Blind Pulse Train Detection via Self-Convolution

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Abstract

We show that the self-convolution of a horizontally polarized pulse train with a constant pulse repetition frequency (PRF) is the same as its autocorrelation, only shifted in time, provided that the pulses are symmetric, making the waveform amenable to blind detection even in the presence of a constant Doppler shift.

Self-Convolution = Autocorrelation

Consider the pulse train $\mathfrak{z}(z) = z^0 + z^1 + z^2$.
Its autocorrelation is given by:

$$R_{\mathfrak{z}\mathfrak{z}}(z) = \left(z^0 + z^1 + z^2\right) \cdot \left(z^0 + z^{-1} + z^{-2}\right)$$
$$R_{\mathfrak{z}\mathfrak{z}}(z) = z^{-2} + 2z^{-1} + 3 + 2z^1 + z^2$$

Its self-convolution is given by:

$$S_{\mathfrak{z}\mathfrak{z}}(z) = \left(z^0 + z^1 + z^2\right) \cdot \left(z^0 + z^1 + z^2\right)$$
$$S_{\mathfrak{z}\mathfrak{z}}(z) = z^0 + 2z^1 + 3z^2 + 2z^3 + z^4$$

We see the relation:

$$S_{\mathfrak{z}\mathfrak{z}}(z) = z^2 R_{\mathfrak{z}\mathfrak{z}}(z)$$

For a train of order N $\mathfrak{z}(z) = z^0 + z^1 + \cdots + z^N$ we have:

$$S_{\mathfrak{z}\mathfrak{z}}(z) = z^N R_{\mathfrak{z}\mathfrak{z}}(z)$$

If we allow a non-zero pulse width p(z) and spread the train by a positive value m, we get:

$$S_{xx}(z) = z^{mN} R_{pp}(z) R_{\mathfrak{z}\mathfrak{z}}(z^m)$$

provided that p(z) is symmetric, i.e. that $p(z) = p(z^{-1})$.

In practice, this spreading operation could be performed by a Doppler shift, and so we see that self-convolution is a viable method for constant PRF symmetric pulse train detection even in the presence of a Doppler shift.

Self-Convolution Detector

We assume we receive a signal $x(t) = z(t) + w(t)$ where $w(t) \sim \mathcal{N}(0, \sigma^2)$. The self-convolution of the digitized x[n] can be represented as:

$$S_{xx}[n] = S_{zz}[n] + 2S_{zw}[n] + S_{ww}[n]$$

This leads to the following detection problem:

$$H_0: \quad S_{xx}[n] = S_{ww}[n]$$
$$H_1: \quad S_{xx}[n] = S_{zz}[n] + 2S_{zw}[n] + S_{ww}[n]$$

For a given threshold γ, the probability of false alarm can be expressed as:

$$P_{FA} = \frac{1}{(M-1)!} \sum_{k=0}^{M-1} \frac{(M+k-1)!}{2^{(M+k)} k! (M-k-1)!} \Gamma\left(M-k, \frac{\gamma}{2\sigma^2}\right)$$

where $\Gamma(M-k, \frac{\gamma}{2\sigma^2})$ is the upper incomplete gamma function, and $M = \frac{L}{4}$ with L being the number of samples of x[n] used for self-convolution.

