Innovative Applications of O.R.

Markdown money contracts for perishable goods with clearance pricing

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Abstract

It is common in practice that retailers liquidate unsold perishable goods via clearance pricing. Markdown money is frequently used between manufacturers and retailers in such a supply chain setting. It is a form of rebate from a manufacturer to subsidize a retailer’s clearance pricing after the regular season. Two forms of markdown money are percent markdown money, in which the markdown money is limited to only a certain percentage of the retail price markdown, and quantity markdown money, which is essentially a buyback contract or returns policy with a rebate credit paid to the retailer for each unsold unit after the regular season. We show both forms of markdown money contracts can coordinate the supply chain and we discuss their strengths and limitations.

Keywords:
Supply chain management
Markdown money
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Supply chain contracts

1. Introduction

The volatile market of perishable goods (e.g., fashion apparel, consumer electronics, personal computers, toys, books, and CDs) is featured by uncertain demand, long lead time, and a short selling season. The retailer initially purchases products from the manufacturer before the selling season begins and has a great chance of facing overstock. Since perishable goods will lose value in the eyes of the customer after the regular season, it is common in practice that retailers liquidate excess inventory via clearance pricing (Forest et al., 2003; Kratz, 2005; Rozhon, 2005).

Clearance pricing is an important price promotion tool for the retailer to enhance sales of unsold units after the regular selling season. Since the retailer makes less money from clearance sales than the regular season, it is common in practice that large retailers (e.g., May, Federated, Kohl’s, Saks, and J.C. Penney) demand a rebate called markdown money (or markdown allowance) from the manufacturer to subsidize their clearance sales (Kratz, 2005; Rozhon, 2005). Markdown money is prevalent in industries selling perishable goods, e.g., fashion apparel, cosmetics and fragrances, toys, specialty status, and over-the-counter medications (Tsay, 2001).

One form of markdown money is quantity markdown money (QMM), in which the manufacturer pays a rebate credit to the retailer for each unsold unit at the end of the regular selling season. QMM contracts are also known as buyback contracts or returns policies in the literature when the retailer, not the manufacturer, salvages overstock at the end of the regular selling season (Cachon, 2003, p. 242).

Another form of markdown money called percent markdown money (PMM), in which the markdown money paid to the retailer is a certain percentage of the retail price markdown, i.e., the difference between the regular selling price and clearance price. For example, Rozhon (2005) reports that in the fashion industry, a strong retailer may demand as much as 100% of their retail price markdowns from a supplier. The main difference between a PMM and a QMM contract is that the rebate depends on the end of season clearance price in a PMM contract whereas the rebate is specified at the start of the season in a QMM (or buyback) contract. If the end of season clearance price is known at the start of the season when the contract terms are set, then PMM and QMM contracts are identical.

The main purpose of this paper is to address the following issues between the manufacturer and the retailer regarding the markdown money contract in practice: (1) the impact of the magnitude of the markdown money and (2) the relative performance of PMM and QMM contracts. Regarding the first issue, if the manufacturer’s markdown money is too little, then the retailer will order fewer products than the manufacturer would like; however, if the retailer demands too much markdown money, then the manufacturer’s performance may be hurt because of the large payment to the retailer. For example, an executive for one of the best-known apparel makers reports that if the manufacturer refuses the markdown money proposed by the retailers, then they will order 5% percent less than usual. On the other hand, two major clothing companies, Kellwood and Jones, have warned of lower earnings in part because of post-holiday markdown money payments to...
the retailer (Rozhon, 2005). Thus, it is important to know whether or not the QMM and PMM contracts can improve supply chain performance and allow both parties to benefit. Regarding the second issue, there is a vast literature on the QMM contracts (Cachon, 2003; Lariviere, 1999), but this work assumes that the end of season salvage value (e.g., clearance price) is exogenous. In practice, clearance price is commonly affected by market conditions observed during the selling season, in which case PMM and QMM contracts have different impacts on decisions and expected profits. We seek to understand the merits of each type of contract when clearance price is endogenous.

Cachon and Kök (2007) propose and analyze a clearance pricing model. Their work lends insight into how a retailer can apply the powerful framework of the newsvendor model, which relies on an exogenous salvage value assumption, in settings where salvage value depends on the retailer’s end-of-season clearance price decision. This paper builds on the clearance pricing model of Cachon and Kök (2007), but rather than addressing the question of how to estimate salvage value when clearance price is endogenous, we investigate the relative performance of two types of markdown money contracts. In summary, the previous research on perishable goods treats markdown money and clearance pricing separately. This paper draws on both these literatures to evaluate the relative performance of markdown money contracts.

This paper is organized as follows. In Section 2, we briefly review the relevant literature. In Section 3, we present our model preliminaries and analyze supply chain models under a general clearance demand function. To identify additional insights, in Section 4, we analyze supply chain models under a special linear clearance demand function. Finally, in Section 5, we summarize our results and identify research opportunities for future study.

2. Literature review

The literature related to this paper can be divided into two categories: papers on the newsvendor problem and its extensions and papers on supply chain contracts.

The single-period newsvendor model has been well studied in the literature (see Porteus, 1990; Lee and Nahmias, 1993; Khouja, 1999; Petruzzi and Dada, 1999 for more detailed reviews). In those models, it is commonly assumed that the salvage value for excess inventory is fixed and exogenously given. Recently, a number of attempts have been made to extend the understanding of the newsvendor model. The paper most closely related to our research is Cachon and Kök (2007), who study a newsvendor model with clearance pricing for the leftover inventory at the end of the selling season. In contrast with the traditional newsvendor model, they treat the salvage value (or clearance price) as a decision variable and focus on methods for estimating salvage value in the newsvendor problem.

Supply chain contract models have received much attention from researchers recently. In this section, we only focus on returns policies and markdown money, which are most closely related to this research. We refer to Cachon (2003) for reviews of other types of supply chain contracts.

