Channel Coordination for a Supply Chain with a Risk-Neutral Manufacturer and a Loss-Averse Retailer*

Charles X. Wang†
School of Management, State University of New York at Buffalo, Buffalo, NY 14260,
e-mail: cxwang@buffalo.edu

Scott Webster
Whitman School of Management, Syracuse University, Syracuse, NY 13244,
e-mail: stwebste@syr.edu

ABSTRACT

This article considers a decentralized supply chain in which a single manufacturer is selling a perishable product to a single retailer facing uncertain demand. It differs from traditional supply chain contract models in two ways. First, while traditional supply chain models are based on risk neutrality, this article takes the viewpoint of behavioral principal–agency theory and assumes the manufacturer is risk neutral and the retailer is loss averse. Second, while gain/loss (GL) sharing is common in practice, there is a lack of analysis of GL-sharing contracts in the supply chain contract literature. This article investigates the role of a GL-sharing provision for mitigating the loss-aversion effect, which drives down the retailer order quantity and total supply chain profit. We analyze contracts that include GL-sharing-and-buyback (GLB) credit provisions as well as the special cases of GL contracts and buyback contracts. Our analytical and numerical results lend insight into how a manufacturer can design a contract to improve total supply chain, manufacturer, and retailer performance. In particular, we show that there exists a special class of distribution-free GLB contracts that can coordinate the supply chain and arbitrarily allocate the expected supply chain profit between the manufacturer and retailer; in contrast with other contracts, the parameter values for contracts in this class do not depend on the probability distribution of market demand. This feature is meaningful in practice because (i) the probability distribution of demand faced by a retailer is typically unknown by the manufacturer and (ii) a manufacturer can offer the same contract to multiple noncompeting retailers that differ by demand distribution and still coordinate the supply chains.

Subject Areas: Buyback Contract, Gain/Loss Sharing, Loss Aversion, Supply Chain Contracts and Incentives, and Supply Chain Coordination.

INTRODUCTION

The volatile market of manufactured goods with short life cycles (such as fashion apparel, electronics, personal computers, toys, and books) is characterized by

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†Corresponding author.
uncertain demand, a short selling season, and long lead times. In this setting, the retailer typically has to purchase products from the manufacturer before the season begins and has no opportunity to replenish during the selling season.

One of the commonly used supply chain contracts for short-life-cycle products is a buyback contract, wherein the retailer receives a buyback credit from the manufacturer for unsold goods at the end of the selling season. For example, buyback contracts are common in the publishing industry, where between 30% and 35% of new hardcover books are returned by the booksellers to the publisher (Cachon & Terwiesch, 2006; Chopra & Meindl, 2007). In the apparel industry, department stores (e.g., Federated, Dillard’s, Saks, Kohl’s, and J.C. Penney) receive a buyback credit (known as markdown money in the industry) from the manufacturers (e.g., Tommy Hilfiger, Liz Claiborne, Ralph Lauren, and the Jones Apparel Group) to subsidize their clearance sales (Kratz, 2005; Rozhon, 2005). In the toy industry, DoodleTop offers retailers a buyback contract with each new toy it introduces to the market (Leccese, 1993). In the personal computer industry, Hewlett Packard and IBM have implemented buyback contracts to help distributors better manage their inventory (Anonymous, 2001). In the electronics industry, manufacturers (e.g., Intel) provide returns policies to their distributors (e.g., Arrow OEM Computing Solution), which in turn extend such returns privileges to the original equipment manufacturers (Spiegel, 2002; Roos, 2003).

It is well known that, if both manufacturer and retailer are risk neutral and maximize their own profits, double marginalization, a phenomenon first identified by Spengler (1950) for a deterministic demand model, will prevail. That is, the retailer will order less than the integrated supply chain’s optimal inventory level. We refer to this supply chain phenomenon as inventory understocking. Because inventory understocking is system suboptimal, supply chain contracts, which have drawn much attention from researchers, are used to provide incentives (e.g., buyback credit) to adjust the relationship of supply chain partners so as to coordinate the supply chain. That is, the supply chain contracts are used to drive the total expected profit of the decentralized supply chain toward that achieved under an integrated system. In this article, we refer to this practice as supply chain coordination.

Traditional supply chain contract models are based on risk neutrality, where managers make decisions to maximize expected profits. In practice, however, there are examples of order quantity decisions that are not consistent with expected profit maximization (e.g., Kahn, 1992; Fisher & Raman, 1996; Patsuris, 2001). In view of this, some researchers have called for models that deviate from the assumption of risk neutrality. For example, Anupindi (1999) points out that relaxing some restraints such as risk neutrality in order to represent more realistic settings will be very useful in understanding supply chain behaviors. Tsay, Nahmias, and Agrawal (1999) call for future research in analysis of supply chain contracts when various players are allowed to have objective functions other than profit maximization. Wu, Roundy, Storer, and Martin-Vega (1999) state that, in order for planning or decision models to capture the dynamics of a manually operated supply chain system, consideration of different aspects of human behavior is a crucial and challenging task.

This article differs from those that examine traditional supply chain contract models in two ways. First, to answer the recent calls for research on the behavioral
issues in supply chain contracts, this article takes the viewpoint of Kahneman and Tversky’s (1979) prospect theory rather than risk neutrality to describe the retailer’s decision-making behavior. A key feature of prospect theory is loss aversion, meaning that people are more averse to losses than they are attracted to same-sized gains, and the perception of a loss or gain is tied to a reference point. Loss aversion is both intuitively appealing and well supported in finance, economics, marketing, and organizational behavior (Rabin, 1998; Camerer, 2001). For example, there are economic field tests supporting loss aversion in financial markets (Benartzi & Thaler, 1995), life savings and consumptions (Bowman, Minehart, & Rabin, 1999), labor supply (Camerer, Babcock, Loewenstein, & Thaler, 1997), marketing (Putler, 1992), real estate (Genesove & Mayer, 2001), and organizational behavior (Fiegenbaum & Thomas, 1988; Fiegenbaum, 1990; Lehner, 2000). Specifically, this article takes the viewpoint of behavioral agency theory, in which the principal (e.g., a large manufacturer) is risk neutral because it can diversify its assets across multiple firms, and the agent (e.g., a small retailer) is loss averse because its security of doing business and income are tied to the principal (Wiseman & Gomez-Mejia, 1998). In view of this, we study a supply chain composed of a single risk-neutral manufacturer and a single loss-averse retailer.

Second, while supply chain contracts have received much attention from researchers recently, there is a lack of research on the gain/loss (GL)-sharing contract. A GL-sharing contract specifies that the upstream party (i.e., the manufacturer) must share some or all of the downstream party’s (i.e., the retailer’s) gains or losses. GL-sharing provisions are common in insurance and banking industries (Smith & Bickelhaupt, 1981; Rochet & Tirole, 1996) and some supply chain settings such as third-party logistics and outsourcing/procurement (Min, 2002; Ericson, 2003; Leganza, 2004). To fill this research gap, we investigate the role of a GL-sharing provision for mitigating the loss-aversion effect and improving supply chain performance.

Our analytical and numerical results lend insight into when and how a manufacturer should use GL sharing and/or buyback credit provisions in a contract with a loss-averse retailer. We contrast the performance of a price-only contract with a GL-sharing and buyback (GLB) contract and its two special cases—a pure GL contract and a pure buyback contract. First, we find that consideration of retailer loss aversion in contract design can significantly improve manufacturer profit in settings where demand uncertainty is high and the total contribution margin of the supply chain is low. Second, we show that, in some settings, it is not possible to coordinate the supply chain without a buyback credit provision. Third, we identify a special class of distribution-free coordinating GLB contracts. If the wholesale price is exogenously given by the market, then there always exists a class of GLB contracts with higher expected manufacturer profit than the coordinating buyback contract. And, more significantly, the coordinating parameters of contracts in this class do not depend on the retailer’s perception of market demand. This characteristic is unique among the contracts we consider and is meaningful in practice because (i) a manufacturer is unlikely to be able to accurately predict the probability distribution of demand as perceived by the retailer and (ii) even if the manufacturer knows the probability distribution with precision, a dependency between contract parameter values and the probability distribution means that a single contract cannot be
offered to multiple retailers while simultaneously maximizing expected supply chain profit.

This article is organized as follows. First, we review the relevant literature. Second, we introduce the integrated supply chain model as a benchmark and a bilateral supply chain model under a wholesale price-only contract without coordination. Third, we describe a GLB contract and alternative contracts that are special cases (i.e., buyback, GL, and price-only). Fourth, we investigate the relative contract performance through analytical properties and numerical studies. Fifth, we summarize the main managerial implications of our results. Finally, we offer concluding comments and suggest opportunities for future research.

RELATED SUPPLY CHAIN CONTRACT LITERATURE

Supply chain contract models have received much attention from researchers recently. In this section, we focus on buyback contracts, which are most closely related to this research. We refer to Cachon (2003) and Tsay et al. (1999) for reviews of other types of supply chain contracts.

As discussed earlier, buyback contracts, which are also known as returns policies, are common in the distribution of perishable commodities with uncertain demand, such as books, magazines, newspapers, recorded music, computer hardware and software, greeting cards, and pharmaceuticals (Padmanabhan & Png, 1995). Pasternack (1985) is the first to study a buyback contract. He shows that the two extremes, full returns with full buyback credit and no returns, are system suboptimal, but the supply chain can be coordinated by an intermediate buyback contract policy (e.g., partial returns with full buyback credit or full returns with partial buyback credit). He also identifies a class of buyback contracts with coordinating parameters that are independent of market demand. Kandel (1996) extends Pasternack (1985) to a price-sensitive stochastic demand model and concludes that the supply chain cannot be coordinated by buyback contracts without retail price maintenance (i.e., allowing the manufacturer to dictate the retail price). Emmons and Gilbert (1998) study a price-sensitive multiplicative model of demand uncertainty for catalog goods and demonstrate that uncertainty tends to increase the retail price. They also show that, under certain conditions, a manufacturer can increase her profit by offering a buyback contract. Webster and Weng (2000) take the viewpoint of a manufacturer selling to a single retailer and describe risk-free buyback contracts. Donohue (2000) studies buyback contracts in a supply chain model with multiple production opportunities and improving demand forecasts. Taylor (2002) incorporates a buyback contract with a target sales rebate contract to coordinate the supply chain when demand is sensitive to the retailer sales effort.