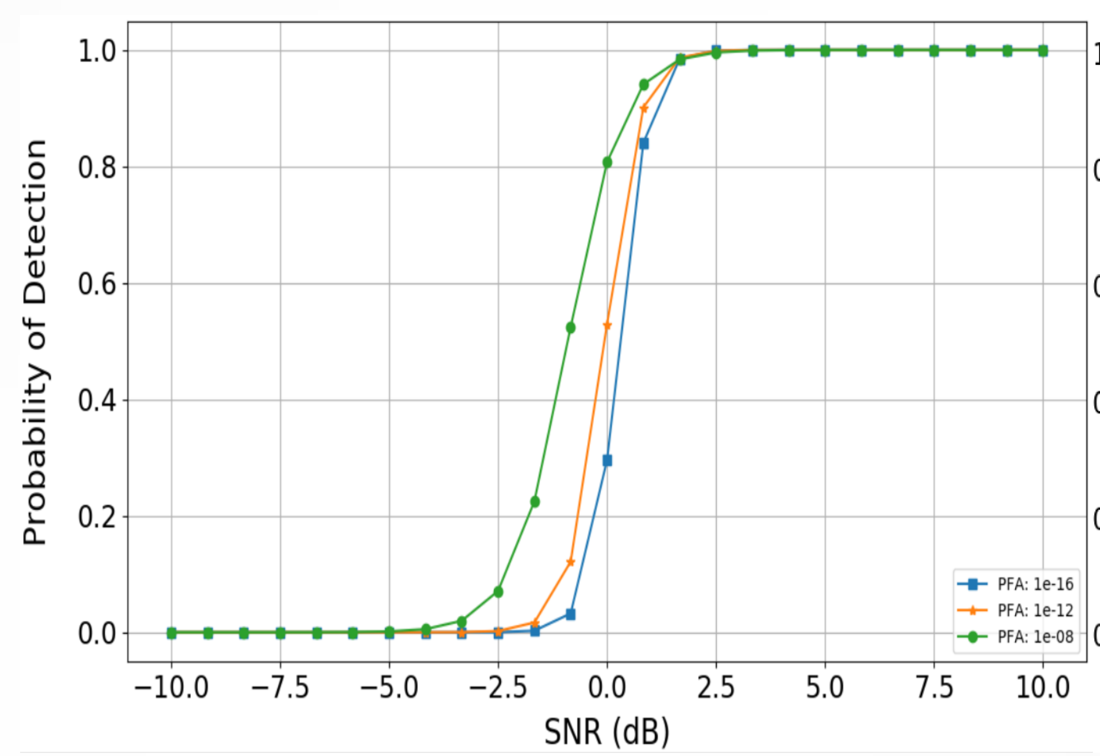

Fig. Self-Convolution

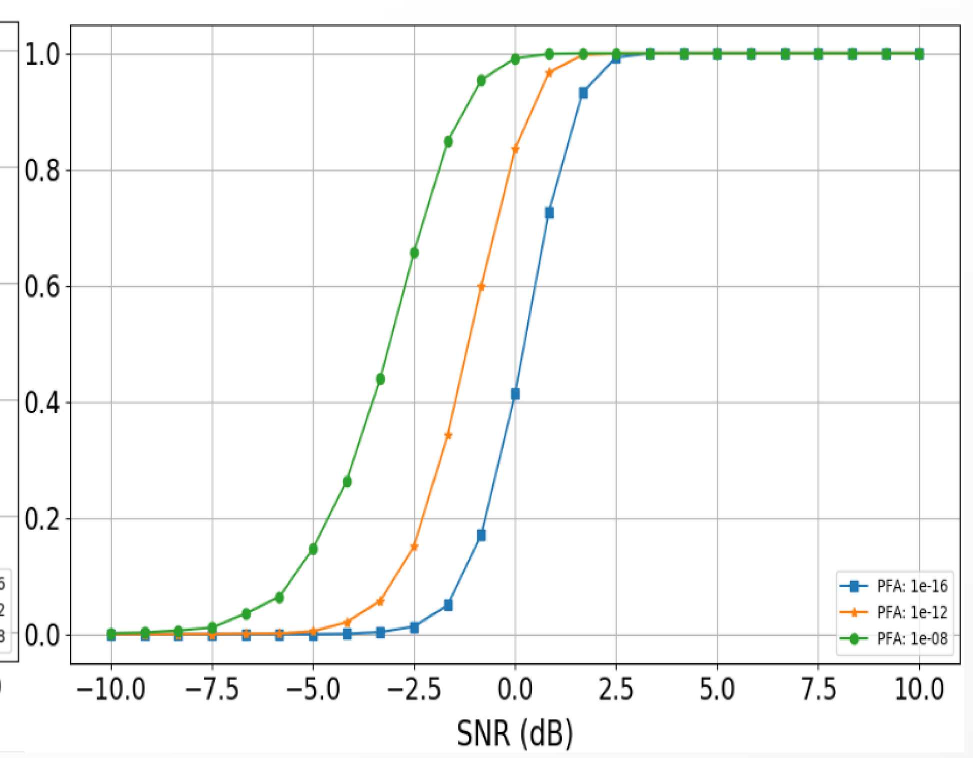

Fig. Matched Filter

We see that detecting x[n] using self-convolution is more difficult than when using a matched filter, however, we gain blind detection and a robustness to a constant Doppler shift.

PRF Estimation

Once we have detected the waveform, we can estimate its PRF using a logarithmic frequency matched filter. In sequence notation, the logarithmic frequency power spectrum of the baseband x[n] is given by:

$$|X|^2(z) = |P(z)|^2 \cdot z^{-log(f_{PRF})} \sum_{k=1}^{\infty} z^{-log(k)}$$

If we define our matched filter as:

$$M(z) = \sum_{k=1}^{\infty} z^{-log(k)}$$

then the autocorrelation can be expressed as:

$$R_{XM}(z) = |X|^2(z) \cdot M^*(z^{-1})$$

The estimated PRF is therefore :

