Robust Attitude Estimation from Uncertain Observations of Inertial Sensors using Covariance Inflated Multiplicative Extended Kalman Filter

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Abstract—This paper presents an attitude estimation method from uncertain observations of inertial sensors, which is highly robust against different uncertainties. The proposed method of covariance inflated multiplicative extended Kalman filter (CIMEKF) takes advantage of non-singularity of covariance in MEKF as well as a novel covariance inflation approach to fuse inconsistent information. The proposed covariance inflation approach compensates the undesired effect of magnetic distortion and body acceleration (as inherent biases of magnetometer and accelerometer sensors data, respectively) on the estimated attitude. Moreover, the CI-MEKF can accurately estimate the gyro bias. A number of simulation scenarios are designed to compare the performance of the proposed method with the state of the art in attitude estimation. The results show the proposed method outperforms the state of the art in terms of estimation accuracy and robustness. Moreover, the proposed CI-MEKF method is shown to be significantly robust against different uncertainties such as large body acceleration, magnetic distortion, and errors in the initial condition of the attitude.

Index Terms—Attitude Estimation, IMU, MEKF, Covariance Inflation, Sensor Fusion, Motion Tracking.

I. INTRODUCTION

Attitude estimation from inertial sensors is vastly used for motion tracking and control purposes in applications ranging from daily activity monitoring [1]–[3] to autonomous navigation of unmanned vehicles [4], robotics and augmented reality [5]. Inertial measurement unit (IMU) are usually used for this purpose that includes three axis accelerometer, gyroscope and magnetometer. Among IMU sensors, accelerometer and particularly magnetometer are subjected to some uncertainties. This entails serious technical challenges that limit the accuracy level of attitude estimation methods [6], [7].

Various methods have been used for IMU based attitude estimation such as extended Kalman filter [8]–[11], multiplicative extended Kalman filter (MEKF) [12], [13] also known as indirect Kalman filter [14], [15], unscented Kalman filter and nonlinear filters [16], [17], as well as optimization based methods like steepest decent [18].

Attitude estimation methods are often composed of two steps. In the prediction step, starting from an initial estimation, the attitude is usually calculated by integrating the gyroscope output. However, this estimation diverges due to cumulative integration error as well as gyroscope bias. The attitude can also be estimated from other sensor’s measurement that can be used in the second step to update the prediction in the first step. For instance, in spacecraft attitude estimation, attitude sensors such as star-trackers provide high fidelity attitude data to estimate gyro bias [12], [13], [16]. However, attitude estimation with the help of IMU consisting of accelerometer and magnetometer along with gyro is more complex as neither of these sensors provide high fidelity attitude information. Accelerometer and magnetometer are desired to measure the Earth’s gravity and the magnetic field, respectively. Thus, in the presence of body acceleration and magnetic distortion, their attitude estimation error can become large. Therefore, the fusion of accelerometer and magnetometer information with gyroscope output is one of the main challenges in any IMU-based attitude estimation algorithm. All these factors prohibit the straightforward use of various methods [12], [13], [16] for applications involving attitude estimation using a single IMU.

Different workarounds have been used to mitigate this uncertainty issue of accelerometer and magnetometer data, which can be categorized into four approaches. In the first approach, Sabatini et al. [8]–[10] considers the magnetometer and/or accelerometer biases as augmented state variables and estimate them along with the attitude. However, it suffers from observability issue and as a result, the estimated biases become erroneous and the overall estimation accuracy decreases [10].

This technique has also been complimented with some thresholding criteria to apply the update step only at instances in which the errors between predicted and measured values of accelerometer and magnetometer are negligible [8]–[10], [17]. Such criteria may discard the accelerometer or magnetometer data for an uncertain period of time during which the cumulative error may significantly increase. Moreover, functionality of these thresholding criteria often depends on availability of an accurate estimation of the attitude that itself is prone to cumulative error. This indicates a risk of getting trapped inside a viscous circle that lead to large errors. Vector selection strategy proposed by Zhang et al. [19] is one of the recent works in this category, which attempts to address this issue.

Suh [14] has proposed a new approach to compensate the effect of body acceleration by adaptive estimation of its covariance and incorporating it into the update step. While it is effective, this approach suffers from an inherent time delay that limits its robustness. This is because the adaptive method used for covariance estimation requires at least a few measurement samples. In the final approach, some complementary measures...
are taken to isolate the roll and pitch angles from magnetic distortion and divert all the adverse effect of magnetic distortion to yaw angle [14], [15], [18].