Returns policies, also known as buyback contracts, allow the retailer to return a certain amount of unsold goods to the manufacturer at the end of the selling season for a partial rebate credit. Returns policies are common in the distribution of perishable commodities with uncertain demand, such as books, magazines, newspapers, recorded music, computer hardware and software, greeting cards, and pharmaceuticals (Padmanabhan and Png, 1995). Pasternack (1985) is the first to study a returns policy. He shows that both full returns with full rebate credit and no returns are system suboptimal. The supply chain can be coordinated by an intermediate returns policy, e.g., partial returns with full rebate credit. Kandel (1996) studies two extreme contract schemes for allocation of responsibility for unsold inventory in a supply chain: the consignment contract and the no-return contract. He also shows if demand is stochastic and price-sensitive, then the supply chain cannot be fully coordinated by returns policies without retail price maintenance (i.e., allowing the manufacturer to dictate the retail price). Emmons and Gilbert (1998) study a price-sensitive multiplicative model of demand uncertainty for catalog goods and demonstrate that uncertainty tends to increase the retail price. They also show that under certain conditions, a manufacturer can increase her profit by offering a returns policy. Webster and Weng (2000) take the viewpoint of a manufacturer selling a short life-cycle product to a single retailer and describe risk-free returns policies that, when compared to no returns, the retailer’s expected profit is increased and the manufacturer’s realized profit is ensured to be at least as large as when no returns are allowed. Donohue (2000) studies returns policies in a supply chain model with multiple production opportunities and improving demand forecasts. Lee et al. (2000) study dynamic optimal price protection policies for products subject to price reductions due to obsolescence (e.g., personal computers), which closely resemble returns policies. Taylor (2002) incorporates a buyback contract with a target sales rebate contract to coordinate the supply chain when demand is sensitive to the retailer sales effort. Krishnan et al. (2004) study a decentralized supply chain with retailer promotional effort. They show that returns policies alone will reduce the retailer’s promotional incentives and adversely affect supply chain profits. However, under certain conditions, coupling returns policies with other channel mechanisms (e.g., promotional cost sharing) can coordinate the supply chain. Su and Zhang (2005) study a decentralized supply chain with strategic customers, who anticipate the seller’s future clearance sales at a fixed salvage price and choose the best purchasing time to maximize their expected surplus. They show how contractual arrangements can be used to improve supply chain performance. Finally, Wang and Webster (2007) study a decentralized supply chain in which a single risk-neutral manufacturer is selling a perishable product to a single loss-averse retailer. They investigate a returns policy with a gain/loss sharing provision to coordinate the supply chain. As pointed out by Cachon (2003), the name of returns policy or buyback contract is somewhat misleading since it implies physical returns of overstock at the end of the selling season, which only happens when the manufacturer’s salvage value is higher than the retailer’s. If the retailer’s salvage value is higher than the manufacturer, then the retailer liquidates overstock and the manufacturer credits the retailer for those units, which is often referred to as markdown money contract (Tsay, 2001). To our knowledge, past research on buyback and markdown money contracts assumes that salvage value (e.g., clearance price) is exogenous. However, there are many settings where a retailer’s salvage value is not fixed, but is based on a clearance price that is influenced by the observed demand during the selling season. In this paper we use the clearance pricing model of Cachon and Kök (2007). We extend the supply chain contract literature by allowing for endogenous salvage, and we find that the way in which markdown money is calculated (e.g., PMM versus QMM) leads to meaningful differences in performance.

3. Supply chain models with a general clearance demand function

In Section 3.1, we begin our analysis by investigating a vertically integrated firm that owns both manufacturer and retailer
and acts as a central planner for the supply chain. This centralized control setting provides a first-best solution that maximizes total supply chain profit. We then investigate a decentralized supply chain under a wholesale price-only contract. In Sections 3.2 and 3.3, we study the decentralized supply chain with the PMM contract and QMM contract, respectively.

3.1. Integrated and decentralized supply chains models with a wholesale price-only contract


We consider an integrated firm selling a perishable product with a selling season divided into two periods, a regular season \(T_1\) and a clearance period \(T_2\). At the beginning of \(T_1\), the integrated firm produces \(q\) units of a single item at a quantity independent unit production and delivery cost \(c\) and sells at a unit retail price \(p_1 > c\) in \(T_1\). Like most of the supply chain contract models we assume the retail price \(p_1\) in \(T_1\) is fixed (e.g., Pasternack, 1985; Kandel, 1996; Lariviere, 1999; Lee et al., 2000; Tsay, 2001; Taylor, 2002; and Cachon and Kók, 2007). This assumption is practically reasonable if the retail market is highly competitive (e.g., the retailer acts as a price taker).

Let \(q \in [0, \infty)\) be the realized demand in \(T_1\). Let \(F(q)\) be the strictly increasing and differentiable distribution function of demand and let \(f(q)\) be the density function. If realized demand \(\xi\) is higher than the order quantity \(q\), then all sales are lost without additional penalty; if realized demand \(\xi\) is lower than \(q\), then the leftover inventory \(L(q, \xi) = (q - \xi)^+\) will be carried over to the clearance period \(T_2\). We let \(I(q) = \int_0^q (q - \xi) f(\xi)d\xi\) be the expected leftover inventory.

Clearance demand \(D_q(p_2, \xi)\) in \(T_2\) is a deterministic function of the clearance price \(p_2\) and the realized demand \(\xi\) in \(T_1\). \(D_q(p_2, \xi)\) is non-negative, differentiable, and decreasing in \(p_2\). Hence, the inverse demand function exists, \(p_2(q, \xi)\), where \(s_2(q)\) is actual sales in \(T_2\). We assume revenue in \(T_2\), \(s_2(p_2, \xi)\), is concave in \(s_2\) for all \(\xi\); and \(p_2(q) < p_1\) for all \(q\), where \(p_2(q) = \arg \max_{p_2} (p_2D_q(p_2, q))\). We assume \(D_q(p_2, \xi)\) is monotone in \(\xi\) for all \(p_2\).

Let \(\hat{R}_2(s_2, \xi) = s_2p_2(s_2, \xi)\) be the unconstrained revenue function. Let \(s_2(\xi)\) be the unconstrained optimal sales in \(T_2\)

\[
\frac{\partial R_2(q, \xi)}{\partial s_2} = \begin{cases} 
2\xi & 0 < \xi < s_2(q), \\
I(q, \xi)p_2(l(q, \xi), \xi) & \xi < s_2(q), \\
0, & q < \xi.
\end{cases}
\]

where \(s_2(\xi)\) and \(\hat{\xi}(q)\) are defined in Cachon and Kók (2007). More specifically, if realized demand is lower than \(\hat{\xi}(q)\), then the firm will only liquidate some leftover inventory at a clearance price \(p_2 < p_1\) and hold back (destroy) the rest; if realized demand is higher than \(\hat{\xi}(q)\) but lower than \(\xi(q)\), then the firm will liquidate all leftover inventory; if realized demand is higher than \(\xi(q)\), then the firm will liquidate all leftover inventory at the regular season retail price \(p_1\); finally, if realized demand is higher than \(q\), then there is no clearance sales.

Then, we can express the integrated firm’s expected profit as follows:

\[
\Pi(q) = -cq + R_1(q) + R_2(q),
\]
6. At the end of $T_2$, the retailer salvages the inventory he holds back and the manufacturer pays the retailer a percentage $\gamma$ of revenues lost due to markdowns during $T_2$ and due to leftover inventory that is salvaged at the end of $T_2$.

The retailer’s clearance revenue function is slightly different from the integrated firm because of the introduction of the PMM contract, under which the retailer is able to get additional compensation from the manufacturer for his clearance sales and inventory holdbacks. Similarly, we can express the retailer’s revenue function, $R_2(q, \gamma, \xi)$, given realized demand $\xi$ in $T_1$, as follows:

$$R_2(q, \gamma, \xi) = \begin{cases} (1 - \gamma)\xi_2(\xi)p_2(\xi_2(\xi), \xi) + \gamma p_1 I(q, \xi), & 0 \leq \xi \leq \xi(q), \\ (1 - \gamma)l(q, \xi)p_2(l(q, \xi), \xi) + \gamma p_1 I(q, \xi), & \xi(q) < \xi \leq \xi(q), \\ l(q, \xi)p_1, & \xi(q) < \xi \leq q, \\ 0, & q < \xi. \end{cases}$$

Then, we can express the retailer’s expected profit under the PMM contract as follows:

$$\Pi(q, w, \gamma) = -wq + R_1(q) + R_2(q, \gamma).$$

where $R(q)$ is defined in (2) and

Finally, by the Implicit Function Theorem, we have

$$\frac{dq}{dy} = \frac{\partial^2 \Pi(q, w, \gamma)}{\partial y^2} / \frac{\partial^2 \Pi(q, w, \gamma)}{\partial y}$$. 

