Wage Volatility and Changing Patterns of Labor Supply

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June 2015

Abstract

Over the past few decades, the skilled-unskilled hours differential in US men increased while the skill premium rose. This contrasts with studies suggesting a fairly small labor supply elasticity. We explore wage volatility to resolve this discrepancy. Using the PSID, we document that skilled men experienced larger increases in wage volatility with rising shares of persistent shocks than unskilled men. Feeding in these wage processes, our general equilibrium incomplete markets model replicates the increased hours differential. We find that hours adjustment is important for self-insurance in the short run, whereas precautionary savings play the dominant role in the long run.

Keywords: Skill Premium, Wage Volatility, Labor Supply by Skill Group, Precautionary Savings

JEL Classifications: E24, J22, J24, J31

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* We are grateful to Mark Aguiar, Mark Bils, Yongsung Chang, Dirk Krueger, and participants at the Wegmans conference held at the U of Rochester in 2009, Midwest Macro Meetings 2011, annual conference of the Canadian Economics Association 2011, Midwest Economics Association Annual Meeting 2013, NY/Philadelphia Workshop on Quantitative Macroeconomics 2013, Shanghai Macro Workshop 2014, European Meetings of the Econometric Society 2014, Yonsei U, Cleveland Fed, Seoul National U, Sogang U, Korea U, Sungkyunkwan U, and West Virginia U for their valuable comments and suggestions. Kyooho Kwon and Jinhee Woo provide excellent research assistance. This work was partially supported by Research Resettlement Fund for the new faculty of Seoul National University. All errors remain ours.
1 Introduction

Over the past four decades in the U.S., male hours worked of college graduates (skilled) relative to non-college graduates (unskilled) increased. This pattern is in line with recent literature on U.S. trends in hours worked by skill group. For example, based on five waves of American Time Use Surveys (ATUS), Aguiar and Hurst (2007) show that less-educated men reduced their time spent on total market work relative to those with a college degree or higher over 1965-2003. Additionally, Costa (2000) and Santos (2014) find a similar pattern from the Current Population Survey (CPS) using wage as a measure of skill.\(^1\) Given that the relative wages of skilled to unskilled men rose substantially during the period, the increased skilled-unskilled hours differential is in contrast with empirical evidence on wage elasticity of labor supply.\(^2\) In a recent survey, Saez et al. (2012) writes: “With some notable exceptions, the profession has settled on a value for this elasticity close to zero for prime-age males.”\(^3\)

This paper explores the changing patterns of second moment of wages, i.e. increased wage volatility, to resolve this discrepancy. As wages become more volatile under incomplete markets, individuals work longer hours as well as accumulate more precautionary savings for self-insurance. Flodén (2006) shows that an increase in future wage uncertainty raises current labor supply due to a precautionary motive in a simple two-period model. Pijoan-Mas (2006) also emphasizes the impact of wage volatility on labor supply decisions, by showing that both the level of precautionary savings and labor supply are higher in an economy with incomplete markets than in one with complete markets. If skilled men were exposed to greater increases in wage volatility than unskilled men, then this channel can potentially explain the observed rise in skilled-unskilled hours differential.

Many studies such as Gottschalk and Moffitt (1994), Gottschalk and Moffitt (2012), and Heathcote et al. (2010) find that volatility of residual wages and earnings in the U.S. has increased for the past few decades. In this paper, we take a step forward by estimating the time path of wage volatility separately for skilled and unskilled men using the data from the Panel Study of Income Dynamics (PSID) and document that there are marked differences in the trends of the second moment–as well as the first moment–of wages across skill groups for the same period. Our estimation shows that i) wage volatility (measured by the variance of residual wages) has risen for both skill groups between 1967 and 2000, consistent with the findings in the related literature and

\(^1\)Costa (2000) shows that the length of a work day by higher-wage earners has increased relative to that of lower-wage earners between 1973 and 1991 in the CPS. Santos (2014) confirms that this finding holds for the CPS data extended to 2006.

\(^2\)See, among others, Autor et al. (2008).

\(^3\)Estimating this elasticity is a challenging task, given that it is difficult to measure purely exogenous wage changes in the data, and it is often the case that workers are not able to freely adjust their work hours. Ashenfelter et al. (2010) exploit a panel data set of taxi drivers (considered to be free from the two problems stated above) and estimates a negative elasticity of male labor supply. They view that this finding suggests that the income effect is larger than the substitution effect in the long run. Given their result, the increase in skilled-unskilled hours differential is more puzzling. For a survey of the literature on wage elasticity of labor supply, see Keane (2011) and Saez et al. (2012).
ii) skilled men have experienced much larger increases in their wage volatility compared to the unskilled, which is a new finding.\textsuperscript{4} We also find that the rise in wage volatility for skilled men was largely due to persistent wage shocks, while unskilled men experienced increasing shares of transitory wage shocks in their residual wages for the period.\textsuperscript{5}

We ask whether the changes in wage volatility can account for the increased hours difference in spite of the rising skill premium. To this end, this paper develops a general equilibrium incomplete markets model with heterogeneous agents of different skill levels. Agents face uninsurable idiosyncratic productivity shocks, which consist of a persistent and a transitory component drawn from a skill-specific distribution. The initial steady state of the model is calibrated to the 1967 U.S. economy. We then feed in estimated wage processes from the PSID data to quantify how much the absolute and relative increase in wage volatility can explain widening gap between skilled and unskilled hours.

The model can replicate the increasing pattern of the skilled-unskilled hours differential in U.S. men observed from the data. In our model economy, workers increase their hours worked in the short run in response to higher wage volatility. This effect is much stronger for skilled workers because they are exposed to larger increases in wage volatility than the unskilled. However, as skilled men accumulate a buffer stock of precautionary savings over time, they can afford to reduce work hours. The wealth accumulation by skilled men also eventually decreases the equilibrium real interest rate, causing unskilled men to increase their hours while lessening savings. Consequently, the short-run increase in the skilled-unskilled hours differential is reversed in the long run. This implies that hours adjustment is important for self-insurance in the short run, whereas precautionary savings play a dominant role in the long run. We also find that the relative composition of persistent versus transitory wage shocks rather than the \textit{total} level of wage volatility is critical for individual labor supply decisions. Persistent wage shocks are crucial in explaining the observed trends in hours worked by skill type, whereas the quantitative impact of transitory wage shocks on the trends is fairly small.

This paper is closely related to Heathcote et al. (2010), who explore the implications of observed changes in the wage structure (including a rising skill premium, a declining gender gap, and the increasing overall residual wage dispersion) for cross-sectional inequality in hours, earnings, and consumption. Our work differs from theirs in that we focus on the differences in the evolution of wage volatility \textit{across different skill groups}. We explore the quantitative effects of the differences in increased wage volatility on the evolution of the relative hours worked of skilled to unskilled

\textsuperscript{4}We are aware that a few recent studies including Sabelhaus and Song (2010) use large administrative data sets to estimate earnings process and claim that earnings variability fell in recent decades. However, we follow the existing literature that exploits survey data to reach a conclusion that earnings (or wage) volatility has increased, and investigate the process further by skill group in this paper.

\textsuperscript{5}Although examining the causes of the rising wage volatility is outside the scope of this study, we view deunionization and increasing adoption of performance-pay contracts in the U.S. labor market as important factors behind the increases in wage volatility. Particularly, performance-pay wage contracts are likely to be responsible for a greater increase in wage volatility among skilled men than unskilled men, as Lemieux et al. (2009) suggest.
men. Another contribution of our study is that it attempts to distinguish the impact of total wage volatility on hours worked from that of the relative composition of persistent versus transitory shocks. The results show that the shock composition is quantitatively important in accounting for the evolution of hours by skill type.

We can also relate our paper to Erosa et al. (2014) and Castro and Coen-Pirani (2008) who examine differences in male labor supply by skill groups. However, Erosa et al. (2014) focus on their life-cycle features to study aggregate labor supply responses to changes in economic environment. Their main purpose is to quantify the aggregate elasticity of labor supply to wage changes and evaluate the importance of extensive versus intensive margins in this elasticity. Instead, our study aims to explain changing patterns of relative hours worked using differences in the evolution of wage volatility across skill type. Our work also differs from Castro and Coen-Pirani (2008) in that they explore demand-side factors to explain changes in business cycle fluctuations of hours by skill groups, whereas we consider supply-side factors to address the trends in hours by skill levels.

Several recent studies, including Santos (2014), Elsby and Shapiro (2012), and Michelacci and Pijoan-Mas (2008), have tried to explain phenomena related to our motivating facts using a different mechanism. These studies all explore the effects of current experience/hours on future labor market outcomes as potential explanations for the phenomena, although the details of the mechanism vary. Santos (2014) considers a model in which current work hours affect future productivity in order to explain the increase in the correlation between hours and wages for the last quarter of the 20th century in the U.S. He shows that in the CPS data, this dynamic effect has become stronger for higher wage quintiles, whereas it has weakened for lower wage quintiles, leading to the recent rise in the hours-wages correlation. Elsby and Shapiro (2012) focus on the extensive margin of labor supply and explore changes in the return to experience as driving forces behind the changing patterns of employment/nonemployment rates by education. They find that the return to experience for high-school dropouts has fallen significantly since the 1970s, which contributed to a downward trend in their employment rate. Michelacci and Pijoan-Mas (2008) exploit a search-matching framework to explain the divergence in hours worked between the U.S. and Continental Europe. They assume that current hours of work affect the future probability of getting outside job offers. In this framework, greater wage inequality makes outside offers more attractive, inducing agents to work longer hours. Relative to these studies, we propose an alternative mechanism based on a self-insurance motive. Our work shows that due to this motive, the evolution of the second moment of wages plays a more important role over the first moment in explaining the recent changes in U.S. male hours worked by skill groups.

The remainder of this paper is organized as follows. Section 2 describes the stylized facts on changes in the U.S. wage structure and the trends in hours worked by skill group. In section 3, we describe a general equilibrium model with heterogeneous agents and incomplete markets. We then describe the calibration procedure in section 4. Section 5 presents main quantitative results and

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6 Castro and Coen-Pirani (2008) attempt to explain increased cyclicality of skilled hours since the mid-1980s and propose changes in capital-skill complementarity as potential explanations.
Section 6 discusses several sensitivity analyses. Section 7 then concludes the paper.

2 Data

This section documents how the U.S. wage structure and hours worked have changed by skill type over the past few decades. Our data on the first moment of wages and hours worked are drawn from the Current Population Survey (CPS) March Supplements over the period 1967-2000. Since a panel dimension is required to estimate time-varying variances of persistent and transitory wage shocks, we also exploit the Panel Study of Income Dynamics (PSID). In order to avoid issues associated with changing selection into labor force participation among women, we restrict our attention to men. Detailed sample selection criteria are provided in Appendix A-1.

