Chapter 1 Exercises

1. Reviewing the calculator keys.
   Calculate the following values and check your results against those given. If your calculator doesn't accept a calculation, try to determine a reason for that error:
   
a. \(2^2\)  Calculate as \(2^2\) or \(2 \times 2\).

   b. \(2^{2^2}\). Clearly there are two ways to do this: \((2^2)^2\) or \(2^{(2^2)}\). Does it matter which way you choose?

   c. \(3^3\)  27

   d. But what about \(3^{3^3}\)?

   \[
   (3^3)^3 = 19683 \\
   3^{(3^3)} = 7.625597485e12 = 7625597485000
   \]

   *This answer is in scientific notation which is discussed in the footnote on page 42 of Inside Your Calculator.*

   Argue for the order you think \(x^y^z\) should be calculated.

   e. \(25^{-1}\)  .04

   *You can do this calculation with the \(x^{-1}\) key or by 1/25.*

   f. \(\sqrt{2}\)  1.414213562

   g. \(\sqrt[3]{2}\)  1.189207115

   h. \(\frac{\sqrt{2}}{2}\)  Calculate as \(2^{1/3}\): that is \(2 \times (1/3)\).  1.259921050

   i. \(\frac{\sqrt{2}}{2}\)  1.189207115

   *Notice that your answers to g and i are the same.*

   j. \(e\)  Calculate as \(e^1\)  2.718281828

   k. \(e – 2.718281828\)  4.59E-10 = .0000000000459

   *This means that to 12 digits, \(e = 2.718281828459\) and those 1828s do not repeat again.*

   l. \(\cos 30^\circ\)  .8660254038

   m. \(517\cos 37^\circ/\sin 53^\circ\)  517

   n. \(\cos^2 30^\circ\)  \*This notation means \((\cos 30^\circ)^2\)*  .75

   o. \(\cos^{-1} .5\)  Use the \(\cos^{-1}\) key.  60°

   *This is the inverse cosine: \(\cos^{-1} .5\) is the angle whose cosine is .5*

   p. \(\cos^{-1} 1.2\)

   q. \(\tan^{-1} (1/2)\)  26.56505118°

   r. \(\frac{\tan^{-1}(1)}{2}\)  22.5°

   s. \(\tan^{-1} (\sqrt{2}/2)\)  35.26438968°

   t. \(\frac{\tan^{-1}\sqrt{2}}{2}\)  27.36780516°
Inside Your Calculator

2. Your Calculator's Stored Values
   a. Use the technique of page 9 to determine the value of \( \pi \) stored in your calculator.
   b. Use the same technique to determine the value your calculator stores for \( \sqrt{2} \).

3. The Accuracy of GPS Devices
   Pages 13 and 14 of this chapter bring up the remarkable accuracy of GPS devices. You have seen that these devices locate positions to the thousandth of a minute. We want to see how many feet .001' would represent along a line of latitude. (The distance along a line of longitude would be even less except at the equator.) Here is how to proceed:
   a. Use \( C = 2\pi r \) with \( r = 4000 \) miles to calculate the circumference of our planet Earth.
   b. Convert this to feet by multiplying by 5280, the number of feet in a mile.
   c. Now you want to determine what fraction of this distance is .001 minute. To do this first divide your answer in b by 360. This gives you the length of 1° of the Earth's circumference in feet.
   d. Next divide by 60 to determine the length of 1' of circumference in feet.
   e. Finally multiply by .001 to determine how long .001' is in feet.
   f. Recall that the accuracy reading for my GPS accuracy was 18 feet. By comparing with your answer in e, determine approximately what accuracy that represents in minutes.

4. Black Boxes
   Here are some simple Black Box results. Give a formula that calculates the values in each 2nd column from its partner in the 1st. Give a sixth value based on your formula.
   a. | x | y |
      |---|---|
      | 1 | 1 |
      | 2 | 3 |
      | 3 | 5 |
      | 4 | 7 |
      | 5 | 9 |
   b. | x | y |
      |---|---|
      | 1 | 7 |
      | 2 | 4 |
      | 3 | 1 |
      | 4 | -2|
      | 5 | -5|
   c. | x | y |
      |---|---|
      | 1 | 2 |
      | 2 | 6 |
      | 3 | 12|
      | 4 | 20|
      | 5 | 30|
   d. | x | y |
      |---|---|
      | 1 | 6 |
      | 2 | 24|
      | 3 | 60|
      | 4 | 120|
      | 5 | 210|
   e. | x | y |
      |---|---|
      | 1 | 0 |
      | 2 | 0 |
      | 3 | 0 |
      | 4 | 0 |
      | 5 | 0 |
   f. It is important that you understand that given any finite sequence of values you can make up a function to give you any next value. To see this consider exercise 5e. You probably...
suggested that all values in the right column will be zero. Thus the function would be\( y = 0 \). But consider the following formula: \( y = (x-1)(x-2)(x-3)(x-4)(x-5) \). Since any product containing one 0 factor gives a zero result, this function would give the values in 5e. For \( x = 5 \), for example, you have \( y = (5-1)(5-2)(5-3)(5-4)(5-5) = 43210 = 0 \). But when \( x = 6 \), there are no longer any zero factors. Now you have \( y = (6-1)(6-2)(6-3)(6-4)(6-5) = 54321 = 120 \). What would be the value for \( x = 7 \)?

