

## Chapter 9:

### Exercises 9-1

1. a, e
3. c

### Exercises 9-2

1.
 

1.	1.	$\forall x(Fx \vee Gx)$	P
	2.	$\forall x \sim Gx$	P
	3.	<b>SHOW</b> $Fb$	DD
-----			
	4.	$Fb \vee Bg$	1, $\forall E$
	5.	$\sim Gb$	2, $\forall E$
	6.	$Fb$	4, 5, $\vee E$
-----			
  
3.
 

3.	1.	$Fa \ \& \ Ga$	P
	2.	<b>SHOW</b> $\exists zGz$	DD
-----			
	3.	$Ga$	1, $\&E$
	4.	$\exists zGz$	3, $\exists I$
-----			
  
5.
 

5.	1.	$\exists xMx \vee \sim \exists yPy$	P
	2.	$\sim(Pq \rightarrow Mg)$	P
	3.	<b>SHOW</b> $\exists xMx$	DD
-----			
	4.	$Pq \ \& \ \sim Mg$	2, $\sim \rightarrow$
	5.	$Pq$	4, $\&E$
	6.	$\exists yPy$	5, $\exists I$
	7.	$\sim \sim \exists yPy$	6, DN
	8.	$\exists xMx$	1, 7, $\vee E$
-----			
  
7.
 

7.	1.	$\forall x \forall y [(Jx \ \& \ Ky) \rightarrow (My \ \& \ Nx)]$	P
	2.	$Ja \ \& \ Kb$	P
	3.	<b>SHOW</b> $\exists y(Mb \ \& \ Ny)$	DD
-----			
	4.	$\forall y [(Ja \ \& \ Ky) \rightarrow (My \ \& \ Na)]$	1, $\forall E$
	5.	$(Ja \ \& \ Kb) \rightarrow (Mb \ \& \ Na)$	4, $\forall E$
	6.	$Mb \ \& \ Na$	2, 5, $\rightarrow E$
	7.	$\exists y(Mb \ \& \ Ny)$	6, $\exists I$
-----			
  
9.
 

9.	1.	$\exists x(Kx \ \& \ Mx) \rightarrow \forall yGy$	P
	2.	$\forall x \forall y [Nx \rightarrow (Kx \ \& \ My)]$	P
	3.	$Np$	P
	4.	<b>SHOW</b> $Gm$	DD
-----			
	5.	$\forall y [Np \rightarrow (Kp \ \& \ My)]$	2, $\forall E$
	6.	$Np \rightarrow (Kp \ \& \ Mp)$	5, $\forall E$
	7.	$Kp \ \& \ Mp$	3, 6, $\rightarrow E$
-----			

8.	$\exists x(Kx \ \& \ Mx)$	7, $\exists I$
9.	$\forall yGy$	1, 8, $\rightarrow E$
10.	$Gm$	9, $\forall E$

Exercises 9-3

1.	1.	$\forall x(Ax \rightarrow Bx)$	P
	2.	$\exists yAy$	P
	3.	<b>SHOW</b> $\exists zBz$	DD
-----			
	4.	$Aa$	2, $\exists E$
	5.	$Aa \rightarrow Ba$	1, $\forall E$
	6.	$Ba$	4, 5, $\rightarrow E$
	7.	$\exists zBz$	6, $\exists I$
-----			

3.	1.	$\exists y(Ga \ \& \ Fy)$	P
	2.	$\forall x\forall y[(Gx \ \& \ Fy) \rightarrow (Mx \ \& \ Sy)]$	P
	3.	<b>SHOW</b> $Ma \ \& \ \exists zSz$	DD
-----			
	4.	$Fa \ \& \ Gb$	1, $\exists I$
	5.	$\forall y[(Ga \ \& \ Gy) \rightarrow (Ma \ \& \ Sy)]$	2, $\forall E$
	6.	$(Ga \ \& \ Fb) \rightarrow (Ma \ \& \ Sb)$	5, $\forall E$
	7.	$Ma \ \& \ Sb$	4, 6, $\rightarrow E$
	8.	$Ma$	7, $\&E$
	9.	$Sb$	7, $\&E$
	10.	$\exists zSz$	9, $\exists I$
	11.	$Ma \ \& \ \exists zSz$	8, 10, $\&I$
-----			

5.	1.	$\exists x(Fx \rightarrow Ga)$	P
	2.	<b>SHOW</b> $\forall xFx \rightarrow Ga$	CD
-----			
	3.	$\forall xFx$	ACD
	4.	<b>SHOW</b> $Ga$	DD
-----			
	5.	$Fb \rightarrow Ga$	1, $\exists E$
	6.	$Fb$	3, $\forall E$
	7.	$Ga$	5, 6, $\rightarrow E$
-----			

