FAITHFUL COMPACT QUANTUM GROUP ACTIONS ON CONNECTED COMPACT METRIZABLE SPACES

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Abstract. We construct faithful actions of quantum permutation groups on connected compact metrizable spaces. This disproves a conjecture of Goswami.

1. Introduction

Compact quantum groups were introduced by Woronowicz in [14, 15]. They are noncommutative analogues of compact groups. Among all literatures related to compact quantum groups, one particularly interesting topic is the compact quantum group actions on commutative or non-commutative unital C*-algebras (from the viewpoint of non-commutative topology, that means actions on commutative or non-commutative compact spaces). The actions of compact quantum groups are the natural generalizations of actions of compact groups. It was Podleś who first formulated the concept of compact quantum group actions, then established some basic properties [7]. Later, Wang introduced the quantum permutation groups [12] and showed that they are the universal compact quantum groups acting on finite spaces. After that, many interesting actions are studied (see [1–7] and the references therein). But so far, all known (commutative) compact spaces admitting genuine faithful compact quantum group actions are disconnected. In [6], Goswami showed that there is no genuine faithful quantum isometric action of compact quantum groups on the Riemannian manifold G/T where G is a compact, semisimple, centre-less, connected Lie group with a maximal torus T, and conjectured that the quantum permutations on (disconnected) finite sets are the only possible faithful actions of genuine compact quantum groups on classical spaces. In this paper, we construct faithful actions of quantum permutation groups on

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connected compact metrizable spaces and disprove Goswami's conjecture.

The paper is organized as follows. In the next section we recall some basic definitions and terminologies related to compact quantum groups and their actions. Then in section 3, we construct faithful quantum permutation group actions on connected compact metrizable spaces.

2. Preliminaries

In this section, we recall some definitions about compact quantum groups. See [7, 11, 12, 14, 15] for more details.

Throughout this paper, the notation $A \otimes B$ for two unital C*-algebras A and B stands for the minimal tensor product of A and B.

For a $*$-homomorphism $\beta : B \to B \otimes A$, use $\beta(B)(1 \otimes A)$ to denote the linear span of the set $\{\beta(b)(1_B \otimes a) | b \in B,\ a \in A\}$ and $\beta(B)(B \otimes 1)$ to denote the linear span of the set $\{\beta(b_1)(b_2 \otimes 1_A) | b_1, b_2 \in B\}$.

Denote by $\mathbb{C}$ the set of complex numbers. For a compact Hausdorff space X and a unital C*-algebra A, denote by $C(X, A)$ the C*-algebra of continuous functions mapping from X to A. Especially, we write $C(X, \mathbb{C})$ as $C(X)$. Use ev_x to denote the evaluation functional on $C(X)$ at the point $x \in X$.

Definition 2.1 (Definition 1.1 in [15])**.** A compact quantum group is a unital C*-algebra A with a unital $*$-homomorphism $\Delta : A \to A \otimes A$ such that

(1) $(\Delta \otimes id)\Delta = (id \otimes \Delta)\Delta$;
(2) $\Delta(A)(1 \otimes A)$ and $\Delta(A)(A \otimes 1)$ are dense in $A \otimes A$.

If A is non-commutative, we say that (A, Δ) is a *genuine* compact quantum group. A unital C*-subalgebra Q of A is called a *compact quantum quotient group* of (A, Δ) if $\Delta(Q) \subseteq Q \otimes Q$, and $\Delta(Q)(1 \otimes Q)$ and $\Delta(Q)(Q \otimes 1)$ are dense in $Q \otimes Q$. That is, $(Q, \Delta|_Q)$ is a compact quantum group [11, Definition 2.9]. If $Q \neq A$, we call Q a *proper compact quantum quotient group.*

Definition 2.2 (Definition 1.4 in [7])**.** An action of a compact quantum group (A, Δ) on a unital C*-algebra B is a unital $*$-homomorphism $\alpha : B \to B \otimes A$ satisfying that

(1) $(\alpha \otimes id)\alpha = (id \otimes \Delta)\alpha$;
(2) $\alpha(B)(1 \otimes A)$ is dense in $B \otimes A$.

When A is non-commutative, we call α a *genuine* compact quantum group action.

We say that α is *faithful* if there is no proper compact quantum quotient group Q of A such that α induces an action α_q of Q on B satisfying $\alpha(b) = \alpha_q(b)$ for all b in B [12, Definition 2.4].

If A acts on $C(X)$ for a compact Hausdorff space X, we say that A acts on X.

