t) - \rho_{ee}(t)W_A(t)|Z(t)\} \le c^M \\ &- V\rho_{ee}^{opt}E\{W_A^\omega(t)\} + VE\{\rho_{ee}(t)W_A^\omega(t)\}.\end{aligned} \quad (48)$$

By taking iterated expectation and using telescoping sums over $t \in \{1, \ldots, T\}$ in (48), and then we obtain

$$\begin{aligned}&E\{L(Z(T))\} - E\{L(Z(1))\} \le T(c^M - V\rho_{ee}^{opt}E\{W_A^\omega(t)\}) \\ &+ V\sum_{t=1}^{T}E\{\rho_{ee}(t)R_A^\omega(t)\} + V\sum_{t=1}^{T}E\{R_A(t) \\ &- \rho_{ee}(t)W_A(t)|Z(t)\}.\end{aligned} \quad (49)$$

Dividing (49) by VT and using the fact that $E\{L(Z(T))\} \ge 0$, we acquire

$$\begin{aligned}&\frac{1}{T}\sum_{t=1}^{T}E\{R_A(t) - \rho_{ee}(t)W_A(t)\} \ge -\frac{E\{L(Z(1))\}}{VT} \\ &\quad - \frac{c^M}{V} + \rho_{ee}^{opt}E\{W_A^\omega(t)\} - E\{W_A^\omega(t)\}\frac{1}{T}\sum_{t=1}^{T}E\{\rho_{ee}(t)\}.\end{aligned} \quad (50)$$

Taking the limit as $T \to \infty$, and after some manipulations, then we obtain

$$\rho_{ee} \ge \rho_{ee}^{opt} - \frac{c^M/E\{W_A^\omega(t)\}}{V} \ge \rho_{ee}^{opt} - \frac{c^M/W_{A_{min}}}{V}. \quad (51)$$

Similarly, plug (33) and (34) into (47), take iterated expectation and use telescoping sums over $t \in \{1, \cdots, T\}$, then

$$\begin{aligned}&E\{L(Z(T))\} - E\{L(Z(1))\} + \varepsilon\sum_{t=1}^{T}\sum_{k=1}^{K}Q_k^D(t) - V\sum_{t=1}^{T}E\{\rho_{ee}(t)W_A^\omega(t)\} \\ &\le T(c^M - V\rho_{ee}^{opt}E\{W_A^\omega(t)\}) + V\sum_{t=1}^{T}E\{R_A(t) - \rho_{ee}(t)W_A(t)\}.\end{aligned} \quad (52)$$

Divide (52) by εT and take limit as $T \to \infty$

$$\overline{Q} = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_{k=1}^{K}Q_k^D(t) \le \frac{c^M + V\{R_{A_{max}} + \rho_{ee}^{opt}(W_{A_{max}} - W_{A_{min}})\}}{\varepsilon}. \quad (53)$$

This finished the proof of Eqs. (35) and (36). □

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