7.	1.	$\exists x\sim Dx$	P
	2.	<b>SHOW</b> $\exists x(Dx \rightarrow \forall yDy)$	DD
-----			
	3.	$\sim Da$	1, $\exists E$
	4.	$\sim Da \vee \forall yDy$	3, $\vee I$
	5.	$Da \rightarrow \forall yDy$	$\rightarrow v$
	6.	$\exists x(Dx \rightarrow \forall yDy)$	5, $\exists I$
-----			

9.	1.	$\exists xQx$	P
	2.	<b>SHOW</b> $\sim\forall x\sim Qx$	ID

3.	$\forall x \sim Qx$	AID
4.	<b>SHOW !</b>	DD
-----		
5.	$Qa$	1, $\exists E$
6.	$\sim Qa$	3, $\forall E$
7.	!	5, 6, !I

11.	1.	$\exists x(Fa \ \& \ Mx)$	P
	2.	$\forall x \forall y[(Fx \ \& \ My) \rightarrow \exists yBy]$	P
	3.	<b>SHOW</b> $\exists zBz$	DD
-----			
	4.	$Fb \ \& \ Ma$	1, $\exists E$
	5.	$\forall y[(Fb \ \& \ My)y \rightarrow \exists wBw]$	2, $\forall E$
	6.	$(Fb \ \& \ Ma) \rightarrow \exists wBw$	5, $\forall E$
	7.	$\exists wBw$	4, 6, $\rightarrow E$
	8.	$Bc$	7, $\exists E$
	9.	$\exists zBz$	8, $\exists I$

Exercise 9-4

1.	1.	$\forall x(Ax \rightarrow Bx)$	P
	2.	$\forall x(Bx \rightarrow Cx)$	P
	3.	<b>SHOW</b> $\forall x(Ax \rightarrow Cx)$	UD
-----			
	4.	<b>SHOW</b> $Aa \rightarrow Ca$	DD
-----			
	5.	$Aa \rightarrow Ba$	1, $\forall E$
	6.	$Ba \rightarrow Ca$	2, $\forall E$
	7.	$Aa \rightarrow Ca$	5, 6, HS

3.	1.	$\forall x(Fx \leftrightarrow Gx)$	P
	2.	$\forall y(Py \rightarrow Gy)$	P
	3.	<b>SHOW</b> $\forall x(Px \rightarrow Fx)$	UD
-----			
	4.	<b>SHOW</b> $Pa \rightarrow Fa$	CD
-----			
	5.	$Pa$	ACD
	6.	<b>SHOW</b> $Fa$	DD
-----			
	7.	$Fa \leftrightarrow Ga$	1, $\forall E$
	8.	$Pa \rightarrow Ga$	2, $\forall E$
	9.	$Ga$	5, 8, $\rightarrow E$
	10.	$Fa$	7, 9, $\leftrightarrow E$

5.	1.	$\forall x(\sim Px \vee Rx)$	P
----	----	------------------------------	---

2.	$\forall x(\sim Dx \rightarrow \sim Rx)$	P
3.	<b>SHOW</b> $\forall x(Px \rightarrow Dx)$	UD
4.	<b>SHOW</b> $Pa \rightarrow Da$	DD
5.	$\sim Pa \vee Ra$	1, $\vee E$
6.	$Pa \rightarrow Ra$	5, $\sim v$
7.	$\sim Da \rightarrow \sim Ra$	2, $\vee E$
8.	$Ra \rightarrow Da$	7, Ctr
10.	$Pa \rightarrow Da$	6, 8, HS

7. 1.	$\exists xFx \rightarrow Ga$	P
2.	<b>SHOW</b> $\forall x(Fa \rightarrow Ga)$	UD
3.	<b>SHOW</b> $Fb \rightarrow Ga$	CD
4.	$Fb$	ACD
5.	<b>SHOW</b> $Ga$	DD
6.	$\exists xFx$	4, $\exists I$
7.	$Ga$	1, 6, $\rightarrow E$

9. 1.	$\forall z(Hz \rightarrow Fz)$	P
2.	$\exists xGx \rightarrow \forall x(Gx \rightarrow Hx)$	P
3.	<b>SHOW</b> $\forall x(Gx \rightarrow Fx)$	UD
4.	<b>SHOW</b> $Ga \rightarrow Fa$	CD
5.	$Ga$	ACD
6.	<b>SHOW</b> $Fa$	DD
7.	$Ha \rightarrow Fa$	1, $\vee E$
8.	$\exists xGx$	5, $\exists I$
9.	$\forall x(Gx \rightarrow Hx)$	2, 8, $\rightarrow E$
10.	$Ga \rightarrow Ha$	9, $\vee E$
11.	$Ha$	5, 10, $\rightarrow E$
12.	$Fa$	7, 11, $\rightarrow E$

9. 1.	$\forall x\forall y(Wxy \rightarrow Ixx)$	P
2.	$\forall x\exists yWxy$	P
3.	<b>SHOW</b> $\forall xIxx$	UD
4.	<b>SHOW</b> $Iaa$	DD
5.	$\exists yWay$	2, $\vee E$

