Philosophy 214/414
Logical Methods in Philosophy
Spring 2008

## Practice Problems for the Third Examination

I. Deontic Logic Symbolizations Symbolize the following sentences into the language of Deontic Propositional Logic. Feel free to use any of the primitive and defined sentential and deontic connectives: $\sim, \rightarrow, \wedge, \vee, \leftrightarrow$, **O**, **P**. Please write the English sentence before symbolizing. Be sure to provide your symbolization scheme. If you think that the English sentence is ambiguous, provide the two most plausible symbolizations.

1. Johnnie may have ice cream or candy.
2. If Suzie ought to pay her property tax, then Mary should pay her income tax.
3. Smith can go to the concert only if he has enough money to buy a ticket.
4. Jones should not give to charity if he does not have enough money to feed himself.
5. If it's legal for Johnson to sell his stock, then it's permissible for her to do so.

II. Deontic Logic Validity and Invalidity

1. Prove that for all DPL wffs $\varphi$, and all classes of deontic standard models C, $\vDash_C \mathbf{O}\varphi\rightarrow\mathbf{P}\varphi$.
2. Prove that for all DPL wffs $\varphi$ and $\psi$, and all classes of deontic standard models C, $\vDash_C\mathbf{O}(\varphi\wedge\psi)\rightarrow\mathbf{O}\varphi\wedge\mathbf{O}\psi$.
3. Prove that for all DPL wffs and all classes of deontic standard models C, $\vDash_C\mathbf{P}(\varphi\vee\psi)\rightarrow\mathbf{P}\varphi\vee\mathbf{P}\psi$.
4. Prove that it is *not* the case that for all DPL wffs $\varphi$, and all classes for deontic standard models C, $\vDash_C \varphi\rightarrow\mathbf{OP}\varphi$.
5. Prove that is it *not* the case that all DPL wffs $\varphi$ and $\psi$, and all classes of deontic standard models C, $\vDash_C\mathbf{O}(\varphi\vee\psi)\rightarrow\mathbf{O}\varphi\vee\mathbf{O}\psi$.

III. Tense Logic Symbolizations Symbolize the following sentences into the language of Tensed Propositional Logic. Feel free to use any of the primitive and defined sentential and temporal connectives: $\sim, \rightarrow, \wedge, \vee, \leftrightarrow$, **F**, **G**, **P**, **H**. Please write the English sentence before symbolizing. Be sure to provide your symbolization scheme. If you think that the English sentence is ambiguous, provide the two most plausible symbolizations.

1. The universe has always been filled with energy.
2. If Smith turns off the stove, then he will have turned off the stove.
3. If John loves Mary, then it will always be the case that he loved her.
4. John had eaten pizza.
5. Smith smoked as Jones played billiards.

IV. Tense Logic Validity and Invalidity

1. Let C be the class of temporal standard models with a <-minimal element. Prove that $\vDash_C\mathbf{H}(\varphi\wedge\sim\varphi) \vee \mathbf{PH}(\varphi\wedge\sim\varphi)$.
2. Let C be the class of temporal standard models with no maximal <-element. Prove that $\vDash_C(\mathbf{G}\varphi\rightarrow\mathbf{F}\varphi)$.

3. Prove that it is not the case that for all classes of temporal standard models C, $\vDash_C \mathbf{G}\varphi \to \varphi$.
4. Prove that it is not the case that for all classes of temporal standard models C, $\vDash_C \mathbf{P}\varphi \to \mathbf{PP}\varphi$.

<u>V. Counterfactuals: Validity and Invalidity</u> For each of the following wffs or schemas, determine whether it is valid or invalid in all **SC models**. If valid, provide a proof of validity. If invalid, provide a proof of invalidity (accompanied by a diagram of your type of invalidating model). Then determine whether each is valid or invalid with respect to all **LC models** and provide an appropriate proof.

