Decentralized Downside Risk Management*

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Abstract

The process of risk management for institutional investors faces two challenges. First, since most institutions are decentralized as opposed to being direct investors in assets, it is difficult to separate the risks of the assets in the portfolio from the risks generated by the investment decisions by the fund management to construct the portfolio. To address this issue, we propose a risk measurement methodology which calculates the risk contributions of individual securities and investment decisions simultaneously. This decomposition is applicable to any decentralized investor as long as its relevant risk measurement statistic can be additively decomposed. Second, statistics used to measure risk may not coincide with institution-specific investment risks, in the sense that the utility employed in asset allocation may be unrelated to the risk measure utilized. For example, an institution may do mean-variance asset allocation, but inconsistently measure the risk of the portfolio using Value at Risk. We apply this methodology to a particular type of decentralized investor, specifically, endowment funds where the relevant risk statistic is the downside risk of returns relative to actual payout levels, plus inflation. We show how downside risk can be decomposed and apply our simultaneous downside risk decomposition empirically on a sample of U.S. endowment funds. We find that an endowment's asset allocation to U.S. Equity, consistent with having the largest weight in the average endowment portfolio, generates about 50% of total endowment returns but almost 100% of total portfolio downside risk. We further find that tactical allocations (or timing) have economically small contributions to both returns and risk. Finally, we find that the allocations to U.S. Fixed Income and to Hedge Funds as well as active investment decisions (except for tactical) contribute positively to returns, while reducing portfolio downside risk. The risk contributions are sensitive to changes in payout levels and an increase in the latter may offset the risk reducing power of active investing.

Keywords: Risk Management, Asset Allocation, Downside Risk, Endowment Funds

1 Introduction

In the context of recent financial meltdowns, the need for institutions to bolster their risk management capabilities has never been greater. Recently, the Federal Reserve Chairman declared that "improvements in banks' risk management will provide a more stable financial system by making firms more resilient to shocks".[1] Recently, however, we learned that many financial institutions were considerably riskier than previously thought and witnessed in the wake of catastrophic market conditions their devastating and costly collapse. Whereas market risk cannot be controlled, the *decision* to take on this risk can be. Thus, the following question arises: has the overall risk of these institutions increased because the markets in which they were invested became riskier, or because the institutions *knowingly* chose to invest in these markets? As the latter risks can be managed, answering this question means identifying at which point in the investment process of a decentralized, complex organization, decisions were taken that led to significant changes in the overall risk of the institution. This paper's aim is to propose a methodology that provides such an answer. Since our methodology is specifically designed for decentralized, or "top-down" investors, it is applicable to many types of institutions such as mutual and hedge funds, funds of funds, pension funds, endowments and foundations.

In decentralized organizations, the portfolio of the institution is a sum of not only the individual securities but also of the investment decisions made by fund management to construct the portfolio. We often find that in stratified investment structures, the interests of the investment staff are not aligned. There may be discrepancies between the utility function of the Chief Investment Officer (CIO) and the utility functions of investment staff of the fund, who are responsible for the implementation of the investment decisions of the CIO. The result is that the management structure itself adds to the overall risk of a portfolio. For example, a U.S. Equity manager may decide to invest in a 130-30 fund in order to enhance her adjusted returns. This manager may be what Leibowitz (2005) calls a "beta grazer": by making the decision to invest in a hedge fund-like product, the manager adds an active component to her portfolio that will be correlated with an index representative of hedge funds. This investment decision may thus *reduce* the diversification effect between hedge

[1] From Ben Bernanke's keynote speech at the 44th Annual Conference on Bank Structure and Competition in Chicago, May 14-16, 2008.

funds and U.S. Equity, and shift the overall risk of the portfolio away from what the CIO may consider optimal.[2] Although the U.S. Equity manager may find the 130-30 investment appropriate for her portfolio, the CIO's utility function may dictate a higher level of portfolio risk and a lower overall exposure to hedge funds. In order to capture such potentially risky behavior, the risk management methodology proposed in this study *simultaneously* addresses the risks contributed by investment decisions as well as by individual securities. We refer to this risk measurement methodology as *simultaneous risk decomposition*.

Utility discrepancies do not stem only from the multilayered nature of investment management, but also from the way risk is measured. In particular, the risk statistics traditionally used to measure risk may not relate to the utility function of the CIO. For example, the main cause of concern for an investor facing a liability – and the most significant source of risk – is that returns will fall short of the fund's required payout. In contrast, risk is traditionally measured as Value at Risk, standard deviation, etc., and these risk measures do not explicitly incorporate an investor's payout obligation in their calculation. In this paper we develop a risk measurement methodology which suits an investor facing a liability. We consider that the relevant risk statistic in this case is the downside risk of the portfolio relative to a minimal acceptable return[3]. There are several reasons that we focus on this particular risk statistic. First, it is a measure of risk appropriate for many types of investors: pension funds facing a liability, university endowments required to meet a payout, hedge funds which have a watermark, mutual funds attempting to outperform a benchmark, or simply funds seeking to preserve their value. Second, asymmetry in the returns of asset classes such as hedge funds require appropriate risk measures. Third, several large university endowments and pension plans now consider downside risk as an alternative to standard deviation in their asset allocation process, which traditionally was a mean-variance framework. To avoid a discrepancy between their utility function (which is of mean-downside risk type), these institutions also need to use downside deviation as a risk measure. However, risk decomposition methodologies are nonexistent for downside risk. For these reasons we propose a risk management methodology based on downside risk.