2.1 Changes in the Wage Structure

We measure skill \( e \) based on one’s educational attainment: define a skilled worker \( s \) as one with a college degree or higher and an unskilled worker \( u \) as one without a college degree. We assume that an individual wage follows a skill-specific process that depends on a time-varying skill price per efficiency unit of labor, years of experience, and a history of persistent and transitory labor productivity shocks. Specifically, the hourly wage of individual \( i \), aged \( a \) at time \( t \) and skill type \( e \in \{ s, u \} \) is assumed to follow

\[
\ln w_{eit} = \beta_e^i + f_e^i(X_{it}) + y_{eit},
\]

where \( \beta_e^i \) is a skill-specific time dummy, \( f_e^i \) is a skill-specific return to experience (assumed to be a cubic polynomial of \( X_{it} \), \( X_{it} \equiv a_{it} - S_i - 5 \) is potential experience with \( S_i \) representing years of schooling, and \( y_{eit} \) is the log wage residual. We assume that the log wage residual \( y_{eit} \) consists of a persistent and a transitory component:

\[
y_{eit} = \mu_{eit}^c + v_{eit}^c + \theta_{eit}^c,
\]

where \( \mu_{eit}^c \) is a persistent component, \( v_{eit}^c \sim (0, \lambda_{eit}^c) \) is a transitory component, and \( \theta_{eit}^c \sim (0, \lambda^\theta) \) is measurement error. The variance \( \lambda_{eit}^c \) of the transitory component \( v_{eit}^c \) is allowed to vary over time, whereas the measurement error’s variance is time-invariant and independent of skill type. The persistent component \( \mu_{eit}^c \) is modelled as an AR(1) process:

\[
\mu_{eit}^c = \rho^c \mu_{eit-1}^c + \eta_{eit}^c,
\]

where \( \rho^c \) is the persistence and \( \eta_{eit}^c \sim (0, \lambda_{eit}^\eta) \) is the persistent wage shock that has a time-varying variance of \( \lambda_{eit}^\eta \). The initial value of persistent component is drawn from a time-invariant, skill-specific distribution: \( \mu_0 \sim (0, \lambda_{eit}^{\mu}) \). We assume that all four variables, \( v_{eit}^c, \theta_{eit}^c, \eta_{eit}^c, \) and \( \mu_0^c \) are orthogonal and i.i.d. across individuals. We focus on time effects in this specification because empirical evidence suggests that time effects rather than cohort effects have been important in explaining rising wage inequality in the U.S. for recent decades (see Heathcote et al. (2005)).
With this statistical model, we estimate the wage processes of skilled and unskilled workers separately in two stages. In the first stage, we run an Ordinary Least Squares (OLS) regression of log hourly wages on a time dummy and a cubic polynomial of potential experience by skill type, as shown in Equation (1). The residuals from these first-stage OLS regressions are exploited to estimate the parameters governing persistent and transitory wage shocks. Since a transitory shock in wages disappears after one period, autocovariances of individual log wage residuals depend solely on the persistent component, which help us to identify a persistent component from a transitory one.\(^7\) In implementing the estimation procedure, we compute sample autocovariances of the residual \(y_{et}^c\) from the first-stage regressions of all possible orders for each age group in every survey year. According to the specifications above, the autocovariances of log residual wages can be written as functions of model parameters including \(\rho_e^c, v_{et}^c, \theta_{et}^c, \eta_{et}^c,\) and \(\mu_0^c\). The parameters are then estimated by minimizing the equally weighted distance between the sample autocovariances and the model counterparts.\(^8\) To disentangle a transitory component from a measurement error, we use an estimate of 0.02 for the measurement error taken from French (2004).

We define the log skill premium as the differential in the mean predicted log wages between skilled and unskilled men:
\[
\ln(\text{skill premium}_t) = \ln \widehat{w}_t^s - \ln \widehat{w}_t^u,
\]
where \(\ln \widehat{w}_t^c\) denotes the average of individual predicted log wages in period \(t\) from the first-stage regressions for skill type \(e\). Figure 1 plots the trends in the male skill premium over 1967-2000, using data from CPS and PSID. Both data show a drop in the skill premium during the 1970s and a sharp rise since the late 1970s, as is consistent with the literature.

Figure 2 presents the time series of the variances of persistent and transitory wage shocks of each skill type, estimated in the second stage. The estimated variance \((\lambda_t^{s,e})\) of a persistent wage shock for skilled men rose substantially for the sample period. In particular, they experienced sharp rises in the variance of their persistent wage component during the 1980s and 1990s. Around 2000, the variance of skilled men’s persistent wage component reached a level greater than 6 times that in 1967. Alternatively, the estimated variance \((\lambda_t^{u,e})\) of a persistent wage shock for unskilled men has doubled between 1967 and 2000. The variance rose mostly during the 1970s and dropped sharply during the latter half of the 1980s.\(^9\)

\(^7\)A large literature in labor and macroeconomics has been devoted to estimating labor income processes. As Krueger et al. (2010) summarize, these studies often found that the estimates of persistent and transitory variances, based on the covariances of the levels of log earnings, are significantly different from those based on their first differences. A recent paper by Daly et al. (2014) attempts to reconcile these different estimates and finds that “outlying” earnings observations either prior to or following a missing observation are misspecified in standard labor income processes used in this literature. However, they find that without dropping such “outlying” observations, one can use the estimates for persistent variances based on the levels of log earnings because the biases are quantitatively small. In section 5, we show that a time-varying persistent wage component drives most of important quantitative results in our model while the impacts of a transitory wage component is quantitatively small.

\(^8\)Details of the estimation procedure are provided in Appendix A-3.

\(^9\)Blundell et al. (2008) estimate time-varying variances of permanent income shocks during the 1980s using the
The estimated variances \((\lambda_{e,u,t}^{e,u})\) of a transitory wage shock also exhibited different patterns by skill groups. Skilled men experienced a mild increase in the variance of a transitory wage shock during the 1980s and faced a sudden increase in its variance starting in the late 1990s. Conversely, the bottom right panel of Figure 2 indicates that the variance of a transitory wage shock of unskilled men increased at a constant pace over the measured period.

These changes in the variances of persistent and transitory wage shocks ultimately raised the volatility of residual wages faced by both skill groups. Figure 3 reports the variance decomposition of log wage residuals by skill type, based on the estimated parameters. In order to show the trend more easily, we normalize the total residual wage variance of skilled men to 1 in 1967. The left panel shows that the total variance of residual wages of skilled men more than doubled over the sample period. The rise in wage volatility occurred mostly during the 1980s and 1990s, mainly due to a rise in volatility of the persistent component. Over the same period, unskilled men also experienced a large increase in the variance of their residual wages. Their total residual wage variance reached 70% higher by 2000 but to a level lower than skilled men. Unlike skilled men, most of the increase in unskilled men’s wage volatility occurred between 1967 and the early 1980s. Comparing both skill groups, we conclude that skilled men experienced a much larger increase in their residual wage volatility (measured as the estimated variance of log wage residuals) for the sample period than unskilled men.

Over the sample period, the composition of a persistent and a transitory wage component has changed substantially by skill type as shown in Figure 4. The trend in the share of the persistent wage component of skilled men was not monotonic: it decreased during the 1970s, but rose sharply during the 1980s and stayed roughly constant at about 0.75 during the 1990s. On the other hand, the persistent component of unskilled men steadily declined as a fraction of wage volatility over the sample period. It was 0.78 in the late 1960s and dropped by 15% point over the sample period. These patterns imply that unskilled men experienced their rise in residual wage volatility through a rising share of the transitory wage component during the sample period. In contrast, skilled men went through a substantial increase in residual wage volatility during the 1980s largely due to the accumulated effects of a rising variance in persistent wage shocks. In summary, compared to unskilled men, skilled men experienced a greater increase in total wage volatility, with a larger fraction of the rise due to the persistent wage component.

One possible explanation behind the greater rise in wage volatility for skilled men than unskilled men is that the increasing share of college graduates caused more recent college graduates to be drawn from the bottom of the ability distribution.\(^{10}\) In this case, the cross-sectional variation of residual wages may widen without any changes in wage volatility faced by individuals. However, we argue that the estimation results are not mainly driven by these changes in labor composition by education. Firstly, the estimation exploits a panel dimension to identify time-series variations.

\[^{10}\text{According to the CPS data, the share of college graduates was about 15% in 1967 and rose to 31% in 2006.}\]
of wage volatility from the cross-sectional variation of wages. Secondly, the fraction of college graduates increased most rapidly during the 1970s and grew only slowly thereafter, whereas wage volatility among skilled men changed little during the 1970s and rose sharply since the early 1980s.\footnote{Carneiro and Lee (2011) examine the effect of distributional changes in the context of the skill premium. Specifically, they computed the time path of skill premium adjusted for changes in quality of new college graduates and compare it with the raw skill premium. They found that the distance between the two increased the most during the 1970s when the share of college graduates rose most rapidly.}

We also examine a possibility that wage volatility appears to have risen due to increased unemployment risks. Considering that wages tend to decline after a spell of unemployment, frequent unemployment spells may increase variability of observed wages. In the PSID data, we did not find any distinct trends in the transition probabilities from employment to unemployment for either skill group, particularly during the 1980s and 1990s. Thus, the estimated rises in residual wage volatility for both skill groups are unlikely due to changes in unemployment risks.

To summarize, there have been substantial changes in the U.S. wage structure between 1967 and 2000. It is well known that the skilled premium rose sharply, in favor of skilled men. On top of that, wage volatility also increased by different magnitudes across skill groups. Our empirical analysis concludes that skilled men were exposed to a greater rise in wage volatility than unskilled men, with a larger fraction of the rise attributable to persistent wage shocks. These changes are summarized in Figure 5, which presents the linear prediction of each time series.

### 2.2 Trends in Hours Worked

In this section, we document the trends in hours worked of U.S. men by skill group using CPS March Supplements. In particular, we focus on changes in labor supply at the intensive margin. We make this choice because an increase in wage volatility causes employed workers to raise their work hours as an insurance mechanism, whereas it is unlikely to affect the extensive margin of labor supply. Agents decide whether to participate in the labor market by comparing the costs and benefits of employment. An increase in wage volatility reduces the expected value of employment, while providing agents who lack a buffer stock of savings with a stronger incentive to work for an insurance. These offsetting effects appear to mitigate the effect of rising wage volatility on the extensive margin of labor supply. Moreover, the participation margin is closely related to unemployment shocks and retirement, which are outside the scope of this paper.\footnote{There are many previous studies on changes in the extensive margin of labor supply in the U.S. (see Juhn and Potter (2006), Juhn (1992), Juhn et al. (2002), and Elsby and Shapiro (2012)). These studies document that nonemployment rate of U.S. men fell significantly over the past few decades with it concentrated among less-skilled men. They also find that the early retirement rate has increased and unemployment duration has become longer. Suggested explanations for the phenomena include a reduction in real wages of less-skilled men, an expansion of disability benefits program, and a decline in return to experience.}

The left panel of Figure 6 plots the trends in the average weekly hours (annual hours worked divided by 52 weeks) of employed men by skill type over the 1967-2000 period. The average weekly
hours of both skilled and unskilled men were steady during the 1970s and began to increase in the early 1980, the trend of which continued until 2000. The right panel presents the differences in the average weekly hours between skilled and unskilled men over the same period. In the beginning of the sample period, skilled men on average worked about 2 hours more than unskilled men. The hours differential rose to about 3.5 hours in the mid 1990s and declined afterwards. Most of the rise in skilled-unskilled hours differential occurred during the 1980s and the 1990s while the skill premium continued to increase rapidly.\footnote{\textsuperscript{13}}

Assuming that age effects are time-invariant, the rise in the relative hours of skilled to unskilled men may be due to either time effects (skilled-unskilled hours differential has increased over time within every age group) or cohort effect (hours gap between skilled and unskilled men is larger among younger cohorts than older cohorts). We claim that time effects rather than cohort effects have been more important in the rise in the skilled-unskilled hours differential, as is the case for increased wage volatility. To illustrate this point, consider a model in which the skilled-unskilled hours differential ($hd$) depends on age ($a$), time ($t$), and cohort ($c = t - a$), where these three components are additively separable:

$$hd(a, t, t-a) \equiv g_1(a) + g_2(t) + g_3(t-a).$$

As in Heathcote et al. (2005), we compute changes in the hours differential within cohort, within age, and between age groups to identify time and cohort effects. If we track changes in the hours differential of the same cohort $c = t - a$ between $t$ and $t+1$, the change excludes cohort effects and captures age and time effects:

$$\Delta hd^c_{t,t+1} = g_1(a+1) - g_1(a) + g_2(t+1) - g_2(t) = \Delta g_1(a) + \Delta g_2(t).$$

The average of within-cohort changes in the hours differential across different cohorts is then given by $\Delta hd^C_{t,t+1} = \Delta g_1(a) + \Delta g_2(t)$. If the average varies by subperiods, it is attributable to time effects because age effects are time-invariant.

