Chapter 4 Exercises

1. Download Appendix U, read about the program OLDSQRT, then enter or download, and test this program in your calculator.
   (a) Notice that X is set equal to 2 in the second program step. Use the program to confirm that $\sqrt{1.522756} = 1.234$, in this way showing that the program works when $X < 2$. Look ahead in the program to see why it works for numbers between 1 and 2 as well as others.
   (b) Unlike the programs later in the chapter like SQRTG10 on page 65, OLDSQRT asks for a value of D, the number of decimal places you wish reported. Modify OLDSQRT to give you ten digit accuracy (note that this means an integer with nine decimal digits) whose first line is simply: Prompt X.
   (c) Revise OLDSQRT so that you can use it for any positive real number value for N. You will need to modify N in your program to fit the requirement that it be between 1 and 100, in the process determining and keeping track of the correct decimal position, then correct the value produced by the original program at the end.

2. Average or Arithmetic Mean
   (a) Program your calculator to calculate the average of three numbers. You know that the average of N numbers is the sum of those numbers divided by N. For example, the average of a, b and c is $(a + b + c)/3$.
   (b) Program your calculator to calculate the average of N numbers. To do this, you will need to enter N, and then use a For loop governed by N to enter and sum, as S, the individual values. When you exit the For loop you will need only calculate S/N.
   (c) Develop a program to calculate the average of a number of entries with that number unspecified when you begin the calculation. To do this you will need to include a counter in your While loop to keep track of the number of entries. You can use an arbitrary number, say -1000, to control the end of your series of entered numbers.

3. Geometric Mean or Mean Proportion
   In Chapter 4 you met the Arithmetic Mean and Geometric Mean for two numbers. The Geometric Mean for two numbers, A and B, is $\sqrt{AB}$ and for three numbers, A, B and C, is: $\sqrt[3]{ABC}$. In general, the formula for the Geometric Mean is:
   
   $$G.M. = \sqrt[n]{X_1X_2X_3\ldots X_{N-1}X_N}$$

   (a) Modify your program in 1 (b) to calculate the Geometric Mean for N numbers.
   (b) Modify your program in 1(c) to calculate the Geometric Mean for an unspecified number of entries.

4. Comparing G.M. and A.M.
   (a) Use the programs you developed in 1 and 2 to compare the Arithmetic and Geometric Means for the following sets of values: \{2,3,4\}, \{2,12,13,15\}, \{5,5,5,5,5\}.
   (b) Use your resulting data to confirm the important mathematical inequality (which we proved only for the case of two numbers):
G.M. ≤ A.M.

In other words, see if your results agree with the inequality:

\[ \frac{\sqrt[N]{X_1X_2X_3\ldots X_{N-1}X_N}}{N} \leq \frac{X_1 + X_2 + X_3 + \ldots + X_{N-1} + X_N}{N} \]

5. \( \sqrt[N]{N} = \sqrt[N]{N} \) and \( \sqrt[8]{N} = \sqrt[N]{N} \).
   (a) Use the first of these relationships to modify the \texttt{SQRTG10} program of page 65 to calculate 4th roots.
   (b) Develop a program by modifying \texttt{SQRTG10} that will calculate the \((2^R)\)th root of \(N\) with the first program line: \texttt{Prompt N, R}.
   (c) \( b^{3/4} = \left(\sqrt[4]{b}\right)^3 \). Develop a program by modifying \texttt{SQRTG10} to calculate \( b^{3/4} \) using this relationship.
   (d) \( b^{3/4} = b^{1/2} \times b^{1/4} = \sqrt{b} \times \sqrt[4]{b} \). Develop a program to calculate \( b^{3/4} \) using this relationship.
   (e) Check several values to see that your programs in (c) and (d) give the same answers.

6. The square root programs \texttt{SQRT}, \texttt{SQRTG10}, and \texttt{OLDSQRT} of this chapter and Appendix U all use a process variously described as feedback, iteration or recursion. A first guess is chosen and the guess refined until, by a reasonable mathematical process, the sequence of "guesses" converges on the required answer. This is a very powerful tool that is used widely in computation.
   (a) It can be shown (not here) that the solution of most pairs of linear equations \( y = mx + b \) and \( y = nx + c \), may be found by the following steps:
      1. Solve the equation whose slope has the larger absolute value for \( x \).
      2. Choose a guess for \( x \).
      3. Substitute this value in the equation solved for \( y \).
      4. Substitute the resulting \( y \) value in the equation solved for \( x \).
      5. Repeat steps 3 and 4 until you converge on an answer.

Consider, for example the pair of equations, (1) \( y = 2x - 3 \) and (2) \( y = -x + 3 \). Since \( |2| > |-1| \), solve the first equation for \( x \) giving (1') \( x = (y + 3)/2 \). Now choose an \( x \) value to start the iteration. Assume we choose \( x = 1 \). In equation (2) this gives \( y = 2 \). Taking that \( y \)
to equation (1') gives x = 2.5. Now that value goes to equation (2) giving y = 0.5, and so on. What is happening is pictured in the preceding graph. Notice that the x and y values are getting closer and closer to the correct answer (2,1). Write a program that will continue this process for these two equations, displaying the x and y values as they occur. Run this program until you reach these values.

(b) Use this same technique to solve the pair of equations: y = x/2 + 3 and y = 2x - 2. Note that you can use your program of (a) substituting appropriate equations. Check your solution by solving the pair of equations algebraically.

(c) This method does not always work. Consider, for example, the pair of equations y = 2x – 3 and y = -2x + 4. What happens when you use these equations in your program? Since their slopes have equal absolute value, choose either one to solve for x. Draw a graph to see what is going on.

7. Not only linear equations may be solved by iteration. Here is an example of a fifth degree equation whose single real root we can find by this technique:\[X^5 + 3X^2 - 10 = 0\]. We can rewrite this as \[X^5 = 10 - 3X\] and finally \[X = \sqrt[5]{10 - 3X}\]. Now we can use this form to give successive values of X, using a succession of guesses, \(G_n\), for each calculating a corresponding \(H\)

\[H = \sqrt[5]{10 - 3G}\]

and then letting H serve as the next G. Write a program that will find this root by iteration using this equation. You can quickly see by graphing the function \(f(X) = X^5 + 3X^2 - 10\), or substituting integer values \(X = 0, 1, 2\), that this graph crosses the X axis between 1 and 2. Thus an initial guess of 1 or 2 should work.) While the idea of this process is straightforward, carrying it out is not quite so easy. It is appropriate to use a While loop here with a test like \(\text{abs}(G-H) > 0.00000000005\) but clearly, if you let \(H \rightarrow G\) at the end of the loop, it will immediately terminate. A way around this is to set \(G = 0\) and \(H = 1\) (so that the two are different) before entering the loop and than letting \(H \rightarrow G\) at the beginning of the loop before evaluating your iterative equation.

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