Now we're ready to start programming. Background: Appendix A, pages 179-187:

5. Calculating Formula Results

See pages 181-183 for how to enter and run a program; pages 185-186 for entering and displaying information.

Write separate programs that will calculate:

a. Celsius temperature given the current Fahrenheit temperature. The formula is
\[
C = \frac{5}{9}(F - 32).
\]
Have your program report both F and C values.

b. Fahrenheit temperature given the current Censius temperature. The formula is
\[
F = \frac{9}{5}C + 32.
\]
Have your program report both C and F values. Now run your programs from a and b to compare calculations.

c. The area and volume of a sphere given its radius. The formulas are
\[
A = 4\pi r^2 \quad \text{and} \quad V = \frac{4}{3}\pi r^3.
\]
Have your program report r, A and V.

6. The Cosine Program

a. Enter the program \text{COSDEG} for calculating cosine given on page 9 and compare various values you obtain from it with those in the far less accurate table on page 12. Also compare your results with those you obtain using the \text{cos} key on your calculator.

b. The program \text{COSDEG} includes that strange number 4294967296, which is equivalent to \( 2^{32} \). Can you see any possible relationship between that number and the number of passes through the For loop?

7. Information for a Table.

See page 184 for information about For loops.

Write a program that displays the numbers from 1 to 20, one at a time, together with their squares and square roots. Be sure that your program stops after each triple is displayed. Your program should display:

\[
\begin{align*}
1 & \\
1 & \text{and then, after you press Enter:} \\
1 & \text{etc.}
\end{align*}
\]

1.414213562
8. Basic Operations.  

*See pages 183 and 184 for information about the If control structure and note especially point 4 at the top of the page about entering a test.*

a. Write a program that will accept two numbers, X and Y, and display X + Y.

b. Modify your program to make it display X - Y instead.

c. Now combine those two simple tasks into one program. Have your program accept three numbers, X, Y, and O, the O to determine whether to add or subtract. Let O = 1 represent add and O = 2 subtract. For example, here is how your program might perform:

```
X? 7
Y? 3
O? 2
4
Done
```

d. Important question: Run your program with O = 0. What is the result and why does this happen?

e. Finally, write a program that will again accept three numbers, X, Y, and O, the first two to be operated on and the third determining the operation: 1: add; 2: subtract; 3: multiply; 4: divide.

*This will require several If control structures or a combination of If Then Else controls.*

9. A Cash Register

Simulate a cash register at various levels of development.

a. First, have your program accept a series of item costs and simply display their sum, S.  

*Use a While loop — see pages 184-185 — with an item price of 0 to end your list of entered prices.*

b. Modify your program to calculate the total price with a local sales tax. Use a variable like T = .075 to represent this tax rate so that it can be changed as the tax rate is changed.

c. Assume that some items are taxed and some are not. (You can use 1 for no tax, 2 for tax.) Modify your program so that it displays for each item entered that item's full cost (as you enter it) and finally displays the total bill.

10. Property Depreciation

There are three methods of depreciating the value of property.¹

The simplest is straight-line depreciation. If the value today is X and you wish to depreciate X over Y years, you simply reduce the property value by X/Y each year. Thus a car worth $20,000 today would depreciate over 10 years at $2000 each year. Its subsequent yearly values would then be: $18,000, $16,000, $14,000,..., $2,000, $0.

A second method is called double-declining depreciation. By this method, the depreciation factor each year, D, is 2/Y. Thus, for our previous example, D = 2/10 = 20%. Each year the property value would decline by 20% (or equivalently be worth 80% as much.) In this case, for our example ($20,000 depreciated over 10 years), the successive yearly

---

values would be: $16,000, $12,800, $10,240, ..., $2,684, $2,147. (Note that this method does not go to zero so this last value $2,147 is usually replaced by $0.

A third method is called sum-of-the-years depreciation. By this method, the denominator of the depreciation factor is the sum on the numbers from 1 to Y. This sum is the same as:

\[ D = \frac{Y(Y + 1)}{2} \]

By this method, each year the amount depreciated is the original cost, X, times the number of years remaining divided by D. In other words the first year the amount depreciated would be: \( XY/D \), the second year \( X(Y-1)/D \), the third year \( X(Y-2)/D \), etc. In this case, again for our example ($20,000 depreciated over 10 years), the successive yearly values would be: $16,363.63, $13090.90, $10,181.81, ..., $363.63, $0.

a. Write separate programs (DEP1, DEP2 and DEP3) to calculate the remaining value for successive years by each of these methods. Check your program against the values given in the example.

b. Combine those three programs into a single program that calculates the yearly value by each method and reports them together.

c. Choose an amount to be depreciated and the number of years for the depreciation to run, then use your program developed in b to make a table of remaining values for comparison. (If you have property mortgaged or if you are paying off a loan, you can use your personal data for this problem.)