\textbf{Proposition 2.} The retailer’s optimal order quantity $q^*$ under the PMM contract is unique and satisfies the following first-order condition:

$$p_1 - w - (1 - \gamma) \left( p_F(q^*) - \int_{l(q, \xi)}^{\xi(q)} \frac{\partial}{\partial \xi} \left( l(q, \xi)p_2(l(q, \xi), \xi) \right) dF(\xi) \right) = 0. \quad (6)$$

Furthermore, $q^*$ is increasing in $\gamma$.

\textbf{Proof.} After taking the first and second derivatives of $\Pi(q, w, \gamma)$ expressed in (5) with respect to $q$, we get

$$d\Pi(q, w, \gamma)/dq = p_1 - w - (1 - \gamma) \left( p_F(q) - \int_{l(q, \xi)}^{\xi(q)} \frac{\partial}{\partial \xi} \left( l(q, \xi)p_2(l(q, \xi), \xi) \right) dF(\xi) \right)$$

and

$$d^2\Pi(q, w, \gamma)/dq^2 = (1 - \gamma) \left( \int_{l(q, \xi)}^{\xi(q)} \frac{\partial^2}{\partial \xi^2} \left( l(q, \xi)p_2(l(q, \xi), \xi) \right) dF(\xi) \right) - (1 - \gamma) \left( p_1 - \frac{\partial}{\partial \xi} \left( l(q, \xi)p_2(l(q, \xi), \xi) \right) \right) \right|_{l(q, \xi)}^{\xi(q)}$$

$$f(q(q)) + f(q(q)) - (1 - \gamma) \frac{\partial}{\partial \xi} \left( l(q, \xi)p_2(l(q, \xi), \xi) \right) \right|_{l(q, \xi)}^{\xi(q)}$$

Thus, $\Pi(q, w, \gamma)$ is concave in $q$, which implies there must exist a unique optimal retailer order quantity $q^*$ that satisfies the first-order condition, $d\Pi(q^*, w, \gamma)/dq = 0$, i.e., Eq. (6).

\textbf{Proposition 2.} shows that under the PMM contract, the retailer’s total expected profit function is concave and there exists a unique optimal order quantity that maximizes the retailer’s expected profit. In addition, the higher the manufacturer’s markdown money percentage, the more the retailer would like to order from the manufacturer.

\textbf{Proposition 3.} Consider the set of PMM contracts with

$$\gamma_w = \frac{w - c}{p_1 - c}. \quad (7)$$

(i) The retailer orders the integrated supply chain optimal order quantity $q^0$, i.e., those contracts can coordinate the supply chain.

(ii) The manufacturer’s expected profit is $\Pi(q^0, w, \gamma_w) = \gamma_w \Pi(q^0)$.

(iii) The set of PMM contracts $(w, \gamma_w)$ can arbitrarily allocate integrated supply chain profit between the manufacturer, who receives share $\gamma_w$, and the retailer, who receives share $1 - \gamma_w$.

(iv) $\gamma_w$ is increasing in $w$ and decreasing in both $p_1$ and $c$.

(v) For any clearance price $p_2 \leq p_1$, the markdown money $M^* = \gamma_w(p_1 - p_2) < \gamma_w$, less than the wholesale price $w$.

\textbf{Proof.} (i) Let $(w, \gamma_w)$ be the channel coordinating PMM contract under which the retailer’s optimal order quantity $q^*$ is the same as the integrated supply chain’s optimal order quantity $q^0$, i.e., $q^* = q^0$. Then $q^*$ must satisfy $d\Pi(q^*, w, \gamma_w)/dq = 0$. From the first-order condition (3), we have

$$p_1 - c - p_1 F(q^*) - \int_{l(q, \xi)}^{\xi(q)} \frac{\partial}{\partial \xi} \left( l(q, \xi)p_2(l(q, \xi), \xi) \right) dF(\xi)$$

+ $\int_{l(q, \xi)}^{q} p_1 dF(\xi) = 0.$

For any clearance price $p_2 \leq p_1$, the markdown money $M^* = \gamma_w (p_1 - p_2) < \gamma_w$, less than the wholesale price $w$.
increase w from c to \( p_1 \), then the manufacturer’s percentage of the total supply chain profit \( \gamma_w \) will increase from 0% to 100%, i.e., such PMM contracts can arbitrarily allocate supply chain profit between the retailer and the manufacturer.

(iv) The proof is straightforward so we omit it.

(v) Observe that \( M' /w = \gamma_w(p_1 - p_2) / w = (w - c)(p_1 - p_2) / (p_1 - p_2) = w \). Since \( p_1 > w \), we have \( \gamma_w < 1 \), which implies \( M' /w < 1 \), i.e., \( M' < w \).

\[ \text{Proposition 3(i)} \text{– (iii)} \text{ altogether imply that the PMM contract with } (w, \gamma_w) \text{ coordinates the supply chain and, through alternative choices of } w \text{ (or } \gamma \text{ equivalently), can arbitrarily allocate supply chain profit between the manufacturer and retailer. The relationship among the PMM coordinating contract parameters is simple and suitable for interpretations that, while clear in hindsight from the expressions, may not be obvious before-hand. From (7), the set of PMM coordinating parameters can be expressed in terms of either wholesale price or markdown money percentage: } w \text{ and } \gamma_w = (w - c)/(p_1 - c), \text{ or equivalently, } \gamma \text{ and } \gamma_w = c + \gamma(p_1 - c).

Interestingly, the manufacturer’s percentage of the supply chain profit is exactly the same as the markdown money percentage. Thus, a higher coordinating markdown money percentage will result in a higher (lower) manufacturer’s (retailer’s) percentage of the supply chain profit. At the first glance, this result seems counterintuitive since a higher markdown money percentage means a larger markdown money payment from the manufacturer to the retailer. However, from (7), we see that a higher coordinating markdown money percentage also means that the manufacturer charges a higher wholesale price to the retailer, which results in a higher manufacturer profit from the retailer’s order to offset the markdown money payment.

Let \( \text{Exp}[I] \) and \( \text{Var}[I] \) be the mean and variance of the supply chain’s profit. (I made this change because \( I \) was used for supply chain profit above.) Then the mean and variance of the manufacturer’s profit under the coordinating PMM contract \((w, \gamma_w)\) can be expressed as \( \gamma_w \text{Exp}[I] \) and \( \gamma_w^2 \text{Var}[I] \), respectively. Similarly, the mean and variance of the retailer’s profit under the coordinating PMM contract \((w, \gamma_w)\) can be expressed as \((1 - \gamma_w) \text{Exp}[I] \) and \( (1 - \gamma_w)^2 \text{Var}[I] \), respectively. The expressions highlight a clear risk-return relationship in the PMM contract: as \( \gamma_w \) increases, both the expected profit and the risk from uncertain demand in terms of the variance of profit shifts from the retailer to the manufacturer.