In addition to risk neutrality, some researchers have studied supply chain contracts based on the traditional principal–agency theory that incorporates risk aversion. Agrawal and Seshadri (2000) investigate a supply chain composed of a risk-neutral distributor and risk-averse retailers for a short-life-cycle product, where the distributor offers a risk intermediation contract with a fixed fee, a wholesale price for each unit sold, and a buyback price for each unit unsold, in order to induce the risk-averse retailers to order their profit-maximizing quantities. Plambeck and Zenios (2000) study a dynamic principal–agent model with a physical structure of a
Markov decision process that is different from the newsvendor model setting in our article. They show that the risk-neutral principal can design an optimal payment scheme to induce the risk-averse agent’s actions for profit maximization. While those works are built on traditional principal–agency theory that incorporates risk aversion, this article draws on behavioral principal–agency theory that incorporates loss aversion. Loss aversion stems from the field of psychology and can be traced to the work of Kahneman and Tversky (1979). It is distinguished from risk aversion by the presence of a reference point that determines whether a payoff is perceived as a loss or a gain, and by an abrupt change in the slope of the utility function at the reference point. We refer interested readers to Wiseman and Gomez-Mejia (1998) and references therein for the detailed discussion of the difference between the traditional and behavioral principal–agency theory.

THE INTEGRATED AND DECENTRALIZED SUPPLY CHAIN

We begin by investigating a vertically integrated firm in which a large and diversified risk-neutral manufacturer owns its own retail channel and acts as a central planner for the supply chain. This centralized control setting provides us with a benchmark solution that maximizes total expected supply chain profit.

At the beginning of the selling season, the integrated firm produces $Q$ units of a single item at a quantity-independent cost $c$ per unit of production and delivery and sells these units at a unit retail price $p > c$ during the selling season. The selling season demand $X$ is a random variable with PDF $f(x)$ and CDF $F(x)$ defined over the continuous interval $I \in [0, \infty)$. We let $\bar{F}(x) = 1 - F(x)$ denote the tail distribution. As with most of the traditional newsvendor models, we assume $F(x)$ is differentiable, invertible, and strictly increasing over $I$. If the realized demand $x$ is higher than $Q$, then the retailer loses the opportunity to make a profit on $x - Q$ units. We assume there is no additional cost (e.g., loss of goodwill) on unsatisfied demand (such as in Lariviere & Porteus, 2002; Netessine & Zhang, 2005). If the realized demand $x$ is lower than $Q$, then the integrated firm salvages $Q - x$ unsold products at a unit value $v < c$. The expected total system profit is

$$E[\pi(X, Q)] = \int_{0}^{Q} [px + v(Q - x)]f(x)\,dx + \int_{Q}^{\infty} pQf(x)\,dx - cQ. \quad (1)$$

The production quantity that maximizes expected total system profit is

$$Q^* = F^{-1}\left(\frac{p - c}{p - v}\right). \quad (2)$$

Now consider a decentralized supply chain composed of a large independent manufacturer and a small independent retailer. We assume the manufacturer, as a large diversified firm, is risk neutral, and we assume the retailer, as a smaller and less diversified firm, is loss averse. At the beginning of the selling season, the retailer orders $Q$ units of a single item from the manufacturer at a wholesale price $w$, where the manufacturer’s production and delivery cost is $c < w$ and the unit retail price is $p > w$. Let $W_0$ denote the retailer’s reference level (i.e., initial wealth)
at the beginning of the selling season. We assume gain (or loss) is perceived if the final wealth after the selling season is higher (or lower) than the initial wealth. We define the retailer’s loss-aversion utility function to be piecewise linear as follows:

\[
U(W) = \begin{cases} 
W - W_0 & W \geq W_0 \\
\lambda(W - W_0) & W < W_0,
\end{cases}
\]  

(3)

where \( \lambda \geq 1 \) is defined as the loss-aversion level and \( W \) is the retailer’s final wealth after the selling season. If \( \lambda = 1 \), then the retailer is risk neutral. If \( \lambda > 1 \), then there exists a slope change at the reference level, and higher values of \( \lambda \) imply higher levels of loss aversion. Without loss of generality, we normalize \( W_0 = 0 \). This piecewise-linear form of loss-aversion utility function does not preserve the diminishing sensitivity property in prospect theory. However, because of its simplicity, it is commonly used in the literature (Kahneman & Tversky, 1979; Schweitzer & Cachon, 2000; Barberis & Huang, 2001).

Assuming that the retailer realizes a unit salvage value \( v < w \) for each unsold unit, the retailer’s expected profit function under this price-only contract is

\[
E[\pi_r(X, Q, w)] = \int_0^Q [px + v(Q - x)]f(x)\,dx + \int_Q^{\infty} pQf(x)\,dx - wQ.  
\]  

(4)

We let

\[
q(Q, w) = (w - v)Q / (p - v)  
\]  

(5)

denote the retailer’s breakeven selling quantity function. If realized demand relative to \( Q \) is too low (i.e., \( x < q(Q, w) \)), then the retailer faces losses. If realized demand is more than \( q(Q, w) \), then the retailer faces gains. After mapping the retailer’s expected profit function (4) into its utility function (3), we can express the retailer’s expected utility \( E[U(\pi_r(X, Q, w))] \) as follows:

\[
E[U(\pi_r(X, Q, w))] = (\lambda - 1) \int_0^{q(Q, w)} [px + v(Q - x) - wQ]f(x)\,dx \\
+ E[\pi_r(X, Q)].  
\]  

(6)

From (6), we see the loss-averse retailer’s expected utility under the price-only contract is the expected profit plus the expected loss, biased by a factor of \( \lambda - 1 \). If \( \lambda = 1 \), then the retailer is risk neutral and the first term in (6) vanishes.

After taking the first and second derivatives of (6) with respect to \( Q \), we get

\[
dE[U(\pi_r(X, Q, w))] / dQ = -(\lambda - 1)(w - v)F(q(Q, w)) \\
+ p - w - (p - v)F(Q) 
\]
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and

\[
d^2 E[U(\pi_r(X, Q, w))] / dQ^2 = -(\lambda - 1) \frac{(w - v)^2}{p - v} f(q(Q, w)) - (p - b) f(Q) < 0.
\]

Hence, the retailer’s expected utility function under the price-only contract is concave. Because \(dE[U(\pi_r(X, \inf I, w))] dQ > 0\) and \(dE[U(\pi_r(X, \sup I, w))] dQ < 0\), the retailer’s optimal order quantity under the price-only contract \(Q^*_\lambda\) is a unique and finite stationary point in \(I\) and satisfies the following condition:

\[
-(\lambda - 1)(w - v) F(q(Q^*_\lambda, w)) + [p - w - (p - v) F(Q^*_\lambda)] = 0. \tag{7}
\]

If the retailer is risk neutral, then it follows from (7) that the retailer’s optimal order quantity \(Q^*_0\) is unique and satisfies the following first-order condition:

\[
(p - w) \tilde{F}(Q^*_0) - (w - v) F(Q^*_0) = 0, \tag{8}
\]

which reduces to the standard newsvendor result:

\[
Q^*_0 = F^{-1}\left(\frac{p - w}{p - v}\right).
\]

A comparison of (2), (7), and (8) shows the two sources of inventory understocking in the decentralized supply chain. The first source, which is reflected in the first term of (7), is due to loss aversion. The second source, which is reflected in the second term of (7), is due to double marginalization. Both of these factors decrease the optimal order quantity, so that for \(\lambda > 1\), \(Q^*_\lambda < Q^*_0 < Q^*\). The reduction of the retailer’s order quantity leads to a lower expected profit for the decentralized supply chain with a loss-averse retailer than for either the integrated system or the decentralized supply chain with a risk-neutral retailer.

THE GLB CONTRACT

In this section, we investigate the role of the GLB contract for supply chain coordination, using the risk-neutral integrated firm’s solution as the benchmark. Because loss aversion on the part of the retailer drives down the order quantity and the total expected supply chain profit, there are competitive pressures for the large and diversified risk-neutral manufacturer to design a contract that aligns the retailer’s decision-making behavior with supply chain profit maximization.

The GLB contract \((w, \gamma, \beta, b)\) specifies that, in addition to paying the manufacturer the unit wholesale price \(w\), the retailer (i) receives \(b \in [v, w)\) from the manufacturer for each unsold unit at the end of the selling season and either (ii) shares a fraction \(\beta \in [0, 1)\) of his gain with the manufacturer or (iii) is reimbursed a fraction \(\gamma \in [0, 1]\) of its loss by the manufacturer. We assume the parameters \(p, c, v,\) and \(\lambda\) are exogenous and \(p, v,\) and \(\lambda\) are known to both the retailer and
the manufacturer. The timing of the supply chain events with a GLB contract is as follows:

- Prior to the selling season, the manufacturer acts as the supply chain leader and offers the retailer (possibly after negotiation) a GLB contract \((w, \gamma, \beta, b)\).
- The retailer responds by placing an order for \(Q\) units of the product with the manufacturer at the unit wholesale price \(w\).
- Production takes place and all finished goods are delivered to the retailer before the season begins.
- Regular season demand is realized.
- The retailer receives \(b\) from the manufacturer for each unsold unit at the end of the selling season, the manufacturer receives salvage value \(v\) for each unsold unit, and the retailer’s gain or loss is computed. The retailer shares a fraction \(\beta\) of its gain with the manufacturer or is reimbursed a fraction \(\gamma\) of its loss by the manufacturer.

We define \(q(Q, w, b) = (w - b)Q / (p - b)\) as the retailer’s breakeven selling quantity function, \(L(X, Q, w, b) = \int_0^{q(Q, w, b)} [px + b(Q - x) - wQ] f(x) dx\) as the retailer’s expected loss function, and \(G(X, Q, w, b) = \int_0^Q [px + b(Q - x) - wQ] f(x) dx + \int_0^{\infty} (p - w)Q f(x) dx\) as the retailer’s expected gain function.