[2]In our empirical analysis we document that adding *more* hedge funds to the portfolio in fact *reduces* the total downside risk.

[3]This risk statistic is also referred to as *target semi-deviation*.

To illustrate empirically, we apply our simultaneous risk decomposition methodology to portfolios of actual U.S. endowment funds from the 2005 National Association of College and University Business Officers (NACUBO). Consistent with our understanding that most endowments are traditional investors, holding about 43% of their portfolio in U.S. Equity, we find that the long term allocation to this asset class generates most of the returns and downside risk. Our findings further indicate that the allocation to U.S. Equity generates only about 45% of a typical endowment's portfolio returns, but 94% of the portfolio downside risk. It is the long term allocation to U.S. Fixed Income and to alternative asset classes such as Hedge Funds that reduce the downside risk of the portfolio, while at the same time contributing positively to the overall return.

The risk reduction effect of Hedge Funds is a surprising finding of our analysis. An endowment fund's average allocation to hedge funds is 11.23%, about half of that to U.S. Fixed Income (which is 19.95%). Each asset class contributes to returns in equal proportions (about 15%) and reduces risk (by as much as 8% in the case of Hedge Funds). This finding debunks the somewhat traditional belief that hedge funds are high-risk assets. In a similar vein, we find that the active component of an endowment's portfolio *reduces* overall downside risk. This evidence is suggestive of good security selection skills of university endowment managers.

Furthermore, we study the sensitivity of our risk decomposition to the payout level. We find that varying the payout level has the potential of changing not only the magnitude of the risk contributions of investment decisions and portfolio holdings, but also our conclusions about whether those portfolio components add to overall risk or reduce it. For example, holding cash *reduces* the overall downside risk of the portfolio if the payout level is low. However, for higher payout levels, holding cash increases the difficulty of achieving the required payout level. Thus, cash *adds* to the overall risk of the portfolio. This example illustrates how risk management can give different answers when different risk statistics are applied, and highlights the importance of customizing the ways in which investors measure risk. Another important finding from our our analysis is that the risk reducing power of active management disappears as payout levels increase. Typically endowment payouts are a fixed proportion of the moving average of the fund value. Therefore, as returns fall payout ratios for these institutions will rise and consequently active investing, which does not contribute significantly to performance, will no longer reduce portfolio risk.

Our paper contributes to literature in several ways. First, by providing a risk management methodology for *decentralized* investors, it complements studies such as van Binsbergen et al. (2008) that address portfolio optimization rules for such investors. By proposing a method to simultaneously evaluate the risk contributed by investment decisions and individual securities, we extend the literature that deals with risk decomposition across securities and separately across investment decisions.[4] Second, we complement the literature dedicated to mean-downside risk portfolio construction and equilibrium asset pricing, such as Hogan and Warren (1974), Ang et al. (2006) or Morton et al. (2006), by proposing a decomposition method for downside risk. This extends the well known decomposition methodologies for homogenous risk statistics such as Value at Risk, standard deviation and expected shortfall to downside deviation from a fixed return. By providing an intuitive interpretation to this formula we add to the works of Gouriéroux et al. (2000), who provides an intuitive understanding of the decomposition of Value at Risk and of Scaillet (2002), who offers a similar approach for expected shortfall. By arguing for downside risk as a risk statistics we respond to arguments such as those of Fung and Hsieh (2002) who point out potential problems in measuring the risk of hedge funds portfolios as standard deviation. Finally, having proposed a risk management methodology that, in our view, is suitable to endowment funds, we apply it to actual data for U.S. endowments. We thus contribute to the literature on endowment funds, such as Lerner et al. (2007), Lerner et al. (2008), Brown et al. (2008) and Dimmock (2008). However, unlike these papers which analyze issues related to the *performance* of university endowments, our paper is focused on the analysis of the sources of *risk* in endowment portfolios.

The remainder of the paper is organized as follows. Section 2 outlines the risk methodology we propose, consisting of the simultaneous decomposition of risk across investment decisions as well as individual securities. Section 3 presents the mathematics of the downside risk decomposition. Section 4 applies this methodology to endowments from the NACUBO database. Section 5 concludes.

[4] See Pearson (2002) for a review.