The particular profit split and allocation of supply chain risk may depend upon the firms’ relative bargaining power. If the manufacturer (retailer) is more powerful and would like to coordinate the supply chain, then one can expect a high (low) markdown money percentage \( \gamma_w \) in the PMM contract. The value of \( \gamma_w \) raises another issue in the PMM contract. If \( \gamma_w \) is very high (e.g., close to 1), then the retailer’s expected profit is quite small (e.g., nearly zero). Thus, in absolute terms, a deviation from the supply chain optimal order quantity \( q^* \) imposes little penalty on the retailer but has significant effect on the manufacturer’s profit. Under this situation, the coordinating PMM contract may not be enforceable especially if the retailer feels slighted and orders a suboptimal quantity in order to retaliate against the manufacturer. On the other hand, if \( \gamma_w \) is small, then a deviation from the supply chain optimal order quantity \( q^* \) imposes large penalty on the retailer but has less of an effect on the manufacturer’s profit. Under this situation, the coordinating PMM contract is more enforceable, i.e., there is no incentive for the retailer to retaliate against the manufacturer by ordering a suboptimal quantity.

Note that the channel coordinating PMM contract parameters are independent of both regular season demand distribution and the clearance revenue function (see (7)). Consequently, a manufacturer can offer a single PMM contract to multiple non-competing retailers with the same regular selling price but different demand distributions and clearance revenue functions. This property is helpful for avoiding antitrust issues that may arise when contract terms vary by customer. For example, Kirkpatrick (2001) reports that independent bookstores have accused Barnes & Noble and Borders of striking preferential deals with publishers that include more generous returns policies.

\[ \text{Proposition 3(iv)} \text{ is intuitive and says that the higher the wholesale price, the higher the channel coordinating markdown money percentage; the higher the retail price and production cost, the lower the channel coordinating markdown money percentage. We should note that the manufacturer’s actual markdown money } M \text{ paid to the retailer for each unit of leftover inventory depends upon the realized demand in the regular season and the retailer’s actual clearance price. If realized demand in the regular season is too low so that the leftover inventory is too high, then the retailer has to liquidate overstock at a low clearance price. This will result in higher manufacturer’s markdown money. However, as Proposition 3(v) shows, the actual unit markdown money } M \text{ can never be higher than the wholesale price } w. \]

3.3. Decentralized supply chain with a QMM contract

In this section, we investigate the role of the QMM contract on the supply chain coordination. The QMM contract \((w, m)\) specifies that the manufacturer charges the retailer a unit wholesale price \( w \) and pays the retailer markdown money \( m < w \) for each unsold unit after the regular season but at a lower clearance price (and salvage value) in \( T_2 \). As noted above, the QMM contract is essentially a returns policy since it is the retailer whole salvage the unsold products from \( T_1 \) at a clearance price in \( T_2 \).

Similarly, we can express the retailer’s expected profit under the QMM contract as follows:

\[
R_2(q, m, z) = \begin{cases} \hat{s}_2(z) p_2(\hat{s}_2(z), z) + ml(q, z), & 0 \leq z \leq \hat{z}(q), \\ \hat{s}_2(z)p_2(\hat{z}(q), z) + ml(q, z), & \hat{z}(q) < z \leq q, \\ 0, & q < z. \end{cases}
\]

Then, we can express the retailer’s expected profit under the QMM contract as follows:

\[
\Pi(q, w, m) = -wq + R_1(q) + R_2(q, m),
\]

where

\[
R_2(q, m) = \int_0^{\hat{z}(q)} \hat{s}_2(z)p_2(\hat{s}_2(z), z)dF(z) + \int_\hat{z}(q) ^ q \hat{s}_2(z)p_2(l(q, z), z)dF(z) + \int_0^q p_1l(q, z)dF(z) + \int_0^q ml(q, z)dF(z).
\]

\[ \text{Proposition 4. The retailer’s profit-maximizing optimal order quantity is determined by some } q^* \text{ that solves the following first-order condition: } \]

\[
p_1 - w - (p_1 - m)F(q^*) + \int_{(q^*)} ^ {q^*} \frac{\partial}{\partial q}(l(q^*, z)p_2(l(q^*, z), z))dF(z) + \int_{(q^*)} ^ {q^*} p_1dF(z) = 0.
\]

\[ \text{Proof. After taking the first-derivative of } \Pi(q, w, m) \text{ expressed in (8) with respect to } q, \text{ we get } \]
\[ d\Pi_r(q, w, m)/dq = p_1 - w - (p_1 - m)F(q) + \int_{0}^{q} \frac{1}{f(q)} \cdot I(q, \xi)p_2(l(q, \xi))dF(\xi) + \int_{0}^{q} p_1 dF(\xi). \]

Therefore, the optimal order quantity \( q^* \) must satisfy \( d\Pi_r(q, w, m)/dq = 0 \), i.e., Eq. (9).

We wish to note that compared with the PMM contract which can always coordinate the supply chain, the QMM contract is more restrictive and may not be able to coordinate the supply chain if the solution to the first-order condition (9) is not unique. We next focus only on the coordinating QMM contract when (9) has a unique solution \( q^* \).

**Proposition 5.** If \( q^m \) is unique, then consider the set of QMM contracts with

\[ m_w = (w - c)/F(q^m). \tag{10} \]

(i) The retailer orders the integrated supply chain’s optimal order quantity \( q^* \), i.e., those contracts can coordinate the supply chain.

(ii) The manufacturer’s expected profit \( \Pi_m(q^*, w, m_w) \) is increasing in \( w \) and the retailer’s expected profit \( \Pi_r(q^*, w, m_w) \) is decreasing in \( w \).

(iii) The set of QMM contracts \((w, m_w)\) can arbitrarily allocate integrated supply chain profit between the manufacturer and the retailer.

(iv) \( m_w \) is increasing in \( p_1 \) but may increase or decrease in \( c \).

(v) \( m_w < w \).

**Proof.** (i) The proof is similar to Proposition 3(i) so we omit it.

(ii) The manufacturer's expected profit under the coordinating QMM contract \((w, m_w)\)

\[ \Pi_m(q^*, w, m_w) = (w - c)q^* - m_w \int_{0}^{q^*} l(q^*, \xi)dF(\xi). \tag{11} \]

After taking the first-derivative of \( \Pi_m(q^*, w, m_w) \) with respect to \( w \), from (11) we get

\[ d\Pi_m(q^*, w, m_w)/dw = q^* - \frac{1}{F(q^*)} \int_{0}^{q^*} l(q^*, \xi)dF(\xi) > 0 \tag{12} \]

and expression (12) also implies \( d\Pi_m(q^*, w, m_w)/dw < 0 \).