For any positive integer n, let A_n be the universal C*-algebra generated by a_{ij} for $1 \le i, j \le n$ under the relations

$$a_{ij}^* = a_{ij} = a_{ij}^2, \quad \sum_{i=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ij} = 1.$$

Let $\Delta_n : A_n \to A_n \otimes A_n$ be the $*$-homomorphism satisfying that

$$\Delta_n(a_{ij}) = \sum_{k=1}^{n} a_{ik} \otimes a_{kj}.$$

Then (A_n, Δ_n) is a compact quantum group and is called a *quantum permutation group* [12, Theorem 3.1]. Moreover, A_n is a genuine compact quantum group when $n \ge 4$ [13, the example preceding Theorem 6.2].

Let $X_n = \{x_1, x_2, \ldots, x_n\}$ be the finite space with n points. Define e_i for $1 \le i \le n$ to be the function on X_n such that $e_i(x_j) = \delta_{ij}$ for $1 \le j \le n$. There is an action α_n of A_n on X_n given by

$$\alpha_n(e_i) = \sum_{k=1}^{n} e_k \otimes a_{ki}$$

for all $1 \le i \le n$ [12, Theorem 3.1].

3. Main Results

Let Y be a compact Hausdorff space. Note that $C(X_n \times Y) \cong C(X_n) \otimes C(Y)$. For C*-algebras $A_1, A_2, \ldots, A_n$, we use $\sigma_{23} : A_1 \otimes A_2 \otimes A_3 \otimes \cdots \otimes A_n \to A_1 \otimes A_3 \otimes A_2 \otimes \cdots \otimes A_n$ to denote the operator flipping the 2nd and 3rd components.

Lemma 3.1. *There exists an action α of the quantum permutation group A_n on $X_n \times Y$ given by*

$$\alpha(\sum_{i=1}^{n} e_i \otimes f_i) = \sum_{i=1}^{n} \sum_{k=1}^{n} e_k \otimes f_i \otimes a_{ki},$$

where every $f_i \in C(Y)$.

Proof. The map $\alpha : C(X_n) \otimes C(Y) \to C(X_n) \otimes C(Y) \otimes A_n$ is well-defined since every element of $C(X_n) \otimes C(Y)$ can be expressed uniquely as $\sum_{i=1}^{n} e_i \otimes f_i$ for $f_i \in C(Y)$, and a routine calculation shows that α is a $*$-homomorphism. We also have

$$\begin{aligned}(\alpha \otimes id)\alpha(e_i \otimes f) &= \sum_{k=1}^{n}\sum_{l=1}^{n} e_l \otimes f \otimes a_{lk} \otimes a_{ki} \\ &= (id \otimes \Delta_n)\alpha(e_i \otimes f).\end{aligned}$$

Thus $(\alpha \otimes id)\alpha = (id \otimes \Delta_n)\alpha$. Moreover, note that $\alpha = \sigma_{23}(\alpha_n \otimes id)$. Since α_n is an action of A_n on X_n, $\alpha_n(C(X_n))(1 \otimes A_n)$ is dense in $C(X_n) \otimes A_n$. It follows easily that $\alpha(C(X_n) \otimes C(Y))(1 \otimes A_n)$ is dense in $C(X_n) \otimes C(Y) \otimes A_n$. This proves that α is an action of A_n on $X_n \times Y$. □

Remark 3.2. Suppose that compact quantum groups $(\widetilde{A}_1, \widetilde{\Delta}_1)$ and $(\widetilde{A}_2, \widetilde{\Delta}_2)$ act on B_1 and B_2 respectively. Then $(\widetilde{A}_1 \otimes \widetilde{A}_2, \sigma_{23}(\widetilde{\Delta}_1 \otimes \widetilde{\Delta}_2))$ is a compact quantum group and acts on $B_1 \otimes B_2$ naturally [10, Theorem 2.1]. Lemma 3.1 also follows from this by taking $(\widetilde{A}_1, \widetilde{\Delta}_1) = (A_n, \Delta_n)$ and $\widetilde{A}_2 = \mathbb{C}$.

Let Y_1 be a closed subset of Y. We define an equivalence relation $\sim$ on $X_n \times Y$ as follows. For y', y'' in Y and x', x'' in X_n, two points (x', y') and (x'', y'') in $X_n \times Y$ are equivalent if one of the following is true:

(1) $y' = y'' \in Y_1$;
(2) $y' = y''$ and $x' = x''$.