2 Decentralized Portfolio Decomposition

In an attempt to generate additional performance, either by market timing or by security selection, the CIO of an investment company such as a pension fund, university endowment, hedge fund or mutual fund would typically delegate the responsibility to invest in various asset classes to managers who specialize in those respective asset classes. Consequently, there are multiple steps by which the portfolio of the institution is constructed, i.e., the institution is decentralized as described in van Binsbergen et al. (2008). In such funds, a Board together with senior fund management allocate the portfolio to a variety of asset classes on an infrequent basis. After agreeing upon these broad investment directions with the Board, senior management of the endowment has the ability to adjust the actual portfolio away from the broad, long term allocations. This reallocation may be the result of an active decision to time the markets or a result of specific market conditions which may make it difficult to rebalance the weights of less liquid asset classes. Asset class managers who report to the CIO then implement these decisions within their respective asset class. These managers either select securities directly or invest through external or internal managers. In the latter case, the asset class managers specify the benchmarks against which the performance of the external managers in their portfolio is judged. These benchmarks may not be identical to the indices that are representative of the broad asset class in which they are invested. Thus several strata of decentralization emerge. First, the CIO may allocate the portfolio in a manner that does not maximize the utility of the Board. Second, the asset class managers may choose to invest differently from their asset class benchmarks, either through external managers or though direct holdings. Finally, the external managers may depart from the benchmarks against which they are evaluated. These levels of decentralization have the potential to reduce the utility function of the Board and it is important to identify which strata, and/or what securities increase or decrease the risk of the portfolio relative to the case where the institution is centrally managed.

In the following section we model the typical "top-down" portfolio structure that characterizes decentralized investors and decompose the portfolio relative to both decision strata and individual securities simultaneously.

2.1 The general case

A portfolio is usually thought of as a combination of securities. As explained above, we expand this definition by adding that a portfolio is also a combination of investment decisions that determine the weights of the securities.[5] In a "top-down" organization, these weights are the result of a stratified investment decision process. Thus, we start the decomposition by recognizing that the portfolio is a sum of the returns of the portfolios generated by each investment management stratum. Thus, for a portfolio with returns P we write:

$$P = D^1 + D^2 + ... + D^K, \tag{1}$$

where $D^1, ..., D^K$ are portfolio returns generated by the investment decisions $1, ..., K$. We call this the *investment decision decomposition* of the portfolio because it reflects the stratified nature of the decentralized investment process. Specific to the investment decisions decomposition of the portfolio is that each portfolio with returns $D^1, ..., D^K$ is obtained by maximizing a utility function specific to the investment decision making strata $1, ..., K$. For example, the portfolio D^1 maximizes the utility of the most senior investment decision maker, D^2 is the correction to D^1 made by the subsequent decision maker so that $D^1 + D^2$ maximizes the more junior decision maker's utility function, and so on.

Denoting the return of an individual security i from portfolio D^k by $S_{k,i}$, the returns of each investment decision making strata D can be further disaggregated as

$$D^k = w_1^k S_{k,1} + ... + w_{n_k}^k S_{k,n_k}, \; k = 1, \; ..., \; K. \tag{2}$$

We refer to this as to the *asset decomposition* of our portfolio returns. Here, "asset" refers to the asset weights $w_i^k, \; i = 1, \; ..., \; n_k$ that are obtained at the *same* investment management level within the organization.

It is important to note the fundamental difference between the investment decision and the asset decompositions. While the former decomposes the portfolio along the structure of the organization, which needs not be optimal, is usually given and in most cases cannot be modified, the latter decomposes portfolios constructed by maximizing the utility functions of

[5] Here, we understand securities as a subset of all the available securities considered by the management of the fund in question.

the investment decision maker at each investment stratum. At any given level of the investment management structure, these portfolios may be changed by the investment manager responsible for their construction. The act of modifying these portfolios in order to satisfy various risk considerations represents active risk management.

Summing the returns from the investment decision strata (the investment decision decomposition in different columns) with the returns of the portfolios within the strata (the asset decomposition in different rows), the return of the portfolio can be written as

$$P = \begin{array}{c} w_1^1 S_{1,1} \\ + \\ \cdot \\ \cdot \\ \cdot \\ + \\ w_{n_1}^1 S_{1,n_1} \end{array} + \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} + \begin{array}{c} w_1^K S_{K,1} \\ + \\ \cdot \\ \cdot \\ \cdot \\ + \\ w_{n_K}^K S_{K,n_K} \end{array} \tag{3}$$

This type of return decomposition has precedent. For example, similar to our investment decision decomposition, Daniel et al. (1997) decompose the returns of a portfolio according to whether they were generated by passive, long term investing, market timing or security selection. Brinson et al.(1986) and Blake et al. (1999) propose a similar decomposition for pension plans. As it is the case with our asset decomposition, other authors (see Pearson (2002)) decompose portfolio risk into the risk contributions of individual securities. Our approach is novel, however, as it performs the two distinct decompositions (investment decision and asset) *simultaneously.* As a convention, in tables throughout this paper we shall present the asset decomposition across rows while presenting the investment decision decomposition down columns.

2.2 An example from endowment funds

We continue by particularizing this decomposition for the case of endowment funds, which are a perfect example of decentralized investors. The structure of such a fund is outlined graphically in Figure 1. In the discussion which follows we describe the decision process by which an endowment fund arrives at its portfolio.