In this paper, a method of covariance inflated multiplicative extended Kalman filter (CI-MEKF) is proposed based on MEKF [12] and is benefited from a new covariance inflation approach. The non-singularity of state error covariance of MEKF [12], [13] enables our method to effectively use covariance inflation to reject observation uncertainty. The performance of the proposed CI-MEKF method is compared with the state of the art in IMU based attitude estimation. Our proposed method demonstrates a higher accuracy as well as a fast convergence rate in presence of body acceleration and magnetic distortion. Moreover, it shows a high robustness against gyro biases and errors in attitude initial condition.

II. METHOD

To have consistent notations, we first present the conventions used in this paper to manipulate quaternion, as the main variable representing the attitude. We then describe the sensor models, and finally the proposed CI-MEKF method.

A. Quaternion Representation of Attitude & Motion Dynamics

In order to determine the attitude of the sensor’s body frame (B with x, y, z bases) with respect to the inertial frame (N with X, Y, Z bases), one can use a unit quaternion \( q = [q_0 \ \hat{q}^T]^T \), in which \( q_0 \) is the scalar part and \( \hat{q} = [q_1 \ q_2 \ q_3]^T \) is the vector part of the quaternion, as described by (1).

\[
p_b = q^{-1} \otimes p_N \otimes q
\]  

where, \( p \) is an arbitrary vector expressed in the body \((p_b)\) and inertial frame \((p_N)\). Symbols \( \otimes \) and \( \otimes^{-1} \) denote the quaternion multiplication and quaternion inverse operators, respectively.

We can also rewrite (1) in the matrix form of (2).

\[
p_b = C(q) p_N
\]  

where, \( C(q) \equiv 2 \begin{bmatrix} q_0^2 + q_1^2 - \frac{1}{2} & q_1q_2 + q_0q_3 & q_1q_3 - q_0q_2 \\ q_1q_2 - q_0q_3 & q_0^2 + q_2^2 - \frac{1}{2} & q_2q_3 + q_0q_1 \\ q_1q_3 + q_0q_2 & q_2q_3 - q_0q_1 & q_0^2 + q_3^2 - \frac{1}{2} \end{bmatrix} \)

\[
\dot{q} = \frac{1}{2} q \otimes \omega
\]  

Equation (4) is the rate of change of \( q \) that is used to predict the future value of \( q \) using its current value and \( \omega \), where \( \omega = [\omega_x \ \omega_y \ \omega_z]^T \) is the instantaneous angular velocity expressed in the body frame. For implementation purposes, the matrix form of (4) is obtained as (5) using quaternion algebra.

\[
\dot{q} = \frac{1}{2} \Omega(\omega) q, \quad \Omega(\omega) = \begin{bmatrix} 0 & -\omega^T \\ \omega & -[\omega \times] \end{bmatrix}
\]  

For an arbitrary vector \( v = [v_1 \ v_2 \ v_3]^T \), \( [v \times] \) is the outer product tensor which is defined by (6).

\[
[v \times] = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}
\]  

B. Sensors Model

The model of three inertial sensors used for attitude estimation are described in this section. The sensor model for gyroscope is given by (7), where \( y_g \) is the gyro output. \( v_g \) is the measurement noise modeled as a zero-mean i.i.d. Gaussian process with \( \sigma_g \) standard deviation and \( b_g \) is the gyro bias which is modeled as a random walk process.

\[
y_g = \omega + b_g + v_g
\]  

Accelerometer and magnetometer provide body frame observations from Earth’s gravitational and magnetic fields. Their calibrated models is represented by (8) and (9), respectively.

\[
y_a = C(q)(-g_N + a_N) + v_a
\]  

\[
y_m = C(q)\omega (I - r^2) m_N + b_m + v_m
\]  

The accelerometer measures the gravitational field of earth \( g_N \) as well as the body acceleration \((a_N)\) caused by relative motion of the sensor with respect to the inertial frame. \( a_N \) is a time varying vector and is regarded as a disturbance in this context. Finally \( v_a \) and \( v_m \) are the accelerometer and magnetometer’s measurement noise that are modeled as zero-mean i.i.d. Gaussian processes with standard deviations of \( \sigma_a \) and \( \sigma_m \), respectively.

In equation (9), the magnetic filed of Earth is denoted by \( m_N \) that is a constant vector at each geographical location. The calibrated model of the magnetometer compensates the soft-iron effect. However, it is still subjected to two types of uncertainties. The first one is the hard Iron effect caused by ferromagnetic objects approaching the sensor, which is modeled by a time-varying bias vector \((b_m)\). Another uncertainty is the weakening effect on \( m_N \) represented by the coefficient \( \Gamma \), which is due to surrounding ferromagnetic infrastructure. Unlike \( \Gamma \) that is almost constant inside a building, changes in \( b_m \) is usually abrupt and intermittent particularly in indoor applications [20], [21]. Combination of \( b_m \) and \( \Gamma \) cause a magnetic distortion \( d_N = b_m - r^2 m_N \) which is regarded as a disturbance in our filter design.