Lemma 3.3. *The quotient space $X_n \times Y/\sim$ is compact and Hausdorff.*

Proof. For convenience, denote $X_n \times Y$ by Z. The compactness of $Z/\sim$ follows from the compactness of Z. To show that $Z/\sim$ is Hausdorff, it suffices to show that the subset $R := \{(z_1, z_2) \in Z^2 | z_1 \sim z_2\}$ of Z^2 is closed [8, Theorem 8.2]. Let (z_1, z_2) be in $Z^2 \backslash R$. Thus $z_1 \nsim z_2$. Use (x', y') and (x'', y'') to denote z_1 and z_2 respectively.

Case 1. If $y' \neq y''$, then there exist two open subsets U and V of Y such that $y' \in U$, $y'' \in V$ and $U \cap V = \emptyset$. Thus $(X_n \times U) \times (X_n \times V)$ is an open neighborhood of (z_1, z_2) and is disjoint with R.

Case 2. If $y' = y''$, then $x' \neq x''$, and $y' \notin Y_1$. Since Y_1 is closed and Y is compact Hausdorff, there exists an open subset U of Y containing y' and U is disjoint with Y_1. Consequently $(\{x'\} \times U) \times (\{x''\} \times U)$ is an open neighborhood of (z_1, z_2) and is disjoint with R.

Combining Case 1 and Case 2, we obtain that $Z^2 \backslash R$ is open. Hence R is closed and $Z/\sim$ is Hausdorff. □

Lemma 3.4. *If an element F of $C(X_n \times Y)$ satisfies that $F(x_i, y') = F(x_j, y')$ for some y' in Y and all $1 \le i, j \le n$, then $\alpha(F)(x_k, y') = F(x_j, y')1_{A_n}$ for all $1 \le j, k \le n$.*

Proof. Note that F can be written as $\sum_{i=1}^n e_i \otimes f_i$ where $f_1, ..., f_n$ are in $C(Y)$. If $F(x_i, y') = F(x_j, y')$, then $f_i(y') = f_j(y')$. We obtain that

$$\begin{aligned}\alpha(F)(x_k, y) &= (ev_k \otimes ev_y \otimes id)\alpha(\sum_{i=1}^n e_i \otimes f_i) \\ &= \sum_{i=1}^n (ev_k \otimes ev_y \otimes id)\alpha(e_i \otimes f_i) \\ &= \sum_{i=1}^n \sum_{l=1}^n (ev_k \otimes ev_y \otimes id)(e_l \otimes f_i \otimes a_{li}) \\ &= \sum_{i=1}^n f_i(y)a_{ki}\end{aligned}$$

for any $y \in Y$ and $1 \le k \le n$. Since $f_i(y') = f_j(y')$ for all $1 \le i, j \le n$, and $\sum_{i=1}^n a_{ki} = 1_{A_n}$ for all $1 \le k \le n$, we get

$$\alpha(F)(x_k, y') = \sum_{i=1}^n f_i(y')a_{ki} = f_j(y') \sum_{i=1}^n a_{ki} = f_j(y')1_{A_n} = F(x_j, y')1_{A_n}$$

for all $1 \le j, k \le n$. This completes the proof. □

Note that $C(X_n \times Y/\sim)$ is a C*-subalgebra of $C(X_n \times Y)$.

Proposition 3.5. *When the action α is restricted on $C(X_n \times Y/\sim)$, it induces an action $\widetilde{\alpha}$ of A_n on $X_n \times Y/\sim$.*

Proof. We first prove the following:

$$\alpha(C(X_n \times Y/\sim)) \subseteq C(X_n \times Y/\sim) \otimes A_n. \qquad (1)$$

Since $C(X_n \times Y/\sim) \otimes A_n \cong C(X_n \times Y/\sim, A_n)$ and $C(X_n \times Y) \otimes A_n \cong C(X_n \times Y, A_n)$, an element c of $C(X_n \times Y) \otimes A_n$ belongs to $C(X_n \times Y/\sim) \otimes A_n$ if and only if $(ev_k \otimes ev_y \otimes id)(c) = (ev_l \otimes ev_y \otimes id)(c)$ for all $1 \le k, l \le n$ and $y \in Y_1$.

Therefore, to prove (1), it suffices to show that

$$(ev_k \otimes ev_y \otimes id)\alpha(F) = (ev_l \otimes ev_y \otimes id)\alpha(F)$$

for all $1 \le k, l \le n$, $y \in Y_1$ and F in $C(X_n \times Y/\sim)$.