6.	Wab		5, $\exists E$
7.	$\forall y(Way \rightarrow Iaa)$		1, $\forall E$
8.	$Wab \rightarrow Iaa$		7, $\forall E$
9.	Iaa		6, 8. $\rightarrow E$
-----			

11.	1.	$\forall x(Lxa \vee Lxb)$	P
	2.	$\forall x(\exists yLxy \rightarrow Hx)$	P
	3.	<b>SHOW</b> $\forall xHx$	UD
-----			
	4.	<b>SHOW</b> Hc	DD
-----			
	5.	Lca $\vee$ Lcb	1, $\forall E$
	6.	$\exists yLcy \rightarrow Hc$	2, $\forall E$
	7.	<b>SHOW</b> Lca $\rightarrow \exists yLcy$	CD
-----			
	8.	Lca	ACD
	9.	<b>SHOW</b> $\exists yLcy$	DD
-----			
	10.	$\exists yLcy$	8, $\exists I$
-----			
	11.	<b>SHOW</b> Lcb $\rightarrow \exists yLcy$	CD
-----			
	12.	Lcb	ACD
	13.	<b>SHOW</b> $\exists yLcy$	DD
-----			
	14.	$\exists yLcy$	12, $\exists I$
-----			
	15.	$\exists yLcy$	5, 7, 11, SC
	16.	Hc	6, 15, $\rightarrow E$
-----			

Exercises 9-5

1. 4. WRONG Existential Exploitation can't be used on line 1 because the existential quantifier is not the main logical operator of line 1. The arrow is.
5. WRONG Universal Exploitation can't be used on line 4 because the Universal quantifier is not the main logical operator of line 4. The arrow is.

1.	$\exists xFx \rightarrow \forall xFx$	P
2.	Fa	P
3.	<b>SHOW</b> Fb	DD
-----		
4.	$\exists xFx$	2, $\exists I$
5.	$\forall xFx$	1, 4, $\rightarrow E$
6.	Fb	5, $\forall E$
-----		

3. 4. WRONG Universal Exploitation can't be used on line 1 because the Universal quantifier is not the main logical operator of line 1. The arrow is.
5. WRONG An assumption can only be made on the next line after an appropriate SHOW line. Line 4 is not a SHOW line.
7. WRONG Existential Exploitation can't be used on line 4 because the existential quantifier is not the main logical operator of line 4. The arrow is. In addition, existential exploitation requires new name; "a" has already occurred in the derivation.
8. WRONG Existential Exploitation requires a new name; "a" already occurs in the derivation.
11. WRONG Existential Exploitation can't be used on line 10, because the existential quantifier is not the main logical operator of line 10. The tilde is. In addition, Existential Exploitation requires a new name; "a" already occurs in the derivation.

1.	$\forall x \exists y Fxy \rightarrow \sim \exists x Fxx$	P
2.	$\exists x \forall y Fyx$	P
3.	SHOW $\sim Faa$	ID
-----		
4.	Faa	AID
5.	SHOW !	DD
-----		
6.	$\exists x Fxx$	4, $\exists I$
7.	$\sim \sim \exists x Fxx$	6, DN
8.	$\sim \forall x \exists y Fxy$	1, 7, MT
9.	SHOW $\forall x \exists y Fxy$	UD
-----		
10.	SHOW $\exists y Fby$	DD
-----		
11.	$\forall y Fyc$	2, $\exists E$
12.	Fbc	11, $\forall E$
13.	$\lceil y Fby$	12, $\exists I$
-----		
14.	!	8, 9, !I
-----		

5. 5. WRONG You cannot box and cancel using UD here. To box and cancel using UD you must have an instance of the universal sentence you are trying to show where the name that replaces the quantifier's variable is new to the derivation; "a" already occurs in the derivation. Line 6 should have had a name other than "a" instead of "a".
13. WRONG The annotation should be 11, 12,  $\rightarrow E$ .
17. WRONG There is no such rule as Universal Introduction. To derive line 17, a Universal Derivation should have been

started earlier in the derivation.