1. $(\varphi \Box\!\!\to (\psi \vee \chi)) \to [(\varphi \Box\!\!\to \psi) \vee (\varphi \Box\!\!\to \chi)]$
2. $(P \Box\!\!\to Q) \to (\sim Q \to \sim P)$
3. $\sim(\varphi \Box\!\!\to \psi) \to \Diamond\varphi$
4. $\sim(P \Box\!\!\to Q) \to (P \Box\!\!\to \sim Q)$
5. $[(P \Box\!\!\to (Q \Box\!\!\to R)] \to [(P \wedge Q) \Box\!\!\to R]$
6. $[(P \Box\!\!\to Q) \wedge (Q \Box\!\!\to R)] \to (P \Box\!\!\to R)$
7. $[(P \Box\!\!\to Q) \wedge (P \wedge Q \Box\!\!\to R)] \to [P \Box\!\!\to R]$
8. $[(\varphi \wedge \psi) \Box\!\!\to \chi] \to [(\varphi \Box\!\!\to (\psi \Box\!\!\to \chi)]$
9. $\Box(P \to Q) \to (P \Box\!\!\to Q)$
10. $\sim\Diamond\varphi \to (\varphi \Box\!\!\to \psi)$

<u>VI. MFOPL: Symbolization</u> Symbolize the following sentences into the language of MFOPL. Feel free to use both the primitive and defined symbols of MFOPL (including $\wedge$, $\vee$, $\Diamond$, etc.). If the sentence is ambiguous, give the two most plausible symbolizations, and indicate which (if either) is the most plausible. (It could be that they are equally plausible.) Be sure to specify your symbolization scheme.

1. Everything that is made of sugar can dissolve.
2. Some mathematicians might not have been rational.
4. Every body is necessarily divisible. (Assume that an object is divisible iff it is possible that something divides it. Symbolize using "F: *a* divides *b*")
5. Nine is necessarily odd. ('Nine' is a proper name.)
6. Everything is necessarily identical with itself.
7. Twain could not have failed to be (identical with) Clemens. ('Twain' and 'Clemens' should be symbolized with individual constants. And use the identity symbol!)
8. It's impossible for George W. Bush to be Bill Clinton. (Again, symbolize both names using individual constants, and use identity.)
9. Every human necessarily has a parent, but every human who is a parent could have failed to be a parent. (F: *a* is a parent of *b*. H: *a* is human.)
10. Every bachelor is necessarily unmarried, and Hefner is a bachelor, but it's not necessary that Hefner be unmarried.

<u>VII. More symbolization</u> Symbolize the following sentences into the language of MFOPL. Feel free to use both the primitive and defined symbols of MFOPL (including $\wedge$, $\vee$, $\Diamond$, etc.). Consider how the sentence should be symbolized using both possibilist and actualist quantifiers. When

you need to symbolize 'exists', symbolize using both the existence predicate E and the existential quantifier ∃, and when you do so, state whether the resulting symbolization is true if the quantifiers are possibilist or actualist. (That is, for each symbolization, consider whether it is true at some world in some CD model and whether it is true at some world in some VD or VD+ model.) If the sentence is ambiguous (due to a scope ambiguity), give the two most plausible symbolizations, and indicate which (if either) is the most plausible. (It could be that they are equally plausible.) Be sure to specify your symbolization scheme.

1. Everything exists necessarily.
2. Necessarily, everything exists.
3. Everything is such that, necessarily, it exists.
4. There are things that do not exist.
5. Something doesn't exist.
6. There are things that do not possibly exist.
7. Something is such that it is impossible that it exists.
8. Nothing is possibly a god, but it is possible that there are gods.
9. Socrates is essentially human. (Assume that an object is essentially F iff: necessarily, if it exists, then it is F.)
10. Every human is essentially human.
11. If the teacher of Alexander exists, then the teacher of Alexander is necessarily a teacher of Alexander. (Do this problem only if you know how to symbolize definite descriptions using Russell's method: 'The F is G' ⇒ '$\exists x(Fx \wedge \forall y(Fy \rightarrow y=x) \wedge Gy)$'.)