1. The Asset Allocation decision The asset allocation decision is the process by which an endowment selects the long term asset class mix that best suits its long term objectives. The responsibility of this decision belongs to the Board of the endowment. The result of this process is a set of weights which, combined with the proper asset class benchmarks, forms the endowment's *policy portfolio.* If the returns of the relevant asset class benchmarks are respectively r_1^P, ..., r_N^P and the fund invests in the proportions denoted by w_1^P, ..., w_N^P in each of these asset classes, then the returns of the *policy portfolio* are:

$$R^P = \begin{array}{c} w_1^P r_1^P \\ + \\ ... \\ + \\ w_N^P r_N^P \end{array} . \tag{4}$$

For example, we assume that the fund XYZ, with \$100 million in assets, invests in two main asset classes, U.S. Equity and Inflation Hedge (summary statistics of these asset classes' returns are presented in Table 2). The performance benchmark for U.S. Equity is the Russell 3000 Index, and for Inflation Hedge, the Merrill Lynch Inflation Linked Notes Index. In this first step of the investment management process, that of asset allocation, the Board of the XYZ fund agrees on a policy portfolio B characterized by the weights $w_{useq}^P = 50\%$ and $w_{inflink}^P = 50\%$.

2. The Tactical Asset Allocation decision The CIO of an endowment fund may elect to diverge from the long term allocation targets set by the Board in an attempt to "market time", or as a result of specific market conditions which may make it difficult to rebalance the weights of less liquid asset classes. That is, instead of investing w_i^P in asset class i, the endowment CIO may choose to allocate w_i to the respective asset class, thus changing the policy weight w_i^P by $w_i^T := w_i - w_i^P$. The contribution of the *tactical* component to the endowment's returns is:

$$R^T = \begin{array}{c} w_1^T r_1^P \\ + \\ \dots \\ + \\ w_N^T r_N^P \end{array}. \tag{5}$$

This expression captures the return that is achieved by over- or underweighting the policy portfolio weights in an effort to increase the returns or change the risk profile of the fund.

To exemplify, we assume that the CIO of the XYZ fund considers the long term assumptions shown in Table 2 unrealistic, as she expects equities to decline in the short term. Consistent with her investment viewpoint, the CIO makes the tactical decision to invest $w_{useq} = 40\%$ and $w_{inflink} = 60\%$. Each asset class is managed internally by an asset class manager. After the CIO determines the tactical asset allocation, the U.S. Equity manager receives \$40 million to invest, while the Inflation Hedge manager receives \$60 million.

3. Benchmark selection decision After the Board determines the weights w to various asset classes, for each asset class i the implementation of the actual weights w_i falls to the investment manager of the respective asset class. The asset class manager may choose to invest in either external or internal managers, or directly in individual securities. These managers, or securities, with returns denoted by r_j^i, $j = 1, \dots, M_i$, are evaluated in turn against more specific benchmarks, with returns denoted by $r_j^{B,i}$. The investment decisions made by the asset class managers contribute to the overall returns of the fund by steering the fund's allocation from the passive indices r_i^P that are representative of each asset class toward the more specific (and thus different) indices $r_j^{B,i}$. Assuming that in each asset class i, the external/internal manager or individual security j, whose performance is evaluated against the benchmark $r_j^{B,i}$, receives the weight $w_{i,j}$, the contribution R^B of the choice of internal benchmarks to the overall portfolio return is given by:

$$R^B = \begin{array}{c} \left(\sum_{j=1}^{M_1} w_{1,j} r_j^{B,1} - w_1 r_1^P\right) \\ + \\ \ldots \\ + \\ \left(\sum_{j=1}^{M_N} w_{N,j} r_j^{B,N} - w_N r_N^P\right) \end{array}, \quad (6)$$

where for each asset class i we have that $\sum_{j=1}^{M_i} w_{i,j} = w_i$. That is, the individual weights of the securities chosen in asset class i sum to the total weight assigned to the asset class in the portfolio.

For our example of the fund XYZ, we assume that instead of investing in the Russell 3000 Index (which is combination of approximatively 92% of the Russell 1000 Index and 8% of the Russell 2000 Index), the U.S. Equity manager makes the active decision to invest $20 million in a large cap external manager and $20 in a small cap manager. The U.S. Equity manager prefers to evaluate the large cap manager against the S&P 500 Index, and the small cap manager against the Russell 2000 Index. In the investment guidelines, the U.S. Equity manager is allowed some flexibility on how closely each external manager follows its prescribed benchmark.

Similarly, the Inflation Hedge manager makes an active investment decision by choosing to invest in a combination of TIPS and commodities rather than investing 100% of his $60 million allocation solely in TIPS. He benchmarks the TIPS investment against the policy benchmark for the Inflation Hedge asset class, which is the Merill Lynch Inflation Linked Notes Index and the commodities manager against the Goldman Sachs Commodities Index. A summary of the fund's portfolio is presented in Table 1.

4. The Active (Security Selection) decision After each internal or external manager security is assigned a benchmark the responsible investment manager (whether internal or external) makes the actual investment. This investment will generate returns that differ from its benchmark. The same is true for investments in individual securities, whose returns may also be different from those of their corresponding benchmarks. This differential represents the active security selection return component in an endowment's portfolio. If the actual returns of the security j of asset class i with a portfolio weight of $w_{i,j}$ are $r_{i,j}$, then the *active* or security selection contribution to the fund's returns is:

$$R^{A} = \sum_{i=1}^{N} \sum_{j=1}^{M_i} w_{i,j}(r_{i,j} - r_j^{B,i}). \tag{7}$$