1.	$\forall x(Fx \rightarrow Hax)$	P																																																						
2.	$\forall x(\exists yHxy \rightarrow Kx)$	P																																																						
3.	$\forall y(\exists xKx \rightarrow Jy)$	P																																																						
4.	$\forall xJx \rightarrow \forall zGz$	P																																																						
5.	<b>SHOW</b> $\forall x(Fx \rightarrow Gx)$	UD																																																						
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">6.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>SHOW</b> <math>Fb \rightarrow Gb</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">CD</td> </tr> <tr> <td>7.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb</math></td> <td style="text-align: right; border-right: 1px dashed black;">ACD</td> </tr> <tr> <td>8.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>SHOW</b> <math>Gb</math></td> <td style="text-align: right; border-right: 1px dashed black;">DD</td> </tr> <tr> <td colspan="3" style="border-top: 1px dashed black; padding-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">9.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>-SHOW</b> <math>\forall xJx</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">UD</td> </tr> <tr> <td colspan="3" style="border-top: 1px dashed black; padding-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">10.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>SHOW</b> <math>Jc</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">DD</td> </tr> <tr> <td colspan="3" style="border-top: 1px dashed black; padding-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">11.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb \rightarrow Hab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\forall E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Hab</math></td> <td style="text-align: right; border-right: 1px dashed black;">7, 11, <math>\rightarrow E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\exists I</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay \rightarrow Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, <math>\forall E</math></td> </tr> <tr> <td>15.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, 14, <math>\rightarrow E</math></td> </tr> <tr> <td>16.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx</math></td> <td style="text-align: right; border-right: 1px dashed black;">15, <math>\exists I</math></td> </tr> <tr> <td>17.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx \rightarrow Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, <math>\forall E</math></td> </tr> <tr> <td>18.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">16, 17, <math>\rightarrow E</math></td> </tr> </table> </td> </tr> </table> </td> </tr> <tr> <td>19.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\forall zGz</math></td> <td style="text-align: right; border-right: 1px dashed black;">4, 9, <math>\rightarrow E</math></td> </tr> <tr> <td>20.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Gb</math></td> <td style="text-align: right; border-right: 1px dashed black;">19, <math>\forall E</math></td> </tr> </table> </td> </tr> </table>			6.	<b>SHOW</b> $Fb \rightarrow Gb$	CD	7.	$Fb$	ACD	8.	<b>SHOW</b> $Gb$	DD	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">9.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>-SHOW</b> <math>\forall xJx</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">UD</td> </tr> <tr> <td colspan="3" style="border-top: 1px dashed black; padding-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">10.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>SHOW</b> <math>Jc</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">DD</td> </tr> <tr> <td colspan="3" style="border-top: 1px dashed black; padding-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">11.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb \rightarrow Hab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\forall E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Hab</math></td> <td style="text-align: right; border-right: 1px dashed black;">7, 11, <math>\rightarrow E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\exists I</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay \rightarrow Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, <math>\forall E</math></td> </tr> <tr> <td>15.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, 14, <math>\rightarrow E</math></td> </tr> <tr> <td>16.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx</math></td> <td style="text-align: right; border-right: 1px dashed black;">15, <math>\exists I</math></td> </tr> <tr> <td>17.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx \rightarrow Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, <math>\forall E</math></td> </tr> <tr> <td>18.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">16, 17, <math>\rightarrow E</math></td> </tr> </table> </td> </tr> </table> </td> </tr> <tr> <td>19.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\forall zGz</math></td> <td style="text-align: right; border-right: 1px dashed black;">4, 9, <math>\rightarrow E</math></td> </tr> <tr> <td>20.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Gb</math></td> <td style="text-align: right; border-right: 1px dashed black;">19, <math>\forall E</math></td> </tr> </table>			9.	<b>-SHOW</b> $\forall xJx$	UD	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">10.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>SHOW</b> <math>Jc</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">DD</td> </tr> <tr> <td colspan="3" style="border-top: 1px dashed black; padding-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">11.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb \rightarrow Hab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\forall E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Hab</math></td> <td style="text-align: right; border-right: 1px dashed black;">7, 11, <math>\rightarrow E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\exists I</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay \rightarrow Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, <math>\forall E</math></td> </tr> <tr> <td>15.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, 14, <math>\rightarrow E</math></td> </tr> <tr> <td>16.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx</math></td> <td style="text-align: right; border-right: 1px dashed black;">15, <math>\exists I</math></td> </tr> <tr> <td>17.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx \rightarrow Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, <math>\forall E</math></td> </tr> <tr> <td>18.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">16, 17, <math>\rightarrow E</math></td> </tr> </table> </td> </tr> </table>			10.	<b>SHOW</b> $Jc$	DD	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">11.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb \rightarrow Hab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\forall E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Hab</math></td> <td style="text-align: right; border-right: 1px dashed black;">7, 11, <math>\rightarrow E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\exists I</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay \rightarrow Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, <math>\forall E</math></td> </tr> <tr> <td>15.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, 14, <math>\rightarrow E</math></td> </tr> <tr> <td>16.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx</math></td> <td style="text-align: right; border-right: 1px dashed black;">15, <math>\exists I</math></td> </tr> <tr> <td>17.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx \rightarrow Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, <math>\forall E</math></td> </tr> <tr> <td>18.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">16, 17, <math>\rightarrow E</math></td> </tr> </table>			11.	$Fb \rightarrow Hab$	1, $\forall E$	12.	$Hab$	7, 11, $\rightarrow E$	13.	$\exists yHay$	12, $\exists I$	14.	$\exists yHay \rightarrow Ka$	2, $\forall E$	15.	$Ka$	13, 14, $\rightarrow E$	16.	$\exists xKx$	15, $\exists I$	17.	$\exists xKx \rightarrow Jc$	3, $\forall E$	18.	$Jc$	16, 17, $\rightarrow E$	19.	$\forall zGz$	4, 9, $\rightarrow E$	20.	$Gb$	19, $\forall E$
6.	<b>SHOW</b> $Fb \rightarrow Gb$	CD																																																						
7.	$Fb$	ACD																																																						
8.	<b>SHOW</b> $Gb$	DD																																																						