Let F be in $C(X_n \times Y/\sim)$. Then F can be written as $\sum_{i=1}^n e_i \otimes f_i$ for $f_i \in C(Y)$ satisfying that $f_i(y) = f_j(y)$ for all $1 \le i, j \le n$ and

$y \in Y_1$. By Lemma 3.4, we have

$$(ev_k \otimes ev_y \otimes id)\alpha(\sum_{i=1}^{n} e_i \otimes f_i) = f_j(y)1_{A_n} = (ev_l \otimes ev_y \otimes id)\alpha(\sum_{i=1}^{n} e_i \otimes f_i)$$

for all $y \in Y_1$ and $1 \leq j, k, l \leq n$. This proves (1).

Next we verify the density condition, that is, $\alpha(C(X_n \times Y/\sim))(1 \otimes A_n)$ is dense in $C(X_n \times Y/\sim) \otimes A_n$.

It is enough to show that $F \otimes a$ is in the closure of $\alpha(C(X_n \times Y/\sim))(1 \otimes A_n)$ for all F in $C(X_n \times Y/\sim)$ and a in A_n. Denote $F \otimes a - \alpha(F)(1 \otimes a)$ by G. Note that F can be written as $\sum_{i=1}^{n} e_i \otimes f_i$ for $f_i \in C(Y)$ satisfying that $f_i(y) = f_j(y)$ for all $1 \leq i, j \leq n$ and $y \in Y_1$. By Lemma 3.4, we have $\alpha(F)(x_i, y) = F(x_j, y)1_{A_n}$ for all x_i, x_j in X_n and y in Y_1. Thus $G|_{X_n \times Y_1} = 0$. For arbitrary $\varepsilon > 0$, let U be an open subset of Y containing Y_1 and satisfying that $\|G(x_i, y)\| < \varepsilon$ for all $(x_i, y) \in X_n \times U$. By Urysohn's Lemma, there exists an f in $C(Y)$, such that $f|_{Y_1} = 0$, $f|_{Y \backslash U} = 1$ and $0 \leq f \leq 1$. Denote $1 \otimes f \in C(X_n \times Y)$ by H_ε. Then $H_\varepsilon|_{X_n \times Y_1} = 0$ and $H_\varepsilon(x_i, y) = 1$ for all x_i in X_n and y in $Y \backslash U$. It follows from Lemma 3.4 that $(\alpha(H_\varepsilon)G - G)(x_i, y) = 0$ for all $x_i \in X_n$ and $y \in Y \backslash U$. Since $0 \leq H_\varepsilon \leq 1$, for $(x_i, y) \in X_n \times U$, we have

$$\|(\alpha(H_\varepsilon)G - G)(x_i, y)\| \leq \|\alpha(H_\varepsilon) - 1\| \|G(x_i, y)\| < \varepsilon.$$

Hence $\|\alpha(H_\varepsilon)G - G\| < \varepsilon$. Moreover, since α is an action of A_n on $X_n \times Y$, we have that $\alpha(C(X_n \times Y))(1 \otimes A_n)$ is dense in $C(X_n \times Y) \otimes A_n$. So there exist $F_i \in C(X_n \times Y)$ and $a_i \in A_n$ for $1 \leq i \leq m$ where m is a positive integer such that $\|G - \sum_{i=1}^{m} \alpha(F_i)(1 \otimes a_i)\| < \varepsilon$. It follows from $0 \leq H_\varepsilon \leq 1$ that $\|\alpha(H_\varepsilon)G - \alpha(H_\varepsilon) \sum_{i=1}^{m} \alpha(F_i)(1 \otimes a_i)\| < \varepsilon$. Hence

$$\begin{aligned}
&\|F \otimes a - \alpha(F)(1 \otimes a) - \sum_{i=1}^{m} \alpha(H_\varepsilon F_i)(1 \otimes a_i)\| \\
&= \|G - \sum_{i=1}^{m} \alpha(H_\varepsilon F_i)(1 \otimes a_i)\| \\
&\leq \|G - \alpha(H_\varepsilon)G\| + \|\alpha(H_\varepsilon)G - \sum_{i=1}^{m} \alpha(H_\varepsilon F_i)(1 \otimes a_i)\| < 2\varepsilon.
\end{aligned}$$

Note that $H_\varepsilon F_i|_{X_n \times Y_1} = 0$ for all $1 \leq i \leq m$. Thus $H_\varepsilon F_i$ is in $C(X_n \times Y/\sim)$ for all $1 \leq i \leq m$. It follows that $\alpha(F)(1 \otimes a) + \sum_{i=1}^{m} \alpha(H_\varepsilon F_i)(1 \otimes a_i)$ is in $\alpha(C(X_n \times Y/\sim))(1 \otimes A_n)$. Since $\varepsilon > 0$ is arbitrary, we conclude that $F \otimes a$ is in the closure of $\alpha(C(X_n \times Y/\sim))(1 \otimes A_n)$. This completes the proof. □

Remark 3.6. In fact, in the proof of Proposition 3.5, we can prove the density condition more briefly by using [9, Remark 2.3]. But this method is more involved and uses the Hopf $*$-subalgebra of A_n.