Instead, following the hours gap between skilled and unskilled men within the same age group $a$ between $t$ and $t+1$ sorts out age effects and reflects the sum of time and cohort effects:

$$\Delta hd^a_{t,t+1} = g_2(t+1) - g_2(t) + g_3(t+1-a) - g_3(t-a) = \Delta g_2(t) + \Delta g_3(t-a).$$

Note that the correlation between within-cohort changes and within-age changes is mainly determined by the strength of time effects. Taking the average of within-age changes across different age groups yields $\Delta hd^a_{t,t+1} = \Delta g_2(t) + \Delta g_3(t-a)$.

Lastly, variations in the skilled-unskilled hours differential between ages $a$ and $a+1$ in period $t+1$ indicate the difference between age and cohort effects, while removing time effects:

$$\Delta hd^{a+1}_{a,a+1} = g_1(a+1) - g_1(a) - (g_3(t+1-a) - g_3(t-a)) = \Delta g_1(a) - \Delta g_3(t-a).$$

\textsuperscript{13}Male work hours vary by age: prime-age male work more than the young and the old. However, the trends in hours by skill type are not mainly driven by changing age composition. Holding the age distribution of the sample the same as that in 1967 changes the trends (based on the changing age distribution) little.
The average between-age group change is given by 
\[ \Delta h_{d+1}^{t+1} = \Delta g_1(a) - \Delta g_3(t-a). \] Any variations in the average between-age group changes across different subperiods can then be ascribed to time-varying cohort effects.

Table 1 presents the average annual changes in the skilled-unskilled hours differential within cohort, within age, and between age groups (within-period) for various subperiods. The results show that within-cohort changes in the relative labor supply are statistically different across subperiods. However, between-age groups variations are not statistically different from zero except for the first decade in the sample. Given that the age effects are time-invariant, whereas both time and cohort effects are time-varying, the results imply that time effects are strong and cohort effects are not.

Figure 7 also confirms the same point. The left panel presents within-cohort variations in hours differential across age groups, the slopes of which represent both age and time effects. If there are little time effects, we should observe age-hours differential profiles with the same slope for every cohort, although the level of the hours differential may vary by cohort. However, the figure displays much steeper age-hours differential profiles between ages 25 and 45 for earlier cohorts than for more recent cohorts, which is attributable to strong time effects.

In addition, since both within-cohort and within-age changes in the skilled-unskilled hours differential have time effects in common, the correlation between the two should be strong if the time effects are strong. The correlation is very close to one as shown in the bottom of Table 1. To the contrary, the correlation between within-age and between-age variations is 0.21, implying that cohort effects have been weak. These results suggest that time effects rather than cohort effect have been more important in the increasing trends of the skilled-unskilled hours differential.

Do the time effects differ by age? The right panel of Figure 7 presents the time series of skilled-unskilled hours differential for various age groups. Taking the dominant time effects described above, changes in the skilled-unskilled hours differential over time within each age group are mainly attributable to time effects. The figure suggests that the time effects do not vary much by age. The hours differential stayed roughly constant during the 1970s and increased during the 1980s and 1990s across all age groups. In light of this, our study abstracts from life-cycle features of hours and focuses on how hours worked by skill level change over time in response to rising wage volatility.

3 The Model

To understand how our empirical findings are connected, we develop a general equilibrium heterogeneous agent model with incomplete markets. In the model, we consider two skill types of workers that face idiosyncratic productivity shock drawn from skill-specific distributions.
3.1 Households

There are two types of single-person households—skilled and unskilled—in this economy. We denote the skill type by $e$ where $e \in \{s, u\}$ and the population share of skilled households is given by $\pi$.\textsuperscript{14} Households are endowed with one unit of time in each period. Each household faces a skill-specific process for his idiosyncratic productivity. In the beginning of a period, a household of skill type $e$ draws persistent and transitory components ($\mu^e$ and $\nu^e$, respectively) of his productivity, $x^e$, from his type-specific distribution. After observing his productivity draw, the household makes his labor supply decision. He provides $hx^e$ efficiency units of labor and gets paid $w^e$ per efficiency unit of labor, where $h$ is hours worked chosen by this household. There is a minimum requirement for the number of hours such that each household should work at least $\tilde{h}$.

In this economy, only a risk-free asset is available for saving and no borrowing is allowed, which make markets incomplete. Households insure themselves against low productivity shocks through both precautionary savings and labor supply decisions.

At the end of the period, households face a survival probability of $\gamma$: a fixed fraction $\gamma$ of the population survives to the next period and the remainder is replaced by newborn households. We make this choice because it allows the model to generate reasonable predictions on hours and saving without an explicit life-cycle, which we exclude to highlight the time effects in hours. Out of these newborn households, a $\tilde{\pi}$ fraction is assumed to be born as skilled, thus the evolution of total number of skilled workers is given by $\pi' = \gamma \pi + (1 - \gamma) \tilde{\pi}$. Unclaimed assets of deceased households are collected and redistributed to all households in a lump-sum transfer, $T$.

Each household maximizes the expected lifetime utility from a stream of consumption $c_t$ and non-working hours or leisure $1-h_t$:

$$
\mathbb{E} \sum_{t=0}^{\infty} (\beta \gamma)^t u(c_t, h_t),
$$

$$
u(c, h) = \frac{c^{1-\sigma}}{1 - \sigma} + \psi^e \frac{(1 - h)^{1-\nu}}{1 - \nu} \text{ and } \beta, \gamma \in (0, 1),
$$

where $\beta$ is the discount factor, $\psi^e$ is a skill-specific constant, $\sigma$ is the inverse of elasticity of intertemporal substitution with respect to consumption, and $\nu$ is the same but with respect to leisure. Consumption and leisure are assumed to be additively separable in a constant relative risk aversion (CRRA) utility function, which is common for both skill types. In period $t$, households choose consumption $c_t$, hours of work $h_t$, and asset holdings $a_{t+1}$ for the next period subject to

$$
c_t + a_{t+1} \leq a_t(1 + r_t) + w^e_t h_t x^e_t + T_t,
$$

$$
c_t \geq 0, a_{t+1} \geq 0, h_t \in [h, \tilde{h}],
$$

\textsuperscript{14}We abstract from educational choice by households to focus on differences in labor supply by skill level. According to the CPS March supplements, the fraction of individuals with a college degree has increased rapidly until 1970s and the increase has slowed down afterwards. The extent that this change in acquiring a college education affects labor supply decisions is unaccounted for in our analysis, thus, our quantitative results may be biased.
where \( r_t \) is real interest rate on the risk-free asset and \( w^e_t \) is real wage per efficiency unit of labor for skill type \( e \). In order to incorporate a skill premium into the model, we consider two different real wages by skill types. The logarithm of the idiosyncratic productivity \( x^e_t \) of skill type \( e \) is specified as the sum of a persistent and a purely transitory component (\( \mu^e_t \) and \( \nu^e_t \), respectively), given by

\[
\ln x^e_t = \mu^e_t + \nu^e_t,
\]

where \( \nu^e_t \sim N(0, \lambda^e_{\nu}) \). The persistent component is modelled as an AR(1) process:

\[
\mu^e_t = \rho^e \mu^e_{t-1} + \eta^e_t,
\]

where \( \eta^e_t \sim N(0, \lambda^e_{\eta}) \). These specifications allow for differences by skill types in the persistence of productivity shocks and in the variances of both persistent and transitory components. The variances of both persistent and transitory productivity shocks are time-varying to capture changing patterns in wage volatility seen from the PSID data. We assume that a household born at period \( \tau \) starts his lifespan with a persistent productivity shock drawn from a type-specific and time-invariant initial distribution, defined as \( \mu^e_\tau \sim N(0, \lambda^e_{\mu}) \). The recursive formulation of a household problem of skill type \( e \) is then given by:

\[
V^e(a, \mu^e, \nu^e) = \max_{c \geq 0, a' \geq 0, h \in \mathcal{H}} \left\{ u(c, h) + \beta \gamma \mathbb{E} V^e(a', \mu^{e'}, \nu^{e'}) \right\},
\]

subject to

\[
\begin{align*}
    c + a' & \leq a(1 + r) + w^e_h x^e + T, \\
    \ln x^e & = \mu^e + \nu^e, \\
    \nu^e & \sim N(0, \lambda^e_{\nu}), \\
    \mu^{e'} & = \rho^e \mu^e + \eta^e, \quad \eta^e \sim N(0, \lambda^e_{\eta}), \\
    \mathcal{H} & = [h, \bar{h}].
\end{align*}
\]

### 3.2 Production

We consider a representative firm that produces output using a standard Cobb-Douglas technology with competitive factor markets. The firm’s production function is given by

\[
F(K_t, H_t, z_t) = z_t K_t^\alpha H_t^{1-\alpha},
\]

where \( K_t \) is aggregate capital and \( H_t \) is aggregate labor input. The aggregate labor \( H_t \) is a CES aggregator of efficiency hours worked by skilled \( S_t \) and unskilled \( U_t \) households:

\[
H_t = \left\{ \chi_t U_t^\phi + (1 - \chi_t) S_t^\phi \right\}^{\frac{1}{\phi}}, -\infty < \phi \leq 1,
\]

where \( \frac{1}{1-\phi} \) denotes the elasticity of substitution between the two types of labor inputs. The parameter \( \chi_t \) is introduced to capture skill-biased technical changes over time. A decrease in \( \chi_t \) makes
efficiency units of hours worked by skilled workers more productive relative to those of unskilled workers, leading to a rise in the skill premium.

The assumption of competitive factor markets imply
\[ r_t + \delta = F_K(K_t, H_t, z_t) = z_t \alpha (K_t/H_t)^{\alpha-1}, \]
\[ w^s_t = F_S(K_t, H_t, z_t) = z_t (1-\alpha)(1-\chi_t)(K_t/H_t)^{\alpha} H_t^{1-\phi} S_t^{\phi-1}, \]
\[ w^u_t = F_U(K_t, H_t, z_t) = z_t (1-\alpha) \chi_t (K_t/H_t)^{\alpha} H_t^{1-\phi} U_t^{\phi-1}, \]
where \( \delta \) is the depreciation rate of physical capital. Therefore, the skill premium \( \omega \) is given by:
\[ \omega_t \equiv \frac{w^s_t}{w^u_t} = \frac{1-\chi_t}{\chi_t} \left( \frac{S_t}{U_t} \right)^{\phi-1}. \]

### 3.3 Competitive Equilibrium

A competitive equilibrium consists of individuals’ optimal policies, aggregate inputs, prices, and a law of motion for the distribution \( G_{t+1} = T_t(G_t) \) such that:

i) Given \( \{w^s_t, w^u_t, r_t\}_{t=0}^{\infty} \), households of skill type \( e \) optimally choose \( c_t(e, a_t, \mu_t, v_t; G_t) \), \( a_{t+1}(e, a_t, \mu_t, v_t; G_t) \), and \( h_t(e, a_t, \mu_t, v_t; G_t) \) that are consistent with households problem specified above,

ii) The firm chooses \( K_t, S_t, \) and \( U_t \) to maximize profits,

iii) The labor market for each skill type clears:
\[ S_t = \int h_t(s, a_t, \mu_t, v_t; G_t) \exp(\mu_t + v_t) \, dG_t(s, a_t, \mu_t, v_t) \]
\[ U_t = \int h_t(u, a_t, \mu_t, v_t; G_t) \exp(\mu_t + v_t) \, dG_t(u, a_t, \mu_t, v_t) \]

iv) The capital market clears:
\[ K_t = \sum_e \int a_t \, dG_t(e, a_t, \mu_t, v_t) \]

v) The goods market clears:
\[ \sum_e \int c_t(e, a_t, \mu_t, v_t; G_t) \, dG_t(e, a_t, \mu_t, v_t) = F(K_t, H_t, z_t) + (1-\delta)K_t - K_{t+1} \]

vi) Individual and aggregate behaviors are consistent.