C. Attitude Estimation Algorithm

The proposed method is based on MEKF quaternion based attitude estimation method first introduced by Lefferts et al. [12]. In the context of attitude estimation, regular Kalman filter is referred to as an additive extended Kalman filter (AEKF) [11]. Unlike AEKF, where the error is defined as the difference between the true and estimated quaternion, the error in MEKF \( \delta q \) is defined as the multiplication of the estimated quaternion inverse \((\hat{q}^{-1})\) and the true quaternion \((q)\) as shown in (10).

\[
\delta q = \hat{q}^{-1} \otimes q = q = \hat{q} \otimes \delta q
\]  

The multiplicative error can be further expressed as \( \delta q = [\delta q_0 \ \delta q^T]^T \), where \( \delta q_0 \) and \( \delta q = [\delta q_1 \ \delta q_2 \ \delta q_3]^T \) are its scalar and vector parts, respectively.

In addition to the prediction and update states that are accustomed in any kalman filter, the proposed CI-MEKF method includes an extra step of \textit{Covariance Inflation} prior
to the update step. The main goal of covariance inflation is to enhance the performance of MEKF in presence of uncertainties such as time-varying biases in observations. The three steps of CI-MEKF are discussed in the following section.

1) Process Model & Prediction Step: According to (5), one can estimate \( \hat{q} \) (the expected value of \( q \)) by integrating (11) over time. For one sampling time period (\( T \)), this estimation is provided by (12) that is obtained by zero-hold assumption on \( \hat{\omega} \), where \( \hat{\omega} \) is the estimated angular velocity from gyroscope.

\[
\dot{\hat{q}} = \frac{1}{2} \hat{\dot{q}} \otimes \hat{\omega} \tag{11}
\]

\[
\hat{q}_k = e^{\frac{1}{2} \Omega (\hat{\omega}^T T) \hat{q}_{k-1}} \tag{12}
\]

Taking the derivative of the right side of (10) with respect to time and then substituting \( \hat{q} \) and \( \dot{\hat{q}} \) with (4) and (11), one can describe the dynamics of the multiplicative error as (13).

\[
\frac{d}{dt} \delta q = \frac{1}{2} (\delta q \otimes \hat{\omega} - \hat{\omega} \otimes \delta q) + \frac{1}{2} \delta q \otimes (\omega - \hat{\omega}) \tag{13}
\]

Using quaternion algebra, (13) can be described in the matrix form of (14), where \( \Delta \omega = \omega - \hat{\omega} \).

\[
\frac{d}{dt} \begin{bmatrix} \delta q_0 \\ \delta q_1 \\ \delta q_2 \\ \delta q_3 \end{bmatrix} = \begin{bmatrix} -[\hat{\omega} \times] \delta q - \frac{1}{2} \Delta \omega^T \delta q \\ -[\hat{\omega} \times] \delta q - \frac{1}{2} \Delta \omega^T \delta q \\ -[\hat{\omega} \times] \delta q - \frac{1}{2} \Delta \omega^T \delta q \end{bmatrix} \tag{14}
\]

For \( ||\delta q|| < \ll 1 \) and \( ||\Delta \omega|| \ll ||\hat{\omega}|| \), (14) is simplified as (15).

\[
\frac{d}{dt} \begin{bmatrix} \delta q_0 \\ \delta q_1 \\ \delta q_2 \\ \delta q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}
\]

According to (15), \( \delta q_0 \) must be a constant. Since \( \delta q \) remains considerably small, in MEKF \( \delta q_0 = 1 \cdot ||\delta q||^2 \) is assumed to be equal to one. Therefore, \( \delta q \) would be the only parameter describing the state error corresponding to the quaternion.

Moreover, an accurate estimation of \( \hat{\omega} \) is required to satisfy the \( ||\Delta \omega|| \ll ||\hat{\omega}|| \) assumption. To this end, the gyro bias \( b_g \) must be estimated according to (7). Assuming such an estimation is available as \( \hat{b}_g \), one can obtain \( \hat{\omega} \) using (16).

\[
\hat{\omega} = \hat{y}_g - \hat{b}_g \tag{16}
\]

To estimate the gyro bias, \( b_g \) is modeled as a random walk process, for which \( \frac{d}{dt} b_g = w_b \) and \( w_b \) is an i.i.d. zero-mean Gaussian processes with \( \sigma_b^2 I_{3x3} \) covariance matrix. Accordingly, \( \delta q \) is augmented by \( b_g \) to establish the state vector of MEKF. Equation (17) represents the MEKF state vector and its corresponding state transition equation.

