

Homework Assignment #9

Due at the beginning of class on Wednesday, March 26

I. Validity Show that all instances of the following schemas are valid with respect to the corresponding class of standard models.

1. $\diamond(\varphi \vee \psi) \rightarrow (\diamond\varphi \vee \diamond\psi)$ *any class*
2. $\diamond\Box\varphi \rightarrow \Box\Diamond\varphi$ *equivalence class*

II. Invalidity

3. Prove that there are instances of schema 4 that are invalid with respect to the class of serial euclidean models.

III. Validity and Invalidity

4. Let Refl be the class of reflexive models. Determine whether the following claim is true. If it is true, prove that it is. If it is not, prove that it is not.
For all wffs φ , $\models_{\text{Refl}} \diamond(\Box\varphi \rightarrow \varphi) \rightarrow \diamond(\varphi \rightarrow \Box\varphi)$

IV. Canonical Models

5. Consider Theorem 4.27 on p. 179.

THEOREM 4.27. TRUTH LEMMA Let S be a normal system of modal logic, and let $M_S = \langle W_S, R_S, V_S \rangle$ be the canonical model for S . Let V_{M_S} be the valuation function for M_S . Then for every MPL wff φ , and every world $\Delta \in W_S$, $V_{M_S}(\varphi, \Delta) = 1$ iff $\varphi \in \Delta$.

The proof of this theorem uses induction on the complexity of wffs on the following condition C (see p. 179).

- C. For every world $\Delta \in W_S$, $V_{M_S}(\varphi, \Delta) = 1$ iff $\varphi \in \Delta$.

To use induction on the complexity of wffs, we show (a) all sentence letters satisfy this condition; (b) for all wffs φ , if φ satisfies this condition, then so does $\sim\varphi$; (c) for all wffs φ and ψ , if φ and ψ satisfy this condition then so does $(\varphi \rightarrow \psi)$; and (d) for all wffs φ , if φ satisfies this condition, then so does $\Box\varphi$. We can then conclude by induction that all wffs satisfy the condition.

For this problem, prove the Base Case and the Negation Case. (See p. 179.)