### 4 Calibration

We select a set of parameters using standard values used in related literature. The model period is one year. We set the preference parameter \( \sigma \) to 1.5, based on the common estimates for risk aversion
between 1 and 2 in the literature (see Attanasio (1999)). The parameter $\nu$ is set to match the Frisch elasticity of hours of 0.4, consistent with most micro estimates. Following Aiyagari (1994) and Prescott (1986), we assign 0.36 and 0.08 to the capital share $\alpha$ in the production function and the depreciation rate $\delta$ of capital. There is a large literature on estimating the elasticity of substitution between skilled and unskilled workers and most estimates fall between 1.4 and 2. We use 1.5 as the elasticity of substitution between skilled and unskilled men by setting the parameter $\phi$ to $1/3$. The maximum hours of work $\bar{h}$ is set at 5840 hours per year (16 hours per day times 365 days). The minimum hours of work $h$ is set to match 800 hours per year, which was used as a cutoff for a moderate attachment to the labor force in Prescott et al. (2009).

Another set of time-invariant parameters is calibrated so that the model can replicate target data moments in 1967, which we assume to be the initial steady state. The discount factor, $\beta$, is picked so that the equilibrium real interest rate in the initial steady state matches an annual real interest rate of 0.04. The parameters $\psi^s$ and $\psi^u$ in the utility function are chosen to match the average weekly hours worked of skilled and unskilled men in the initial steady state, which are 43.5 and 41.3 hours, respectively. We set the survival probability $\gamma$ to equal 0.972 so that the average life span of workers is 35 years.

The last set of parameters are allowed to vary over time. This includes the parameter $\chi_t$ governing the skill-biased technical change, which is chosen to match the linear prediction of the skill premium $\omega_t$ in the CPS data over the sample period, which is shown in the first panel of Figure 5. We also calibrate the total factor productivity $z_t$ by normalizing output $y_t$ to 1 in every period. In regards to the parameters that determine the two components of idiosyncratic productivity, $x_t^e$, we take a linear trend in each phase for our point estimates from the PSID data, which are presented in the last two panels of Figure 5. The population share $\pi_t$ of skilled men is taken from the CPS. Table 2 summarizes the values of parameters used for our model economy.

---

15This value for the relative risk aversion implies that the intertemporal elasticity of subsitution is less than 1 (i.e., the income effect is greater than the substitution effect). In section 6, we implement a sensitivity analysis by varying the parameter $\sigma$ and find that our quantitative results are still valid with $\sigma = 1$, which is consistent with micro evidence suggesting the long-run wage elasticity of labor supply close to zero.

16According to Macurdy (1981), most micro estimates for the elasticity range between 0.1 and 0.5.

17Katz and Murphy (1992) find that this elasticity is about 1.4 based on CPS March Supplements for the period 1967-1973. Autor et al. (2008) show that the elasticity is estimated to be about 1.6 using the CPS data extended to 2005. Heckman et al. (1998) and Card and Lemieux (2001) obtain the estimates of 1.5 and 2, respectively.

18We make this choice to isolate the direct effect of rising wage volatility on hours from the level effect of larger output caused by increases in factor inputs. As an alternative, we solve the model by normalizing the TFP $z_t$ to 1 in every period and find that the short-run quantitative results differ little. In the long-run, there are some level effects at work, yet the long-run skilled-unskilled hours differential is the same as the benchmark outcome. These results are available upon request.
5 Results

In this section, we present our main results. The economy is initially at steady state in 1967. We then assume that the economy experiences unexpected changes in the variances of wage shocks as well as in the skill premium between 1967 and 2000. Specifically, in the beginning of 1967, workers in the economy receive a new set of complete information that the first and second moments of their wage process (i.e. the skill premium and wage volatility) are gradually changing until 2000. After 2000, the economy gradually converges to a new steady state.

This section begins with the initial steady state results. It then follows the transitional dynamics of hours worked and precautionary savings by skill group in response to the changing wage structure.\(^\text{19}\) We also examine the main mechanism through which the changes in the wage structure affect the evolution of hours.

5.1 Initial Steady State

We solve for the initial steady state allocations of the model economy by targeting the U.S. wage structure in 1967. The parameter for the skill-biased technology, \(\chi\), is chosen to match the U.S. skill premium in 1967. The variances of persistent and transitory wage shocks for each skill type are their point estimates in 1967. Lastly, the total factor productivity is set to normalize the aggregate output to 1 with the fraction of skilled men fixed at 0.1541. Table 3 reports these values.

The initial steady state allocations are summarized in Table 4. Skilled men face a 47% higher real wage per efficiency unit of labor and work 5% longer hours than unskilled men. In combination, skilled men on average earn 57% more wage income than unskilled men.

Skilled men also face larger volatility in their residual wages than unskilled men in the initial steady state. The unconditional variance of the persistent wage component \(\lambda_\eta/(1 - \rho^2)\) for skilled men is 21% larger than that for unskilled men mostly because the persistence parameter is larger for skilled men. The volatility of transitory wage shocks \(\lambda_\upsilon\) is also slightly larger for skilled men than for unskilled men. With these differences, skilled men holds 20% more assets and enjoy 48% more consumption than unskilled men in the initial steady state.

5.2 Transitional Dynamics

5.2.1 Hours Worked

Our main exercise is to find transition dynamics of the economy in response to the changes in the wage structure. In our benchmark economy, there are three gradual changes in the wage structure from 1967 to 2000: i) the skill premium; ii) the variances of persistent and transitory wage shocks, and iii) the skill composition in the labor force.\(^\text{20}\) We assume that households have perfect foresight

\(^{19}\)The computation algorithm to solve the model is described in Appendix A-4.

\(^{20}\)Changes in the skill premium comes from changes in skill biased technology \(\chi\) and changes in the skill composition \(\pi\). Changes in the skill composition also has an indirect effect on the relative wages through the general equilibrium
about the changes in the wages and the skill composition. After 2000, the wage structure stays unchanged and the economy gradually converges to a new steady state. Figure 8 presents how hours by skill group evolve along the transition to the new steady state in our benchmark model. Due to the lack of business cycle features that affect both skill groups evenly, the levels of hours generated by the model do not mimic those in the data. However, taking the difference in hours between the two skill groups eliminates the business cycle component, justifying a comparison of the evolution of the skilled-unskilled hours differential in both the model and data.

While the wage structure changes, skilled men continue to increase their hours worked with the most rapid rise occurring in the 1980s. Given that the relative wages of skilled men continued to increase for most of the sample period, the rise in hours worked by skilled men is attributable to the increasing wage volatility. The variance of the persistent wage component seems to play an especially important role in the evolution of hours because both skilled men’s hours and the variance of their persistent wage shocks increased most rapidly during the 1980s. In 2000, skilled men work about 2 more hours per week than in 1967. However, the upward trend in skilled men’s hours is reversed starting in 2000 when wage volatility stops increasing. As skilled men accumulate a buffer stock of precautionary savings over time, they gradually reduce their work hours thanks to wealth effects. In the new steady state, their weekly hours of work is smaller than that in the initial steady state.

Unskilled men also increase their hours worked for the initial decade when they experience a rise in the variance of persistent wage shocks. As unskilled men fully anticipate that the skill premium will rise after the late 1970s, they also have strong incentives to increase their work hours for the initial decade during which they gain from the temporary drop in the skill premium. In the late 1970s, however, they begin to reduce weekly hours as the share of transitory wage shocks in total wage volatility rises and that of persistent wage shocks declines steadily. This pattern of changing labor supply hints that the relative composition of persistent and transitory wage components in residual wages may have significant effects on individual labor supply decisions. In contrast to their short-run behavior, unskilled men increase their hours worked in the long run. The substantial amount of savings accumulated by skilled men lowers the real interest rate, discouraging unskilled men to save. This general equilibrium effect causes unskilled men to reduce their savings while raising their work hours, compared to the initial steady state.

As skilled men increase their work hours more than unskilled men in the short-run, the model predicts that the difference in weekly hours between skilled and unskilled men rises for the period. Figure 9 shows that this model prediction is qualitatively consistent with its data counterpart. However, the model slightly overstates the rise in the hours differential. This overstatement may be attributable to some missing factors that strengthen income effects for skilled men such as an increasing fraction of skilled men with working spouses. In recent decades, labor force participation rates have risen more markedly among highly-educated women while the correlation between husband’s and wife’s education levels has become stronger (see Greenwood et al. (2014)). It is then
conceivable that a larger fraction of skilled men can afford a reduction in hours thanks to working spouses compared to a few decades ago. Our study abstracts from this channel, potentially overstating the short-run rise in skilled men’s hours worked and therefore the skilled-unskilled hours differential.

Ultimately, the rise in the skilled-unskilled hours differential is a short-run phenomenon. In the long run, precautionary savings play an important role in the evolution of hours worked. As skilled men reduce their hours worked and unskilled men move in the other direction, the hours difference between the two skill groups converges to an even lower level in the new steady state than in the initial steady state.

5.2.2 Precautionary Savings

Aiyagari (1994) established that households facing uninsurable labor productivity shocks accumulate a buffer stock of precautionary savings. Figure 10 shows skilled men accumulate a substantial amount of wealth in the long run in response to rising wage volatility, but this path is not monotone. Skilled men reduce their wealth initially from the drop in the skill premium. This initial dissaving by skilled men also appears to be caused by gains from anticipated rises in the future skill premium. As a result, skilled men’s wealth in the mid 1980s is about 40% lower than that in the initial steady state. Skilled men’s savings pattern is reversed around the mid 1980s when they begin to experience rapid increases in the volatility of their residual wages. From there, they continue to increase their wealth which decreases the real interest rate. During the transition, the composition of skilled workers has monotonically increased from 15.4% in 1967 to 30.4% in 2000. The increase in skilled wealth accumulation then has much stronger effect on the real interest rate over time.

Unskilled men gradually increase their wealth over the first couple of decades. Increased wage volatility causes unskilled men to work more and build a buffer stock of savings. For the same period, skilled men also increase their hours of work. The resulting increase in total labor supply raises the rate of return to capital stock, and the real interest rate rises from 4% in the initial steady state to 5.6% in the late 1980s. However, as the real interest rate declines after the 1980s from wealth accumulation by skilled men, unskilled men are discouraged to save, reducing their wealth until they reach the new steady state.

In order to explore how consistent the model implications on wealth are with their data counterparts, we look at the Survey of Consumer Finances (SCF) from 1983 through 2007. We restrict our sample to households with a working male head (excluding those self-employed) and divide the sample into two skill groups based on the household head’s education. Figure 11 presents the wealth-to-labor income ratios by skill type in the model and the data equivalent over the sample period. Targeting the levels of the wealth-to-labor income ratio by each skill group is outside the scope of our study. Instead, we evaluate our model with a focus on the growth in the wealth-to-labor income ratio. To this aim, we normalize the wealth-to-labor income ratio in 1983 of each skill group to 1 in both the model and data. We find that the evolution of wealth by skill type predicted by
the model is broadly consistent with what we observe in the data. In the data, the wealth-to-labor income ratio of both skill type households increased since the mid 1990s, but it rose more rapidly among households with a skilled head. Since the wage structure stays constant after 2000 in the model, we compare the growth rate of the wealth-to-labor income ratio of both skill types between 1983 and 2001 from the data with that from the model. In the data, the wealth-to-labor income ratio of skilled and unskilled households grew at rates 1.02% and 0.28% per annum, respectively, between 1983 and 2001, which are in line with their model counterparts of 0.90% and 0.35% per annum.

5.3 Understanding the Mechanism

5.3.1 Short-Run vs. Long-Run

The benchmark results show that in response to the changing wage structure, the hours difference between skilled and unskilled men increases in the short run. In this section, we decompose the forces behind the short-run increase in the skilled-unskilled hours differential.