\[
x = \begin{bmatrix} \delta q_0 \\ \delta q_1 \\ \delta q_2 \\ \delta q_3 \\ b_g \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} f_{3x3} \\ 0 & 0 & 0 & \frac{1}{2} f_{3x3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \Phi_1 & \Phi_2 & I_{3x3} & F_k \end{bmatrix} \begin{bmatrix} x_{k-1} \\ P_{k-1} \\ \psi_1 I_{3x3} \\ \psi_2 I_{3x3} \end{bmatrix} + \begin{bmatrix} Q_k \\ \psi_3 I_{3x3} \end{bmatrix} \tag{17}
\]

According to (7), \( v_g \) is assumed to be a i.i.d. zero-mean Gaussian process with \( \sigma_v^2 I_{3x3} \) covariance matrix.

In order to propagate the error covariance matrix for a sampling period of \( T \), (18) is obtained by integrating (17) over time. The parameters of (18) are described in (19).

\[
\hat{x}_k = \begin{bmatrix} 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ \Phi_1 & \Phi_2 & I_{3x3} & F_k \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & I_{3x3} \end{bmatrix} \hat{x}_{k-1} + \begin{bmatrix} P_{k-1} \\ F_k P_{k-1} \end{bmatrix} \begin{bmatrix} F^T_k + Q_k \\ \psi_1 I_{3x3} \\ \psi_2 I_{3x3} \end{bmatrix} \tag{18}
\]

The closed form solution of (12) and (19) are used here [14]. In MEKF, the first three elements of \( x_k^c \) corresponding to \( \hat{q} \) are set to zero at the prediction step since the estimated quaternion (\( \hat{q} \)) obtained from (11) is assumed to be unbiased.

2) Observation Model & Update Step: The filter states and the error covariance are corrected twice during the update step of CI-MEKF, first using accelerometer’s output and then magnetometer’s. Since \( q = \hat{q} \otimes \delta q \) and we use inverse rotation to obtain (3), \( C(q) \) can be factorized as (20).

\[
C(q) = C(\delta q) C(\hat{q}) \tag{20}
\]

Considering that the second-order terms are negligible \( (\delta q_i \delta q_j \approx 0) \), \( C(\delta q) \) can be approximated as (21) using (3).

\[
C(\delta q) = \begin{bmatrix} 1 & -2\delta q_3 & 2\delta q_2 \\ -2\delta q_3 & 1 & -2\delta q_1 \\ 2\delta q_2 & -2\delta q_1 & 1 \end{bmatrix} + \Delta^2(\delta q) \approx I - 2[\hat{\delta} q \times] \tag{21}
\]

Let \( p_n \) be a known constant vector in the inertial frame, which is observed by a sensor as \( (p_n^w) \) in the body frame with measurement error \( (v) \). One can substitute (21) in (20) and then in (2) to obtain (22) and eventually the observation model of (23).

\[
p_n^w = \hat{p}_B + 2[\hat{p}_B \times] \hat{\delta} q + v, \quad \hat{p}_B = C(\hat{q}) p_N \tag{22}
\]

where \( y_k = H_k x_k + v_k, \quad H_k = [2[\hat{p}_B \times] 0_{3x3}] \tag{23} \)

the difference between sensor measurement of the known vector (either acceleration or magnetic field) and its estimated value in the body frame \( (p_n^w - \hat{p}_B) \).

The observer model of (23) is used to update the filter state from the accelerometer and magnetometer observations. We use can use (23) for accelerometer by substituting \( p_n^w = y_a \) and \( p_N = -g \). In this case, the observer model error would be \( v = v_a + a_B \) according to (8), where \( a_B = C(q) a_w \) is the body acceleration expressed in body frame. Similarly, one can substitute \( p_n^w = y_m \), \( p_N = m_N \), in (23) for the magnetometer. The observer model error will be \( v = v_m + d_m \) where \( d_m = C(q) d_N \) is the magnetic distortion expressed in body frame.