(iii) From (10), if \( w = c \), then \( m_w = 0 \) and \( \Pi_m(q^*, w, m_w) = 0 \); if \( w = p_1 \) then since \( m_w < w = p_1 \), by Proposition 5(v), we have \( \Pi_m(q^*, w, m_w) < 0 \) and \( \Pi_m(q^*, w, m_w) > \Pi_0(q^*) \). Since from Proposition 5(ii), \( d\Pi_m(q^*, w, m_w)/dw > 0 \), there must exist a wholesale price \( w_1 < p_1 \) such that \( \Pi_m(q^*, w, m_w) = \Pi_0(q^*) \). Thus, the set of contracts specified in (10) can arbitrarily allocate supply chain profit between the manufacturer and the retailer.

(iv) It follows from (10) that \( dw_m/dw = 1/F(q^m) > 0 \). Observe from (3) that \( dp_1/dp_1 > 0 \), it follows that \( dm_w/dp_1 = (w - c)/F(q^m) > 0 \). Similarly, \( dp_0/dp_1 < 0 \). The sign of \( dm_w/dc \) can be positive or negative.

(v) Since the term \( \int_{0}^{q^*} (l(q^*, \xi)p_2(l(q^*, \xi))dF(\xi)) + \int_{0}^{q^*} p_1 dF(\xi) \) in (3) is strictly positive, we have \( F(q^m) > (p_1 - c)p_1 > (w - c)/w \). Therefore, \( m_w = (w - c)/F(q^m) < w \). □

If the retailer's optimal order quantity under the QMM contract is unique, then Proposition 5(i)-(iii) altogether imply that the QMM contract with \((w, m_w)\) coordinates the supply chain and, through alternative choices of \( w \) (or \( m \) equivalently), can arbitrarily allocate supply chain profit between the manufacturer and retailer. The relationship among the QMM coordinating contract parameters is not as simple as the PMM contract, but can still be expressed in terms of either wholesale price or markdown money:

\[ w \text{ and } m_w = (w - c)/F(q^m), \text{ or equivalently, } m \text{ and } m_w = mF(q^m) + c. \]

The effect of increases in the contract parameters \( w \) and \( m_w \) is similar to the effect of increases in \( w \) and \( \gamma_w \) in a PMM contract; the manufacturer’s share of total expected supply chain profit is increasing in \( w \) and \( \gamma_w \), and the retailer's share of total expected supply chain profit is decreasing in \( w \) and \( \gamma_w \). Accordingly, we see the same type of risk-return relationship that arises in a QMM contract: as \( w \) and \( m_w \) increase, both the expected profit and the risk from uncertain demand shifts from the retailer to the manufacturer.

Compared with the PMM contract, we see from (10) that the optimal markdown money in a QMM contract depends upon the regular season demand distribution. For a fixed \( q^m \), a higher probability of stocking out will lead to higher markdown money paid to the retailer. Thus, one disadvantage of the QMM contract is that the manufacturer cannot offer a uniform channel coordinating QMM contract to multiple non-competing retailers.

**Proposition 5(iv) and (v) describes relationships among the optimal markdown money and other price/cost parameters, i.e., markdown money is increasing in wholesale price, decreasing in regular retail price, and less than the wholesale price.**

### 4. Supply chain models with a linear clearance demand function

To gain additional insight, we next investigate a supply chain with a linear clearance demand function. We assume clearance demand \( D_z(p_2, \zeta) \) in \( T_2 \) is a combination of a linear demand function \( d_2(p_2) \) and a multiplicative shock \( x(\zeta) \), i.e.,

\[ D_z(p_2, \zeta) = d_2(p_2)x(\zeta), \tag{13} \]

where \( d_2(p_2) \) is in the form of

\[ d_2(p_2) = a - bp_2, \tag{14} \]

with \( a > 0 \) and \( b > 0 \). The parameter \( a \) represents the size of the clearance market and \( b \) represents the price sensitivity of clearance demand. The linear demand curve \( d_2(p_2) \) is common in the marketing and supply chain literature and has empirical support (e.g., Lillien et al., 1992; Monroe, 1990; Trivedi, 1998; Corbett et al., 2004). Similar to Cachon and Kók (2007), we assume the multiplicative shock function \( x(\zeta) = \mu(\zeta) \), where \( \mu \) is the mean demand in \( T_1 \). The case of \( \mu(\zeta) = \zeta \) means that demands in the regular season and clearance period are independent and appeal to two distinct market segments, e.g., the firm practices clearance pricing through a separate channel such as an online website (e.g., J.C. Penny and Gap) or a discount specialist (e.g., T.J. Maxx in the apparel industry). The case of \( \mu(\zeta) = \zeta \) means that regular season and clearance period demands are positively correlated, i.e., regular season demand is a good indicator of higher clearance demand.

From (13) and (14), the inverse demand function is

\[ p_2(s_2, \zeta) = \frac{1}{b} \left( a - s_2/(x(\zeta)) \right), \tag{15} \]

where \( s_2 \) is the clearance demand in \( T_2 \). In addition, we see that \( D_z(p_2, \zeta) = 0 \) if \( p_2 \geq p_{max} = a/b \) and \( D_z(p_2, \zeta) = \zeta x(\zeta) \) if \( p_2 = 0 \). We assume \( p_{max} < p_1 \), i.e., \( a/b < p_1 \), so that \( s_2 > 0 \), \( p_2(s_2, \zeta) < p_2 \). This assumption essentially captures the characteristic of perishable goods, i.e., the product will lose value in the eyes of the customer after the regular selling season.
It follows from (15) that \( dp_2(s_2, \xi)/da > 0 \) and \( dp_2(s_2, \xi)/db < 0 \), i.e., the larger the clearance demand pool a and the smaller the price sensitivity b, the higher the clearance price. Let

\[
\hat{R}_2(s_2, \xi) = s_2 p_2(s_2, \xi) - \frac{1}{b} \left( a s_2 - \frac{s_2^2}{x(\xi)} \right)
\]

be the unconstrained revenue function. Since \( \hat{R}_2(s_2, \xi) \) is quadratic, there exists a unique optimal sales quantity \( s_2(\xi) = ax(\xi)/2 \) in \( T_2 \) that maximizes \( \hat{R}_2(s_2, \xi) \), and the corresponding maximal clearance revenue is \( \hat{R}_2(s_2, \xi) = a^2 x(\xi)/4b \).