Consider a one-time permanent change in the wage structure in Table 5 that corresponds to actual changes in wages experienced by both skill groups from 1967 to 2000. We investigate how workers respond in the short run to the change in the wage structure in four steps: (i) SP: workers experience the rise in skill premium only; (ii) SR1: workers are permanently exposed to increased variances of both persistent and transitory wage shocks with no other changes in the wage structure; (iii) SR2: workers anticipate the changes in the variances of persistent and transitory wage shocks in the next period with no current changes. Workers, however, do not anticipate the skill premium to change.; and (iv) SR: workers face the new wage structure in the current period. Both the first and the second moment of wages have changed. One can view SR as combining SP with SR1. For all of above four exercises, we hold market prices (the real interest rate and the price of aggregate labor $H$) constant to isolate the contemporaneous effect of changes in the wage structure from (future) endogenous changes in savings and the general equilibrium effect. To contrast these short-run responses with the long-run outcome, we also present a new steady state (SS2) of the model economy with the wage structure of 2000, where the wealth distribution and prices are adjusted accordingly.

Table 6 shows that in response to the rise in the skill premium (SP), skilled men reduce their hours worked due to a dominant income effect, whereas unskilled men increase their hours marginally. This causes hours differential between skilled and unskilled men to decline from 2.20 hours per week in the initial steady state to 1.84 hours per week in the short-run. In the bottom panel we report average wealth (measured relative to GDP) of each skill group. Savings do not respond much to the rise in the skill premium for either skill group. Since the rise in the skill premium is a permanent change, both skill groups can adjust their future consumption without changing their savings. This exercise confirms that the rise in the skill premium is not a main driving force behind the short-run increases in the skilled-unskilled hours differential and the accelerated wealth accumulation by both skill groups observed in the U.S. data.
When the variances of residual wages increase without a change in the skill premium (SR1), both skilled and unskilled men raise their hours worked in the short run. Skilled men, however, increase their hours more than unskilled men do because they experience a greater rise in wage volatility and derive less disutility from work than unskilled men ($\psi_s < \psi_u$). This causes the hours differential between the two skill groups to rise from 2.20 hours in the initial steady state to 3.39 hours in SR1. Exposed to permanently larger variations in their labor productivity, both skilled and unskilled men accumulate more wealth as a buffer stock with the change more pronounced among skilled men.

The effects of anticipated increases in future wage volatility (SR2) on current hours worked and savings are even greater than the SR1 effects. Both skill groups increase their weekly hours remarkably to prepare for larger variations in their wage income in the next period. The skilled-unskilled hours differential soars up to 5.26 hours per week while the gap in wealth between the two skill types rises to 0.89. If wage volatility continues to show a gradual increase over time and households fully anticipate the changes, the SR2 effect is quantitatively important in explaining the evolution of hours.

The SR exercise combines the SP with the SR1 by changing both the first and the second moments of wages simultaneously for skilled and unskilled men. As is the case in the SR1, households anticipate the new wage structure to persist, excluding the SR2 effect. The result indicates the quantitative importance of rising wage volatility over increases in the skill premium in changes of hours worked and savings. The insurance motive from the larger variations in residual wages induce skilled men to work longer hours in spite of the rise in the skill premium. Consequently, skilled men’s weekly hours rise to 44.69 hours per week, which raises the hours difference between the two skill types by 0.88 hours compared to the initial steady state. Changes in savings of both skill groups are mainly affected by the rise in wage volatility, thus the results are similar to those in the SR1.

These short-run effects of the changing wage structure on hours worked are reversed in the long run (SS2). Skilled men increase their wealth substantially in response to more volatile wages, almost doubling their wealth in the new steady state compared to the initial steady state. Due to wealth effects, skilled men work about 1 hour less in the long run than in the initial steady state. On the other hand, wealth accumulation by skilled men lowers the real interest rate in the capital market. Since the lower interest rate disincentivizes unskilled men to save, they have less wealth and work longer hours in the new steady state than in the initial steady state. This lowers the skilled-unskilled hours differential to 1.02 hours per week in the long run.

### 5.3.2 General Equilibrium Effect

The short-run effects of the changes in the wage structure on hours worked differ from their long-run effects because of the general equilibrium effect. Investigating the general equilibrium effect along the transition to the new steady state helps us to better understand the evolution of hours worked by skill type. To this aim, we proceed by solving a partial equilibrium (PE, hereafter) model where
the real interest rate is held constant throughout the period. We then compare the results with the benchmark general equilibrium (GE) outcome. The PE model shares the same paths of the TFP $z_t$ and the skill-biased parameter $\chi_t$ with the GE model, so any difference in the results between the two is attributable solely to the evolution of the real interest rate.

The transition dynamics of key macroeconomic variables in both the GE and the PE models are presented in Figure 12. Note that in the benchmark model, the real interest rate initially soars upwards and then begins a downward trend around the early 1990s. Without a rise in the real interest rate, both skilled and unskilled men anticipate much less capital income in PE than in GE. This causes both skill groups to initially increase their work hours more rapidly than in the benchmark model. Both types of households also accumulate wealth more quickly when facing a constant real interest rate, which enables them to reduce their hours of work much sooner than in the benchmark model. The wealth effect on hours worked is more pronounced among unskilled men than skilled men not only because skilled men derive less disutility from work than unskilled men, but also because skilled men face larger volatility in their wages, with a greater fraction through a persistent component. Consequently, the skilled-unskilled hours differential rises by a larger amount in the short run for a constant real interest rate than in the GE model.

Recall that in the benchmark model, wealth accumulation by skilled men pushes down the real interest rate in the long run, discouraging unskilled men to save. This causes unskilled men to have less wealth and work longer hours in the new steady state, relative to the initial steady state. This phenomenon does not occur in PE. Both skilled and unskilled men accumulate greater levels of wealth in the long run, allowing them to work shorter hours and consume more in the new steady state compared to the GE results. As it turns out that this wealth effect on hours worked is stronger for unskilled men than skilled men, the hours differential between the two skill groups converges to a larger value in the new steady state with PE than in GE.

The exercise in this subsection implies that a constant real interest rate amplifies the effect of rising wage volatility on hours worked. Without an anticipated rise in the real interest rate, households initially increase their hours of work by a larger amount in response to increased wage volatility. However, households accumulate wealth more rapidly for a constant real interest rate and thus can afford an even larger reduction in hours worked in the long run. In contrast, the general equilibrium effect allows households to adjust their hours gradually while spreading out the changes over a longer span of time. Lastly, the general equilibrium effect reduces the level of total precautionary savings in the economy.

6 Discussion

There are various elements of our benchmark model that affect the response of hours to rising wage volatility. We begin by investigating how important the changing skill premium has been in explaining the evolution of hours. It is then followed by an attempt to distinguish the effect of total wage volatility on hours worked from the effect of wage shock composition (persistent vs.
transitory). Lastly, we study how robust the main quantitative results are to the perfect foresight assumption and the degree of relative risk aversion.

6.1 Effect of the Changing Skill Premium

In section 5.3.1, we study short-run impacts of a one-time permanent increase in the skill premium on hours work of both skill groups. However, the skill premium actually dropped sharply before rising over three decades since the late 1970s. How did this time path of the skill premium affect the evolution of hours and savings of both skill groups during the transition period? Would the skilled-unskilled hours differential have increased in the short run even if there had been no changes in the skill premium? To answer these questions, we implement a counterfactual exercise where the skill premium is held constant and thus neither skill group gains from the rise in their relative wages.

Figure 13 presents transitional dynamics of various macroeconomic variables with a time-invariant skill premium. Without a change in the skill premium, skilled men raise their work hours more rapidly than in the benchmark economy. Since skilled men gain from the constant skill premium initially relative to the benchmark model where the skill premium drops sharply, they have incentives to work less than in the benchmark results. However, they become worse off when the skill premium stays constant even after the late 1970s during which it actually rose at a rapid pace. This makes them work more with the constant skill premium than in the benchmark model. It turns out that the latter effect dominates the former, and skilled men’s work hours rise in the short run more with the constant skill premium than in the benchmark model. Skilled men still dissave initially because of the anticipated increase in the real interest rate, yet it occurs for a shorter period and by a smaller amount than in the benchmark economy.

With the constant skill premium, unskilled men do not increase their work hours as much as in the benchmark model. As unskilled men face no future drop in their wages relative to skilled men, the income effect causes them to work less. As a result, the skilled-unskilled hours differential rises to 4.3 hours per week in the early 1980s. Unskilled men’s wage gain due to the constant skill premium also makes unskilled men maintain a lower level of wealth in the short run.

As the skill premium stays the same, skilled men accumulate less wealth in the long run and work longer hours due to the wealth effect compared to the benchmark economy. Unskilled men’s hours change little. Thus, the hours differential between skilled and unskilled men in the new steady state is larger than the benchmark results.

6.2 The Importance of Shock Composition

For the sample period, not only the total wage volatility changed substantially for both skill groups but also the composition of persistent versus transitory wage shocks. It is then natural to ask which is quantitatively more important in explaining the trends in hours: the total wage volatility
or the wage shock composition? In order to answer this question, we implement a couple of counterfactual exercises in which skilled men are exposed to the same changes in either the level of total wage volatility (CF1) or the shock composition (CF2) as unskilled men. Holding the time series of TFP $z_t$ and the parameter $\chi_t$ the same as in the benchmark economy, we investigate how skilled men adjust their hours of work in response to these counterfactual changes in wage volatility.

Figure 14 presents how skilled men respond to the same increase in the total wage volatility as unskilled, holding the shock composition the same as in the benchmark model. The result shows that skilled men work less hours during the transition than in the benchmark model, because of the smaller increase in their total wage volatility. However, it is intriguing that hours increase most rapidly when the total wage volatility does not experience the largest rise. In this counterfactual exercise, skilled men are assumed to face the most rapid increase in the total wage volatility during the 1970s and the early 1980s as is clear in Figure 3. However, skilled men’s hours increase more rapidly during the 1980s and 1990s when their persistent wage component forms an increasing share of the total wage volatility. It appears that the shock composition is quantitatively more important in explaining the changes in hours worked than total wage volatility.

This smaller rise in the total wage volatility also weakens skilled men’s precautionary savings motive. Skilled men increase their wealth less aggressively and thus work more in the long run than in the benchmark. Due to much weaker general equilibrium effect, the real interest rate declines less in the long run, which induces unskilled men to accumulate more savings and work less than in the benchmark case. As a result, the skilled-unskilled hours differential in the new steady state is larger than in the benchmark model.

Suppose now that skilled men experience the same changes in the composition of persistent and transitory wage shocks as unskilled men, holding the path of the total wage volatility the same as in the benchmark model. Figure 15 reports that with increased wage volatility more through transitory shocks, skilled men’s hours worked rise until the early 1980s almost as much as in the benchmark model. However, the trend is reversed in the early 1980s, whereas they continue to rise for the same period in the benchmark economy. Given that the total wage variance increased most remarkably during the 1980s and 1990s, the drop in skilled men’s hours in this counterfactual exercise appears to be caused by the changes in the shock composition of unskilled men. Note that unskilled men experienced a rapid decline in the share of persistent wage shocks in the variance of their wages since the early 1980s. Holding the total wage variance constant at its benchmark level, skilled men could afford a larger reduction of hours worked during the 1980s and 1990s than in the benchmark model because a smaller fraction of wage shocks has a persistent effect on their productivity. These transition dynamics confirm that the composition of persistent versus transitory wage shocks is

Note that skilled and unskilled men differ in the degree of disutility from working and in their fraction of wealth due to labor income. These differences between the two skill groups affect the quantitative impacts of the level of wage uncertainty, and its composition, on the trend of skilled-unskilled hours differentials we present using these counterfactual experiments. Therefore, we suggest readers to focus on the relative importance of the two channels instead of the absolute quantitative impact of each channel.
quantitatively more important in hours worked than the total wage variance.