At the update step, Kalman filter equations of (24) are used to correct the cumulative error of the prediction step.

\[
\begin{align*}
\hat{y}_k &= y_k - H_k \hat{\chi}_k \\
S_k &= H_k P_k H_k^T + R_k \\
K_k &= P_k H_k^T S_k^{-1} \\
x_k &= x_{k-1} + K_k \hat{y}_k \\
P_k &= (I - K_k H_k) P_k
\end{align*} \tag{24}
\]

The information of the estimated \( \hat{\delta} q \) is used to update \( \hat{q} \) via (25). According to the right side of (10), the new estimation will be closer to the true \( (q) \).

\[
\hat{q} \leftarrow \frac{1}{||\hat{\delta} q||} \hat{\delta} q, \quad \hat{q} \leftarrow \hat{q} \otimes \delta q, \quad \hat{q} \leftarrow \hat{q} \tag{25}
\]

The main issue of implementing this method is the determination of the error covariance \( (R_k) \) in (24). One can simply assume that \( R_k = R_a \) for accelerometer and \( R_k = R_m \) for magnetometer. However, this simplistic assumption ignores uncertainties such as time varying biases due to body accel-
Inflated Measured ellipsoid of Predicted ellipsoid of residual ellipsoid Residual error ($\varepsilon$) uncertainties such as magnetometer in (28). In the presence of observation between $y_1$ and $y_2$, the Mahalanobis distance (Fig. 1) as shown in (27), while the second one, $y_{h2}$, utilizes the measured output ($y_k$) as indicated in (28).

$$y_{h1} = N(H_k \hat{x}_k, H_k P_k^{-1} H_k^T)$$

$$y_{h2} = N(y_k, R_k)$$

$y_{h2}$ depends on the observation model error ($v_k$) which by assumption include only measurement noise ($v_m$) or $v_a$. This implies that $R_k = R_a$ for accelerometer and $R_k = R_m$ for magnetometer in (28). In the presence of observation uncertainties such as $a_B$ and $d_B$, this assumption is no longer valid and therefore the two hypothesis conflict.

As shown in Fig.1-a, the ellipsoids of $y_{h1}$ and $y_{h2}$ will not overlap in the case of hypotheses conflict. For two hypotheses with independent Gaussian distribution, one can use Mahalanobis distance ($r$) to examine a potential conflict between them [22]. Since, $E[y_{h1} \cdot y_{h2}] = \bar{y}_k$ and $E[(y_{h1} - y_{h2}) (y_{h1} - y_{h2})^T] = S_k$, the Mahalanobis distance between $y_{h1}$ and $y_{h2}$ can be calculated by (29).

$$r = \sqrt{\bar{y}_k^T S_k^{-1} \bar{y}_k}$$

For instance, the probability that $y_{h1}$ and $y_{h2}$ conflict when $r > 1$ and $r > 3$ is 0.20 and 0.97, respectively. These values are approximated by 3 DOF Chi-Square quantile function and can be regarded as a weak and strong inconsistency, respectively.

In order to properly fuse information of the two hypotheses, we need to resolve this inconsistency issue. For this purpose, one can inflate the covariance of less confident hypothesis. This technique is coined as covariance inflation (CI) in the literature. CI has been extensively used in different domains such as ensemble Kalman filtering for data assimilation [23] as well as simultaneous localization and mapping (SLAM) [24]. These applications mostly deal with model uncertainty and therefore enlarge $y_{h1}$ covariance by inflating the state covariance ($P_k$). In tracking problems with uncertain observations like ours, the uncertainty mostly stems from $y_{h2}$ due to unknown time-varying biases in observation model error (such as $a_B$ and $d_B$ in our case). As a result, to address the inconsistency issue, we enlarge $y_{h2}$ covariance by inflating the residual error covariance ($S_k$).

For this purpose, we have implemented two different covariance inflation approaches. The generalized covariance union (GCU) introduced by Reece and Roberts [25] is adopted to develop the first CI approach. Although GCU is originally designed to address multiple-hypothesis tracking problem, the closed form solution provided by GCU is applicable to our problem as well. The implemented GCU-CI method is presented in (30).

$$S_k = \beta (H_k P_k^{-1} H_k^T + \bar{y}_k \bar{y}_k^T) + R_k, \quad \beta = \begin{cases} 2, & r > 1 \\ \frac{(1+r)^2}{1+r^2}, & r \leq 1 \end{cases}$$

In this approach, we inflate $S_k$ in the direction of residual error $\bar{y}$ such that the inflated covariance ellipsoid tightly embraces the estimated ellipsoid as shown in Fig. 1-b. Comparing the new $S_k$ ellipsoid (Fig. 1-b) with that of Kalman filter (Fig. 1-a), we can see that unlike Kalman filter, the ellipsoid of the new $S_k$ embraces the $y_{h1}$ ellipsoid even when residual error is considerable. Using Sherman-Morrison formula, one can verify that after GCU-based CI, the new Mahalanobis distance between two hypotheses would be always less than one.

