Practice Problems for Exam 1

I. Well-formed formulas and grammatical trees
For each of the expressions below, say whether (a) it is a wff of PL, (b) it is an abbreviation of a wff of PL, or (c) it is neither of these. If (a), provide a grammatical tree. If (b), give its unabbreviated form and give a tree for that. If (c), say briefly why it is not a wff.

1. \( \neg\neg (P \land T) \)
2. \( \neg (M \land Z) \)
3. \( Q \equiv_{PL} Q \)
4. \( (S \land Q) \lor (T \land P) \)
5. \( (\neg (\psi \land \chi)) \land ((\neg \psi) \land (\neg \chi)) \)
6. \( P \in \{P, Q\} \)

II. Validity and Semantic Consequence

7. Prove that for all wffs \( \psi \) and \( \chi \), \( \equiv_{PL} (\neg \varphi \land \neg \chi) \land (\psi \land \varphi) \)
   (This theorem is part of the proof of the Soundness Theorem. A truth-table may appear as part of your proof, but you must explain why it helps show the above theorem.)
8. Prove that it is not the case that for all wffs \( \varphi \) and \( \psi \), \( \equiv_{PL} (\varphi \lor \neg \chi) \land (\neg \varphi \lor \neg \psi) \)
   (The most straightforward way to prove this is to take an instance and show that it is false in some assignment. Please fully specify the assignment [specify the truth-value that it assigns to every sentence letter].)
9. Prove Theorem 2.12 (a)-(h) on p. 34.
10. Prove that it is not the case that for all sets of wffs \( \Gamma \) and \( \Delta \), if \( \Gamma \) is satisfiable and \( \Delta \) is satisfiable, then \( \Gamma \lor \Delta \) is satisfiable.
   (Hint: The most straightforward proof specifies two satisfiable sets, and proves that no assignment can satisfy the union of them.)
11. Prove that for all wffs \( \varphi \), if \( \varphi \) is unsatisfiable, then \( \equiv_{PL} \neg \varphi \).
12. Prove that for all wffs \( \varphi \) and \( \psi \), and all sets of wffs \( \Gamma \), if either \( \equiv_{PL} \neg \varphi \) or \( \equiv_{PL} \psi \), then \( \equiv_{PL} (\varphi \lor \psi) \).
   (Hint: Use separation of cases, which is the following inference rule: “From (If \( A \) then \( C \)) and (if \( B \) then \( C \)), infer (If either \( A \) or \( B \), then \( C \)).” To apply it here, first state that you will use separation of cases. Then prove that if \( \equiv_{PL} \neg \varphi \), then \( \equiv_{PL} (\varphi \lor \psi) \). Next prove that if \( \equiv_{PL} \psi \), then \( \equiv_{PL} (\varphi \lor \psi) \). Then infer, by separation of cases, that if either \( \equiv_{PL} \neg \varphi \) or \( \equiv_{PL} \psi \), then \( \equiv_{PL} (\varphi \lor \psi) \).
III. Theorems and Syntactic Consequence

13. Prove that $-\varphi_{\text{pl}}(\varphi\psi)$. Do not use the Deduction Theorem.
14. Prove that $(R-P)-(T-S)_{\text{pl}}(-P-R)-(T-S)$. You may use the Deduction Theorem.
15. Prove Theorem 2.18 (a)-(f), p. 43 in the book.
16. Prove that for all wffs $\varphi$ and $\psi$, and all sets of wffs $\Gamma$ and $\Delta$, if $\Gamma_{\text{pl}}\varphi$ and $\Delta_{\text{pl}}(\varphi\psi)$, then $\Gamma\cup\Delta_{\text{pl}}\psi$.
17. Prove that it is not the case that for all wffs $\varphi$ and all sets of wffs $\Gamma$, if $\Gamma\cup\{\varphi\}$ is inconsistent, then $\Gamma\cup\{-\varphi\}$ is consistent.
   (Hint: find an instance, and prove that it is an instance.)
18. Prove that for all wffs $\varphi$ and $\psi$, and sets of wffs $\Gamma$ and $\Delta$, if $\Gamma_{\text{pl}}(-\varphi\neg\psi)$ and $\Delta_{\text{pl}}\psi$, then $\Gamma\cup\Delta_{\text{pl}}\varphi$. 