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">9.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>-SHOW</b> <math>\forall xJx</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">UD</td> </tr> <tr> <td colspan="3" style="border-top: 1px dashed black; padding-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">10.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>SHOW</b> <math>Jc</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">DD</td> </tr> <tr> <td colspan="3" style="border-top: 1px dashed black; padding-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">11.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb \rightarrow Hab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\forall E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Hab</math></td> <td style="text-align: right; border-right: 1px dashed black;">7, 11, <math>\rightarrow E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\exists I</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay \rightarrow Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, <math>\forall E</math></td> </tr> <tr> <td>15.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, 14, <math>\rightarrow E</math></td> </tr> <tr> <td>16.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx</math></td> <td style="text-align: right; border-right: 1px dashed black;">15, <math>\exists I</math></td> </tr> <tr> <td>17.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx \rightarrow Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, <math>\forall E</math></td> </tr> <tr> <td>18.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">16, 17, <math>\rightarrow E</math></td> </tr> </table> </td> </tr> </table> </td> </tr> <tr> <td>19.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\forall zGz</math></td> <td style="text-align: right; border-right: 1px dashed black;">4, 9, <math>\rightarrow E</math></td> </tr> <tr> <td>20.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Gb</math></td> <td style="text-align: right; border-right: 1px dashed black;">19, <math>\forall E</math></td> </tr> </table>			9.	<b>-SHOW</b> $\forall xJx$	UD	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">10.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>SHOW</b> <math>Jc</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">DD</td> </tr> <tr> <td colspan="3" style="border-top: 1px dashed black; padding-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">11.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb \rightarrow Hab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\forall E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Hab</math></td> <td style="text-align: right; border-right: 1px dashed black;">7, 11, <math>\rightarrow E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\exists I</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay \rightarrow Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, <math>\forall E</math></td> </tr> <tr> <td>15.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, 14, <math>\rightarrow E</math></td> </tr> <tr> <td>16.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx</math></td> <td style="text-align: right; border-right: 1px dashed black;">15, <math>\exists I</math></td> </tr> <tr> <td>17.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx \rightarrow Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, <math>\forall E</math></td> </tr> <tr> <td>18.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">16, 17, <math>\rightarrow E</math></td> </tr> </table> </td> </tr> </table>			10.	<b>SHOW</b> $Jc$	DD	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">11.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb \rightarrow Hab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\forall E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Hab</math></td> <td style="text-align: right; border-right: 1px dashed black;">7, 11, <math>\rightarrow E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\exists I</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay \rightarrow Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, <math>\forall E</math></td> </tr> <tr> <td>15.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, 14, <math>\rightarrow E</math></td> </tr> <tr> <td>16.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx</math></td> <td style="text-align: right; border-right: 1px dashed black;">15, <math>\exists I</math></td> </tr> <tr> <td>17.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx \rightarrow Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, <math>\forall E</math></td> </tr> <tr> <td>18.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">16, 17, <math>\rightarrow E</math></td> </tr> </table>			11.	$Fb \rightarrow Hab$	1, $\forall E$	12.	$Hab$	7, 11, $\rightarrow E$	13.	$\exists yHay$	12, $\exists I$	14.	$\exists yHay \rightarrow Ka$	2, $\forall E$	15.	$Ka$	13, 14, $\rightarrow E$	16.	$\exists xKx$	15, $\exists I$	17.	$\exists xKx \rightarrow Jc$	3, $\forall E$	18.	$Jc$	16, 17, $\rightarrow E$	19.	$\forall zGz$	4, 9, $\rightarrow E$	20.	$Gb$	19, $\forall E$												
9.	<b>-SHOW</b> $\forall xJx$	UD																																																						
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">10.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><b>SHOW</b> <math>Jc</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">DD</td> </tr> <tr> <td colspan="3" style="border-top: 1px dashed black; padding-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">11.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb \rightarrow Hab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\forall E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Hab</math></td> <td style="text-align: right; border-right: 1px dashed black;">7, 11, <math>\rightarrow E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\exists I</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay \rightarrow Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, <math>\forall E</math></td> </tr> <tr> <td>15.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, 14, <math>\rightarrow E</math></td> </tr> <tr> <td>16.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx</math></td> <td style="text-align: right; border-right: 1px dashed black;">15, <math>\exists I</math></td> </tr> <tr> <td>17.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx \rightarrow Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, <math>\forall E</math></td> </tr> <tr> <td>18.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">16, 17, <math>\rightarrow E</math></td> </tr> </table> </td> </tr> </table>			10.	<b>SHOW</b> $Jc$	DD	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">11.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb \rightarrow Hab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\forall E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Hab</math></td> <td style="text-align: right; border-right: 1px dashed black;">7, 11, <math>\rightarrow E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\exists I</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay \rightarrow Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, <math>\forall E</math></td> </tr> <tr> <td>15.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, 14, <math>\rightarrow E</math></td> </tr> <tr> <td>16.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx</math></td> <td style="text-align: right; border-right: 1px dashed black;">15, <math>\exists I</math></td> </tr> <tr> <td>17.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx \rightarrow Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, <math>\forall E</math></td> </tr> <tr> <td>18.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">16, 17, <math>\rightarrow E</math></td> </tr> </table>			11.	$Fb \rightarrow Hab$	1, $\forall E$	12.	$Hab$	7, 11, $\rightarrow E$	13.	$\exists yHay$	12, $\exists I$	14.	$\exists yHay \rightarrow Ka$	2, $\forall E$	15.	$Ka$	13, 14, $\rightarrow E$	16.	$\exists xKx$	15, $\exists I$	17.	$\exists xKx \rightarrow Jc$	3, $\forall E$	18.	$Jc$	16, 17, $\rightarrow E$																								
10.	<b>SHOW</b> $Jc$	DD																																																						
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">11.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Fb \rightarrow Hab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\forall E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Hab</math></td> <td style="text-align: right; border-right: 1px dashed black;">7, 11, <math>\rightarrow E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\exists I</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yHay \rightarrow Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, <math>\forall E</math></td> </tr> <tr> <td>15.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ka</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, 14, <math>\rightarrow E</math></td> </tr> <tr> <td>16.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx</math></td> <td style="text-align: right; border-right: 1px dashed black;">15, <math>\exists I</math></td> </tr> <tr> <td>17.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists xKx \rightarrow Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, <math>\forall E</math></td> </tr> <tr> <td>18.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Jc</math></td> <td style="text-align: right; border-right: 1px dashed black;">16, 17, <math>\rightarrow E</math></td> </tr> </table>			11.	$Fb \rightarrow Hab$	1, $\forall E$	12.	$Hab$	7, 11, $\rightarrow E$	13.	$\exists yHay$	12, $\exists I$	14.	$\exists yHay \rightarrow Ka$	2, $\forall E$	15.	$Ka$	13, 14, $\rightarrow E$	16.	$\exists xKx$	15, $\exists I$	17.	$\exists xKx \rightarrow Jc$	3, $\forall E$	18.	$Jc$	16, 17, $\rightarrow E$																														
11.	$Fb \rightarrow Hab$	1, $\forall E$																																																						
12.	$Hab$	7, 11, $\rightarrow E$																																																						
13.	$\exists yHay$	12, $\exists I$																																																						
14.	$\exists yHay \rightarrow Ka$	2, $\forall E$																																																						
15.	$Ka$	13, 14, $\rightarrow E$																																																						
16.	$\exists xKx$	15, $\exists I$																																																						
17.	$\exists xKx \rightarrow Jc$	3, $\forall E$																																																						
18.	$Jc$	16, 17, $\rightarrow E$																																																						
19.	$\forall zGz$	4, 9, $\rightarrow E$																																																						
20.	$Gb$	19, $\forall E$																																																						