Then, the retailer’s expected utility under the GLB contract is

\[
E[U(\pi_r(X, Q, w, \gamma, \beta, b))] = \left[\lambda(1 - \gamma) - (1 - \beta)\right]L(X, Q, w, b) + (1 - \beta)E[\pi_r(X, Q, w, b)]
\]

and the retailer’s expected profit is

\[
E[\pi_r(X, Q, w, \gamma, \beta, b)] = (\beta - \gamma)L(X, Q, w, b) + (1 - \beta)E[\pi_r(X, Q, w, b)],
\]

where

\[
E[\pi_r(X, Q, w, b)] = L(X, Q, w, b) + G(X, Q, w, b)
\]

is the retailer’s expected profit under the buyback contract. Given the retailer’s order quantity \(Q\), we can express the manufacturer’s expected profit function as follows:

\[
E[\pi_m(X, Q, w, \gamma, \beta, b)] = (w - c)Q + \gamma L(X, Q, w, b) + \beta G(X, Q, w, b) - \int_0^Q (b - v)(Q - x) f(x) dx.
\]

It follows from (1), (10), and (12) that \(E[\pi(X, Q)] = E[\pi_r(X, Q, w, \gamma, \beta, b)] + E[\pi_m(X, Q, w, \gamma, \beta, b)]\).
The following four properties help characterize the supply chain behavior under the set of GLB contracts in which the manufacturer’s loss sharing fraction and the retailer’s gain sharing fraction satisfy $\gamma \in [0, 1 - \frac{1}{\lambda}]$. All proofs are located in the Appendix.

**Property 1.** If $\gamma \in [0, 1 - \frac{1}{\lambda}]$, then the retailer’s utility-maximizing order quantity $Q^*_{\lambda-1}$ is finite and unique and $Q^*_{\lambda-1}$ satisfies
\[
-[\lambda(1 - \gamma) - (1 - \beta)](w - b)F(Q^*_{\lambda-1}, w, b) + (1 - \beta)[p - w - (p - b)F(Q^*_{\lambda-1})] = 0. \tag{13}
\]

**Property 2.** If $\gamma \in [0, 1 - \frac{1}{\lambda}]$, then $Q^*_{\lambda-1}$ is increasing in $\gamma$ and $b$ and decreasing in $\beta$, $\lambda$, and $w$.

**Property 3.** If $\gamma \in [0, 1 - \frac{1}{\lambda}]$, then there exists a unique buyback credit $b^*_{\gamma,\beta,w} \in [v, w]$ that can coordinate the supply chain, and $b^*_{\gamma,\beta,w}$ satisfies
\[
-[\lambda(1 - \gamma) - (1 - \beta)](w - b^*_{\gamma,\beta,w})F(Q^*, w, b^*_{\gamma,\beta,w}) + (1 - \beta)[p - w - (p - b^*_{\gamma,\beta,w})F(Q^*)] = 0, \tag{14}
\]
where
\[Q^* = F^{-1} \left( \frac{p - c}{p - v} \right).\]

**Property 4.** If $\gamma \in [0, 1 - \frac{1}{\lambda}]$, then $b^*_{\gamma,\beta,w}$ is increasing in $\lambda$, $\beta$, and $w$, and decreasing in $\gamma$.

The proceeding properties address only the set of GLB contracts with $\gamma \in [0, 1 - \frac{1}{\lambda}]$, because in this range we are guaranteed concavity of the retailer’s expected utility function. A natural question arises as to whether a GLB contract with $\gamma \in (1 - \frac{1}{\lambda}, 1]$ permits coordination of the supply chain. To facilitate our analysis on the GLB contract with $\gamma \in (1 - \frac{1}{\lambda}, 1]$, we define a function
\[g(Q) = f(Q)/f(q(Q, w, b)) \tag{15}\]
and make the following assumption:

**Assumption (A).** $g(Q)$ is constant, increasing, decreasing, or first increasing and then decreasing in $Q$.

The function $g(Q)$ represents the ratio of the probability density at the order quantity $Q$ to the probability density at the breakeven quantity $q(Q, w, b)$. If $g(Q) > 1$, then demand is more likely to fall near the order quantity $Q$ than the breakeven quantity $q(Q, w, b)$, whereas $g(Q) < 1$ implies demand is more likely to fall near $q(Q, w, b)$ than $Q$. The following lemma shows that assumption (A) is not very restrictive and is satisfied by commonly used probability distributions.
Lemma 1. Assumption (A) holds for uniform, normal, gamma, and exponential distributions.

The following properties help characterize the supply chain behavior under the set of GLB contracts with \( \gamma \in (1 - \frac{1-\beta}{\lambda}, 1] \).

Property 5. Assuming (A) holds under the GLB contract with \( \gamma \in (1 - \frac{1-\beta}{\lambda}, 1] \), the retailer’s utility-maximizing order quantity \( Q^{*}_{\lambda - 1} \) is finite and unique and \( Q^{*}_{\lambda - 1} \) satisfies the first-order condition (13). Furthermore, \( Q^{*}_{\lambda - 1} \) is increasing in \( \gamma \) and decreasing in \( \lambda \) and \( \beta \) but may increase or decrease in \( w \) and \( b \).

Property 6. Assuming (A) holds under the GLB contract with \( \gamma \in (1 - \frac{1-\beta}{\lambda}, 1] \),

(i). if \( F(q(Q^{*}, w)) \leq \frac{(w-c)}{w-v} \), then for any \( \gamma \in (1 - \frac{1-\beta}{\lambda}, 1] \), there always exists a buyback credit \( b^{*}_{\gamma, \beta, w} \in (v, w) \) that can coordinate the supply chain and that satisfies (14);

(ii). if \( F(q(Q^{*}, w)) > \frac{(w-c)}{w-v} \), then for any \( \gamma \in (1 - \frac{1-\beta}{\lambda}, \tilde{\gamma}] \), there always exists a buyback credit \( b^{*}_{\gamma, \beta, w} \in (v, w) \) that can coordinate the supply chain and that satisfies (14), where

\[
\tilde{\gamma} = 1 - \left( \frac{1 - \beta}{\lambda} \right) \left( \frac{(w-v)F(q(Q^{*}, w)) - (w-c)}{(w-v)F(q(Q^{*}, w))} \right). 
\]

(16)

Recall that \( q(Q^{*}, w) \) is defined in (5) as the retailer’s breakeven order quantity at the supply chain’s optimal quantity \( Q^{*} \), under the price-only contract. According to assumption (A), Property 6(i) shows that, if the retailer’s probability of facing losses at the supply chain’s optimal quantity \( Q^{*} \) is relatively high, then for any loss sharing fraction \( \gamma \in (1 - \frac{1-\beta}{\lambda}, 1] \), there always exists a buyback credit \( b^{*}_{\gamma, \beta, w} \in (v, w) \) that can coordinate the supply chain. Property 6(ii) shows that if the retailer’s probability of facing losses at \( Q^{*} \) is relatively low, then the channel coordinating buyback credit \( b^{*}_{\gamma, \beta, w} \in (v, w) \) always exists if the loss sharing fraction is not more than a threshold value (i.e., \( \gamma < \tilde{\gamma} \)), but may not exist if the manufacturer shares too much of the retailer’s losses (i.e., \( \gamma > \tilde{\gamma} \)). For example, under the uniform distribution with \( X \in [0, 200] \), if \( p = 10 \), \( w = 9 \), \( c = 8 \), \( v = 0 \), \( \lambda = 2 \), and \( \beta = .1 \), we find that there is no coordinating buyback credit in the GLB contract for any \( \gamma > \tilde{\gamma} \approx 0.83 \).

COMPARISON OF BUYBACK, GL, AND A SPECIAL CLASS OF GLB CONTRACTS

The GLB contract with four parameters \( (w, \gamma, \beta, b) \) generalizes a few forms of supply chain contracts in practice: (i) if \( \beta = \gamma = 0 \) and \( b = v \), then the GLB contract reduces to a price-only contract with a single parameter \( (w) \), which has been discussed in the previous section; (ii) if \( \beta = \gamma = 0 \) and \( b > v \), then the GLB contract reduces to a buyback contract with two parameters \( (w, b) \); and (iii) if \( b = v \), then the GLB contract reduces to a GL-sharing contract with three parameters.
(w, γ, β). This section presents several corollaries and properties of buyback, GL, and a special class of GLB contracts and compares their performance under a fixed wholesale price.

Buyback Contract

**Corollary 1.** Given a wholesale price \( w \), there always exists a unique buyback credit \( b^*_w \in (v, w) \) such that the buyback contract \((w, b^*_w)\) can coordinate the supply chain, where \( b^*_w \) satisfies

\[ -(\lambda - 1)(w - b^*_w)F(q(Q^*, w, b^*_w)) + [p - w - (p - b^*_w)F(Q^*)] = 0. \] (17)

Corollary 1 shows that the set of buyback contracts \((w, b^*_w)\) can always coordinate the supply chain with a loss-averse retailer. Because the loss-averse retailer orders less than a risk-neutral retailer, the coordinating buyback credit \( b^*_w \) for the supply chain with a loss-averse retailer is larger than the coordinating buyback credit \( b^*_0w = p - (p - v)(p - w)/(p - c) \) for the supply chain with a risk-neutral retailer. From (17), we see that the coordinating buyback credit \( b^*_w \) is dependent on the retailer’s demand distribution. Hence, the manufacturer is unable to offer a uniform coordinating buyback contract to multiple noncompeting loss-averse retailers with different demand distributions. Note that, if \( \lambda = 1 \), then because \( F(Q^*) = (p - c)/(p - v) \), (17) reduces to \( b^*_w = b^*_0w \), agreeing with Pasternack (1985), which shows that the manufacturer can offer a uniform coordinating buyback contract to multiple risk-neutral retailers.

GL Contract

**Corollary 2.** (i) No GL contract with \( \gamma \in [0, 1 - \frac{1 - \beta}{\lambda}] \) can coordinate the supply chain. (ii) Assuming (A) holds, the supply chain can be coordinated under the GL contract if and only if \( F(q(Q^*, w)) \geq \left(\frac{w - v}{w - c}\right) \) and the coordinating loss sharing fraction \( \gamma^*_{w, \beta} \) satisfies \( \gamma^*_{w, \beta} = \tilde{\gamma} \), where \( \tilde{\gamma} \) is defined as in (16).