Let \( \hat{\xi}(q) = \max(0, \tilde{\xi}_0) \), where \( \tilde{\xi}_0 \) is uniquely determined by solving \( q - \tilde{\xi}_0 = s_2(\tilde{\xi}_0) \). If realized demand \( \xi \) in \( T_1 \) satisfies \( \xi > \hat{\xi}(q) \), then the leftover inventory in \( T_1 \) must be less than the optimal sales quantity in \( T_2 \), i.e., \( q - \xi < s_2(\xi) \), and the retailer will liquidate all \( q - \xi \) units of leftover inventory via clearance pricing. If \( \xi < \hat{\xi}(q) \), then \( q - \xi > s_2(\xi) \), and the retailer will only liquidate \( s_2(\xi) \) units of leftover inventory via clearance pricing. Therefore, we can express the clearance revenue function \( R_2(q, \xi) \) in \( T_2 \) as follows:

\[
R_2(q, \xi) = \begin{cases} \frac{1}{b} \left( a(q - \xi) - \frac{(q - \xi)^2}{x(\xi)} \right) & \text{if } \xi > \hat{\xi}(q), \\ a^2 x(\xi)/4b & \text{otherwise}. \end{cases}
\]

From (17), we can express expected profit functions of the integrated firm and the independent retailer under the wholesale price-only contract as follows:

\[
\Pi(q) = (p_1 - c)q - p_1 l(q) + \int_0^q \frac{a^2 x(\xi)}{4b} \, df(\xi) + \frac{1}{b} \int_0^q \left( a(q - \xi) - \frac{(q - \xi)^2}{x(\xi)} \right) \, df(\xi) \, dx,
\]

\[
\Pi_r(q) = (p_1 - w)q - p_1 F(q) + \int_0^q \frac{a^2 x(\xi)}{4b} \, DF(\xi) + \frac{1}{b} \int_0^q \left( a(q - \xi) - \frac{(q - \xi)^2}{x(\xi)} \right) \, DF(\xi) \, dx.
\]

**Proposition 6.** (i) With the linear clearance demand function, the integrated firm’s optimal stocking level \( q^0 \) and the independent retailer’s optimal order quantity \( q' \) are unique and satisfy the following first-order conditions:

\[
p_1 - c - p_1 F(q^0) + \frac{1}{b} \int_0^q \left( a - \frac{2(q - \tilde{\xi}(q))}{x(\tilde{\xi}(q))} \right) \, df(\tilde{\xi}(q)) = 0,
\]

\[
p_1 - w - p_1 F(q') + \frac{1}{b} \int_0^q \left( a - \frac{2(q - \tilde{\xi}(q))}{x(\tilde{\xi}(q))} \right) \, DF(\tilde{\xi}(q)) = 0.
\]

(ii) Both \( q^0 \) and \( q' \) are increasing in \( a \) and decreasing in \( b \).

**Proof.**

(i) After taking the first derivative of (18) with respect to \( q \), we get:

\[
\frac{d\Pi(q)}{dq} = p_1 - c - p_1 F(q) + \frac{1}{b} \left( \frac{a^2 x(\tilde{\xi}(q))}{4} - a(q - \tilde{\xi}(q)) + \frac{(q - \tilde{\xi}(q))^2}{x(\tilde{\xi}(q))} \right) f(\tilde{\xi}(q)) \hat{\xi}(q) + \frac{1}{b} \int_0^q \left( a - \frac{2(q - \tilde{\xi}(q))}{x(\tilde{\xi}(q))} \right) \, df(\tilde{\xi}(q)).
\]

Recall that \( \hat{\xi}(q) = \max(0, \tilde{\xi}_0) \). If \( \hat{\xi}(q) = 0 \), then \( \hat{\xi}(q) = 0 \). If \( \hat{\xi}(q) = \tilde{\xi}_0 \), then since \( q - \tilde{\xi}_0 = s_2(\tilde{\xi}_0) = ax(\tilde{\xi}_0)/2 \), we have

\[
\frac{a^2 x(\tilde{\xi}(q))}{4} - a(q - \tilde{\xi}(q)) + \frac{(q - \tilde{\xi}(q))^2}{x(\tilde{\xi}(q))} \quad 0.
\]

Therefore, for any \( \hat{\xi}(q) = \max(0, \tilde{\xi}_0) \), we can rewrite (21) as follows:

\[
\frac{d\Pi(q)}{dq} = (p_1 - c - p_1 F(q) + \frac{1}{b} \int_0^q \left( a - \frac{2(q - \tilde{\xi}(q))}{x(\tilde{\xi}(q))} \right) \, df(\tilde{\xi}(q)).
\]

After taking the second derivative of (23) with respect to \( q \), we get:

\[
\frac{d^2\Pi(q)}{dq^2} = \left( p_1 - \frac{a}{b} \right) f(q) - \frac{2}{b} \int_0^q \frac{1}{x(\tilde{\xi}(q))} \, df(\tilde{\xi}(q)) + \left( a - \frac{2(q - \tilde{\xi}(q))}{x(\tilde{\xi}(q))} \right) f(\tilde{\xi}(q)) \hat{\xi}(q).
\]

Similarly, if \( \hat{\xi}(q) = 0 \), then \( \hat{\xi}(q) = 0 \). If \( \hat{\xi}(q) = \tilde{\xi}_0 \), then \( a - \frac{2(q - \tilde{\xi}(q))}{x(\tilde{\xi}(q))} \) = 0. By the assumption of \( a \leq bp_1 \), we must have

\[
\frac{d^2\Pi(q)}{dq^2} = \left( p_1 - \frac{a}{b} \right) f(q) - \frac{2}{b} \int_0^q \frac{1}{x(\tilde{\xi}(q))} \, df(\tilde{\xi}(q)) < 0.
\]

which implies \( \Pi(q) \) is concave. Therefore, the optimal order quantity \( q^0 \) must be uniquely determined by the first-order condition (20). Similarly, the independent retailer’s optimal order quantity \( q' \) must be uniquely determined by the first-order condition (21).

(ii) By the Implicit Function Theorem, from (20), we have

\[
\frac{dq^0}{da} = \frac{d^2\Pi(q^0)/dq^2}{-d^2\Pi(q^0)/dq^2} > 0,
\]

\[
\frac{dq^0}{db} = \frac{d^2\Pi(q^0)/db^2}{-d^2\Pi(q^0)/db^2} < 0.
\]

Similarly, we can prove \( dq^0/da > 0 \) and \( dq^0/db < 0 \).

**Proposition 6(ii)** characterizes the integrated supply chain’s optimal stocking level \( q^0 \) and the independent retailer’s optimal order quantity \( q' \) under the linear clearance demand function and **Proposition 6(ii)** says that the larger the size of the clearance market, the higher the integrated firm’s optimal inventory stocking level and the independent retailer’s optimal order quantity; the more price-sensitive the clearance demand is, the lower the integrated firm’s optimal inventory stocking level and the independent retailer’s optimal order quantity.

We next investigate how the linear demand parameters \( (a, b) \) interact with the channel coordinating PMM and QMM contracts. From **Proposition 3**, we know that the channel coordinating PMM contract parameter is independent of both \( a \) and \( b \) in the linear clearance demand function. However, from **Proposition 5**, we know that the channel coordinating QMM contract depends upon the clearance demand parameters. The nature of this dependency is shown in the following proposition.

**Proposition 7.** The coordinating markdown money \( m_w \) in the QMM contract is decreasing in \( a \) and increasing in \( b \).