In points 1-4 we have described the decision making components D for a top-down, stratified investment process as is the case for the typical endowment fund. We refer to the decomposition of each of these components as the asset decomposition of a portfolio. The set of decision making components forming the investment decision decomposition of the fund's portfolio is represented simply by:

$$R = R^{P} + \ R^{T} + \ R^{B} + \ R^{A}. \tag{8}$$

In this case formula (3) specializes to the following:

$$R = \begin{array}{c} \text{Asset} \\ \text{Alloc.} \\ \hline \\ w_1^P r_1^P \\ + \\ \ldots \\ + \\ w_N^P r_N^P \end{array} + \begin{array}{c} \text{Tactical} \\ \text{Alloc.} \\ \hline \\ w_1^T r_1^P \\ + \\ \ldots \\ + \\ w_N^T r_N^P \end{array} + \begin{array}{c} \text{Benchmark} \\ \text{Selection} \\ \hline \\ \left(\sum_{j=1}^{M_1} w_{1,j} r_j^{B,1} - w_1 r_1^P\right) \\ + \\ \ldots \\ + \\ \left(\sum_{j=1}^{M_N} w_{N,j} r_j^{B,N} - w_N r_N^P\right) \end{array} + \begin{array}{c} \text{Active} \\ \text{Alloc.} \\ \hline \\ w_{1,1}(r_{1,1} - r_1^{B,1}) \\ + \\ \ldots \\ w_{1,M_1}(r_{1,M_1} - r_{M_1}^{B,1}) \\ + \\ \ldots \\ \ldots \\ + \\ w_{N,1}(r_{N,1} - r_1^{B,N}) \\ + \\ \ldots \\ w_{N,M_N}(r_{N,M_N} - r_{M_N}^{B,N}) \end{array} \tag{9}$$

The advantage of simultaneously analyzing the investment decision as well as the asset decomposition is apparent when we measure the risk of the entire portfolio. Individual security contributions can, in fact, be correlated with the returns contributions of investment decisions. Such relationships cannot be captured unless we evaluate the risk of the portfolio whollistically.

In our example, in order to establish the effect of each of these investment decisions on the XYZ portfolio, we use historical data for the policy indices (Russell 3000 index for U.S. Equity and the Merrill Lunch Inflation Linked Index for the Inflation Hedge), as well as for the external manager benchmarks (Merill Lynch Inflation Linked Notes Index for TIPS, the Goldman Sachs Commodities Index for commodities, the S&P 500 Index for the large cap U.S. Equity and the Russell 2000 Index for small cap U.S. Equity). For the external managers (who create the active component of the portfolio), we make the assumption that their added value (relative to the benchmarks they are assigned) are independent of each other and of the benchmarks, and normally distributed. The capital market assumptions applied to the assets of XYZ are presented in Table 2. The return contributions of each component to the overall expected return of the fund, 11.78%, are presented in Panel A of Table 3.

From Panel A of Table 3 we see that the largest contribution to the portfolio's return comes from the tactical decision to overweight the allocation to the Inflation Hedge asset class. At the same time the worst contributor to performance is the tactical decision to underweight U.S. Equity. Overall, however, the tactical allocation makes a positive contribution to returns (equal to -0.84% + 5.12%).

This is in contrast with what we would obtain if we decompose the portfolio solely across the asset classes in which the fund is invested, namely, U.S. Equity and Inflation Hedge. In the policy portfolio of the fund, the main return generator is the U.S. Equity asset class: it generates 4.19% of the returns, while only 2.56% come from Inflation Hedge. However, if we were to decompose the portfolio solely across asset classes and not consider investment decisions, then the main return generator would be the Inflation Hedge asset class: it generates 7.90% of the returns, as compared to only 3.88% generated by the U.S. Equity.

Having decomposed the returns, we now turn to the decomposition of risk. We proceed with the general risk decomposition formula and illustrate numerically on the fund XYZ.

3 Downside Risk Decomposition

In this study we define risk as semi-deviation from a target and refer to it as downside risk for brevity. The concept of downside risk is not novel in finance. Markowitz (1959, 1991) argues that it is natural for an investor to prefer a low risk to the downside, instead of a low variance (upside and downside) of portfolio returns, but he also recognizes the computational challenges associated with the use of downside risk.[6] One may question the usefulness of downside risk when returns are symmetrically distributed (e.g., when the returns are normally distributed) or when there is no payout obligation. In those specific cases, minimizing downside risk from the expected return and minimizing the standard deviation are equivalent. Moreover, by the law of large numbers, well diversified portfolios have return distributions approximating normal and the normal distribution in particular is symmetric.[7] The assumption of normality becomes problematic, however, when used to assess the risk of asset classes such as hedge funds which may not have symmetric distributions (Brooks and Kat (2002)). For hedge funds, using standard deviation as a measure of risk may also yield results that differ from the case of traditional assets such as U.S. Equity (Fung and Hsieh (2002)). More importantly, in bad times correlations among assets increase (Ang and Chen (2002)), which violates one of the key assumptions of the central limit theorem (that of independence). It is conceivable, therefore, that the overall portfolio returns of even well diversified portfolios are asymmetric, thus measuring the downside risk of endowment returns may offer special insight. Use of downside risk as a risk criteria – in place of variance – has also been suggested by Hogan and Warren (1974) and by Bawa and Lindenberg (1977), who develop a downside risk-based CAPM model. Van Harlow (1991) studies portfolio optimization problems involving the minimization of lower partial moments of returns. More recently, Ang et al. (2006) bring evidence indicating the existence of a downside risk premium that differs from the market premium.