Corollary 2(i) shows that if \( \gamma \in [0, 1 - \frac{1 - \beta}{\lambda}] \), then because the manufacturer’s loss sharing fraction is not enough to eliminate the loss-aversion and double marginalization effects, the supply chain cannot be coordinated. However, if the retailer’s expected utility function is unimodal, then Corollary 2(ii) identifies a necessary and sufficient condition under which the GL contract can coordinate the supply chain. More specifically, if the retailer’s probability of facing losses at \( Q^* \) is relatively high, then the loss sharing fraction \( \gamma^*_{w, \beta} \) can eliminate quantity distortion effects caused by loss aversion and double marginalization. However, from (16), we see that the coordinating loss sharing fraction \( \gamma^*_{w, \beta} = \tilde{\gamma} \) is dependent on the retailer’s demand distribution. Hence, similar to the buyback contract, the manufacturer is unable to offer a uniform coordinating GL contract to multiple loss-averse retailers with different demand distributions. If the retailer’s probability of facing losses at \( Q^* \) is relatively low, then no loss sharing fraction \( \gamma \in [0, 1] \) is sufficient to eliminate the loss-aversion and double marginalization effects.

We conducted a brief numerical study in order to help characterize settings under which a GL contract can and cannot coordinate the supply chain. We fix \( p = 10 \) and \( v = 0 \) and select \( c = \{1, 2, \ldots, 9\} \). We consider both uniformly and normally
Table 1: Ranges of the manufacturer’s wholesale price $w$ and profit margin $m$ for the gain/loss contract to coordinate the supply chain.a

<table>
<thead>
<tr>
<th></th>
<th>CV = .250 (Normal)</th>
<th>CV = .346 (Normal)</th>
<th>CV = .464 (Normal)</th>
<th>CV = .577 (Uniform)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>$w &lt; m &lt; c$</td>
<td>$w &lt; m &lt; c$</td>
<td>$w &lt; m &lt; c$</td>
<td>$w &lt; m &lt; c$</td>
</tr>
<tr>
<td>1</td>
<td>1.001 0.10%</td>
<td>1.006 0.60%</td>
<td>1.027 2.63%</td>
<td>1.112 10.08%</td>
</tr>
<tr>
<td>2</td>
<td>2.003 0.15%</td>
<td>2.032 1.58%</td>
<td>2.13 6.11%</td>
<td>2.500 20.01%</td>
</tr>
<tr>
<td>3</td>
<td>3.014 0.47%</td>
<td>3.104 3.36%</td>
<td>3.354 10.56%</td>
<td>4.286 30.01%</td>
</tr>
<tr>
<td>4</td>
<td>4.046 1.14%</td>
<td>4.260 6.11%</td>
<td>4.769 16.13%</td>
<td>6.667 40.01%</td>
</tr>
<tr>
<td>5</td>
<td>5.133 2.60%</td>
<td>5.560 10.08%</td>
<td>6.491 22.97%</td>
<td>10 50%</td>
</tr>
<tr>
<td>6</td>
<td>6.327 5.17%</td>
<td>7.109 15.60%</td>
<td>8.731 31.28%</td>
<td>10 40%</td>
</tr>
<tr>
<td>7</td>
<td>7.732 9.47%</td>
<td>9.112 23.18%</td>
<td>10 30%</td>
<td>10 30%</td>
</tr>
<tr>
<td>8</td>
<td>9.584 16.53%</td>
<td>10 20%</td>
<td>10 20%</td>
<td>10 20%</td>
</tr>
<tr>
<td>9</td>
<td>10 10%</td>
<td>10 10%</td>
<td>10 10%</td>
<td>10 10%</td>
</tr>
</tbody>
</table>

aFor example, if $c = 5$ and demand is (truncated) normally distributed demand with $CV = .250$, then a gain/loss contract can coordinate the supply chain as long as $w < 5.133$ (or equivalently, $m < 2.6\%$).

For example, if we assume demand $X \in [0, 200]$ with a mean of $\mu = 100$, implying the coefficient of variation ($CV$) $\approx .577$. For the normal case, we assume demand is a truncated normal random variable (i.e., $X = \max\{Z, 0\}$), where $Z$ is a normally distributed random variable with a mean of $\mu = 100$ and a standard deviation of $\sigma = \{25, 35, 50\}$). The corresponding values for the mean and $CV$ of $X$ are $\{100.0, 100.2, 100.8\}$ and $\{.250, .346, .464\}$.

We conduct a thorough search to identify the range of wholesale prices for which the GL contract can coordinate the supply chain and calculate the corresponding range of manufacturer’s profit margin $m = (w - c)/w$. Table 1 shows that, if the wholesale price is relatively close to the production cost or, alternatively, if the manufacturer’s profit margin is relatively low, then the GL contract can coordinate the supply chain.

**Distribution-Free Coordinating GLB Contracts**

**Corollary 3.** Under the GLB contract with $\gamma = 1 - \frac{1 - \beta}{\lambda}$, there exists a unique buyback credit $b^*_{\gamma, \beta, w}$ that can coordinate the supply chain and that satisfies

$$b^*_{\gamma, \beta, w} = b^0_w = p - (p - v)(p - w)/(p - c) \quad (18)$$

and the manufacturer’s expected profit is increasing in $\beta$.

Corollary 3 identifies a special class of coordinating GLB contracts where the gain- and loss-sharing fractions satisfy $\gamma = 1 - \frac{1 - \beta}{\lambda}$, and the coordinating buyback credit $b^*_{\gamma, \beta, w}$ is the same as the coordinating buyback credit for the risk-neutral retailer $b^0_w$. If the retailer is risk neutral, then we have $\lambda = 1$ and $\gamma = 1 - \frac{1 - \beta}{\lambda} = \beta$. In this case with no loss-aversion effect, the buyback credit eliminates double marginalization, and the GL-sharing fraction influences the allocation of expected profit between firms. If the retailer is loss averse, then we have $\gamma > \beta$. In
this case, the buyback credit eliminates double marginalization, the loss-sharing fraction eliminates the loss-aversion effect, and the gain-sharing fraction influences the allocation of expected profit between firms. Because all contract parameters are independent of market demand, we refer to GLB contracts with parameters set according to Corollary 3 as distribution-free coordinating GLB contracts.

**Property 7.** If the manufacturer can freely set the wholesale price \( w \), then the three forms of coordinating contracts (distribution-free GLB, GL, and buyback) can arbitrarily divide the total supply chain expected profit between the manufacturer and retailer.

One important research question on supply chain contracts is whether or not the contract is sufficiently flexible in dividing the supply chain’s profit between the manufacturer and the retailer. As pointed out in Cachon (2003), if a contract can allocate supply chain profit arbitrarily, then there always exists a coordinating contract under which each firm’s profit is no worse off and at least one firm is strictly better off under the coordinating contract. Property 7 implies that, for any given price-only contract, there exists a range of coordinating contracts where the retailer’s expected profit remains the same and the manufacturer’s expected profit is higher than the price-only contract. This property is meaningful because it exposes a level of flexibility available to a manufacturer when designing a mutually beneficial contract that aligns the loss-averse retailer’s decision with supply chain profit maximization.

**Property 8.** For any fixed wholesale price \( w > c \), (i) there always exists a distribution-free coordinating GLB contract under which the manufacturer’s expected profit is higher than that under the coordinating buyback contract and the price-only contract; (ii) if \( F(q(Q^*, w)) \geq (\frac{w-c}{w-v}) \), then there always exists a coordinating GL contract under which the manufacturer’s expected profit is higher than that under the coordinating buyback contract and the price-only contract.

In practice, it can be difficult for the manufacturer to change the wholesale price for the following reasons. First, if the market is highly competitive, then the manufacturer does not have much control over the wholesale price and essentially acts as a price taker. Second, changing the wholesale price can be costly, and there is empirical evidence that manufacturers are reluctant to change wholesale prices (e.g., Iyer & Bergen, 1997; Cachon, 2003). Property 8(i) says that, when wholesale price is fixed, the manufacturer can design a distribution-free coordinating GLB contract under which it is better off (i.e., higher expected profit) relative to a coordinating buyback contract and a price-only contract. Similarly, Property 8(ii) says that, if the condition in Corollary 2(ii) for the coordinating GL contract is satisfied, then the manufacturer can design a coordinating GL contract under which it is better off relative to a coordinating buyback contract and a price-only contract.
Numerical Study on the Impact of Loss Aversion on Contract Design

The preceding properties help to characterize, in a general way, the roles and uses of GL-sharing and buyback provisions for mitigating the effects of loss aversion and double marginalization. The results indicate that a distribution-free coordinating GLB contract provides comparable or more flexibility for coordinating the supply chain and allocating expected profit relative to other contracts and is unique in that the coordinating parameter values do not depend on market demand. However, the coordinating contract parameters depend on retailer loss aversion, and thus a distribution-free feature GLB cannot be offered uniformly across independent retailers that differ by loss aversion.

We emphasize that, from the manufacturer’s viewpoint when considering contract types and specific parameter values to maximize performance, anything other than a distribution-free contract is likely to be impractical. Manufacturers will generally have difficulty gauging how a retailer perceives the probability distribution of market demand, and a retailer has an incentive to distort this information to gain more favorable contract terms. However, given that a manufacturer offers a distribution-free coordinating contract, there is an important question of when accounting for loss aversion in contract design can make a meaningful difference in profitability. In this subsection, we report results from a numerical study designed to address this question.

Consider two possible contracts. Contract C1 is a distribution-free GLB contract that ignores retailer loss aversion and contract C2 is a distribution-free GLB contract that accounts for retailer loss aversion. For both contracts, the buyback credit is \( b^\nu_0 = p - (p - v)(p - w)/(p - c) \), which as noted earlier, is the value that coordinates the supply chain when the retailer is risk neutral. For C1, we select a gain-sharing fraction \( \beta \in [0, 1] \) and specify the loss-sharing fraction \( \gamma = \beta \). C1 coordinates the supply chain when the retailer is risk neutral, and the manufacturer’s expected share of supply chain profit is \( 1 - (1 - \beta)(p - w)/(p - c) \), regardless of the value of \( \lambda \) (see Lemma 2 in the Appendix).

For C2, we select a gain-sharing fraction \( \beta \in [0, 1] \) and specify the loss-sharing fraction \( \gamma = 1 - \frac{1 - \beta}{\lambda} = \beta + (1 - \beta)(\lambda - 1)/\lambda > \beta \). C2 accounts for retailer loss aversion and coordinates the supply chain. C1 and C2 are identical when the retailer is risk neutral. In addition, the parameter values of both contracts are independent of market demand.