The precautionary savings motive associated with transitory wage shocks turns out to be weaker than with persistent wage shocks, leading to smaller increases in savings of skilled men than in the benchmark model. The real interest rate then declines less because of a weaker general equilibrium effect than in the benchmark model. Thus, unskilled men save more and work less in the long run. On the other hand, skilled men can insure themselves against transitory wage shocks with a smaller amount of savings and therefore can afford more leisure in the long run than in the benchmark economy. Consequently, the skilled-unskilled hours differential is smaller in the long run than in the benchmark case.

6.3 Expectations

In our benchmark model, households have perfect foresight about the evolution of both the first and the second moments of wages. Given anticipated changes in the wage structure, they choose the optimal path of hours worked. To the extent that it is difficult to forecast future wages, the actual evolution of hours may drift away from the benchmark results. This section relaxes the perfect foresight assumption to examine the role of expectations in explaining the evolution of hours of both skill groups and the skilled-unskilled hours differential.

Suppose that households update information about the wage structure in every single period. In the beginning of every period, households observe a new set of the skill premium and the variances of persistent and transitory wage shocks and anticipate that this wage structure will persist. Therefore, any change in these variables is completely unexpected. Figure 16 shows how skilled and unskilled men’s weekly hours evolve over time with this partial information updating. For the initial decade when the skill premium drops and the variance of transitory wage shocks rises, skilled men increase their weekly hours as new information about the wage structure arrives. If no additional changes are to occur in the wage structure, their hours would follow the path indicated by the dotted lines. However, as they observe changes in the wage structure again in the next period, they increase their hours further. Even after the skill premium begins an upward trend in the late 1970s, skilled men continue to increase their weekly hours as the variance of persistent wage shocks rises. Around the early 1990s, skilled men begin to reduce their hours as the effect of a constantly rising skill premium dominates that of increased wage volatility fueled through transitory wage shocks. The downward trend of skilled hours continues until the economy reaches the new steady state. Without perfect foresight, skilled men become more responsive to the rise in wage volatility. Since the continued increase in the skill premium is not anticipated, skilled men attempt to accumulate wealth more rapidly by raising their hours by a larger amount in the short run, relative to the benchmark results. However, the difference in the evolution of hours between this exercise and the benchmark one is not quantitatively large.

In contrast, unskilled hours follow a somewhat different path from the benchmark one. Unskilled men are initially hit by the drop in the skill premium and the rise in the variance of persistent wage shocks, both of which have offsetting effects on their hours. The right panel of Figure 16 indicates
that unskilled men reduce their weekly hours initially, suggesting that the gain from the declining skill premium was a dominant factor. As the skill premium begins to pick up in the late 1970s, unskilled men stop reducing their weekly hours, but this decreasing pattern soon resumes with a decline in the share of persistent wage shocks starting in the early 1980s. After the early 1990s, the skill premium rises above its level in the initial steady state and unskilled men’s persistent wage shocks become more volatile, inducing unskilled men to increase their hours slightly.

The path of unskilled men’s hours with the partial information updating differs from its benchmark counterpart, mostly due to short-sighted beliefs in the first decade. In the benchmark model, unskilled men anticipate the skill premium to rise, so they would rather work more to accumulate savings for the initial decade while the skill premium declines. Without perfect foresight, this dynamic consideration is missing in unskilled men’s hours decision, causing them to reduce their hours due to a dominant income effect in response to a drop in the skill premium.

The benchmark model’s qualitative implication on the short-run time path of the skilled-unskilled hours differential stays valid even if we relax our perfect foresight assumption. Figure 17 reports that the skilled-unskilled hours differential rises in the short run and declines in the long run. With partial information updating, the hours difference increases in the short run slightly more than in the benchmark results. It peaks around the early 1990s, exceeding initial steady state levels by 2.5 hours, about 0.6 hours more than the benchmark result.

6.4 Sensitivity Analysis

The benchmark parameterization for the relative risk aversion parameter ($\sigma = 1.5$) implies that the income effect is greater than the substitution effect. Although this value is standard in the literature, one might argue that it is not consistent with micro evidence on the wage elasticity of labor supply that suggests the elasticity is close to zero for men. To address such concerns, we conduct sensitivity analysis to changing values of $\sigma$ (thus changing the relative magnitude of the income effect to the substitution effect). Specifically, we consider two values, $\sigma = 1$ (log utility) and $\sigma = 3$ for relative risk aversion. For both values of $\sigma$, we recalibrate the time discount factor $\beta$, and skill-specific parameters $\psi^s$ and $\psi^u$ governing utility from non-working time so that the model matches the real interest rate and average weekly hours of skilled and unskilled men in the initial steady state. The recalibrated parameter values are summarized in Table 7.

Figure 18 presents the sensitivity analysis results with log utility. With a lower degree of risk aversion implied by $\sigma = 1$, households raise their work hours only modestly in response to increased wage volatility. Skilled men increase their hours slightly in the short run and can afford a larger reduction in their work hours in the long run, relative to the benchmark results. Unskilled men can also handle the larger variations in their wages without increasing their hours much for the initial decade. Starting in the 1980s, unskilled men reduce their hours more rapidly as the share of persistent wage shocks declines, and also work shorter hours in the long run than in the benchmark model. Despite the large differences in the levels of hours worked between this exercise and the benchmark model, the benchmark result on the skilled-unskilled hours differential carries through...
with log utility, rising in the short run and declining in the long run. The time path of the hours difference between skilled and unskilled men with the log utility differs little from the benchmark result, though it converges to a larger value in the long run, relative to the benchmark result. The long run discrepancy on the hours differential is attributable to a smaller income effect associated with the lower value of $\sigma$. A dominant income effect implies that the increased skill premium causes skilled men to work shorter hours while unskilled men work longer hours. The smaller value of $\sigma$ mitigates these effects, increasing the hours difference in the long run, relative to the benchmark result.

The transition dynamics of hours worked with $\sigma = 3$ are reported in Figure 19. As is the case with the log utility, the benchmark model’s qualitative implications on the skilled-unskilled hours differential are still valid, but the hours differential changes more dramatically. The higher degree of risk aversion causes both skilled and unskilled men to increase their hours substantially in response to the same increase in wage volatility. As the impact of increased variation in wages on skilled men dominates that of unskilled men, the skilled-unskilled hours differential rises in the short run more than in the benchmark model. This larger relative risk aversion also induces skilled men to accumulate a greater amount of savings in the long run, reinforcing the general equilibrium effect. In the long run, unskilled men raise their work hours more than skilled men and thus the hours differential between the two skill groups becomes smaller than in the benchmark model.

7 Conclusion

In the past few decades in the U.S., relative labor supply of skilled men to unskilled men increased while the skill premium rose sharply. This fact seems contradictory with the predictions of previous literature suggesting a zero labor supply elasticity with respect to wage. This paper attempts to explain this discrepancy using wage volatility. Using the PSID, we find that wage volatility increased for both skill groups, with greater increases for skilled men than for unskilled men in recent decades. In contrast to unskilled men, we show that the rise in skilled men’s wage volatility was partly created by a larger share of persistent wage shocks.

To explain these findings, we develop a general equilibrium incomplete markets model where workers receive idiosyncratic labor productivity shocks drawn from skill-specific distributions. By feeding in the skill-specific wage processes estimated from the PSID, the model can replicate the observed trend in hours worked of skilled men relative to unskilled men. The result confirms that a comparatively larger rise in wage volatility for skilled men can explain the increased hours differential between skilled and unskilled men over the transition period. In the long run, however, as skilled men accumulate enough precautionary savings, they are able to reduce their labor supply relative to unskilled men in the new steady state. This implies that hours adjustment we have observed is due to self-insurance in the short run, whereas precautionary savings will play a more important role in the long run. Our study also finds that the variances of persistent wage shocks mainly determines the trends in hours, while the quantitative impact of transitory wage shocks is
fairly small.

What caused wage volatility of skilled men to rise more with an increasing fraction of a persistent wage component than that of unskilled men is outside the scope of this study. Exploring potential explanations behind the phenomenon may help improve our understanding of male labor supply, its evolution, and its macroeconomic implications. We leave this for future work.
Appendix

A-1. Data - Wages and Hours Worked

We use data from Current Population Survey (CPS) March Supplements in 1968 through 2001 to document how the skill premium and hours worked by skill level have evolved over the period of 1967 to 2000. The skill-specific wage process to calibrate the individual productivity shocks is estimated based on the data from Panel Study of Income Dynamics (PSID). We estimate the process over the 1967-2000 period, but use information in surveys after 2000 as well to improve the estimation results. Since the survey is conducted biennially beginning 1997, we exploit a total of 35 surveys from PSID.

CPS and PSID Sample selection: We only include men aged between 25 and 59, who are not a student or disabled, with reported educational attainment. In the CPS, we classify individual employment status based on their annual hours worked and hourly wages in the previous calendar year. Annual hours worked are a product of the previous calendar year’s weeks worked and usual hours worked per week. Hourly wage is obtained by dividing annual labor income by annual hours worked. We classify those who worked more than 800 hours for the past calendar year as employed. Conditional on being employed, one should earn more than half the federal minimum wage per hour and not be self-employed to be included in the sample. We apply the former restriction to exclude extreme outliers and the latter one due to it being difficult to distinguish between labor and capital shares out of an individual’s annual income. From the PSID, we restrict the sample to male heads of household who participated in the labor market last year (i.e., worked at least 260 hours). In both CPS and PSID sample, we exclude those who reported that they worked more than 5840 hours in the past year. We exclude males in oversamples. The resulting sample is an unbalanced panel.

CPS and PSID Variables: In the CPS, we obtain annual hours worked as a product of the past year’s weeks worked and usual hours worked per week. In surveys before 1976, usual hours worked per week in the past year are not available and weeks worked in the past year are coded in intervals. Therefore, we impute both variables for previous surveys using the average weeks worked and the average usual hours worked per week in the same education and weeks worked interval cells in the 1976 survey. Annual earnings in the CPS are income from wages and salaries. In the PSID, we use head of household’s total annual work hours and labor income of head to obtain annual hours worked and hourly wages.

A-2. Data - Savings

In order to examine how household savings has changed in the data, we exploit the Survey of Consumer Finances (SCF) from 1983 to 2007. The SCF is a triennial survey on household finances
and provide detailed information on household income, assets, and liabilities. Using the surveys, we measure household wealth and earnings. Since 1989, the SCF imputes missing data using multiple methods and publish five implicates. We use the first implicate for our analysis.

**SCF Sample selection:** The sample from the SCF data consists of households with a male head who are ages between 25 and 59, and have usual hours worked between 260 and 5840 for his main job. As for the hours restriction, we consider hours in the current main job instead of total hours worked because 1983 and 1986 surveys have information on only the male head’s current main job. We further restrict the sample to households with positive labor income defined by wages and salaries, and zero income from professional practices, business and farm sources in order to exclude households with a self-employed head, except for the 1986 survey. For the 1986 survey, a question on the job status of the head is used to determine whether he is self-employed because this particular source of income is not available. Note that we use the first implicate for all statistics except for the 1983 and 1986 surveys because we do not have multiple implicates for these years. We disaggregate the sample based on the skill level of the male head, and we consider two skill levels: college graduates and non-college graduates.

**SCF Variables:** Wealth is defined as the net worth of the households, that is, assets minus debts. Assets include: residential assets and other real estates; net value of businesses; land contracts and notes; checking accounts; certificates of deposit, and other banking accounts; IRA/Keogh accounts; money market accounts; mutual funds; bonds and stocks; cash and call money at the stock brokerage; all annuities; trusts and managed investment accounts; vehicles; net cash value of life insurance policies; pension assets accumulated in accounts from current main job; and total amount of loans owed to the households and gas leases (except for 1986 survey). Residential assets include the current value of both primary and secondary residences. Surveys in 1983 and 1986 provide gross market value of a primary residence if owned or buying, which we use for analysis. For other years, we further refine the measure by considering the portion of primary residences not used for farming/ranching or investment. For other real estate (timeshares) partially owned by the households, values for only the household’s interest are used for all surveys. For the period of 1983 to 1989, face value of bonds was used for calculation because their market value is not available. We include pension accounts from their current main job only because balances in other pension plans are not available for surveys in 1983 and 1986. Pension accounts from the 1986 survey include thrift-type savings plans only.

