Homework Assignment 5  
Due at the beginning of class on Wednesday, February 20

I. Symbolization In Modal Propositional Logic  Do the following:
(a) Symbolize the following sentences into modal propositional logic. (Write the original English sentence on your answer sheet before your symbolize.) Feel free to use any of the primitive and defined sentential and modal connectives: ¬, →, ∧, ∨, ↔, ◻, □. If you think the sentence is ambiguous (can be symbolized in two or more non-equivalent ways), give the two most plausible symbolizations. If a symbolization scheme is provided below use it. Otherwise, create your own and be sure to present your symbolization scheme.
(b) For each sentence, state an interpretation of the modality (e.g., nomological or deontic) that a reasonable author is likely to have intended.
(c) Please use □(φ→ψ) rather than (φ→ψ) or (φ fish-hook ψ)

1. Clinton could have been born on Mars. (P: Clinton is born on Mars.)
2. Humphrey might have won the 1968 election.
3. Descartes cannot exist if his body does not. (P: Descartes exists. Q: Descartes’s body exists.)
4. Descartes’s thinking is necessary for his existence. (P: Descartes thinks. Q: Descartes exists.)
5. John has to take out the garbage.
6. Sarah can go to law school only by taking out loans. (S: Sarah goes to law school. T: Sarah takes out loans.)
7. Leigh’s analyzing the data is necessary for a realistic forecast. (P: Leigh analyzes the data. Q: A realistic forecast occurs.)
8. It just might be the case that Mary is both sincere and stupid.
9. If Earl knows that there is coffee in his cup, then there must be coffee in his cup.
10. Susan’s getting a ‘B’ on her logic exam is enough for her to get a ‘B’ in her logic course.
11. It’s impossible for Kathryn to be both at home and at work.
12. If God exists, then he necessarily exists.
13. Bill’s passing the driving test is both necessary and sufficient for his getting a driver’s license.
14. That snow is not green is a contingent truth.
15. It’s necessarily true that God might have existed.
II. Truth Values of Abbreviated Wffs in a Model  Use our conventions concerning abbreviations of wffs, and the definition of a valuation function for M (V_M) to show the following.

16. For all wffs \( \phi \) and \( \psi \), and all Leibnizian models M, and all \( w \in W \), \( V_M(\phi \lor \psi, w) = 1 \) iff either \( V_M(\phi, w) = 1 \) or \( V_M(\psi, w) = 1 \).

17. For all wffs \( \phi \), and all Leibnizian models M, and all \( w \in W \), \( V_M(\Diamond \phi, w) = 1 \) iff: there is a \( w' \in W \) such that \( V_M(\phi, w') = 1 \).

III. Validity  You may assume the derived truth conditions for \( \land, \lor, \neg, \), and \( \Diamond \).

18. Prove that for all wffs \( \phi, \neg \Diamond \phi \rightarrow \Diamond \phi \). (Schema 5)
19. Prove that for all wffs \( \phi \) and \( \psi \), \( \equiv \Box(\phi \land \psi) \rightarrow (\Box \phi \land \Box \psi) \).
20. Prove that for all wffs \( \phi \) and \( \psi \), if \( \equiv \Box \phi \rightarrow \psi \), then \( \equiv \Box \phi \rightarrow \Box \psi \)