In GCU-based CI, the covariance is only inflated in the direction of $\bar{y}$. A more reasonable alternative to quantify the new uncertainty of the low-confidence hypothesis ($y_{h2}$) will be inflating $S_k$ in all directions with respect to the residual error ($\bar{y}$). Accordingly, we propose a diagonal covariance inflation approach, where the covariance is inflated in each direction with respect to the square of residual error in that specific direction (Fig. 1-c). Closed-form formulation of this approach to which we will refer as Independent CI is shown in (31).

$$S_k = H_k P_k^{-1} H_k^T + c \text{diag}(\bar{y}_k)^2 + R_k$$

The cardinality of $\bar{y}_k$ is denoted by $c$ which has a value
of 3 in our MEKF method. It can easily be proven that the proposed CI approach will reduce the Mahalanobis distance such that $r < 1$ is always guaranteed. This implies that both of the proposed covariance inflation approaches can resolve the inconsistency between the observation $y_{h,2}$ with the filter prediction $\hat{y}_{h,1}$.

Any of the consistent residual error covariances ($S_k$) represented by (30) and (31) can be used in the update step of Kalman filter instead of original $S_k$ described by (24).

D. Simulated Scenario

We considered the case of indoor human gait monitoring as a testbed to assess the performance of the proposed CI-MEKF method. We assume that the monitoring is done via a single IMU attached to the subject’s ankle. This is a challenging attitude estimation task as it contains large and rapidly changing body acceleration. Moreover, indoor magnetic distortion is particularly large and dynamic in the vicinity of the ground floor [21] and thereby, the IMU on the anklet is subjected to considerably high level of magnetic distortion.

A sample scenario including walking, moving up and down the stairs was considered in which minimum jerk model was used to estimate the foot motion with fifth order polynomials. Some features of natural walking (such as stride length, and maximum height of each step) were used to modify the ankle’s movement. The details of the trajectory generation algorithm can be found in [26]. The trace of the ankle and its Euler angles for the sample scenario are shown in Fig.2.

Fig. 2. a) The trace of the ankle and b) The Euler angles for the example human walking scenario designed in simulation environment

The output of the trajectory generation algorithm was the ground truth position and attitude data. These data were sampled at 100 Hz and were used to simulate the IMU raw data. To consider the effect of magnetic distortion, $d_N$ with abrupt and smooth patterns (shown in Fig. 3-a) were added to magnetometer data according to the sensor model (9). Also, the body acceleration (shown in Fig. 3-b) was extracted from ankle motion and was added to accelerometer data according to sensor model (8). Finally, the true angular velocity was obtained from the time derivative of the ground truth attitude data (Euler angles) which is illustrated in Fig. 3-c.

III. RESULTS

Using this simulated data, we considered six different cases listed in Table I, in which the gyro bias and initial condition of the attitude vary. Case 1 is an ideal case with no gyro bias and no error in attitude initial guess. The gyro bias in the first three cases are assumed to be zero. It is then monotonically increased from moderate bias (case 4) to a very extreme bias (case 6) to examine the robustness of the methods against gyro bias. These experiments are used to compare the performance of the proposed CI-MEKF method with the state of the art. For this purpose, we consider the methods presented by Suh [14], Madgwick et. al [18], [27], and Sabatini [8] to which they will be referred by their first author’s name. It should be noted that there are two implementation of the Madgwick method, one doesn’t consider the gyro bias [18] and the other one does [27]. We alternately utilized [27] instead of [18] for cases with gyro bias and distinguished them by a * superscript.

<table>
<thead>
<tr>
<th>Gyro bias (°/s)</th>
<th>Initial cond. (°)</th>
<th>Gyro bias (°/s)</th>
<th>Initial cond. (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case #</td>
<td>$[\mathbf{b}_g]$</td>
<td>$[\mathbf{d}_g, \mathbf{b}_g, \mathbf{b}_g]$</td>
<td>Case #</td>
</tr>
<tr>
<td>1</td>
<td>[0.0, 0.0, 0.0]</td>
<td>[0 0 0]</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>[0.0, 0.0, 0.0]</td>
<td>[10 10 10]</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>[0.0, 0.0, 0.0]</td>
<td>[20 20 20]</td>
<td>6</td>
</tr>
</tbody>
</table>

The true roll, pitch and yaw angles of the example scenario are equal to zero at the beginning of all experiments, we alternatively change the initial condition of the corresponding filter variables in order to test the sensitivity of the methods to different initial conditions.

The values of each case is used as the basis (mean value) for 50 trials of Monte Carlo simulation. In each Monte Carlo trial, the measurement noises are independently sampled from their corresponding distributions. We use the same measurement noise characteristics as Suh [14], which are $\sigma^2_n = 0.003$, $\sigma^2_n = 0.0056$, and $\sigma^2_m = 0.001$. We then contaminate the IMU data according to sensor models (7)-(9).
At each Monte Carlo trial, the initial condition of the filter variables corresponding to the attitude is sampled from a Gaussian distribution with 5° standard deviation and the mean value listed in Table I. Furthermore, in the last three cases, where the gyro bias is not zero, the \( b_g \) at each Monte Carlo trial is sampled from a Gaussian distribution with \( 1°/s \) standard deviation and the mean value of gyro bias. It is then added to the gyro output according to the sensor model (7).

