

Exercises 11-1

1. $\forall x(Bx \rightarrow x = t)$
3. $\exists x[(Sx \ \& \ Bx) \ \& \ \forall y[(Sy \ \& \ By) \rightarrow x = y]]$
5. $\sim \exists x[Bx \ \& \ (x \neq t \ \& \ Sx)]$
7. $\forall x[(Lx \ \& \ x \neq e) \rightarrow Txe]$
9. $\exists x \forall y([(My \ \& \ Ly) \ \& \ Lyg) \rightarrow x = y]$ or
 $\forall x \forall y([(Mx \ \& \ Lx) \ \& \ Lxg] \ \& \ [(My \ \& \ Ly) \ \& \ Lyg] \rightarrow x = y]$
11. $\forall x[(Lx \ \& \ x \neq a) \rightarrow Tax]$
13. D = People
J①: ① could be jivin'
L①②: ① loves ②
i : me
m : my mother

 $\sim \exists x(x \neq m \ \& \ lxi) \ \& \ Jm$
15. D = Presidents (sc. of the U.S.)
I① : ① has been impeached
c : William Clinton
j : Andrew Johnson

 $\sim \exists x[x \neq j \ \& \ x \neq c] \ \& \ Ix]$

Exercises 11-2

1. $\exists x([(Wx \ \& \ Kxa) \ \& \ \forall y[(Wy \ \& \ Kya) \rightarrow x = y]] \ \& \ Rx)$
3. $\exists x[(Wx \ Lax) \ \& \ \exists y([(My \ \& \ Lyg) \ \& \ \forall z[(Mz \ \& \ Lzg) \rightarrow y = z]] \ \& \ Kxy)]$
5. $\exists x \exists y([(Mx \ \& \ Lgx) \ \& \ \forall z[(Mz \ \& \ Lgz) \rightarrow x = z]] \ \& \ [(Wy \ \& \ Lay) \ \& \ \forall z[(Wz \ \& \ Laz) \rightarrow y = z]]) \ \& \ Kxy]$

Exercises 11-3

1. Your interpretation must have: the thing labeled r is not W
3. Your interpretation must have: no more than two items in its domain and exactly one thing that is F .
5. Your interpretation must have exactly one item in its domain.
7. The sentence on the left is a logical truth. Your interpretation must have at least two items in its domain which are F or at least two items which are not F.
9. Your interpretation must satisfy one of the following two conditions:
 - at least two things are G and at most one is not
 - At least two things fail to be G and at most one is.
11. Your interpretation must have nothing that is G.
13. Your interpretation must have all of the following:
 - at least two items in its domain,
 - nothing Qs itself, and
 - each thing Qs something other than itself.

Exercises 11-4:

1.	1.	SHOW $\forall x \forall y \forall z [(x = y \ \& \ y = z) \rightarrow x = z]$	UD
	2.	SHOW $\forall y \forall z [(a = y \ \& \ y = z) \rightarrow a = z]$	UD
	3.	SHOW $\forall z [(a = b) \ \& \ (b = z) \rightarrow a = z]$	UD
	4.	SHOW $(a = b \ \& \ b = c) \rightarrow a = c$	CD
	5.	$a = b \ \& \ b = c$	ACD
	6.	SHOW $a = c$	DD
	7.	$a = b$	5, &E
	8.	$b = c$	5, &E
	9.	$a = c$	7, 8, =E

3.	1.	$\exists x ([Rx \ \& \ \forall y (Ry \rightarrow x = y)] \ \& \ Nx)$	P
	2.	Rt	P
	3.	SHOW Nt	DD
	4.	$[Ra \ \& \ \forall y (Ry \rightarrow a = y)] \ \& \ Na$	1, $\exists E$
	5.	$Ra \ \& \ \forall y (Ry \rightarrow a = y)$	4, &E
	6.	$\forall y (Ry \rightarrow a = y)$	5, &E
	7.	$Rt \rightarrow a = t$	6, $\forall E$
	8.	$a = t$	2, 7, $\rightarrow E$
	9.	Na	4, &E
	10.	Nt	8, 9, =E

5.	1.	SHOW $\forall x \exists y x = y$	UD
	2.	SHOW $\exists y a = y$	DD
	3.	$a = a$	=I
	4.	$\exists y a = y$	3, $\exists I$

7.	1.	$\exists x \forall y (Ay \leftrightarrow x = y)$	P
	2.	SHOW $\exists x [Ax \ \& \ \forall y (Ay \rightarrow x = y)]$	DD
	3.	$\forall y (Ay \leftrightarrow a = y)$	1, $\exists E$
	4.	$Aa \leftrightarrow a = a$	3, $\forall E$
	5.	$a = a$	=I
	6.	Aa	4, 5, $\leftrightarrow E$
	7.	SHOW $\forall y (Ay \rightarrow a = y)$	UD