Debts include: housing debts, such as mortgages, home equity loans, and lines of credit; other residential property debts; credit card debts; installment loans; loans taken against pensions; margin loans; and other debts. For survey years 1983 and 1986, information on loans taken against pensions and margin loans is not separately available, but such loans are considered to be included in total regular and non-regular debt outstanding.

Income is defined as the sum of various sources of income: wages and salaries; income from professional practices, businesses, and farms sources; income from non-taxable investments; income from other interest income; income from dividends; net gains or losses from the sale of stock,
bonds, or real estate; net rent, trusts, or royalties from any other investment or business; income from unemployment or worker’s compensation; income from child support or alimony; income from food stamps, or other forms of welfare or assistance; income from Social Security or other pensions, annuities, or other disability or retirement programs; and income from other sources. For the 1986 survey, an aggregate household income variable is used because information on each of the components of total income is not available. Labor income is defined as wages and salaries of all household members. For the 1986 survey, a comparable variable is not available, so we compute labor income by summing up the head’s and spouse’s annual wage income.

A-3. Estimation of Wage Processes

As described in section 2.1, following Heathcote et al. (2010), we model the log wage residual $y^e_{it}$ as the sum of persistent and transitory shocks with time-varying variances:

$$ y^e_{it} = \mu^e_{it} + \upsilon^e_{it} + \theta^e_{it} $$

where $\mu^e_{it}$ is a persistent component, $\upsilon^e_{it} \sim (0, \lambda^e_{\upsilon})$ is a transitory component, and $\theta^e_{it} \sim (0, \lambda^\theta)$ is measurement error. The persistent component $\mu^e_{it}$ is assumed to follow an AR(1) process:

$$ \mu^e_{it} = \rho^e \mu^e_{it-1} + \eta^e_{it} $$

where $\rho^e$ is the persistence and $\eta^e_{it} \sim (0, \lambda^e_{\eta})$ is a persistent wage shock whose variance $\lambda^e_{\eta}$ varies over time. The initial value of the persistent component is drawn from a skill-specific distribution: $\mu^e_0 \sim (0, \lambda^e_{\mu})$. We assume that all four variables, $\upsilon^e_{it}$, $\theta^e_{it}$, $\eta^e_{it}$, and $\mu^e_0$ are orthogonal and i.i.d. across individuals.

As we mention in 2.1, we take the estimate of 0.02 from French (2004) for the variance, $\lambda^\theta$, of measurement error. Then, we estimate a parameter vector $\Phi^e$, which includes two time-invariant parameters $\rho^e$ and $\lambda^e_{\mu}$ and a set of time-varying parameters $\{\lambda^e_{\upsilon}, \lambda^e_{\eta}\}_{t=1967}^{2006}$ for each skill group $e \in \{s, u\}$. Since the PSID are available biennially beginning in 1997, we do not have empirical moments for transitory shocks for years 1997 and 1999. In order to resolve this issue, we assume that the cross-sectional variance of log residual wages in these missing years is the average of that in the previous year and in the subsequent year, and identify the variances of transitory wage shock for missing years as is done in Heathcote et al. (2010).

For each sample year $t$, we construct 10-year adjacent age cells from ages 29 to 54 such that, for instance, the age group 29 consists of those aged 25 to 34 years. We then compute the empirical autocovariance, $\tilde{g}^e_{a,t,n}$, of all possible orders for each age/year $(a, t)$ cell in our PSID sample using log wage residuals $\tilde{y}^e_{it}$ from the first-stage regressions:

$$ \tilde{g}^e_{a,t,n} = \frac{1}{I^e_{a,t,n}} \sum_{i=1}^{I^e_{a,t,n}} \tilde{y}^e_{i,t} |_{a_{it}=a} \cdot \tilde{y}^e_{i,t+n} |_{a_{it}=a}, \quad n \geq 0, $$

where $I^e_{a,t,n}$ is the number of observations for $n$th order autocovariance for age/year $(a, t)$ cell in skill group $e$. We then pick the parameters $\tilde{\Phi}^e$ that minimize the equally weighted distance between
this empirical autocovariance matrix and its theoretical counterpart:
\[
\hat{\Phi}^e = \arg\min_{\Phi^e} \left[ \hat{G}^e - G^e(\Phi^e) \right]' I \left[ \hat{G}^e - G^e(\Phi^e) \right],
\]

where \( \hat{G}^e \) is a stacked vector of empirical autocovariances as well as cross-sectional variances for missing years, \( G^e(\Phi^e) \) is the theoretical counterpart, and \( I \) is an identity matrix.

**A-4. Algorithm**

A set of parameters \( \beta, \psi^s, \) and \( \psi^u \) is calibrated to match targets in the initial steady state. We allow the parameters \( \Lambda_t = \{z_t, \chi_t, \pi_t, \lambda^s_t, \lambda^u_t, \lambda^\eta_t, \lambda^\upsilon_t \} \), governing the productivity, the skill premium, the population share of the skilled, and persistent and transitory idiosyncratic productivity, to vary over time.

**Solving for the steady state**

Under a set of parameters for the steady state (including \( \Lambda_* \)),

1. Guess price \( r \). Given this guess, compute \( w^u \) and \( w^s \) (given \( \Lambda_* \)).
2. Solve the value function and get \( h(e, a, \mu, \upsilon) \), \( a'(e, a, \mu, \upsilon) \), \( c(e, a, \mu, \upsilon) \).
3. Generate a sample of population \( N \) over \( (e, a, \mu, \upsilon) \) space. That is, \( N_S = \pi_* N \) sample over \( (a, \mu, v) \) and \( N_U = (1 - \pi_*) N \) sample over \( (a, \mu, v) \).
4. Compute aggregate Statistics:
\[
K = \sum_e \int a dG(e, a, \mu, \upsilon) \\
S = \int h(s, a, \mu, \upsilon) \exp(\mu + \upsilon) dG(s, a, \mu, \upsilon) \\
U = \int h(u, a, \mu, \upsilon) \exp(\mu + \upsilon) dG(u, a, \mu, \upsilon) \\
H = \left\{ \chi U^\phi + (1 - \chi) S^\phi \right\}^{\frac{1}{\phi}} \\
\]
And define
\[
\tilde{r} = z\alpha (K/H)^{\alpha - 1} - \delta
\]
5. Check if \( |r - \tilde{r}| < \epsilon \). If not, update \( r \) and go back to step 1.

**Solving transition economy (Changing parameters over time)**

The economy was originally at the initial steady state \( (*) \). There is a gradual change for \( \Lambda \) from \( \Lambda_* \) to \( \Lambda_{**} \) for periods \( t = t_1, \cdots, t_\tau \), and then the economy converges to the new steady state \( (**) \) at \( t_T (> t_\tau) \).
1. A sequence of parameters is given: \( \{\omega_t, \lambda_t^{s,\eta}, \lambda_t^{s,\nu}, \lambda_t^{u,\eta}, \lambda_t^{u,\nu}\}_t \) \( t = 1 \).

2. Solve two (initial and final) steady states and find a set of parameters \( \{z_t, \chi_t, r_t\}_t \) where \( z_t \) and \( \chi_t \) are determined by an aggregate output \( Y_t \) and skill premium \( \omega_t \) in each steady state respectively. Record the stationary distribution \( G_t(e,a,\mu,\nu) \) and the value function \( V_{*t}^{e}(a,\mu,\nu) \).

3. Guess the sequences of \( \{z_t, \chi_t, r_t\}_t \) \( t = 1 \). Then, the other prices are given by
   \[
   w_t^s = w^s(z_t, \chi_t, r_t; \alpha, \delta, \phi) \\
   w_t^u = w^u(z_t, \chi_t, r_t; \alpha, \delta, \phi) \\
   w_t^u = z_t(1-\alpha)\chi_t \left( \frac{r_t + \delta}{z_t+\alpha} \right)^{\alpha-1} \chi_t + (1-\chi_t) \left( \frac{\chi_t}{\lambda_t^{u,\nu}} \right)^{\phi-1} \frac{1-\phi}{\phi} \\
   w_t^s = w_t^u \cdot \omega_t
   \]

4. Solve for workers decisions backwards and get the decision rules during the transition periods:
   (a) Using the value function at the new steady state: \( V_{*t}^{e}(a,\mu,\nu; \Lambda_{*t}) \),
   (b) In each period during the transition \( (t = t_1, \cdots, t_{T-1}) \), solve
   \[
   V_t^{e}(a,\mu,\nu) = \max_{c,a'} \{ u(c,h) + \beta \gamma \mathbb{E}_t V_{t+1}^{e}(a',\mu',\nu') \}
   \]
   subject to constraints with \( p_t = \{r_t, w_t^u, w_t^s\} \) and \( \Lambda_t \) for \( t = t_1, \cdots, t_{T-1} \).
   Solve the problem above backwards from \( t = t_{T-1}, \cdots, t_1 \) to obtain the decision rules:
   \( c_t(e,a,\mu,\nu), h_t(e,a,\mu,\nu), a'_t(e,a,\mu,\nu) \) and the value function, \( V_t^{e}(a,\mu,\nu) \).

5. Given a distribution at \( t, G_t(e,a,\mu,\nu) \), simulate \( G_{t+1} \) by applying \( c_t(\cdot), h_t(\cdot), \) and \( a'_t(\cdot) \).
   Start from \( t = t_1 \) where we know \( G_{t_1}(\cdot) \) from the distribution of the initial steady state \( G_{*} \).
   (a) First calculate the aggregate statistics:
   \[
   K_t = \sum_e \int_a dG_t(e,a,\mu,\nu) \\
   S_t = \int_s h_t(s,a,\mu,\nu) \exp(\mu+v) dG_t(s,a,\mu,\nu) \\
   U_t = \int_u h_t(u,a,\mu,\nu) \exp(\mu+v) dG_t(u,a,\mu,\nu).
   \]
   (b) Using these aggregate statistics, compute
• an updated share $\tilde{\chi}_t$ (which implies the observed skill premium $\omega_t$) such that
  
  $$
  \tilde{\chi}_t = \frac{(S_t/U_t)^{\phi-1}}{\omega_t + (S_t/U_t)^{\phi-1}},
  $$

• an implied price $\tilde{r}_t$ such that
  
  $$
  \tilde{r}_t = z_t \alpha \left( \frac{K_t}{\tilde{H}_t} \right)^{\alpha-1} - \delta,
  $$

  where

  $$
  \tilde{H}_t = \left\{ \tilde{\chi}_t U_t^\phi + (1 - \tilde{\chi}_t) S_t^\phi \right\}^{\frac{1}{\phi}},
  $$

• and a productivity parameter $\tilde{z}_t$ to normalize the aggregate output such that
  
  $$
  \tilde{z}_t = \frac{z_t K_t^\alpha H_t^{1-\alpha}}{K_t^\alpha \tilde{H}_t^{1-\alpha}}.
  $$

(c) Get an updated distribution $G_{t+1}$ and go back to (5a) until we get $G_T$.