VIII. MFOPL: Validity and Invalidity For each wff or schema, determine whether it is valid or invalid with respect to the class of all CD models. If it is, provide a proof of validity. If not, provide a proof of invalidity (along with a suitable diagram of your countermodel). Then do the same for each wff or schema with respect to the class of all VD models.

1. $\exists x \Diamond Fx \rightarrow \Diamond \exists x Fx$
2. $\Diamond \exists x Fx \rightarrow \exists x \Diamond Fx$
3. $\forall x \Diamond Fx \rightarrow \Diamond \forall x Fx$
4. $\Diamond \forall x Fx \rightarrow \forall x \Diamond Fx$
5. $\exists \alpha \Box \varphi \rightarrow \Box \exists \alpha \varphi$
6. $\Box \exists \alpha \varphi \rightarrow \exists \alpha \Box \varphi$
7. $\forall x \Box Fx \rightarrow \Box \forall x Fx$
8. $\Box \forall x Fx \rightarrow \forall x \Box Fx$
9. $\forall x(Fx \rightarrow \Diamond Gx) \rightarrow \Diamond \forall x(Fx \rightarrow Gx)$
10. $\exists x \Diamond (Fx \wedge Gx) \rightarrow \Diamond \exists x(Fx \wedge Gx)$
11. $\Diamond \exists x(Fx \wedge Gx) \rightarrow (\Diamond \exists x Fx \wedge \Diamond \exists x Gx)$
12. $(\Diamond \exists x Fx \wedge \Diamond \exists x Gx) \rightarrow \Diamond \exists x(Fx \wedge Gx)$

13 $\forall x(Fx \rightarrow \Box Gx) \rightarrow \Box \forall x(Fx \rightarrow Gx)$

14. $\Box \forall x(Fx \rightarrow Gx) \rightarrow \forall x(Fx \rightarrow \Box Gx)$

13. $\forall x Fx \rightarrow Fy$

14. $\forall y(\forall x Fx \rightarrow Fy)$

15. $\varphi_\alpha \rightarrow \exists \beta \varphi_\beta$

16. $\forall \alpha(\varphi_\alpha \rightarrow \exists \beta \varphi_\beta)$

<u>IX. More on MFOPL Validity and Invalidity</u> For each wff or schema, determine whether it is valid or invalid with respect to the class of all CD models. Then provide either a proof of validity or a proof of invalidity (with a diagram of your countermodel). Then do the same for each wff or schema with respect to the class of VD or VD+ models (you will have to use VD+ models if the wff contains the identity predicate or an individual constant).

1. $\exists\beta\ \alpha=\beta$
2. $\forall\alpha\exists\beta\ \alpha=\beta$
3. $\forall x\Box\exists y x=y$
4. $\Box\forall x\exists y x=y$
5. $\forall x\Diamond\exists y x=y$
6. $\exists x\ \Box x=a$.
6. $Fa \rightarrow \exists xFx$
7. $\forall xFx \rightarrow Fa$
8. $\forall xFx \rightarrow (Ea \rightarrow Fa)$ (Hint: in VD+ semantics, 'Ea' is an abbreviation for '$\exists yy=a$'. But not in CD semantics: 'Ea' is not an abbreviation for '$\exists yy=a$' and they are not even equivalent .)
9. $\forall xFx \rightarrow (\exists yy=a \rightarrow Fa)$
10. $\sim Fa \rightarrow \exists x\sim Fx$
11. $\sim Fa \rightarrow (Ea \rightarrow \exists x\sim Fx)$
12. $\sim Fa \rightarrow (\exists yy=a \rightarrow\exists x\sim Fx)$
13. $(\Diamond Fa \wedge \Diamond Ga) \rightarrow \Diamond(Fa \wedge Ga)$
14. $(\Diamond Fa \wedge \Diamond Ga) \rightarrow \exists x\ (\Diamond Fx \wedge \Diamond Gx)$
15. $\Box Fa \rightarrow \exists x\Box Fx$
16. $\Box Fa \rightarrow \Box\exists xFx$
17. $\Diamond Fa \rightarrow \exists x\Diamond Fx$
18. $\Diamond Fa \rightarrow\Diamond\exists xFx$