We now turn to describing the methodology to decompose downside risk.

[6]For example, at page 77 Markowitz (1991) states that "One of the measures considered, the semi-deviation, produces efficient portfolios somewhat preferable to those of the standard deviation".

[7]In the case when the downside risk is calculated not relative to expected returns but to fixed number, minimizing downside risk and expected returns are not equivalent.

3.1 The mathematics of the downside risk decomposition

In this section we illustrate the methodology for measuring the contribution of portfolio components to overall downside risk and how to interpret this decomposition. The downside risk is calculated relative to a *minimal acceptable return level* (MAR).

As Sharpe (2002) notes, risk statistics are seldom additive; he cites the example of variance, which is additive if and only if various components of the portfolio are independent. However, if a measure of risk is homogenous as a function of the portfolio weights,[8] then the risk of the portfolio can be decomposed using the marginal risks of the individual components. Such decompositions hold, for example, for standard deviation or for Value at Risk[9]. We proceed by developing a similar decomposition for downside risk. For a portfolio with returns R, the downside risk from the minimal accepted return MAR is defined as

$$DR(R, MAR) = \sqrt{\mathbb{E}\left[\max(MAR - R, 0)^2\right]}. \tag{10}$$

In order to decompose downside risk, we assume that the portfolio return R is a combination $R = w_1 R^1 + ... + w_N R^N$. Similar to the case of standard deviation or Value at Risk, it is shown in the Appendix that

$$DR\left((w_1 R^1 + ... + w_N R^N), MAR\right) = \sum_{i=1}^{N} w_i \frac{\partial DR}{\partial w_i} + MAR \frac{\partial DR}{\partial MAR}. \tag{11}$$

The portfolio components in this decomposition of downside risk, however, are not additive because of the term containing the marginal risk with respect to the minimal accepted return MAR. In order to obtain an additive decomposition, we denote by $\mathcal{I} = \{t \mid R < MAR\}$ the set of states where portfolio returns are smaller than the minimal acceptable return. For a random variable X, we denote by $\mathbb{E}_{\mathcal{I}}[X] := \mathbb{E}[X \mathbf{1}_{\mathcal{I}}]$, where $\mathbf{1}_{\mathcal{I}}$ is the characteristic function of the set $\mathcal{I}$, that is,

$$\mathbf{1}_{\mathcal{I}} := \begin{cases} 1 & , \text{ if } R < MAR \\ 0 & , \text{ otherwise.} \end{cases} \tag{12}$$

[8]A function $f : \mathbf{R}^N \to \mathbf{R}$ is *homogenous* if and only if $f(\lambda w_1, ..., \lambda w_N) = \lambda f(w_1, ..., w_N)$.
[9]See Pearson (2002) for an exposition on risk decomposition.

With this notation the following is shown in the Appendix:

Proposition 1 *The downside risk from a pre-specified level of return MAR of a portfolio $R = w_1 R^1 + \ ... \ + w_N R^N$ can be decomposed as:*

$$DR\left((w_1 R^1 + ... + w_N R^N), MAR\right) = \sum_{i=1}^{N} w_i \left(\frac{\partial DR}{\partial w_i} + \frac{MAR \cdot \ \mathbb{E}_{\mathcal{I}}[MAR - R^i]}{DR} \right). \quad (13)$$

We observe that this decomposition is different from the typical Value at Risk or standard deviation decomposition. The contribution to risk of each portfolio component i is not just the weighted marginal downside risk multiplied by the weight. Instead, the marginal downside risk must be adjusted with a term dependent on whether or not the *whole* portfolio earns a return greater than MAR. In a state where the whole portfolio does not earn MAR, but a particular component i returns more than MAR on average, then in the downside risk decomposition that component is given more "weight" than just the marginal risk. The difference accounts for that component's additional "diversifying power" or ability to "beat" MAR.

In order to provide an intuitive interpretation for the decomposition of downside risk, we start by noting that the contribution of the ith component to the downside risk from MAR of the portfolio $R = w_1 R^1 + \ ... \ + w_N R^N$ satisfies the following:[10]

$$w_i \left(\frac{\partial DR}{\partial w_i} + \frac{MAR \cdot \ \mathbb{E}_{\mathcal{I}}[MAR - R^i]}{DR} \right) = \frac{1}{DR} \ \mathbb{E}_{\mathcal{I}} \left[(w_i MAR - w_i R^i) \cdot (MAR - R) \right]. \quad (14)$$

Combining equations (13) with (14), we obtain that:

$$DR^2 = \sum_{i=1}^{N} w_i \mathbb{E}_{\mathcal{I}} \left[(MAR - R^i) \cdot (MAR - R) \right]. \quad (15)$$

This is nothing more than a decomposition of the total downside *variance* of the portfolio from the prespecified MAR. From equations (11)-(15) we observe the following:

[10]This result is proved in the Appendix.