Exercises 9-6

1.	1. $\exists xAx$	P																											
	2. $Aa \rightarrow \exists y(Cy \ \& \ Dy)$	P																											
	3. $\exists yAy \rightarrow \forall z(Hz \rightarrow Az)$	P																											
	4. $Ha$	P																											
	5. <b>SHOW</b> $\exists yDy$	DD																											
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 5%; vertical-align: top;">6.</td> <td style="width: 85%; border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ab</math></td> <td style="width: 10%; text-align: right; border-right: 1px dashed black;">1, <math>\exists E</math></td> </tr> <tr> <td>7.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yAy</math></td> <td style="text-align: right; border-right: 1px dashed black;">6, <math>\exists I</math></td> </tr> <tr> <td>8.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\forall z(Hz \rightarrow Az)</math></td> <td style="text-align: right; border-right: 1px dashed black;">3, 7, <math>\rightarrow E</math></td> </tr> <tr> <td>9.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Ha \rightarrow Aa</math></td> <td style="text-align: right; border-right: 1px dashed black;">8, <math>\forall E</math></td> </tr> <tr> <td>10.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Aa</math></td> <td style="text-align: right; border-right: 1px dashed black;">4, 9, <math>\rightarrow E</math></td> </tr> <tr> <td>11.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists y(Cy \ \&amp; \ Dy)</math></td> <td style="text-align: right; border-right: 1px dashed black;">2, 10, <math>\rightarrow E</math></td> </tr> <tr> <td>12.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Cc \ \&amp; \ Dc</math></td> <td style="text-align: right; border-right: 1px dashed black;">11, <math>\exists E</math></td> </tr> <tr> <td>13.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>Dc</math></td> <td style="text-align: right; border-right: 1px dashed black;">12, <math>\&amp;E</math></td> </tr> <tr> <td>14.</td> <td style="border-left: 1px dashed black; border-right: 1px dashed black; padding-left: 5px;"><math>\exists yDy</math></td> <td style="text-align: right; border-right: 1px dashed black;">13, <math>\exists I</math></td> </tr> </table>			6.	$Ab$	1, $\exists E$	7.	$\exists yAy$	6, $\exists I$	8.	$\forall z(Hz \rightarrow Az)$	3, 7, $\rightarrow E$	9.	$Ha \rightarrow Aa$	8, $\forall E$	10.	$Aa$	4, 9, $\rightarrow E$	11.	$\exists y(Cy \ \& \ Dy)$	2, 10, $\rightarrow E$	12.	$Cc \ \& \ Dc$	11, $\exists E$	13.	$Dc$	12, $\&E$	14.	$\exists yDy$	13, $\exists I$
6.	$Ab$	1, $\exists E$																											
7.	$\exists yAy$	6, $\exists I$																											
8.	$\forall z(Hz \rightarrow Az)$	3, 7, $\rightarrow E$																											
9.	$Ha \rightarrow Aa$	8, $\forall E$																											
10.	$Aa$	4, 9, $\rightarrow E$																											
11.	$\exists y(Cy \ \& \ Dy)$	2, 10, $\rightarrow E$																											
12.	$Cc \ \& \ Dc$	11, $\exists E$																											
13.	$Dc$	12, $\&E$																											
14.	$\exists yDy$	13, $\exists I$																											