We compute the percentage increase in expected profits when the manufacturer changes from contract C1, which ignores loss aversion, to contract C2, which accounts for loss aversion. The demand distributions are the same as in the study described earlier. We consider four levels of retailer loss aversion: \( \lambda = \{2, 3, 4, 5\} \). We set \( p = 10, v = 0, w = 9, \) and \( c = \{1, 3, 5, 7, 9\} \). The contribution margins for the supply chain are \( scm = (p - c)/p = \{90\%, 70\%, 50\%, 30\%, 10\%\} \). Schweitzer and Cachon (2000) refer to books, bicycles, and fashion apparel as examples of high-profit products (e.g., \( scm > 50\% \)) and computers as an example of low-profit product (e.g., \( scm < 50\% \)).

Let \( \alpha \) denote the manufacturer’s expected share of supply chain profit under C1 for given values of \( c \) and \( \beta \) and assuming the retailer is risk neutral. C1 will not coordinate the supply chain when the retailer is loss averse. To provide
Table 2: Percentage increase in manufacturer, retailer, and total supply chain expected profit when a coordinating contract is offered to a retailer with loss aversion $\lambda$, relative to a coordinating contract that ignores loss aversion (assume the same allocation of expected supply chain profit).

<table>
<thead>
<tr>
<th>CV (pdf)</th>
<th>$\lambda$</th>
<th>(1, 90%)</th>
<th>(3, 70%)</th>
<th>(5, 50%)</th>
<th>(7, 30%)</th>
<th>(9, 10%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.250</td>
<td>2</td>
<td>0.000%</td>
<td>0.000%</td>
<td>0.009%</td>
<td>0.219%</td>
<td>1.952%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.000%</td>
<td>0.000%</td>
<td>0.036%</td>
<td>0.712%</td>
<td>5.078%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.000%</td>
<td>0.001%</td>
<td>0.076%</td>
<td>1.344%</td>
<td>8.245%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.000%</td>
<td>0.001%</td>
<td>0.127%</td>
<td>2.049%</td>
<td>11.259%</td>
</tr>
<tr>
<td>.346</td>
<td>2</td>
<td>0.000%</td>
<td>0.007%</td>
<td>0.143%</td>
<td>1.111%</td>
<td>5.4097%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.000%</td>
<td>0.027%</td>
<td>0.498%</td>
<td>3.363%</td>
<td>13.949%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.000%</td>
<td>0.058%</td>
<td>0.992%</td>
<td>6.059%</td>
<td>22.812%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.000%</td>
<td>0.099%</td>
<td>1.582%</td>
<td>8.947%</td>
<td>31.556%</td>
</tr>
<tr>
<td>.464</td>
<td>2</td>
<td>0.001%</td>
<td>0.079%</td>
<td>0.738%</td>
<td>3.473%</td>
<td>12.664%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.004%</td>
<td>0.285%</td>
<td>2.404%</td>
<td>10.082%</td>
<td>32.653%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.010%</td>
<td>0.585%</td>
<td>4.576%</td>
<td>17.793%</td>
<td>54.095%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.016%</td>
<td>0.960%</td>
<td>7.064%</td>
<td>26.008%</td>
<td>76.024%</td>
</tr>
<tr>
<td>.577</td>
<td>2</td>
<td>0.010%</td>
<td>0.686%</td>
<td>4.167%</td>
<td>12.126%</td>
<td>25.042%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.038%</td>
<td>2.382%</td>
<td>12.500%</td>
<td>32.446%</td>
<td>61.896%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.085%</td>
<td>4.734%</td>
<td>22.500%</td>
<td>54.845%</td>
<td>100.766%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.148%</td>
<td>7.535%</td>
<td>33.333%</td>
<td>78.081%</td>
<td>140.342%</td>
</tr>
</tbody>
</table>

A fair comparison between C1 and C2, we set the gain-sharing fraction $\beta$ in C2 so that the manufacturer’s expected share of supply chain profit is $\alpha$ (i.e., the same share as when C1 is offered to a risk-neutral retailer). (Note that, because of Lemma 2, the particular choice of $\beta$ in C1 does not affect the results in Table 2 above.)

Table 2 reports the percentage increase in manufacturer, retailer, and supply chain expected profit when a manufacturer offers contract C2 instead of C1. We see that the increase in profit is insignificant for high-profit products (e.g., less than .15% for all cases when $scm = 90\%$). On the other hand, we see double-digit percentage increases in profitability for low-profit products, especially when demand uncertainty is high.

On examination of the detailed results, we find that a key determinant of the degree to which accounting for retailer loss aversion in contract parameters can increase profitability is the probability of retailer loss. This result is consistent with intuition. For example, in terms of our notation, the probability of retailer loss for C1 is $F(q(Q^*_{\lambda-1}w, b^0_0))$ and the order quantity $Q^*_{\lambda-1}$ is less than the supply chain coordinating quantity $Q^*$. When $F(q(Q^*_{\lambda-1}w, b^0_0))$ is very small, then retailer loss aversion plays a minimal role in influencing the retailer order quantity and, accordingly, $Q^*_{\lambda-1} \approx Q^*$. Thus, the introduction of C2, which results in the coordinating order quantity $Q^*$, has minimal impact on profitability. The minimal impact is most pronounced in the upper left of Table 2, where margin is
high ($scm = 90\%$) and demand uncertainty is low ($CV = .250$). In this case, we find $F(q(Q^*_\lambda - 1, w, b^0_w)) \approx 0.026\%$ for $\lambda \in \{2, 3, 4, 5\}$, and the introduction of C2 has virtually no impact on expected profit.

**SUMMARY OF MANAGERIAL IMPLICATIONS**

In this section, we summarize the characteristics of the various contract forms and the main implications of our results for a manufacturer selling to a loss-averse retailer. We defined a GLB contract and examined three special cases that, at least in some instances, can coordinate the supply chain: (i) a coordinating buyback contract, (ii) a coordinating GL contract, and (iii) a distribution-free coordinating GLB contract.

For a coordinating buyback contract, a manufacturer selects the wholesale price $w$, which determines the buyback credit $b^*_w$. Different choices for $w$ lead to different allocations of expected supply chain profit between the two firms.

For a coordinating GL contract, the manufacturer selects the wholesale price $w$ and the gain sharing fraction $\beta$, which together determine the loss-sharing fraction $\gamma^*_w, \beta$, if such a fraction exists. Different choices for $w$ and $\beta$ lead to different allocations of expected supply chain profit between the two firms.

For a distribution-free coordinating GLB contract, the manufacturer selects the wholesale price $w$ and gain-sharing fraction $\beta$. The wholesale price determines the buyback credit $b^0_w = p - (p - v)(p - w)/(p - c)$, and the gain-sharing fraction $\beta$ determines the loss-sharing fraction $\gamma = \beta + (1 - \beta)(\lambda - 1)/\lambda$. Different choices for $w$ and $\beta$ lead to different allocations of expected supply chain profit between the two firms.

If wholesale price is a free variable, the manufacturer has accurate information on how the retailer perceives market demand and if there is one retailer (or nonuniform contracts are viable for multiple retailers), then all contract forms are suitable (i.e., capable of coordinating the supply chain and arbitrarily allocating profit). If wholesale price is exogenous or its range of values is limited, then the latter two contract forms offer an advantage. The additional degree of freedom reflected in the choice of $\beta$ in a distribution-free coordinating GLB contract and a coordinating GL contract is the reason these contract forms dominate a coordinating buyback contract (e.g., see Property 8). However, for some values of $w$, a coordinating GL contract may not exist (e.g., see the condition in Property 8(ii) and the results in Table 1).

With the exception of the distribution-free GLB contract, the coordinating parameter values of the other contract forms depend on the probability distribution of demand. In reality, a manufacturer generally will not have accurate information on how the retailer perceives market demand. And, even if the manufacturer knows the probability distribution with precision, a dependency between contract parameter values and the probability distribution means that a uniform contract cannot be offered to multiple retailers while simultaneously maximizing expected supply chain profit. Accordingly, in the remainder of this section, we focus on distribution-free contracts. We highlight the main contributions of our analysis in the form of guidelines for manufacturers.
In brief, our results suggest that a manufacturer wishing to maximize expected supply chain profit when a retailer is loss averse has two basic alternatives: (i) use a distribution-free coordinating contract that ignores loss aversion or (ii) use a distribution-free coordinating contract that accounts for loss aversion. The first alternative can be used with little loss of profit for products with a high supply chain contribution margin, and the second alternative should be used for products with a low supply chain contribution margin. Each alternative is discussed later.

**Distribution-Free Coordinating Contract That Ignores Loss Aversion**

For a given wholesale price $w$, the buyback credit, which is independent of market demand is $b^0_w = p - (p - v)(p - w)/(p - c)$. If a price-only contract is currently in effect, then with no change in wholesale price, the retailer’s share of expected supply chain profit and level of expected profit will increase (i.e., compared to a price-only contract). The manufacturer’s share of expected supply chain profit will decrease and the manufacturer’s level of expected profit could increase or decrease. Without further adjustments, this contract is likely to be unappealing to the manufacturer. Thus, a manufacturer may either increase the wholesale price and/or introduce GL-sharing fractions $\beta$ and $\gamma = \beta$ so as to achieve a suitable allocation of profit. Note that, when a buyback provision is included in a contract, a GL provision can be introduced with no additional monitoring cost because the manufacturer can deduce the retailer’s gain or loss from the number of buybacks.

This contract form with $\gamma = \beta$ has the advantage of a simple distribution-free expression for the share of expected supply chain profit for each firm, regardless of the level of retailer loss aversion. For example, the manufacturer’s share of expected profit, which is increasing in $\beta$, is $1 - (1 - \beta)(p - w)/(p - c)$. The value of the gain-(and loss-) sharing fraction can be set to a mutually agreeable and beneficial level. While expected supply chain profit will increase if the loss-sharing fraction is increased from $\gamma = \beta$ to $\gamma = \beta + (1 - \beta)(\lambda - 1)/\lambda$, our numerical results indicate that the percentage increase will be minimal for a product with a high supply chain contribution margin (e.g., $(p - c)/p \geq 70\%$; a more precise measure of minimal impact from increasing $\gamma$ is a small probability of retailer loss, $F(q(Q^*_\lambda - 1, w, b^0_w))$, which is influenced by both the margin and demand uncertainty, but margin is easier for a manufacturer to accurately estimate and is a reasonable proxy). In summary, this distribution-free contract that ignores retailer loss aversion can be used for products with high margins with little loss in expected profit.