**Proof.** Recall from (10) that \( m_w = (w - c)F(q^0) \). From **Proposition 6(ii)**, we know \( dq^0/da > 0 \) and \( dq^0/db < 0 \). Therefore, we get:

\[
\frac{dm_w}{da} = -\frac{w - c}{F(q^0)} \left( \frac{dq^0}{da} \right) < 0 \quad \text{and} \quad \frac{dm_w}{db} = -\frac{w - c}{F(q^0)} \left( \frac{dq^0}{db} \right) > 0.
\]

**Proposition 7** says that if the QMM contract can coordinate the supply chain, then the larger the clearance market, the less the manufacturer’s markdown money to coordinate the supply chain. In addition, the more price-sensitive the clearance demand is, the larger the manufacturer’s markdown money. We next express the manufacturer’s expected profit functions under the coordinating PMM and QMM contract as follows:
\[ P_m(q^*, w, \gamma) = \left( w - c \right) q^* - \gamma w p_1 I(q^*) + \frac{\alpha^2 w}{4d} \int_0^\infty x(\xi) dF(\xi) \]
\[ + \frac{\gamma w}{b} \int_0^b \left( a q^* - \xi - \frac{(q^* - \xi)^2}{x(\xi)} \right) dF(\xi), \quad (25) \]
\[ P_m(q^*, w, m_w) = \left( w - c \right) q^* - m_w I(q^*). \quad (26) \]

In practice, it can sometimes be difficult for the manufacturer to change the wholesale price for the following reasons: (1) if the market is highly competitive, then the manufacturer does not have much control of the market price and has to act as a price-taker; (2) the Robinson–Patman Act in the US restricts the manufacturer's ability to sell the same product to different retailers at different wholesale prices, especially when the production cost is the same; and (3) changing the wholesale price can be costly, and there is empirical evidence that manufacturers are reluctant to change wholesale prices (e.g., Lyer and Bergen, 1997 and Cachon, 2003). Bosh and Anand (2007) also provide additional support for exogenous wholesale price in practice. In view of this, our next proposition compares the relative performance of the coordinating PMM and QMM contracts, when demands in T_1 and T_2 are independent, i.e., \( x(\xi) = \mu \).

**Proposition 8.** If \( x(\xi) = \mu \) and \( \xi(q^*) = 0 \), then for a fixed \( w \), the manufacturer (retailer)’s expected profit under the coordinating PMM contract is higher (lower) than that under the coordinating QMM contract if and only if \( CV_{\xi(q^*)} < \frac{\sqrt{2F(q^*)}}{\Gamma(q^*)} \), where \( CV_{\xi(q^*)} \) is the coefficient of variation of \( I(q^*) \).

**Proof.** Let \( A = P_m(q^*, w, \gamma) - P_m(q^*, w, m_w) \) be the difference between the manufacturer’s expected profits under the coordinating PMM and QMM contract, and let \( \mu_{\xi(q^*)} \) and \( \sigma_{\xi(q^*)} \) be the mean and standard deviation of the random variable \( I(q^*) \), respectively. For a fixed \( w \), if \( x(\xi) = \mu \) and \( q^* \leq \mu_2/\mu \), then \( \xi(q) = 0 \). From (25) and (26), we have

\[ A = (m_w - \gamma w p_1 + \frac{\alpha^2 w}{4d})\mu_{\xi(q^*)} - \frac{\gamma w}{b} \int_0^b I(q^*, \xi)^2 dF(\xi). \quad (27) \]

Since \( \sigma_{\xi(q^*)}^2 = \int_0^b I(q^*, \xi)^2 dF(\xi) - \mu_{\xi(q^*)}^2 \), we can rewrite (27) as follows:

\[ A = \left( m_w - \gamma w p_1 + \frac{\alpha^2 w}{4d} \right)\mu_{\xi(q^*)} - \frac{\gamma w}{b} \left( \sigma_{\xi(q^*)}^2 + \mu_{\xi(q^*)}^2 \right) \]

\[ = \mu_{\xi(q^*)}^2 \left( \frac{w - \mu_1\gamma p_1}{\mu_1 - \mu_1} \right) \frac{p_1 - c - \left( p_1 - \frac{\alpha^2 w}{4d} \right) F(q^*)}{\mu_{\xi(q^*)}} - \frac{\mu_{\xi(q^*)}^2}{\mu_1}. \quad (28) \]

From (20) we see that

\[ P_1 - \frac{\alpha^2 w}{4d} \frac{F(q^*)}{\mu_{\xi(q^*)}} = \frac{2}{\mu_1} \mu_{\xi(q^*)}. \]

Therefore, we can rewrite (28) as follows:

\[ A = \left( \frac{w - \mu_1\gamma p_1}{\mu_1 - \mu_1} \right) \frac{2}{\mu_{\xi(q^*)}} - \frac{\mu_{\xi(q^*)}^2}{\mu_1}. \quad (29) \]

It follows from (29) that if \( CV_{\xi(q^*)} < \frac{\sqrt{2F(q^*)}}{\Gamma(q^*)} \), then \( A > 0 \), otherwise, \( A < 0 \).

Recall that the condition \( x(\xi) = \mu \) means that demands in T_1 and T_2 are independent and the condition \( \xi(q^*) = 0 \) means that the retailer’s leftover inventory from T_1 is small enough, i.e., at least less than the unconstrained optimal clearance sales quantity \( \delta_0 \), so that inventory hold-backs will not occur. If both conditions hold, then Proposition 8 identifies a necessary and sufficient condition under which the coordinating PMM contract will result in a higher (lower) manufacturer (retailer) profit than the coordinating QMM contract when the wholesale price is fixed.

Since the analytical result on the relative profits associated with PMM and QMM contracts in Proposition 8 applies when the regular season and clearance period demands are independent (i.e., \( x(\xi) = \mu \)) it would be interesting to compare and contrast results across environments with independent and correlated demands. Accordingly, we conduct a numerical study based upon the following combinations of parameters with a total of 168 scenarios:

- Product profit margin: \( m_1 = (p_1 - c)/p_1 \in \{0.25, 0.50\} \)
- Size of the clearance market: \( a = \{5, 50\} \)
- Maximal clearance price: \( p_{\text{max}} = a/b = \{0.2p_1, 0.6p_1, p_1\} \)
- Random shock function: \( x(\xi) = \{\mu, \mu_1\} \)
- Regular season demand distribution: \( D_1 \sim \{\text{Uniform, Normal, Gamma}\} \)

In each scenario, we fix the production cost \( c = $10 \) and the mean regular season demand \( \mu = 50 \).