3.	1. $\forall x\forall y(Fx \rightarrow \neg Gy)$	P
----	---	---

2.	<b>SHOW</b> $\sim\exists x(Fx \ \& \ Gx)$	ID
-----		
3.	$\exists x(Fx \ \& \ Gx)$	AID
4.	<b>SHOW</b> !	DD
-----		
5.	Fa & Ga	3, $\exists E$
6.	Fa	5, $\&E$
7.	$\forall y(Fa \ \rightarrow \ \sim Gy)$	1, $\forall E$
8.	Fa $\rightarrow \ \sim Ga$	7, $\forall E$
9.	$\sim Ga$	6, 8, $\rightarrow E$
10.	Ga	5, $\&E$
11.	!	9, 10, !I
-----		

5.	1.	$\forall x\exists y(Dx \leftrightarrow Fy)$	P
	2.	<b>SHOW</b> $\forall xFx \rightarrow \forall yDy$	CD
-----			
3.	$\forall xFx$	ACD	
4.	<b>SHOW</b> $\forall yDy$	UD	
-----			
5.	<b>SHOW</b> Da	DD	
-----			
6.	$\exists y(Da \leftrightarrow Fy)$	1, $\forall E$	
7.	Da $\leftrightarrow$ Fb	6, $\exists E$	
8.	Fb	3, $\forall E$	
9.	Da	7, 8, $\leftrightarrow E$	
-----			

7.	1.	$\forall x\forall y(Rx \leftrightarrow Wy)$	P
	2.	$\exists x(Rx \vee Wx)$	P
	3.	<b>SHOW</b> $\forall zRz$	UD
-----			
4.	<b>SHOW</b> Ra	DD	
-----			
5.	Rb $\vee$ Wb	2, $\exists E$	
6.	<b>SHOW</b> Wb $\rightarrow$ Ra	CD	
-----			
7.	Wb	ACD	
8.	<b>SHOW</b> Ra	DD	
-----			
9.	$\forall y(Ra \leftrightarrow Wy)$	1, $\forall E$	
10.	Ra $\leftrightarrow$ Wb	9, $\forall E$	
11.	Ra	7, 10 $\leftrightarrow E$	
-----			
12.	<b>SHOW</b> Rb $\rightarrow$ Ra	CD	
-----			
13.	Rb	ACD	
14.	<b>SHOW</b> Ra	DD	
-----			
15.	$\forall y(Rb \leftrightarrow Wy)$	1, $\forall E$	

16.		Rb ↔ Wb		15, ∨E
17.		Wb		13, 16, ↔E
18.		Ra		6, 17, → E
19.	Ra			5, 6, 12, SC

9.	1.	-SHOW $\sim\exists xLx \leftrightarrow \forall x\sim Lx$	DD
	2.	-SHOW $\sim\exists xLx \rightarrow \forall x\sim Lx$	CD
	3.	$\sim\exists xLx$	ACD
	4.	-SHOW $\forall x\sim Lx$	UD
	5.	-SHOW $\sim La$	ID
	6.	La	AID
	7.	-SHOW !	DD
	8.	$\exists xLx$	6, ∃I
	9.	!	3, 8, !I
	10.	-SHOW $\forall x\sim Lx \rightarrow \sim\exists xLx$	CD
	11.	$\forall x\sim Lx$	ACD
	12.	-SHOW $\sim\exists xLx$	ID
	13.	$\exists xLx$	AID
	14.	-SHOW !	DD
	15.	Lb	13, ∃E
	16.	$\sim Lb$	11, ∨E
	17.	!	15, 16, !I
	18.	$\sim\exists xLx \leftrightarrow \forall x\sim Lx$	2, 10, ↔I