**Distribution-Free Coordinating Contract That Accounts for Loss Aversion**

This second contract alternative is identical to the previous alternative except that the loss-sharing fraction accounts for retailer loss aversion. Specifically, the buyback credit is $b^0_w = p - (p - v)(p - w)/(p - c)$ as above, and for a given gain sharing fraction $\beta$, which could be zero, the loss-sharing fraction is $\gamma = \beta + (1 - \beta)(\lambda - 1)/\lambda$. This contract maximizes expected supply chain profit, and the values of $w$ and/or $\beta$ can be used to influence the allocation of expected profit between the two firms. Our numerical results indicate that the percentage increase in expected supply chain profit when this contract form is used in place of the preceding alternative
can be significant for a product with a low supply chain contribution margin (e.g., \((p - c)/p \leq 30\%\)). In summary, a manufacturer should use this contract form for low-margin products.

**CONCLUSION**

We analyzed a supply chain composed of a risk-neutral manufacturer selling a perishable product to a loss-averse retailer. Past research has shown that profits can significantly increase via contracts that align individual firm interests with interests of the total supply chain, and many contract forms have been studied. Past research has also shown that a buyback contract can coordinate the supply chain with parameters that are independent of market demand (Pasternack, 1985). The independence between coordinating parameter values and market demand is important in practice, not only because it is challenging for the manufacturer to estimate the nature of market demand but also because of difficulties associated with offering different contract terms to different retailers.

We find that the independence between parameters and market demand breaks down in a buyback contract when the retailer is loss averse. We identify a class of distribution-free coordinating GLB contracts, which set the buyback credit to the value that would be offered in a buyback contract to a risk-neutral retailer. The value of the buyback credit is just sufficient to eliminate the effect of double marginalization. The gain- and loss-sharing fractions serve to eliminate the effect of loss aversion and can be used to influence the allocation of profit between manufacturer and retailer. Importantly, neither the value of the buyback credit nor the values of the gain- and loss-sharing fractions depend on market demand. Our analysis sheds light on the relationship between parameter values and allocation of profit as well as when it is, and is not, important to account for retailer loss aversion in contract design.

In summary, our results indicate that coordinating contracts based on the assumption of risk neutrality may result in markedly lower supply chain profit when retailers are loss averse. Manufacturers should consider the impact of loss aversion in contract design along with mitigating provisions such as a GL-sharing clause, especially when dealing with small retailers for which the assumption of risk neutrality is less likely to hold.

Future research should consider a supply chain composed of a risk-neutral manufacturer selling to multiple competing loss-averse retailers. Our analytical and numerical results could be tested empirically by experiments or by surveys of managers in relatively large risk-neutral manufacturing firms and managers in relatively small loss-averse retail firms. Because this article is based on a simple piecewise linear loss-aversion utility function, further research could consider other nonlinear utility functions that better describe the retailer’s decision-making behavior (e.g., concave in gains and convex in losses). Alternatively, one could examine other nontraditional utility functions such as the rank-dependent expected utility function (Quiggin, 1982), in which the weight attached to any outcome of a prospect depends not only on the true probability of that outcome but also on its ranking relative to the other outcomes. Such investigations may lend insight into
how manager preferences contribute to supply chain inefficiency and may lead to new contract designs that help mitigate these effects. Finally, because in this article we assume the manufacturer is larger than the retailer and acts as the supply chain leader, future research should consider a supply chain in which the retailer is larger than the manufacturer and acts as the supply chain leader. [Received: December 2005. Accepted: May 2007.]

REFERENCES


APPENDIX: PROOFS

Proof of Property 1. After taking the first and second derivatives of (9) with respect to \( Q \), we get

\[
dE[U(\pi_r(X, Q, w, \gamma, \beta, b))] dQ
= -[\lambda(1 - \gamma) - (1 - \beta)](w - b)F(q(Q, w, b))
+ (1 - \beta)[p - w - (p - b)F(Q)]
\tag{A1}
\]

and

\[
d^2 E[U(\pi_r(X, Q, w, \gamma, \beta, b))] dQ^2
= -[\lambda(1 - \gamma) - (1 - \beta)](w - b)^2/(p - b)
f(q(Q, w, b))
- (1 - \beta)(p - b)f(Q).
\tag{A2}
\]

Because \( \gamma \in [0, 1 - \frac{1 - \beta}{\lambda}] \), \( \beta \in [0, 1) \), and \( b \in [v, w) \), expression (A2) is negative, which implies \( E[U(\pi_r(X, Q, w, \gamma, \beta, b))] \) is concave. Because \( dE[U(\pi_r(X, \inf I, w, \gamma, \beta, b)]/dQ > 0 \) and \( dE[U(\pi_r(X, \sup I, w, \gamma, \beta, b)]/dQ < 0 \), the optimal order quantity \( Q^*_N \) is a unique and finite stationary point in \( I \) and satisfies (13). □

Proof of Property 2. By the implicit function theorem, from (13), we get

\[
\frac{dQ^*_N}{d\gamma} = \frac{d^2 E[U(\pi_r(X, Q^*_N, w, \gamma, \beta, b))] dQ}{d\gamma}
- \frac{d^2 E[U(\pi_r(X, Q^*_N, w, \gamma, \beta, b))] dQ^2}{d\gamma^2}
= \frac{\lambda(w - b)F(q(Q^*_N, w, b))}{-d^2 E[U(\pi_r(X, Q^*_N, w, \gamma, \beta, b))] dQ^2} > 0,
\]

\[
\frac{dQ^*_N}{db} = \frac{d^2 E[U(\pi_r(X, Q^*_N, w, \gamma, \beta, b))] dQ}{db}
- \frac{d^2 E[U(\pi_r(X, Q^*_N, w, \gamma, \beta, b))] dQ^2}{d^2 Q}
= \frac{[\lambda(1 - \gamma) - (1 - \beta)] (p - w)q(Q^*_N, w, b) f(q(Q^*_N, w, b))}{-d^2 E[U(\pi_r(X, Q^*_N, w, \gamma, \beta, b))] dQ^2} + (1 - \beta)F(Q^*_N) > 0
\]

\[
\frac{dQ^*_N}{d\lambda} = \frac{d^2 E[U(\pi_r(X, Q^*_N, w, \gamma, \beta, b))] dQ}{d\lambda}
- \frac{d^2 E[U(\pi_r(X, Q^*_N, w, \gamma, \beta, b))] dQ^2}{dQ^2}
= \frac{-(1 - \gamma)(w - b)F(q(Q^*_N, w, b))}{-d^2 E[U(\pi_r(X, Q^*_N, w, \gamma, \beta, b))] dQ^2} < 0,
\]
\[
\frac{dQ^*_{\lambda-1}}{dw} = \frac{d^2E[U(\pi_r(X, Q^*_{\lambda-1}, w, \gamma, \beta, b))]}{dQ dw} - \frac{d^2E[U(\pi_r(X, Q^*_{\lambda-1}, w, \gamma, \beta, b))]}{dQ^2} \\
= \frac{-(\lambda(1 - \gamma) - (1 - \beta))[F(q(Q^*_{\lambda-1}, w, b) + q(Q^*_{\lambda-1}, b)f(q(Q^*_{\lambda-1}, w, b))] - (1 - \beta)}{-d^2E[U(\pi_r(X, Q^*_{\lambda-1}, w, \gamma, \beta, b))]/dQ^2} < 0,
\]

\[
\frac{dQ^*_{\lambda-1}}{d\beta} = \frac{d^2E[U(\pi_r(X, Q^*_{\lambda-1}, w, \gamma, \beta, b))]}{dQ d\beta} - \frac{d^2E[U(\pi_r(X, Q^*_{\lambda-1}, w, \gamma, \beta, b))]}{dQ^2} \\
= \frac{-\lambda(1 - \gamma)(w - b)F(q(Q^*_{\lambda-1}, w, b))}{-(1 - \beta)d^2E[U(\pi_r(X, Q^*_{\lambda-1}, w, \gamma, \beta, b))]/dQ^2} < 0.
\]

**Proof of Property 3.** Let

\[
h(b) = -(\lambda(1 - \gamma) - (1 - \beta))(w - b)F(q(Q^*, w, b)) + (1 - \beta)(p - w - (p - b)F(Q^*)].
\]

(A3)

After taking the first derivative of \(h(b)\) with respect to \(b\), we get

\[
dh(b)/db = [\lambda(1 - \gamma) - (1 - \beta)][F(q(Q^*, w, b))
+ \frac{(w - b)(p - w)}{(p - b)^2}f(q(Q^*, w, b))] + (1 - \beta)F(Q^*) > 0.
\]

(A4)

From (A4), we see \(h(b)\) is strictly increasing in \(b\). Note that \(h(w) = (1 - \beta)(p - w)\bar{F}(Q^*) > 0\) and \(h(v) = -[\lambda(1 - \gamma) - (1 - \beta)](w - v)F(q(Q^*, w)) - (1 - \beta)(w - c) \leq 0\). Therefore, there exists a unique rebate credit \(b^*_{\gamma, \beta, w} \in [v, w]\) that satisfies (14).

**Proof of Property 4.** Let

\[
K = [\lambda(1 - \gamma) - (1 - \beta)]\left[F(q(Q^*, w, b^*_{\gamma, \beta, w})
+ \frac{(p - w)q(Q^*, w, b^*_{\gamma, \beta, w})f(q(Q^*, w, b^*_{\gamma, \beta, w}))}{p - b^*_{\gamma, \beta, w}}\right] + (1 - \beta)F(Q^*) > 0.
\]
After taking the first derivatives of (14) with respect to $\lambda$, $\gamma$, $\beta$, and $w$ and applying some algebraic manipulations, we get

$$db^*_{\gamma,\beta,w}/d\lambda = (1 - \gamma)(w - b^*_{\gamma,\beta,w}) F(q(Q^*, w, b^*_{\gamma,\beta,w}))/K > 0,$$

$$db^*_{\gamma,\beta,w}/d\gamma = -\lambda (w - b^*_{\gamma,\beta,w}) F(q(Q^*, w, b^*_{\gamma,\beta,w}))/K < 0,$$

$$db^*_{\gamma,\beta,w}/d\beta = \frac{\lambda(1 - \gamma)(w - b^*_{\gamma,\beta,w}) F(q(Q^*, w, b^*_{\gamma,\beta,w}))}{(1 - \beta)K} > 0,$$

$$db^*_{\gamma,\beta,w}/dw = 1 - \beta + [\lambda(1 - \gamma) - (1 - \beta)] \left[ \frac{F(q(Q^*, w, b^*_{\gamma,\beta,w}) + q(Q^*, w, b^*_{\gamma,\beta,w})f(q(Q^*, w, b^*_{\gamma,\beta,w}))}{K} \right] > 0. \qed$$

**Proof of Lemma 1.**

(i) If demand is uniformly distributed, then $g(Q) \equiv 1$ and $g'(Q) \equiv 0$, which implies assumption (A) holds for uniform distribution. (ii) If demand is exponentially distributed with $f(x) = \theta e^{-\theta x}$, then $g(Q) = e^{-\theta(p-w)/p-b}$. (A5)

From (A5) we see $g'(Q) < 0$, which implies assumption (A) holds for exponential distribution.