CI-MEKF, Suh and Madgwick* are capable of estimating the gyro bias. For these methods the initial value of their corresponding gyro bias variables are set to zero in all the experiments. Furthermore, CI-MEKF, Sabatini and Suh require the initialization of the state covariance matrix \( P \). For this purpose, \( P \) is initially set to \( \text{diag}(0.25 I_{3 \times 3}, 0.01 I_{3 \times 3}) \) in CI-MEKF and Suh methods. For Sabatini method, it is set to \( \text{diag}(0.25 I_{4 \times 4}, 0.04 I_{4 \times 4}, 0.04 I_{4 \times 4}) \), the last two items of which correspond to accelerometer and magnetometer bias variables. These values are selected based on case 6 to ensure that initial condition error of 60° in Euler angles and 5°/s in gyro bias are encompassed by the initial covariance bound.

The mean and standard deviation of the errors of 50 Monte Carlo trials are calculated in terms of attitude RMS error (tracking error) and listed in Table II. Additionally the average steady-state error of gyro bias in the last 10 seconds is reported in the last three columns.

Comparing different methods in Table 2, it can be seen that the proposed CI-MEKF filter leads to the highest average accuracy in all three Euler angles and in all six cases. In case 1 which is the ideal condition, all methods perform well. In case 2 and 3, where we only have initial attitude error, CI-MEKF outperforms all other methods. The other methods demonstrate relatively small tracking error in pitch and roll but significantly large error in yaw. By increasing the gyro bias in case 4 to 6, CI-MEKF can maintain its tracking performance while the other methods fail to converge one after another.

Fig. 4 shows CI-MEKF can accurately follow the ground truth. Among the other methods, only Suh converges and has an accuracy comparable to CI-MEKF filter in roll and pitch angles. Moreover, a faster convergence of the CI-MEKF to the true Euler angle comparing to Suh method can be seen in the zoomed in box. Furthermore, the estimated gyro biases of CI-MEKF and Suh are compared to the true values in Fig. 5.

**TABLE II**

<table>
<thead>
<tr>
<th>Case</th>
<th>Method</th>
<th>Attitude RMS error (Euler angles)</th>
<th>Average error ( (b_g - \hat{b}_g) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CI-MEKF</td>
<td>0.6 ± 0.0</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Suh</td>
<td>0.8 ± 0.1</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Madgwick</td>
<td>2.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Sabatini</td>
<td>1.4 ± 0.7</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td>2</td>
<td>CI-MEKF</td>
<td>0.6 ± 0.0</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Suh</td>
<td>0.8 ± 0.1</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Madgwick</td>
<td>1.5 ± 0.4</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Sabatini</td>
<td>1.6 ± 0.8</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td>3</td>
<td>CI-MEKF</td>
<td>0.6 ± 0.0</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Suh</td>
<td>0.8 ± 0.1</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Madgwick</td>
<td>1.5 ± 0.4</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Sabatini</td>
<td>2.3 ± 1.0</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td>4</td>
<td>CI-MEKF</td>
<td>0.7 ± 1.0</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Suh</td>
<td>0.8 ± 0.1</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Madgwick</td>
<td>4.1 ± 1.2</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Sabatini</td>
<td>11.2 ± 5.7</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td>5</td>
<td>CI-MEKF</td>
<td>0.7 ± 1.0</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Suh</td>
<td>0.8 ± 0.1</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Madgwick</td>
<td>6.1 ± 1.8</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Sabatini</td>
<td>26.8 ± 8.7</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td>6</td>
<td>CI-MEKF</td>
<td>0.7 ± 1.0</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Suh</td>
<td>42.2 ± 14.1</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Madgwick</td>
<td>17.5 ± 4.6</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>Sabatini</td>
<td>89.4 ± 29.6</td>
<td>0.0 ± 0.0</td>
</tr>
</tbody>
</table>

*The algorithm presented in [27] that includes the gyro bias drift compensation is used.

Fig. 5 demonstrates that CI-MEKF method can accurately estimate the gyro bias in all directions, whereas Suh not only fail to estimate the bias in z direction but also has a much lower accuracy in \( x \) and \( y \) directions. Similar pattern can be
seen in all the cases with non-zero gyro bias when comparing the bias estimation error of the two methods listed in Table 2.