Proposition 2 *The relative contribution of component i to the total downside risk of a portfolio with returns R is equal to the relative contribution of component i to the total downside variance of portfolio, that is:*

$$\frac{w_i\left(\frac{\partial DR}{\partial w_i}+\frac{MAR\cdot\,\mathbb{E}_{\mathcal{I}}[MAR-R^i]}{DR}\right)}{DR}=\frac{w_i\mathbb{E}_{\mathcal{I}}\left[(MAR-R^i)\cdot(MAR-R)\right]}{DR^2}. \tag{16}$$

The righthandside of the above formula is easily interpreted as follows. First, note that the righthandside of equation (16) looks very similar to a covariation between the returns of the entire portfolio and the returns of particular components, where the "covariance" is calculated on the set on which the total portfolio does not meet the minimal acceptable return. To further elaborate on our intuition, we observe that on the set $\mathcal{I}$ we have $MAR-R>0$. The sign of the term inside of the expectation on the righthandside of equation (16) is thus determined by whether the returns R^i of the ith component are in excess of MAR and also by how much MAR exceeds the returns R of the entire portfolio. For example, if on the set $\mathcal{I}$ we have that $Prob\,(\mathcal{I}\bigcap\{MAR-R^i>0\})=1$, then the term inside the expectation on the righthandside of (16) will be positive and results in the ith component having a positive contribution to the overall portfolio downside variance and downside risk. Similarly, if on the set $\mathcal{I}$ the component i has returns in excess of MAR (i.e., $Prob\,(\mathcal{I}\bigcap\{MAR-R^i>0\})=0$) then the term inside of the expectation will be negative and the component i will act as a downside variance reducer of the entire portfolio. Researchers have previously provided similar interpretations of the decomposition formulae for other risk statistics, such as Value at Risk (Gouriéroux et a. (2000)) and expected shortfall (Scaillet (2002)).

3.2 Risk Decomposition for the XYZ Fund

We continue by the decomposing the downside risk for the hypothetical fund XYZ described in Section 2.2. We assume that $MAR=5\%$.

The results of the downside risk decomposition - simultaneously across the investment decisions used to build the portfolio (the *investment decision decomposition*) as well as the individual securities held in the portfolio (the *asset decomposition*) − are presented in Panel B of Table 3. The largest contribution to portfolio returns is the tactical decision to overweight the Inflation Hedge asset class, while the largest contribution to risk comes from the asset allocation decision to invest 50% of the portfolio in U.S. Equity. The active decision to invest in Commodities as an inflation hedging asset has the negative effect of reducing the returns by 0.02% (see Panel A of Table 3) but at the same time increasing the downside risk by 1.6% (see Panel B of Table 3). All the external managers add returns to the portfolio while increasing the risk[11].

It is worthwhile to contrast the results of the simultaneous risk decomposition presented in Panel B with what we had obtained if we considered instead the portfolio decomposition across the two asset classes, U.S. Equity and Inflation Hedge. If the risk decomposition is performed *solely* on these asset classes, each has an equal contribution to total downside risk (2.9% of the total of 5.8%). Taking this result in isolation, in conjunction with the fact that Inflation Hedge is an asset class with lower expected returns, the CIO may be incentivized to shift the asset allocation away from the Inflation Hedge asset class. A complete picture yielding different conclusions is presented by our simultaneous decomposition. Inflation hedge has a small contribution of 0.9% to risk in the asset allocation policy, relative to that of 2.6% of U.S. Equity. The primary reason for the high downside risk generated by the manager of the Inflation Hedging asset class is a benchmark mismatch between the benchmark of the entire asset class - the Merrill Lynch Inflation Linked Securities Index - and the benchmark employed for the sub-asset class of commodities. As a third of her her portfolio is invested in Commodities, which are considerably riskier than TIPS, the manager of the Inflation Hedging asset class generates a risk much higher risk than that of her benchmark index. We would have not been able to arrive at this conclusion using the classical risk decomposition across asset classes.

4 Returns and Risk Decomposition of U.S. Endowments

In this section we apply the risk management methodology developed in the previous section to a sample of university and college endowment funds. Our goal is two-fold: one the one

[11]The fact that active investing *increases* the risk is not automatic nor obvious. We modeled the fund XYZ in such a way. Our empirical results, however, document that active investing by U.S. Endowment funds in fact *decreases* overall portfolio risk.

hand to identify the management decisions and assets that contribute the most to the risk that university endowments do not meet their payout obligations and preserve their capital, and on the other hand, whether our results change if the payout level is stressed. We start by describing the data.

4.1 Data

Our data source is the 2005 Endowment Survey, a publication by the National Association of College and University Business Officers (NACUBO). Although the NACUBO study contains a plethora of information, we only use the data on target asset allocations, current asset allocations and actual annual returns. Similar but more extensive data, merged at the time series level, was used by Brown et al. (2008). Various data from NACUBO are also used by Dimmock (2008) and by Lerner et al. (2008).

In 2005, NACUBO divided the investment universe for university and college endowments into 12 different asset classes: U.S. Equity, Non-U.S. Equity, U.S. Fixed-Income, Non-U.S. Fixed-Income, Public Real Estate, Private Real Estate, Hedge Funds, Venture Capital, Private Equity (Buyout), Natural Resources, Cash, and Other Assets. We will refer to the combination of Venture Capital and Private Equity Buyout as the Private Equity. "Other Assets" includes assets that are difficult to classify into any of the other broad asset classes. For a summary of how the weights to these asset classes change over time, as well as for a summary of endowment performance, we refer the reader to Brown et al. (2008).