(iii) If demand is normally distributed with $f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, then $g(Q) = e^{-[Q+q(Q,w,b)-2\mu(Q-q(Q,w,b))]2\sigma^2}$, and

$$g'(Q) = -\left( \frac{p-w}{p-b} \right) \left( \frac{p+w-2b}{p-b} \right) Q - \mu \right)$$

$$\times \left( \frac{e^{-[Q+q(Q,w,b)-2\mu(Q-q(Q,w,b))]2\sigma^2}}{\sigma^2} \right). \quad (A6)$$

From (A6), we see $g'(Q) > 0$ if $Q < \frac{\mu(p-b)}{p+w-2b}$ and $g'(Q) \leq 0$ if $Q \geq \frac{\mu(p-b)}{p+w-2b}$. Hence, assumption (A) holds for normal distribution.

(iv) If demand follows gamma distribution with $f(x) = \frac{\theta^k x^{k-1} e^{-\theta x}}{\Gamma(k)}$, then $g(Q) = \left( \frac{w-b}{p-b} \right)^{1-k} e^{-\theta Q(p-w)/(p-b)}$. (A7)

From (A7), we see $g'(Q) < 0$, which implies assumption (A) holds for gamma distribution. \qed
Proof of Property 5. We first prove the retailer’s expected utility function must be initially concave when \( Q \in (\inf I, \inf I + \delta) \), where \( \delta \) is an arbitrarily small number. First, if \( Q = \inf I \) and \( f(\inf I) > 0 \), then from (A2), we have:

\[
d^2E[U(\pi, (X, \inf I, w, \gamma, \beta, b))] / dQ^2 \\
\leq \left( -[\lambda(1 - \gamma) - (1 - \beta)] \left( \frac{w - b}{p - b} \right)^2 - (1 - \beta) \right) (p - b) f(\inf I) < 0.
\]

(A8)

Because \( E[U(\pi, (X, Q, w, \gamma, \beta, b))] \) is continuous in \( Q \), it must be concave for all \( Q \in (\inf I, \inf I + \delta) \) when \( f(\inf I) > 0 \). Second, if \( Q = \inf I \) and \( f(\inf I) = 0 \), then \( f'(Q) > 0 \) for all \( Q \in (\inf I, \inf I + \delta) \), otherwise, it contradicts the fact that \( F(x) \) is strictly increasing. Hence, \( f(q(Q, w, b)) < f(Q) \) and \( f(Q) > 0 \) for any \( Q \in (\inf I, \inf I + \delta) \). Then from (A2),

\[
d^2E[U(\pi, (X, Q, w, \gamma, \beta, b))] / dQ^2 \\
< \left( -[\lambda(1 - \gamma) - (1 - \beta)] \left( \frac{w - b}{p - b} \right)^2 - (1 - \beta) \right) (p - b) f(Q) < 0,
\]

which implies \( E[U(\pi, (X, Q, w, \gamma, \beta, b))] \) is concave for all \( Q \in (\inf I, \inf I + \delta) \) when \( f(\inf I) = 0 \).

After applying some algebraic manipulations to (A2), we get:

\[
d^2E[U(\pi, (X, Q, w, \gamma, \beta, b))] / dQ^2 \\
= (p - b) f(q(Q, w, b)) \left( -[\lambda(1 - \gamma) - (1 - \beta)] \left( \frac{w - b}{p - b} \right)^2 - (1 - \beta)g(Q) \right).
\]

(A9)

If (A) holds, then from (A9), \( E[U(\pi, (X, Q, w, \gamma, \beta, b))] \) is always concave or first concave and then convex. Because \( dE[U(\pi, (X, \inf I, w, \gamma, \beta, b))] / dQ > 0 \) and \( dE[U(\pi, (X, sup I, w, \gamma, \beta, b))] / dQ < 0 \), \( E[U(\pi, (X, Q, w, \gamma, \beta, b))] \) must be unimodal. Hence, the optimal order quantity \( Q^*_\lambda - 1 \) is a unique and finite stationary point in \( I \) and satisfies (13). Furthermore, because \( E[U(\pi, (X, Q, w, \gamma, \beta, b))] \) is initially concave and increasing, \( Q^*_\lambda - 1 \) must satisfy \( d^2E[U(\pi, (X, Q^*_\lambda - 1, w, \gamma, \beta, b))] / dQ^2 < 0 \). Then it follows from the proof of Property 2 that \( dQ^*_\lambda - 1 / d\gamma > 0 \), \( dQ^*_\lambda - 1 / d\lambda < 0 \), and \( dQ^*_\lambda - 1 / d\beta < 0 \). However, the signs of \( dQ^*_\lambda - 1 / db \) and \( dQ^*_\lambda - 1 / dw \) can be positive or negative. \(\square\)

Proof of Property 6. From the proof of Property 5, if (A) holds, then the retailer’s optimal order quantity \( Q^*_\lambda - 1 \) is unique. Hence, if there exists a buyback credit \( h_{\gamma, \beta, w} \in (v, w) \) that can coordinate the supply chain, it must satisfy \( h(b_{\gamma, \beta, w}) = 0 \). From the proof of Property 3, \( h(w) = (1 - \beta)(p - w)F(Q^*) > 0 \) and \( h(v) = -[\lambda(1 - \gamma) - (1 - \beta)](w - v)F(q(Q^*, w)) - (1 - \beta)(w - c) \).
(i). If \( F(q(Q^*,w)) \leq \frac{w-c}{w-v} \), then \( h(v) < 0 \) for any \( \gamma \in (1 - \frac{1-\beta}{\lambda}, 1] \). Hence, there always exists a buyback credit \( b^*_{\gamma,\beta,w} \in (v, w) \) that can coordinate the supply chain and that satisfies (14).

(ii). If \( F(q(Q^*,w)) > \frac{w-c}{w-v} \), then \( h(v) < 0 \) for any \( \gamma \in (1 - \frac{1-\beta}{\lambda}, 1] \) and \( h(v) \geq 0 \) for any \( \gamma \in (\tilde{\gamma}, 1) \). Hence, for any \( \gamma \in (1 - \frac{1-\beta}{\lambda}, \tilde{\gamma}) \), there always exists a buyback credit \( b^*_{\gamma,\beta,w} \in (v, w) \) that can coordinate the supply chain and that satisfies (14). However, the coordinating buyback credit \( b^*_{\gamma,\beta,w} \in (v, w) \) may not exist if \( \gamma \in (\tilde{\gamma}, 1) \).

Proof of Corollary 1. Similar to the proof of Property 3 by letting \( \gamma = \beta = 0 \).

Proof of Corollary 2. (i) If \( \gamma \in [0, 1 - \frac{1-\beta}{\lambda}] \) and there exists a gain sharing fraction \( \gamma^*_{\beta,w} \) that can coordinate the supply chain, then it must satisfy

\[
-\left(1 - \gamma^*_{\beta,w}\right)(w - v)F(q(Q^*, w)) = 0
\]

(i.e., \( \gamma^*_{\beta,w} = \tilde{\gamma} \)). From (16), we see \( \tilde{\gamma} > 1 - \frac{1-\beta}{\lambda} \), which contradicts \( \gamma \in [0, 1 - \frac{1-\beta}{\lambda}] \). Hence, the GL contract with \( \gamma \in [0, 1 - \frac{1-\beta}{\lambda}] \) cannot coordinate the supply chain.

(ii) From the proof of Property 5, if (A) holds under the GL contract with \( \gamma \in (1 - \frac{1-\beta}{\lambda}, 1] \), then the retailer’s optimal order quantity \( Q^*_\lambda \) is unique. Hence, if there exists a gain sharing fraction \( \gamma^*_{\beta,w} \) that can coordinate the supply chain, then it must satisfy \( \gamma^*_{\beta,w} = \tilde{\gamma} \). If \( F(q(Q^*, w)) \geq \frac{w-c}{w-v} \), then \( \gamma^*_{w,\beta} \in (1 - \frac{1-\beta}{\lambda}, 1] \). If \( F(q(Q^*, w)) < \frac{w-c}{w-v} \), then \( \gamma^*_{w,\beta} > 1 \). Hence, the supply chain can be coordinated if and only if \( F(q(Q^*, w)) \geq \frac{w-c}{w-v} \), and the coordinating loss sharing fraction \( \gamma^*_{w,\beta} \) in \( (1 - \frac{1-\beta}{\lambda}, 1] \) is unique and satisfies \( \gamma^*_{w,\beta} = \tilde{\gamma} \).