6. Check if the guessed sequences $\{z_t, \chi_t, r_t\}_{t=1}^{**}$ are close enough to the model-implied sequences $\{\tilde{z}_t, \tilde{\chi}_t, \tilde{r}_t\}_{t=1}^{**}$ from (5b). If not, update the guesses and go back to step 3.
References


Table 1: Average Annual Changes in Skilled-Unskilled Hours Differential

<table>
<thead>
<tr>
<th></th>
<th>Within Cohort</th>
<th>Within Age</th>
<th>Between Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967–1971</td>
<td>0.0754*</td>
<td>0.0269</td>
<td>0.0540**</td>
</tr>
<tr>
<td></td>
<td>(0.0307)</td>
<td>(0.0306)</td>
<td>(0.0132)</td>
</tr>
<tr>
<td>1972–1976</td>
<td>0.0338</td>
<td>−0.0078</td>
<td>0.0480**</td>
</tr>
<tr>
<td></td>
<td>(0.0496)</td>
<td>(0.0495)</td>
<td>(0.0186)</td>
</tr>
<tr>
<td>1977–1981</td>
<td>0.1731**</td>
<td>0.1614*</td>
<td>0.0099</td>
</tr>
<tr>
<td></td>
<td>(0.0381)</td>
<td>(0.0402)</td>
<td>(0.0230)</td>
</tr>
<tr>
<td>1982–1986</td>
<td>−0.0754*</td>
<td>−0.0847**</td>
<td>0.0068</td>
</tr>
<tr>
<td></td>
<td>(0.0356)</td>
<td>(0.0378)</td>
<td>(0.0166)</td>
</tr>
<tr>
<td>1987–1991</td>
<td>0.1298*</td>
<td>0.1103*</td>
<td>0.0275</td>
</tr>
<tr>
<td></td>
<td>(0.0575)</td>
<td>(0.0560)</td>
<td>(0.0212)</td>
</tr>
<tr>
<td>1992–1996</td>
<td>−0.0437</td>
<td>−0.0635</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>(0.0448)</td>
<td>(0.0456)</td>
<td>(0.0208)</td>
</tr>
<tr>
<td>1997–2001</td>
<td>0.0382</td>
<td>0.0247</td>
<td>0.0214</td>
</tr>
<tr>
<td></td>
<td>(0.0456)</td>
<td>(0.0469)</td>
<td>(0.0209)</td>
</tr>
</tbody>
</table>

|               | 0.9962        | —          | 0.2117      |

Correlation with Within Age

Note: Numbers in parentheses represent standard errors. Coefficients with a * and a ** are statistically significant at a 5% and a 1% level, respectively.
### Table 2: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 1.5$</td>
<td>Relative risk aversion of 1.5</td>
</tr>
<tr>
<td>$\nu = 4.2440$</td>
<td>Frisch elasticity of hours of 0.4</td>
</tr>
<tr>
<td>$\alpha = 0.36$</td>
<td>Capital share (e.g. Aiyagari (1994))</td>
</tr>
<tr>
<td>$\delta = 0.08$</td>
<td>Capital depreciation rate (e.g. Prescott (1986))</td>
</tr>
<tr>
<td>$\phi = 1/3$</td>
<td>Substitution elasticity of 1.5 b/w skill groups (e.g. Heckman et. al. (1998))</td>
</tr>
<tr>
<td>$\gamma = 0.972$</td>
<td>Average work life of 35 years</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Skill Premium of 1.4681</td>
</tr>
<tr>
<td>$z$</td>
<td>Output normalized to 1</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Fraction of skilled workers</td>
</tr>
</tbody>
</table>

#### Parameters taken from the literature

- $\sigma = 1.5$
- $\nu = 4.2440$
- $\alpha = 0.36$
- $\delta = 0.08$
- $\phi = 1/3$
- $\gamma = 0.972$

#### Parameters specific to model economy

- $\beta = 0.9849$ - Real interest rate of 0.04
- $\psi^s = 0.2809$ - Avg. weekly hours of employed skilled men: 43.5
- $\psi^u = 0.3845$ - Avg. weekly hours of employed unskilled men: 41.3
- $\gamma = 0.972$ - Average work life of 35 years
- $\chi$ - Time series of skill premium ($\omega_t$) in CPS data
- $z$ - Normalize output to 1 in every period
- $\pi$ - Skilled share from CPS
- $\rho^s = 0.9859$ - Own estimate for the skilled from PSID
- $\rho^u = 0.9838$ - Own estimate for the unskilled from PSID
- $\lambda^s,\eta = 0.1179$ - Own estimate for the skilled from PSID
- $\lambda^u,\eta = 0.1467$ - Own estimate for the unskilled from PSID
- $\lambda^s,\eta, \lambda^t,\eta, \lambda^u,\eta$ - Own estimates from PSID for 1967 through 2000

### Table 3: Parameters for the Initial Steady State

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi$</td>
<td>0.6700</td>
<td>Skill Premium of 1.4681</td>
</tr>
<tr>
<td>$z$</td>
<td>1.8005</td>
<td>Output normalized to 1</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.1541</td>
<td>Fraction of skilled workers</td>
</tr>
<tr>
<td>$\lambda^s,\eta$</td>
<td>0.0040</td>
<td>Variance of the innovation in Persistent Shock: Skilled</td>
</tr>
<tr>
<td>$\lambda^u,\eta$</td>
<td>0.0038</td>
<td>Variance of the innovation in Persistent Shock: Unskilled</td>
</tr>
<tr>
<td>$\lambda^t,\eta$</td>
<td>0.0137</td>
<td>Variance of Transitory Shock: Skilled</td>
</tr>
<tr>
<td>$\lambda^u,\eta$</td>
<td>0.0132</td>
<td>Variance of Transitory Shock: Unskilled</td>
</tr>
</tbody>
</table>
### Table 4: Initial Steady State Results

<table>
<thead>
<tr>
<th></th>
<th>Skilled (s)</th>
<th>Unskilled (u)</th>
<th>Ratio (s/u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage Rate: w</td>
<td>2.1707</td>
<td>1.4786</td>
<td>1.47</td>
</tr>
<tr>
<td>Weekly Hours: h</td>
<td>43.5</td>
<td>41.3</td>
<td>1.05</td>
</tr>
<tr>
<td>Labor Income: wh</td>
<td>0.9219</td>
<td>0.5887</td>
<td>1.57</td>
</tr>
<tr>
<td>Persistent Volatility: $\lambda^n / (1 - \rho^2)$</td>
<td>0.1429</td>
<td>0.1182</td>
<td>1.21</td>
</tr>
<tr>
<td>Transitory Volatility: $\lambda^v$</td>
<td>0.0137</td>
<td>0.0132</td>
<td>1.04</td>
</tr>
<tr>
<td>Assets: a</td>
<td>3.4904</td>
<td>2.9107</td>
<td>1.20</td>
</tr>
<tr>
<td>Consumption: c</td>
<td>1.0474</td>
<td>0.7077</td>
<td>1.48</td>
</tr>
</tbody>
</table>

### Table 5: Changes in the Wage Structure

<table>
<thead>
<tr>
<th></th>
<th>1967</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skill Premium $\omega_t$ (SP)</td>
<td>1.4681</td>
<td>1.6634</td>
</tr>
<tr>
<td>Skilled Persistent Shock $\lambda_s^{\eta}$ (PS)</td>
<td>0.0040</td>
<td>0.0216</td>
</tr>
<tr>
<td>Unskilled Persistent Shock $\lambda_u^{\eta}$ (PU)</td>
<td>0.0038</td>
<td>0.0103</td>
</tr>
<tr>
<td>Skilled Transitory Shock $\lambda_s^{v}$ (TS)</td>
<td>0.0137</td>
<td>0.1121</td>
</tr>
<tr>
<td>Unskilled Transitory Shock $\lambda_u^{v}$ (TU)</td>
<td>0.0132</td>
<td>0.0963</td>
</tr>
<tr>
<td>Fraction of Skilled $\pi_t$</td>
<td>0.1541</td>
<td>0.3042</td>
</tr>
</tbody>
</table>

### Table 6: Short-Run Analysis

<table>
<thead>
<tr>
<th></th>
<th>Changes</th>
<th>Skilled</th>
<th>Unskilled</th>
<th>Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Future</td>
<td>Equil Prices</td>
<td>(s)</td>
</tr>
<tr>
<td>Weekly Hours Worked</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SR1</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>43.20</td>
</tr>
<tr>
<td>SR2</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>44.94</td>
</tr>
<tr>
<td>SR</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>48.52</td>
</tr>
<tr>
<td>SS2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>42.52</td>
</tr>
<tr>
<td>Wealth (Relative to GDP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SR1</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>3.95</td>
</tr>
<tr>
<td>SR2</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>3.85</td>
</tr>
<tr>
<td>SR</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>4.00</td>
</tr>
<tr>
<td>SS2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>6.20</td>
</tr>
</tbody>
</table>
Table 7: Sensitivity Analysis: Recalibrated Parameter Values

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>$\sigma = 1$</th>
<th>$\sigma = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9849</td>
<td>0.9863</td>
<td>0.9812</td>
</tr>
<tr>
<td>$\psi^s$</td>
<td>0.2809</td>
<td>0.2774</td>
<td>0.2963</td>
</tr>
<tr>
<td>$\psi^u$</td>
<td>0.3845</td>
<td>0.3140</td>
<td>0.7181</td>
</tr>
</tbody>
</table>
Figure 1: Trends in Skill Premium for U.S. Men between 1967 and 2006

Note: Wages of employed men who worked more than 800 hours in the past year.

Figure 2: Estimated Variances of Persistent and Transitory Wage Shocks
Figure 3: Variance Decomposition of Log Wage Residuals

Note: The estimated variance of total wage residual of skilled men is normalized to 1 in 1967.

Figure 4: Trends in the Share of a Persistent Wage Component

Note: The share represents the ratio of the persistent wage component’s variance to the sum of the variances of both persistent and transitory components. Both time series depicted are three-year moving averages.
Figure 5: Summarized Changes in the Wage Structure

- **Skill Premium**
  - Values: 1.3, 1.4, 1.5, 1.6, 1.7, 1.8

- **Variance of Persistent Shock**
  - Values: 0, 0.005, 0.01, 0.015, 0.02, 0.025, 0.03

- **Variance of Transitory Shock**
  - Values: 0, 0.05, 0.1, 0.15

Figure 6: Trends in the Male Hours Worked in the U.S.

- **Average Weekly Hours Worked**
  - Values: 39, 40, 41, 42, 43, 44, 45, 46

- **Hours Difference between Skill Groups**
  - Values: 1.5, 2, 2.5, 3, 3.5
Figure 7: Within-Cohort and Within-Age Variations in Skilled-Unskilled Hours Differential

Note: The left panel presents variations in the skilled-unskilled hours differential within each cohort by age. Cohort is defined by birth year. For instance, “1930s” refer to agents born in the 1930s. The right panel depicts the time series of the hours differential for different age groups.

Figure 8: Benchmark results: Hours worked by skill type

Figure 9: Hours Differences between Skilled and Unskilled Men: Model vs. Data
Figure 10: Transition Dynamics: Benchmark Model: Interest Rate and Wealth

Figure 11: Wealth-to-Labor Income Ratio by Skill Type: Model vs. Data
Figure 12: General Equilibrium Effects in Transition Dynamics

Figure 13: Transition Dynamics with a Constant Skill Premium
Figure 14: Counterfactual 1: Same Increase in Wage Uncertainty as the Unskilled

![Graph showing the comparison of Hours by Skill Group and Hours Difference between BM and CF1 over years 1950 to 2100.](image)

Figure 15: Counterfactual 2: Same Change in Shock Composition as the Unskilled

![Graph showing the comparison of Hours by Skill Group and Hours Difference between BM and CF2 over years 1950 to 2100.](image)

Figure 16: Hours Worked by Skill Type with Partial Information Updating

![Graph showing the comparison of Skilled and Unskilled Hours between Benchmark and Partial Information over years 1950 to 2100.](image)

Note: Each square indicates one-period information update. Dotted lines indicate hypothetical path without any additional information update.
Figure 17: Skilled-Unskilled Hours Differential with Partial Information Updating

![Graph showing skilled-unskilled hours differential with partial information updating.]

Figure 18: Sensitivity Analysis with Log Utility

![Graph showing sensitivity analysis with log utility.]

Figure 19: Sensitivity Analysis with $\sigma = 3$

![Graph showing sensitivity analysis with $\sigma = 3$.]