For the normal distribution, we select a coefficient of variation \( CV = \{0.1, 0.2, 0.3\} \). For the uniform distribution, we choose \( D_1 \sim \{0.1, 0.2, 0.3\} \) with \( CV \approx 0.577 \). For the gamma distribution, we select a \( CV = \{0.25, 0.71, 1\} \). For the case of \( CV = 1 \), we set the \( \alpha \) parameter of the gamma distribution at \( \alpha = 1 \), which reduces to the exponential distribution. From Proposition 3 we know the PMM contract can always coordinate the supply chain. Our numerical results also show that the retailer’s expected profit function under the channel coordinating QMM contract \((w, m_w)\) is unimodal and the retailer’s optimal order quantity \( q^* \) satisfies \( dI(q^*)/dq^* = 0 \), i.e., \( q^* = 0 \). Therefore, the QMM contract can also coordinate the supply chain in our numerical study. We compute the percentage change in the manufacturer’s expected profit under the coordinating PMM contract relative to that under the coordinating QMM contract, i.e.,

\[ \theta = \frac{P_m(q^*, w, \gamma) - P_m(q^*, w, m_w)}{P_m(q^*, w, m_w)} \times 100\%. \quad (30) \]

After plugging (25) and (26) into (30), and noting that the coordinating PMM and QMM contracts parameters satisfy \( m_w = (w - c)/F(q^*) \) and \( y_w = (w - c)/(p_1 - c) \), we rewrite (30) as follows:

\[ \theta = \frac{\mu^2 \gamma w p_1}{\mu_{\xi(q^*)}^2} - \frac{b p_1 - c - \left( p_1 - \frac{\alpha^2 w}{4d} \right) F(q^*)}{\mu_{\xi(q^*)}^2} \times 100\%. \quad (31) \]

Interestingly, from (31) we see that \( \theta \) is independent of the wholesale price \( w \). Our numerical results are reported in Tables 1–3 and rounded up to two decimals.

From Tables 1–3, we find that the coordinating PMM contract results in higher manufacturer expected profits than the coordinating QMM contract for all combinations of the parameter values. This result suggests that for some commonly used distributions such as uniform, normal, gamma, and exponential, the manufacturer will prefer a coordinating PMM contract to a coordinating QMM contract when demands in two periods are independent and correlated.

Our numerical results in Tables 1–3 further show that the manufacturer’s expected profit is significantly higher under the PMM contract when (1) the clearance market size \( a \) is relatively large and the price sensitivity of the clearance demand \( b \) is relatively small, or equivalently, when the maximal clearance price \( p_{\text{max}} = a/b \) is relatively large, and (2) the product profit margin \( m_1 \) is relatively high. For example, when \( a = 50 \), \( p_{\text{max}} = 0.6p_1 \), and \( m_1 = 0.50 \), the manufacturer’s expected profits under the coordinating PMM contract are about 200–600% higher than that under the coordinating QMM contract, for all combinations of other parameter values. However, the differences in the manufacturer’s
expected profits under the coordinating PMM and QMM contracts are insignificant when (1) the clearance market size \( a \) is relatively small and the price sensitivity of the clearance demand \( b \) is relatively large, or equivalently, when the maximal clearance price \( p_{\text{max}} \) is relatively small, and (2) the product profit margin \( m_1 \) is relatively low. For example, when \( a = 5 \), \( p_{\text{max}} = 0.2\), and \( m_1 = 0.25 \), the percentage increases in the manufacturer's expected profits under the coordinating PMM contract relative to that under the coordinating QMM contract are all less than 1% for all combinations of other parameter values.

Such results can be explained by reexamining the manufacturer's expected profit functions under the coordinating PMM and QMM contracts in (25) and (26). Although the manufacturer makes the same profit, i.e., \((w - c)q^0\), in the regular season under both coordinating contracts, the manufacturer's profits under the two contracts become different after the regular season. More specifically, under the PMM contract, from (25) we see that the manufacturer only pays the retailer some unsold unit in the regular season, but also shares the markdown money from (26) we see that the manufacturer only pays the retailer some markdown money \( m_w \) for each unsold unit in the regular season, but does not share any of the retailer's clearance revenue. In other words, the PMM contract allows the manufacturer to continue to make some money in the clearance market whereas the QMM contract does not. When the maximal clearance price is relatively low (e.g., \( p_{\text{max}} = 0.2\)), the difference in the manufacturer expected profits under two contracts is small since the clearance market is less profitable. However, when the maximal clearance price is relatively high, e.g., \( p_{\text{max}} = p_1 \), the difference in the manufacturer expected profits under two contracts is large since the clearance market is more profitable. Similarly, as the product profit margin \( m_1 \) becomes higher, the optimal order quantity \( q^0 \) also becomes larger. This means that there will be more expected unsold inventory
for clearance sales, which in turn results in a larger clearance revenue and a larger difference between PMM and QMM contract manufacturer profits.

5. Conclusion

The volatile market of perishable products is featured by uncertain demand and a short selling season. It is common in practice that retailers liquidate excess inventory via clearance pricing. In this paper we investigate two forms of markdown money contract schemes, i.e., PMM and QMM, for supply chain coordination. We find that the PMM contract can always coordinate the supply chain and arbitrarily divide the supply chain profit between the manufacturer and the retailer, but the QMM contract may not be able to coordinate the supply chain.

Our results provide some managerial implications on the strengths and limitations of the two forms of markdown money contract schemes for coordinating a supply chain with clearance pricing. First, the QMM contract is more restrictive than the PMM due to the fact that (1) if the retailer’s optimal order quantity is not unique, then the QMM contract fails to coordinate the supply chain, and (2) even if a QMM contract can coordinate the supply chain, its parameters depend upon the demand distribution. As pointed out in Cachon (2003), a manufacturer normally sells her product not just to one, but to several retailers and is legally obligated to offer the same contractual terms to their retailers. Hence, it is desirable for the manufacturer to offer a uniform contract to all of her retailers, especially if they only differ in demand. Compared to the QMM contract, the PMM contract can always coordinate the supply chain and its parameters are independent of the demand distribution. Therefore, the manufacturer can offer a uniform PMM contract to multiple retailers.

Second, a PMM coordinating contract has the appealing feature of transparency in allocation of expected supply chain profit. A PMM contract is comprised of two parameters—the wholesale price \( w \) and the percentage of the price markdown \( c \) paid by the manufacturer to the retailer—and the value of \( c \) is the manufacturer’s share of expected supply chain profit \((1 – \gamma)\) paid by the retailer’s share of expected supply chain profit. In practice, retailers and manufacturers usually have different bargaining powers when they negotiate contracts. For example, the retailers in the fashion industry, e.g., May, Federated, Kohl’s, Saks, and J.C. Penney, usually have more bargaining power than the manufacturer, and naturally they would like to have a larger share of the total supply chain profit by selecting a \( \gamma < 50\% \) (and \( 1 – \gamma > 50\% \)) in the coordinating PMM contract. Furthermore, as related to the point above, supply chain profit allocation under a coordinating PMM contract is independent of the demand distribution, which is not the case under a QMM coordinating contract. Profit allocation transparency and insensitivity to the demand distribution can be useful during contract presentation and negotiation.

Third, in practice, it can sometimes be difficult for the manufacturer to change the wholesale price. Our numerical results based upon a linear clearance demand function suggest that the coordinating PMM contract will generally result in a higher manufacturer’s expected profit than the coordinating QMM contract. Therefore, a manufacturer who is inflexible in changing wholesale price should use a PMM contract to coordinate the supply chain and improve her expected profit instead of a QMM contract, especially when the clearance market is highly profitable and the product profit margin is high.

Future research should consider a supply chain comprised of a manufacturer selling to multiple retailers competing in both regular season and clearance period. The relative strengths and limitations of the three forms of supply chain contracts could be tested empirically by experiments or by surveys of managers so that more managerial insights can be obtained into which contract form is preferred to others and why.

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