Proof of Corollary 3. After plugging \( \gamma = 1 - \frac{1-\beta}{\lambda} \) into (14), and applying some algebraic manipulations, we get the coordinating buyback credit \( b^*_{\gamma,\beta,w} = b^0_w \). From (10), we can rewrite the retailer’s expected profit under the coordinating GLB contract with \( \gamma = 1 - \frac{1-\beta}{\lambda} \) and \( b^*_{\gamma,\beta,w} = b^0_w \) as follows:

\[
E[\pi_r(X, Q^*, w, \gamma, \beta, b^0_w)]
= (1 - \beta) \left( -\left(1 - \frac{1}{\lambda}\right) L(X, Q^*, w, b^0_w) + E[\pi_r(X, Q^*, w, b^0_w)] \right).
\]

From (A10), we get

\[
dE[\pi_r(X, Q^*, w, \gamma, \beta, b^0_w)]/d\beta = \left(1 - \frac{1}{\lambda}\right) L(X, Q^*, w, b^0_w)
- E[\pi_r(X, Q^*, w, b^0_w)] < 0,
\]

which implies \( dE[\pi_m(X, Q^*, w, \gamma, \beta, b^0_w)]/d\beta > 0 \).
Proof of Property 7. Under the coordinating distribution-free GLB contract with \(\gamma = 1 - \frac{1-\beta}{\lambda}\), if \(w = c\), then from (18) and (11), we get \(b_{y,\beta,w}^* = b_w^0 = v\) and \(E[\pi_r(X, Q^*, c, v)] = E[\pi(X, Q^*)]\). Therefore, we can rewrite the retailer’s expected profit function (A10) as follows:

\[
E[\pi_r(X, Q^*, c, \gamma, \beta, v)] = (1 - \beta) \left( -\frac{\lambda - 1}{\gamma} L(X, Q^*, c, v) + E[\pi(X, Q^*)] \right) .
\]

(A12)

From (A12), we see if \(\beta = 0\), then \(E[\pi_r(X, Q^*, c, \gamma, \beta, v)] > E[\pi(X, Q^*)]\) and if \(\beta = 1\), then \(E[\pi_r(X, Q^*, c, \gamma, \beta, v)] = 0\). Therefore, we can always find a \(\beta = \beta_0 \in (0, 1)\) at which \(E[\pi_r(X, Q^*, c, \gamma, \beta_0, v)] = E[\pi(X, Q^*)]\). Similarly, if \(w = p\), then \(b_{y,\beta,w}^* = b_w^0 = p\) and \(E[\pi_r(X, Q^*, p, \gamma, \beta_0, p)] = 0\). Because \(E[\pi_r(X, Q^*, w, \gamma, \beta_0, b_{y,\beta,w}^*)]\) is continuous in \(w\), the coordinating distribution-free GLB contract at a fixed \(\beta = \beta_0\) can arbitrarily allocate the expected supply chain profit between the manufacturer and retailer by changing the value of \(w\).

Under the buyback contract, if \(w = c\), then the coordinating buyback credit \(b_w^* \geq b_w^0 = v\) and \(E[\pi_r(X, Q^*, c, b_w^*)] > E[\pi_r(X, Q^*, c, v)] = E[\pi(X, Q^*)]\). If \(w = p\), then from (17), we get \(b_w^* = p\). Therefore, \(E[\pi_r(X, Q^*, p, p)] = 0\). Because \(E[\pi_r(X, Q^*, w, b_w^*)]\) is continuous in \(w\), the coordinating buyback contract can arbitrarily allocate the expected supply chain profit between the manufacturer and retailer by changing the value of \(w\).

Under the GL contract, if \(w = c\), then \(F(q(Q^*, w)) > \frac{w-c}{w-v} = 0\). Therefore, from (16), for any \(\beta \in [0, 1]\), there exists a coordinating loss sharing fraction \(\gamma_{w,\beta}^* = \gamma = 1 - \frac{1-\beta}{\lambda}\) and \(E[\pi_r(X, Q^*, c, v)] = E[\pi(X, Q^*)]\). Therefore, we can rewrite the retailer’s expected profit function (A10) under the GL contract as follows:

\[
E[\pi_r(X, Q^*, c, \gamma_{w,\beta}^*, \beta, v)] = (1 - \beta) \left( -\frac{\lambda - 1}{\lambda} L(X, Q^*, c, v) + E[\pi(X, Q^*)] \right) .
\]

(A13)

From (A13), we see if \(\beta = 0\), then \(E[\pi_r(X, Q^*, c, \gamma_{w,\beta}^*, \beta, v)] > E[\pi(X, Q^*)]\) and if \(\beta = 1\), then \(E[\pi_r(X, Q^*, c, \gamma_{w,\beta}^*, \beta, v)] = 0\). Because \(E[\pi_r(X, Q^*, w, \gamma_{w,\beta}^*, \beta, v)]\) is continuous in \(\beta\), the coordinating GL contract with \(w = c\) can arbitrarily allocate the expected supply chain profit between the manufacturer and retailer by changing the value of \(\beta\).

Proof of Property 8. (i) Consider the distribution-free coordinating GLB contract with \(\gamma = 1 - \frac{1-\beta}{\lambda}\) and \(b_{y,\beta,w}^* = b_w^0\). From (A10), we see that \(E[\pi_r(X, Q^*, w, \gamma, \beta, b_w^0)]\) is decreasing in \(\beta\), and if \(\beta = 1\), then \(E[\pi_r(X, Q^*, w, \gamma, \beta, b_w^0)] = 0\). Therefore, for any \(E[\pi_r(X, Q^*, w, b_w^*)] > 0\) under a coordinating buyback contract, or \(E[\pi_r(X, Q_{\lambda-1}^*, w)] > 0\) under a price-only contract, there exists a \(\beta = \beta_2 \in (0, 1)\) at which \(E[\pi_r(X, Q^*, w, \gamma_2, b_w^0)] < E[\pi_r(X, Q^*, w, b_w^*)] \) and \(E[\pi_r(X, Q_{\lambda-1}^*, w, \gamma, \beta_2, b_w^0)] < E[\pi_r(X, Q_{\lambda-1}^*, w, b_w^*)]\), or equivalently, \(E[\pi_m(X, Q^*, w, \gamma, \beta_2, b_w^0)] > E[\pi_m(X, Q^*, w, b_w^*)] \) and \(E[\pi_m(X, Q_{\lambda-1}^*, w, \gamma, \beta_2, b_w^0)] > E[\pi_m(X, Q_{\lambda-1}^*, w, b_w^*)]\).
(ii) If \( F(q(Q^*, w)) > \left( \frac{w-c}{w} \right) \), then there exists a set of coordinating GL contract with \( \gamma_{w, \beta} = \check{\gamma} \). From (A10), we can rewrite the retailer’s expected profit as follows:

\[
E[\pi_r(X, Q^*, w, \gamma_{w, \beta}, \beta, v)] = (1 - \beta) \left( -\frac{1}{\lambda} \left( \lambda - 1 + \frac{(w-c)}{(w-v)F(q(Q^*, w))} \right) \times L(X, Q^*, w, v) + E[\pi_r(X, Q^*, w, v)] \right).
\]

From (A14), we see \( E[\pi_r(X, Q^*, w, \gamma_{w, \beta}, \beta, v)] \) is decreasing in \( \beta \), and if \( \beta = 1 \), then \( E[\pi_r(X, Q^*, w, \gamma_{w, \beta}, \beta, v)] = 0 \). The rest of the proof is similar to the proof of Property 8(i). \( \square \)

**Lemma 2.** For a GLB contract with parameters \( b = b_0^w = p - \frac{(p-v)(p-w)}{p-c} \), \( \beta \in [0, 1) \), and \( \gamma = \beta \), the manufacturer’s expected profit as a fraction of the expected supply chain profit is

\[
\alpha = 1 - (1 - \beta)(p-w)/(p-c).
\]

**Proof of Lemma 2.** Under the GLB contract with \( \gamma = \beta \) and \( b_{\gamma, \beta, w} = b_0^w \), the retailer’s optimal order quantity \( Q_{\lambda-1}^* \) is characterized by (13). Then from (1), we can express the integrated supply chain’s expected profit at \( Q_{\lambda-1}^* \) as follows:

\[
E[\pi(X, Q_{\lambda-1}^*)] = (p-c)Q_{\lambda-1}^* - \int_0^{Q_{\lambda-1}^*} (p-v)(Q_{\lambda-1}^* - x)f(x)dx.
\]

Similarly, from (10), we can express the retailer’s expected profit at \( Q_{\lambda-1}^* \) as

\[
E[\pi_r(X, Q_{\lambda-1}^*, w, \gamma, \beta, b_0^w)]
\]

\[
= (1 - \beta)\left( p-w\right)\left( p-c\right)Q_{\lambda-1}^* - \int_0^{Q_{\lambda-1}^*} (p-v)(Q_{\lambda-1}^* - x)f(x)dx.
\]  

Because \( b = b_0^w = p - \frac{(p-v)(p-w)}{p-c} \), we have \( p - b_0^w = \frac{(p-v)(p-w)}{p-c} \). Hence, we can rewrite (A16) as follows:

\[
E[\pi_r(X, Q_{\lambda-1}^*, w, \gamma, \beta, b_0^w)]
\]

\[
= (1 - \beta)\left( \frac{p-w}{p-c} \right)\left( p-c\right)Q_{\lambda-1}^* - \int_0^{Q_{\lambda-1}^*} (p-v)(Q_{\lambda-1}^* - x)f(x)dx.
\]

\[
= (1 - \beta)\left( \frac{p-w}{p-c} \right)E[\pi(X, Q_{\lambda-1}^*)].
\]  

(A17)
Therefore, it follows from (A15) and (A17) that the manufacturer’s expected profit as a fraction of the expected supply chain profit is

\[ \alpha = 1 - \frac{E[\pi(X, Q^*_L, w, y, b^N, \lambda)]}{E[\pi(X, Q^*_L, \lambda)]} = 1 - (1 - \beta)\left(\frac{p-w}{p-c}\right). \]

Charles X. Wang is an assistant professor of operations management at the State University of New York at Buffalo School of Management. He received a PhD in operations management and a MS in computer engineering from Syracuse University and a bachelor’s degree in mechanical engineering from Tongji University in Shanghai, China. His research focuses on supply chain contracts and coordination, risk management and behavioral issues in supply chains, B2B electronic commerce, and operations and marketing interface. He has published articles in *International Journal of Production Economics* and *Supply Chain Management: An International Journal*. He teaches the MBA core course in operations management and graduate courses in supply chain management.

Scott Webster is a professor at the Syracuse University Whitman School of Management. He received a PhD in operations management and decision sciences from Indiana University, and his undergraduate degree is in mathematics and statistics from Miami University. His research focuses on improving competitiveness through logistics. He has written a textbook on supply chain management and he has published articles in *IIE Transactions, Journal of Operations Management, Management Science, Manufacturing and Service Operations Management, Operations Research, and Production and Operations Management*, among others. He has worked in industry in the areas of consulting and finance, and he currently serves as associate editor for *Decision Sciences*. He teaches undergraduate and graduate classes in supply chain management.