$$f_{PRF} = e^{\text{argmax}\{R_{XM}(log(f))\}}$$

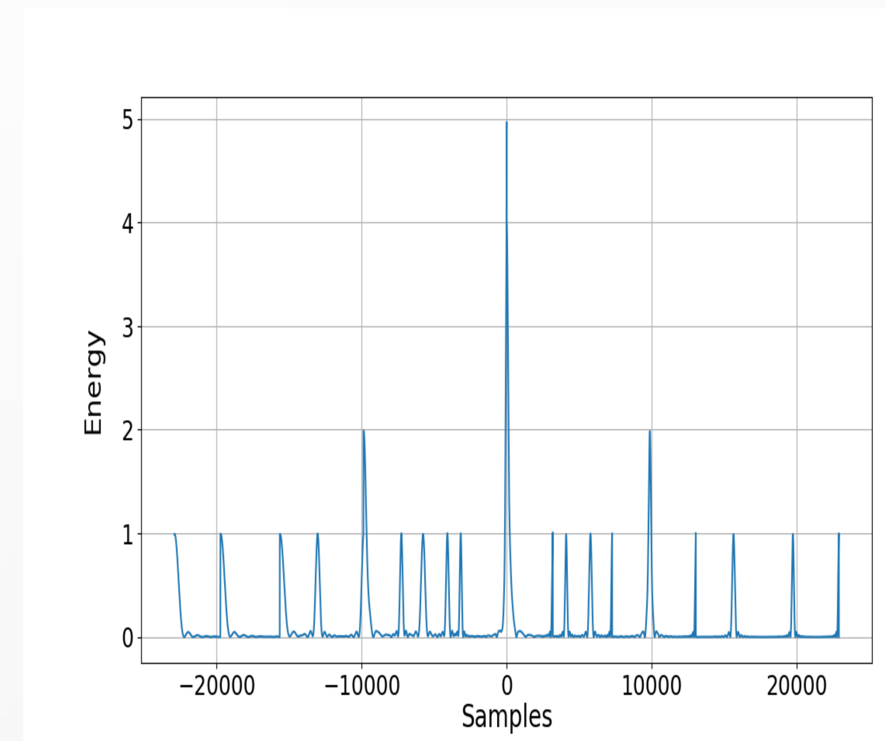

Fig. Matched Filter Result

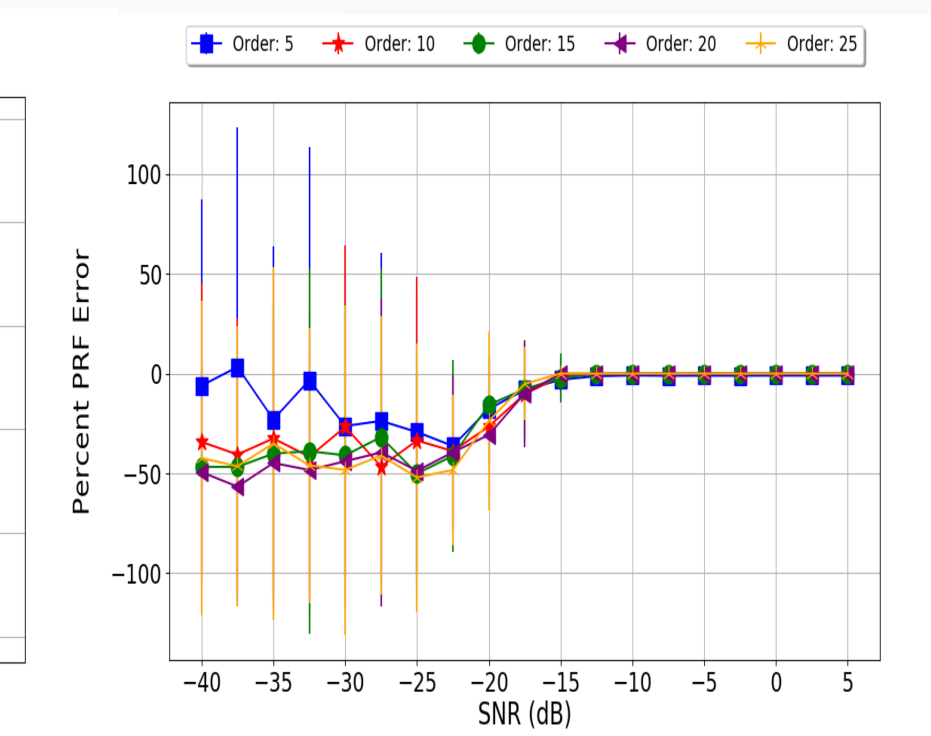

Fig. PRF Estimation Accuracy

We see that PRF estimation is accurate above -15 dB SNR.

Conclusion

Future work will be focused on extending these results to more general pulse trains, such as pulse trains with missing samples, jittered pulse trains, and trains with non-symmetric pulse shapes.