Exercises 9-7

1.	1.	$\forall x\forall y[Fxy \rightarrow (Gx \vee Hy)]$	P
	2.	Frs & $\sim Hs$	P
	3.	SHOW Gr	DD
	4.	Frs	2, &E
	5.	$\forall y[Fry \rightarrow (Gr \vee Hy)]$	1, ∨E
	6.	Frs $\rightarrow (Gr \vee Hs)$	5, ∨E

7.	$Gr \vee Hs$	4, 6 $\rightarrow E$
8.	$\sim Hs$	2, $\&E$
9.	$Gr$	7, 8, $\vee E$

3.	1.	$\exists x \exists y (Pxy \vee Pyx)$	P
	2.	<b>SHOW</b> $\exists x \exists y Pxy$	DD
	-----		
	3.	$\exists y (Pay \vee Pya)$	1, $\exists I$
	4.	$Pab \vee Pba$	3, $\exists I$
	5.	<b>SHOW</b> $Pab \rightarrow \exists x \exists y Pxy$	CD
	-----		
	6.	$Pab$	ACD
	7.	<b>SHOW</b> $\exists x \exists y Pxy$	DD
	-----		
	8.	$\exists y Pay$	6, $\exists I$
	9.	$\exists x \exists y Pxy$	8, $\exists I$
	-----		
	10.	<b>SHOW</b> $Pba \rightarrow \exists x \exists y Pxy$	CD
	-----		
	11.	$Pba$	ACD
	12.	<b>SHOW</b> $\exists x \exists y Pxy$	DD
	-----		
	13.	$\exists y Pby$	11, $\exists I$
	14.	$\exists x \exists y Pxy$	13, $\exists I$
	-----		
	15.	$\exists x \exists y Pxy$	4, 5, 10, SC

5.	1.	<b>SHOW</b> $\exists x \forall y Fxy \rightarrow \forall y \exists x Fxy$	CD
	-----		
	2.	$\exists x \forall y Fxy$	ACD
	3.	<b>SHOW</b> $\forall y \exists x Fxy$	UD
	-----		
	4.	<b>SHOW</b> $\exists x Fxa$	DD
	-----		
	5.	$\forall y Fby$	2, $\forall E$
	6.	$Fba$	5, $\forall E$
	7.	$\exists x Fxa$	6, $\exists I$

7.	1.	$\forall x \forall y (Bxy \vee Byx)$	P
	2.	<b>SHOW</b> $\forall x Bxx$	UD
	-----		
	3.	<b>SHOW</b> $Baa$	ID
	-----		
	4.	$\sim Baa$	AID
	5.	<b>SHOW</b> !	DD
	-----		

6.				$\forall y(Bay \vee Bya)$				1, $\forall E$
7.				$Baa \vee Baa$				6, $\forall E$
8.				$Baa$				4, 7, $\forall E$
9.				!				4, 8, $!I$

9.	1.	$\forall x\forall y\forall z[(Cxy \ \& \ Cyz) \rightarrow Cxz]$	P
	2.	$\forall x\forall y(Cxy \rightarrow Cyx)$	P
	3.	<b>SHOW</b> $\forall x(\sim Cxx \rightarrow \forall y\sim Cxy)$	UD

4.				<b>SHOW</b> $\sim Caa \rightarrow \forall y\sim Cay$				CD
5.				$\sim Caa$				ACD
6.				<b>SHOW</b> $\forall y\sim Cay$				UD
7.				<b>SHOW</b> $\sim Cab$				ID
8.				$Cab$				AID
9.				<b>SHOW</b> !				DD
10.				$\forall y(Cay \rightarrow Cya)$				2, $\forall E$
11.				$Cab \rightarrow Cba$				10, $\forall E$
12.				$Cba$				8, 11, $\rightarrow E$
13.				$Cab \ \& \ Cba$				8, 12, $\&I$
14.				$\forall y\forall z[(Cay \ \& \ Cyz) \rightarrow Caz]$				1. $\forall E$
15.				$\forall z[(Cab \ \& \ Cbz) \rightarrow Caz]$				14, $\forall E$
16.				$(Cab \ \& \ Cba) \rightarrow Caa$				15, $\forall E$
17.				$Caa$				13, 16, $\rightarrow E$
18.				!				5, 17, $!I$

11.	1.	<b>SHOW</b> $\forall x\exists y(Rxy \leftrightarrow Rxx)$	UD
	2.	<b>SHOW</b> $\exists y(Ray \leftrightarrow Raa)$	DD
	3.	<b>SHOW</b> $Raa \rightarrow Raa$	CD
	4.	$Raa$	ACD
	5.	<b>SHOW</b> $Raa$	DD
	6.	$Raa$	4, R
	7.	$Raa \leftrightarrow Raa$	3, $\leftrightarrow I$
	8.	$\exists y(Ray \leftrightarrow Raa)$	7, $\exists I$