

Homework Assignment #14

Due at the beginning of class on April 30, 2008

I. Truth Values of MFOPL Wffs in CD Models

Consider the following CD model M and assignment g for M.

$M = \langle W, R, D, I \rangle$, where

$D = \{ d^*, d\# \}$

$W = \{ w_1, w_2 \}$

$R = \{ \langle w_1, w_1 \rangle, \langle w_1, w_2 \rangle, \langle w_2, w_2 \rangle, \langle w_2, w_1 \rangle \}$ (This is R^2 , the total relation on R).

I is the function such that

a. $I(a) = d^*$

$I(b) = d\#$

and for all other constants α , $I(\alpha) = d^*$.

b. $I(F, w_1) = \{ d^*, d\# \}$

$I(G, w_1) = \{ \langle d^*, d\# \rangle, \langle d^*, d^* \rangle \}$

$I(F, w_2) = \{ d^* \}$

$I(G, w_2) = \{ \langle d^*, d^* \rangle, \langle d\#, d\# \rangle \}$

and for all other predicates π and $w \in W$, $I(\pi, w) = \{ \}$.

g is the function such that

$g(x) = d^*$

$g(y) = d\#$,

and for all other variables α , $g(\alpha) = d^*$.

For the following wffs, state whether it is true or false at the specified world in M under g. Then prove that it does have that truth value, by going step-by-step through the rules for the denotation and valuation functions.

1. Gay is true/false at w_1 in M under g.
2. \Box Gay is true/false at w_1 in M under g.
3. $\exists x \Box Gxb$ is true false at w_1 in M under g.
4. $\Box \exists x Gxb$ is true/false at w_1 in M under g.

Continued . . .

II. Validity in CD Models Prove the following theorems.

5. *The Converse Barcan Formula*: for all variables α and all MFOPL wffs φ and every class C of CD models, $\models_C \Box \forall \alpha \varphi \rightarrow \forall \alpha \Box \varphi$.
6. *The Necessity of Distinctness*: for every class C of CD models, $\models_C \forall x \forall y (x \neq y \rightarrow \Box x \neq y)$.
7. For every class C of CD models, $\models_C \forall x \Box \exists y (y = x)$.

III. Invalidity in CD Models For the following problems, be sure to fully describe your invalidating model.

8. Let T be the class of total CD models. Prove that it is *not* the case that for all variables α and all wffs φ , $\models_T \forall \alpha \Diamond \varphi \rightarrow \Diamond \forall \alpha \varphi$. (Be sure to specify what α and φ are in your chosen instance of the schema.)

IV. Validity in VD Models Prove the following theorems.

9. For every class C of VD models, $\models_C \Box \forall x \exists y (y = x)$.

V. Invalidity in VD Models Prove the following theorems. Be sure to fully describe your invalidating model.

10. It is not the case that for every class C of VD models, $\models_C \forall x \Box \exists y (y = x)$. (Compare with #13 and #15 above.)
11. *The Converse Barcan Formula*: It is not the case that for all variables α and all MFOPL-wffs φ and every class C of VD models, $\models_C \Box \forall \alpha \varphi \rightarrow \forall \alpha \Box \varphi$. (Compare with #11 above.)