To further compare the performance of the proposed CI-MEKF with Suh, we consider case 4 which has a moderate bias term. The attitude error and $3\sigma$ bound of the two methods are compared in Fig. 6. Moreover, their average RMS error over the 50 Mote Carlo trials is shown in Fig. 7. For this purpose, the RMS error is calculated for a one second moving window with no overlap and then averages over all the Monte Carlo trials. It can be seen in Fig. 7 that CI-MEKF has less RMS error in most time windows. The Suh performance is slightly lower but still comparable in roll and pitch angles. However, it fails to correctly estimate yaw angle in presence of magnetic distortion.

\[
\delta q = \begin{bmatrix} \delta q_x \\ \delta q_y \\ \delta q_z \\ \delta q_w \end{bmatrix}
\]

\[
\Delta \omega = \begin{bmatrix} \Delta \omega_x \\ \Delta \omega_y \\ \Delta \omega_z \end{bmatrix}
\]

\[
|\Delta \omega \times \delta q|
\]

In the first two cases, where there is no gyro bias and initial attitude error is small, the accuracy of the GCU-based CI is slightly higher than Independent CI. However, in the present of gyro bias GCU-based CI not only has lower accuracy, but also fails to converge in case 5 and 6.

In fact, the performance of the two CI techniques can be interpreted based on the assumptions upon which they are developed. In GCU-based CI, the covariance is inflated only in the direction of residual error which implies that we maintain our confidence in the observation (magnetometer/accelerometer) in other directions perpendicular to the one’s of the residual error. However, in the presence of conflict between a new observation and its predicted value, one can infer that the observation is most probably biased. Therefore, the confidence on the whole probability distribution should be doubted. This can be compensated by inflating the covariance in all directions (similar to independent CI) and not just some specific directions (GCU-Based CI). Thereby, the independent CI technique is deemed more appropriate since it enlarge the covariance ellipsoid in all directions with respect to the magnitude of the residual error. This can be the main reason that independent CI showed the highest level of robustness against uncertainties.

Covariance inflation technique enables CI-MEKF method to adjust the uncertain observations (from magnetometer and accelerometer) such that they become consistent with the filter’s current state variables on which we have a higher confidence. There are two reasons for the state variables to be more trustworthy. First, they convey the memory of previous observations. Second, they incorporate new uncertain observations into the current state variables by filtering them through a relatively accurate process model. Therefore, CI technique helps the filter’s state variables to approach their true values and not to easily deviate from them. This results in CI-MEKF outperforming the state of the art in attitude estimation, which is evident in the results.

It should be noted that, in all of our comparison studies (Table II), we only presented the results of CI-MEKF with independent covariance inflation. To further analyze the effect of covariance inflation choice, we compare the performance of GCU-based and Independent CI in Fig. 8.

In the first two cases, where there is no gyro bias and initial attitude error is small, the accuracy of the GCU-based CI is slightly higher than Independent CI. However, in the present of gyro bias GCU-based CI not only has lower accuracy, but also fails to converge in case 5 and 6.

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A covariance inflation based MEKF (CI-MEKF) is proposed for attitude estimation from inertial sensors. Benefiting from the non-singular error covariance of MEKF [12], [13], CI-MEKF can trust on the uncertainty boundaries of the estimated filter states. This enables CI-MEKF to detect observation uncertainties, e.g., if the accelerometer or magnetometer measurement is inconsistent with the filter predictions due to disturbances in the measurement. We used covariance inflation to enlarge the residual error covariance in order to make the measurement consistent with the Kalman filter predictions.

Two different approaches are considered for covariance inflation, where the first approach is inspired from generalized covariance union [25] and the new approach is introduced in this paper as Independent CI in order to increase the robustness of CI-MEKF. In comparison with the state of the art, the CI-MEKF method demonstrates the highest estimation accuracy of the Euler angles in presence of body acceleration and magnetic distortion. It also estimates the gyro bias in all directions with a high accuracy.

Comparing two covariance inflation approaches, a trade-off between accuracy and robustness is observed. Moving from GCU-based to Independent CI, one will experience a slight decrease in the estimation accuracy by a significant increase in robustness of the method. The Independent CI-MEKF is highly robust against parametric uncertainties such as large gyro bias and initial condition error of the attitude while the GCU may diverge in such cases. An strategy that can be pursued as a future extension of the CI-MEKF filter is to use hybrid covariance inflation, where a combination of CI approaches are used together to simultaneously enhance the robustness and accuracy of CI-MEKF. The effect of measurement noise and stationary conditions (no movements) on the accuracy and the robustness of the filter cab also be investigated further. Moreover, future application of the proposed independent covariance inflation can include other sensor fusion problems with uncertain observation such as GPS, INS and SLAM.

REFERENCES

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