Chapter 6 Exercises

1. Determine the following values:
   (a) \( \log 10,000,000 \)     (b) \( \ln e^3 \)     (c) \( \lg 32 \)     (d) \( \log .01 \)     (e) \( \ln (1/e^5) \)
   (f) \( 10^{\log 5} \)     (g) \( e^{\ln 2} \)     (h) \( 2^{\lg 7} \)     (i) \( \lg (\log 10,000) \)     (j) \( \log (\lg 1024) \)
   (k) \( \log 1 \)     (l) \( \ln 1 \)     (m) \( \lg 1 \)

2. Use the general logarithm division rule to explain why:
   (a) \( \ln (1/e) = -1 \)     (b) \( \log (1/10) = -1 \)     (c) \( \lg (1/2) = -1 \).

3. Everyone should get a feel for the power of a calculator by comparing computations by paper and pencil with carrying out the same computations with the calculator. In this exercise I invite you to do such computations in three ways:
   (a) Multiply 2739 by 8456 (1) by paper and pencil, (2) by logs with your calculator \((\log 2739 + \log 8456 \text{ then take the antilog} \text{ that is, } 10^{x})\), and (3) by calculator.
   (b) Do the same for 29897098 divided by 4678.

4. On page 87, a derivation of the log equation \( \log (xy) = \log x + \log y \) is offered. Use the same technique to derive the other basic calculation formulas:
   (a) \( \log (x/y) = \log x - \log y \)
   (b) \( \log x^y = y \log x \)
   (c) \( \log \sqrt[y]{x} = \frac{\log x}{y} \)

5. We know from one of the calculations on page 93 that \( \log 421.6965034 = 2.625 \). Develop a table of values and trace the program LOGX on pages 100-101 step by step with \( X = 421.6965034 \). You know that your answer should come out 2.625. Your table will look like:

<table>
<thead>
<tr>
<th>X</th>
<th>C</th>
<th>W</th>
<th>M</th>
<th>U</th>
<th>K</th>
<th>V</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. The equation \( \log_b x \times \log_a b = \log_a x \) occurs in the derivation of the change of base formula on page 256. Substituting \( c \) for \( x \) and reordering the factors on the left side of this equation, we have \( \log_b c \times \log_b c = \log_a c \), an equation that is useful in solving many contest problems. Use this equation to find the values of each of the following and then check by using your calculator:
   (a) \( \log_5 6 \times \log_6 7 \) as a log with base 5.     (b) \( \log e \times \ln 100 \)     (c) \( \ln 10 \times \log e \)

7. The formula for the number of decimal digits in the integer \( D \) introduced on page 258 is \( 1 + \text{int}(\log D) \). This formula is also useful in solving contest problems. Use it together with the program LOGGRE1 of page 97 to find the number of digits in:
   (a) \( 23^{35} \) Recall that \( \log 23^{35} = 35 \times \log 23 \).     (b) \( 23^{75} \)
(c) Check your answer in (a) by simply entering $23\sqrt[3]{35}$ and examining the answer in scientific notation.
(d) Why can't you check the answer in (b) by this means?

7. Another use of the binary search technique applied by the program for calculating logarithms is found in Appendix M. Use the third approach to equation solution by binary search on pages 253-254 to solve the equation $X^5 + 3X^2 - 10 = 0$. You will need to enter this equation in the program \textbf{FN} and then enter and run the program \textbf{BISCH4} of page 254, using the values $P = 1$ and $Q = 2$. (This equation was also solved by iteration in Exercise 7 of Chapter 4. Compare your answers.)

8. In Appendix P instructions are given for construction of a table of base ten logarithm values for the integers 1 to 10. At the end of this Appendix on page 264 suggested factors are given for the numbers from 11 to 19.
(a) Use these factors to fill in the two digit table of logs.
(b) Extend the table for values from 21 to 25.
(c) Calculate and add values for 35, 45, 55, 65 and 75 to your table.

9. Use the values given in Appendix P and those you have calculated to construct a slide rule as suggested in this appendix and on pages 198-200 of Appendix C to construct a slide rule. If possible use a decimal ruler and heavy stock 8 ½” by 11” paper for this task.
(a) Along a longer edge of your paper mark off ten inches.
(b) Multiply the values in the table of Appendix P and exercise 8 by 10 to give you values to mark off and label them along this line.
(c) Along the opposite edge of the paper construct a similar log rule.
(d) Cut these rules out as suggested in the diagram and use them for C and D log scales.

(e) Use your scales as on page 199 to find the following products:
   (1) $25 \times 5$  
   (2) $120 \times 25$  Recall that 120 is the same as 1.2 or 12 on your rule.
   (3) $45 \times 8$  To do this recall that 1 and 10 are the same. You can set the right end of your rule at 45 and read the product opposite the 8.
10. If possible, find a real slide rule and use it to calculate values like those of Exercise 9 (e). See how much accuracy you can get using just the C and D scales of such a rule by seeking the following products and comparing them with calculator answers:

(a) \(125 \times 35\)  
(b) \(75 \times 45\)  
(c) \(132 \times 355\)  
(d) \(87 \times 87\)

(e) What do you notice about the accuracy of multiplying the numbers in (c) and (d)?