Division with Large Numbers

The program BIGDIV is for dividing very large numbers that your calculator could not handle otherwise; for example, numbers with divisors, dividends or quotients containing as many as 200 digits. It does so by using matrices and processing with groups of four digits in each cell (thus allowing plenty of room for carrying). One exception: the first group in the dividend (or numerator) is a double group; that is, a group with 5 to 8 digits, thus insuring that the first group division will produce a partial quotient.

Here is the form of the division:

\[
\begin{array}{c}
\, \\
Q \\
D ) N \\
P \\
R \\
\end{array}
\]

The numbers appear as follows in the matrix:

- row 1: N
- row 2: D
- row 3: R
- row 4: P
- row 5: Q

PROGRAM: BIGDIV

'Part I Getting Information into the Calculator

:Prompt N,D,T 'N the number of dividend groups,
'D divisor groups, and T output groups
:max(N,T+D)+1→M
:{5,M}→dim([A])
:For(I,1,N)
: Prompt X
: X→[A](1,I) 'Fill in the dividend groups
:End
:For(I,1,D)
: Prompt Y
: Y⇒[A](2,I)
:End
:For(I,1,M)
:'These eleven lines zero the
'remaining matrix entries in case
'data remains from previous
'calculations
:
: 0⇒[A](3,I)
: 0⇒[A](4,I)
: 0⇒[A](5,I)
:End
:For(I,D+1,M)
: 0⇒[A](2,I)
:End
:For(I,N+1,M)
: 0⇒[A](1,I)
:End

'Part 2 Processing

'It is important to understand that this process
'mirrors standard long division except as noted below,
'each group replacing a single digit in the usual
'algorithm.

:1⇒J
:For(I,1,D)
: [A](1,I)⇒[A](3,I)   'This step, moving digits from
'the dividend to the remainder
'differs from the usual algorithm but
'only to make the first step in the
'process like succeeding ones.
:End
:Lbl 1
:int([A](3,1)/[A](2,1))⇒[A](5,J)   'This step
'corresponds to the first step in the
'usual long division algorithm:
'obtaining the first group in the
'quotient.
Inside Your Calculator

:Lbl 2
:For(I,1,D)
: \[A\](5,J)\*[A](2,I)\rightarrow[A](4,I)\quad"Multiplying the
'dividend group by the divisor
'groups."
:End

'These products will usually contain more than
'four digits so it is necessary to carry extra
digits to the next group to the left. That is
'the role of the next five lines.

:For(I,D,2,-1)
: \text{int}(\[A\](4,I)*10^{-4})\rightarrow C
: \[A\](4,I)-C*10^4\rightarrow[A](4,I)
: \[A\](4,I-1)+C\rightarrow[A](4,I-1)
:End

'Now recall how, in long division, your quotient
'might have been too large and you had to reduce
'it, returning to do the division again. The
'following nine lines checks for this, sending
'you on if there is no problem but reducing your
'quotient and sending you back if there is.

:For(I,1,D)
: If[A](3,I)>[A](4,I)
: \quadGoto 3
: If[A](3,I)<[A](4,I)
: \quadThen
: \quad[A](5,J)-1\rightarrow[A](5,J)
: \quadGoto 2
: \quadEnd
:End
:Lbl 3

'Here we check to see if we are done.

:J+1\rightarrow J
:If J>T
: \quadGoto 4 \quad"where answers are printed out."
'At this point you have determined a correct quotient group and you are ready to move on to repeat the algorithm to determine subsequent groups. To do so you must determine the remainder left from your prior step and divide that by your divisor. The following seven lines borrow if necessary to prepare for subtraction.

:For(I,D,2,-1)
: If [A](3,I)<[A](4,1)
: Then
:  [A](3,I-1)-1→[A](3,I-1)
:  [A](3,I)+10^4→[A](3,I)
: End
:End

'Now we carry out the subtraction, but again in a minor difference from the usual algorithm, we shift the resulting groups to the left so that they fill the digits from the left (with the first group again with more than four digits.)

:([A](3,1)-[A](4,1))*10^4+[A](3,2)-[A]4,2)→[A](3,1)
'Obtaining the first 5-8 digit group.

:For(I,2,D-1)
: [A](3,I+1)-[A](4,I+1)→[A](3,I)
:End
:[A](1,J+D-1)→[A](3,D) 'Bringing down the next group.
: Goto 1 'To repeat the algorithm steps.

'Part 3 Print out the quotient.

:Lbl 4
:For(I,1,T)
: Disp [A](5,I)
: Pause
:End
An Example

Suppose we wish to calculate to 24 digits accuracy the quotient of: 1,413,121,110,987,654,321/123,456,789,100

First, we separate the dividend into four digit groups working from right to left and seeing to it that the leftmost group has more than four digits:

\[
\begin{align*}
1413121 & \quad 1109 & \quad 8765 & \quad 4321 \\
\end{align*}
\]

We do the same to the divisor:

\[
\begin{align*}
1234 & \quad 5678 & \quad 9100 \\
\end{align*}
\]

We're now ready to run the program BIGDIV.

\[
\begin{align*}
N=\,\,4 \quad & \text{Number of groups in the dividend} \\
D=\,\,3 \quad & \text{Number of groups in the divisor} \\
\begin{align*}
T=\,\,6 & \quad \text{Number of groups in the quotient (24 digits)} \\
X=\,\,1413121 \\
X=\,\,1109 \\
X=\,\,8765 \\
X=\,\,4321 \\
Y=\,\,1234 \\
Y=\,\,5678 \\
Y=\,\,9100 \\
\end{align*}
\end{align*}
\]

'After a pause, the calculator will print:

\[
\begin{align*}
1144 \\
6281 \\
938 \\
8967 \\
268 \\
4453 \\
\text{Done}
\end{align*}
\]

You must place the decimal point in the answer:

\[
11,446,281.0938896702684453
\]