8.	SHOW $Ab \rightarrow a = b$	CD
9.	Ab	ACD
10.	SHOW $a = b$	DD
11.	Ab $\leftrightarrow a = b$	3, $\forall E$
12.	$a = b$	9, 11, $\leftrightarrow E$
13.	$Aa \ \& \ \forall y(Ay \rightarrow a = y)$	6, 7, $\&I$
14.	$\exists x[Ax \ \& \ \forall y(Ay \rightarrow a = x)]$	13, $\exists I$

1.	$\exists x[Ax \ \& \ \forall y(Ay \rightarrow x = y)]$	P
2.	SHOW $\exists x \forall y(Ay \leftrightarrow x = y)$	DD
3.	$Aa \ \& \ \forall y(Ay \rightarrow a = y)$	1, $\exists E$
4.	Aa	3, $\&E$
5.	$\forall y(Ay \rightarrow a = y)$	3, $\&E$
6.	SHOW $\forall y(Ay \leftrightarrow a = y)$	UD
7.	SHOW $Ab \leftrightarrow a = b$	DD
8.	$Ab \rightarrow a = b$	5, VD
9.	SHOW $a = b \rightarrow Ab$	CD
10.	$a = b$	
11.	SHOW Ab	DD
12.	Ab	4, 10, $=E$
13.	$Ab \leftrightarrow a = b$	8, 9, $\leftrightarrow I$
14.	$\exists x \forall y Ay \leftrightarrow x = y$	6, $\exists I$

9.	1.	$\forall x \exists y Gxy$	P
	2.	$\sim \exists x Gxx$	P
	3.	$\sim \exists x \exists y x \neq y$	P
	4.	SHOW !	DD
	5.	$\exists y Gay$	1, $\forall E$
	6.	Gab	5, $\exists E$
	7.	$\forall y \sim \exists y x \neq y$	3, QN
	8.	$\sim \exists y a \neq y$	7, $\forall E$
	9.	$\forall y \sim a \neq y$	8, QN
	10.	$\sim a \neq b$	9, $\forall E$
	11.	$a = b$	10, DN
	12.	Gaa	6, 11, $=E$
	13.	$\exists x Gxx$	12, $\exists I$
	14.	!	2, 13, $!I$

11.	1.	$\exists x \forall y x = y$	P
	2.	$\exists x Fx$	P
	3.	$\exists x Dx$	P
	4.	$\sim \exists x (Fx \ \& \ Dx)$	P
	5.	SHOW !	DD

	6.	Fa	2, $\exists E$
	7.	Db	3, $\exists E$
	8.	Fa & Db	6, 7, &I
	9.	$\forall y c = y$	1, $\exists E$
	10.	c = a	9, $\forall E$
	11.	c = b	9, $\forall E$
	12.	a = b	10, 11, =E
	13.	Fa & Da	8, 12, =E
	14.	$\exists x (Fx \ \& \ Dx)$	13, $\exists I$
	15.	!	4, 14, !I

13. 1. $\forall x x = a$ P
 2. ~~SHOW~~ $\forall x Fx \leftrightarrow Fa$ DD

	3.	SHOW $\forall x Fx \rightarrow Fa$	CD
	4.	$\forall x Fx$	ACD
	5.	SHOW Fa	DD
	6.	Fa	4, $\forall E$
	7.	SHOW $Fa \rightarrow \forall x Fx$	CD
	8.	Fa	ACD
	9.	SHOW $\forall x Fx$	UD
	10.	SHOW Fb	DD
	11.	b = a	1, $\forall E$
	12.	Fb	8, 11, =E
	14.	$\forall x Fx \leftrightarrow Fa$	3, 7, $\leftrightarrow E$

15. 1. $\forall x (x = a \vee x = b)$ P
 2. SHOW $\forall x Fx \leftrightarrow (Fa \ \& \ Fb)$ DD

	3.	SHOW $\forall x Fx \rightarrow (Fa \ \& \ Fb)$	
	4.	$\forall x Fx$	ACD
	5.	SHOW (Fa & Fb)	DD

6.				Fa				4, $\forall E$
7.				Fb				4, $\forall E$
8.				Fa & Fb				6, 7, &I

9.				SHOW (Fa & Fb) \rightarrow $\forall xFx$				

10.				Fa & Fb				ACD
11.				SHOW $\forall xFx$				UD

12.				SHOW Fc				DD

13.				c = a \vee c = b				1, $\forall E$
14.				SHOW c = a \rightarrow Fc				CD

15.				c = a				ACD
16.				SHOW Fc				DD

17.				Fa				10, &E
18.				Fc				15, 17, =E

19.				SHOW c = b \rightarrow Fc				CD

20.				c = b				ACD
21.				SHOW Fc				DD

22.				Fb				10, &E
23.				Fc				20, 22, =E

24.				Fc				13, 14, 19, SC

25.				$\forall xFx \leftrightarrow (Fa \& Fb)$				3, 9, $\leftrightarrow I$