Theorem 3.7. *If $Y_1 \neq Y$, the action $\widetilde{\alpha}$ of A_n on $X_n \times Y/\sim$ is faithful.*

Proof. Suppose $Y_1 \neq Y$. Take a point y_0 in Y but not in Y_1. Since Y is compact Hausdorff, there exists $f \in C(Y)$ such that $f(y_0) = 1$ and $f|_{Y_1} = 0$. Note that $e_i \otimes f$ is in $C(X_n \times Y/\sim)$ for any $1 \leq i \leq n$. Suppose Q is a compact quantum quotient group of A_n such that $\widetilde{\alpha}$ is an action of Q on $X_n \times Y/\sim$. Then for any $1 \leq k \leq n$,

$$\begin{aligned}(ev_k \otimes ev_{y_0} \otimes id)\alpha(e_i \otimes f) &= (ev_k \otimes ev_{y_0} \otimes id)(\sum_{l=1}^{n} e_l \otimes f \otimes a_{li}) \\ &= f(y_0)a_{ki} = a_{ki}\end{aligned}$$

is in Q. Since i is arbitrarily chosen, we get $a_{ki} \in Q$ for any $1 \leq k, i \leq n$. Thus $Q = A_n$. Therefore $\widetilde{\alpha}$ is faithful. □

An action α of a compact quantum group (A, Δ) on a unital C*-algebra B is called *ergodic* if $\{b \in B | \alpha(b) = b \otimes 1_A\} = \mathbb{C}1_B$.

Proposition 3.8. *If Y contains at least two points, the action $\widetilde{\alpha}$ is not ergodic.*

Proof. Since Y consists of at least two points, there exist a non constant function $f \in C(Y)$. Then $1 \otimes f$ is in $C(X_n \times Y \setminus \sim)$ and not constant. Also

$$\widetilde{\alpha}(1 \otimes f) = 1 \otimes f \otimes 1.$$

This shows that $\widetilde{\alpha}$ is not ergodic. □

Proposition 3.9. *If Y is connected and Y_1 is nonempty, then $X_n \times Y/\sim$ is connected.*

Proof. As before, denote $X_n \times Y$ by Z. Take any nonempty closed and open subset U of $Z/\sim$. Denote by π the quotient map from Z onto $Z/\sim$. It follows that $\pi^{-1}(U)$ is a nonempty, closed and open subset of Z. Since X_n is finite, we obtain that $\pi^{-1}(U) = \bigcup_{x_i \in X'} \{x_i\} \times A_i$ where X' is a nonempty subset of X_n, and every A_i is a nonempty closed and open subset of Y. Since Y is connected, we have $A_i = Y$ for all $x_i \in X'$. Take $y \in Y_1$ and $x_i \in X'$. Let $x_j \in X_n$. Then $\pi(x_j, y) = \pi(x_i, y) \in U$. Thus x_j is in X'. Therefore $X' = X_n$ and $U = Z/\sim$. So $Z/\sim$ is connected. □

By Theorem 3.7 and Proposition 3.9, if we take a nonempty proper closed subset Y_1 of a connected compact Hausdorff space Y, then we get a faithful action of A_n on a compact connected space $X_n \times Y/\sim$. To be more specific, we list some examples of $X_n \times Y/\sim$.

Example 3.10.

(1) If $Y = [0, 1]$ and $Y_1 = \{0\}$, then $X_n \times Y/\sim$ is a wedge sum of n unit intervals by identifying $(x_i, 0)$ to a single point for all $1 \le i \le n$. In this case $X_n \times Y/\sim$ is a contractible compact metrizable space.

(2) If $Y = S^1$ is a circle, and $Y_1 = \{y_0\}$ for some point y_0 in S^1, then $X_n \times S^1/\sim$ will be the n circles touching at a point, which is a connected compact metrizable space whose fundamental group is the free group with n generators.

Remark 3.11. By Theorem 3.7, the quantum permutation group A_n can act on the spaces in Example 3.10 faithfully. When $n \ge 4$, this gives us faithful genuine compact quantum group actions on connected compact metrizable spaces. This disproves the conjecture of Goswami [6] mentioned in the introduction. However, Proposition 3.8 tells us that these faithful genuine compact quantum actions on compact connected spaces are not ergodic. For this reason, we ask the following question:

Question 3.12. Are there any faithful ergodic genuine quantum group actions on compact connected spaces?

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