As performance (by which in this context we understand raw returns) is available at annual frequency, and the set of choices consists of 12 asset classes, we only include those institutions with at least 12 data points – that is, endowments which continuously reported for the period 1994 to 2005. Also we only include the endowments which report actual as well as policy asset allocations. These filters reduce our sample to 281 funds. The summary statistics of the 2005 target asset allocation weights, actual allocation weights and annual returns of these 281 institutions are presented in Table 4.

In order to perform the return decomposition outlined in Section 2.2, it is necessary to specify passive benchmark indices that are representative of the NACUBO asset classes. In doing so, we were careful to select indices that are commonly used by the industry. Summary statistics on these passive indices are presented in Table 5.

The level of granularity in the available data is such that we are unable to identify any internal investment implementation decisions or how these investments are evaluated. Due to this limitation, we are unable to calculate the Benchmark Selection and the Active components of the portfolio separately for each asset class; however, in the next section we shall show that although these return attributions are unknown individually, their sum across all asset classes can in fact be computed.

4.2 Empirical Endowment Return Decomposition

Having outlined our data we can now describe the decomposition of endowment returns in our sample. In order to obtain the downside risk decomposition for an endowment we use the sample counterparts of the expectation terms in equations (11)-(15).[12]

1. The Asset Allocation component As noted in Section 3.1 actual asset allocation weights are available in the data as well as target asset allocation weights (or equivalently policy weights). By combining these weights with the benchmark returns as formula (4) suggests we can calculate the contribution of each asset class allocation to overall endowment returns.

2. The Tactical Asset Allocation component As we have the actual allocation weights and target weights for each endowment as well as the returns for each asset class representative index, we are able to compute the return contribution of each tactical decision to under- or overweight an asset class as in formula (5).

[12] Assume that our sample consists of the returns $(R_t, R_t^1, \ ..., \ R_t^N)_{t=1:T}$. An unbiased sample estimate for the downside risk from a fixed MAR is given by

$$\widehat{DR} = \sqrt{\frac{\sum_{t=1}^{T} \max\left(MAR - R_t, 0\right)^2}{T}}.$$

This differs from the unbiased sample estimator for standard deviation, in the fact that we divide by T, and not by $(T-1)$ under the square root.

3. The Benchmark selection component Unfortunately, data on how internal managers evaluate their investments are not available, thus it is impossible for us to calculate the effect of the internal decisions that apply to the selection of benchmarks.

4. The Active component As we do not have data on how various internal investment are evaluated we cannot calculate the active contribution to overall portfolio returns.

However, from formula (8) the sum of the Benchmark selection and Active return components (i.e., $R^B + R^A$) is equal with the total return minus asset allocation returns and tactical returns. As both internal benchmarking as well as deviations from these benchmarks are active investment decisions, we shall refer to the sum of these former components as "active" as well. Accordingly, we shall consider the sum of the benchmark selection component R^B and the active component R^A, that is, $R^B + R^A$ as the "active component" of the portfolio.

As total downside risk will differ from one endowment to another, we unitize the return and risk contributions. Precisely, for a portfolio $R = w_1 R^1 + ... + w_N R^N$ we calculate the returns contribution of component i as the expectation of $w_i R^i / R$. Similarly, using equation (15), we calculate the relative contribution of the component i as the expectation of:

$$\frac{w_i \left(\frac{\partial DR}{\partial w_i} + \frac{MAR \cdot \, \mathbb{E}_{\mathcal{I}}[MAR - R^i]}{DR} \right)}{DR}.$$

Thus, both returns and risk contributions will add up to 100%. Returns and risk contributions in the sense used here have been previously used, for example by Sharpe (2002).

4.3 Return and Risk Contributions: Results and Discussion

In this section we apply the results outlined in Section 2.2 and decompose the downside risk of each endowment. The first subsection analyzes the risk and return contributions of various asset classes and investment management decisions of university endowments, using their payout ratios plus inflation as target (or minimal acceptable return) for calculating their downside risk.[13] The second subsection analyzed the sensitivity of risk contributions of

[13] Many university endowments are bound to preserve their capital by their charters. We thus find appropriate to use the level of payout plus inflation as a threshold for downside risk.

various asset classes and management decisions with respect to the choice of the downside risk target.

4.3.1 Downside risk from inflation adjusted payout

In this subsection we use $MAR = payout + \ inflation$ as minimal acceptable return, where *payout* is the proportion of the endowment wealth paid in a given year and $inflation$ is the return on the University of Michigan Consumer Price Index. The results are presented in Table 6.

As Table 4 shows, the dominant average asset class of an endowment portfolio is the U.S. Equity, which has an average target weight of 43.31% and an actual average weight of 42.38% . We would expect that U.S. Equity has the largest contribution to the return of an average endowment, as it generates a similar proportion of 43.64% of the return. What is surprising is its contribution to the overall downside risk of the portfolio: U.S. Equity generates 94% of the total downside risk. Even more surprising, despite generating less than 50% of the total endowment returns, Equity overall (U.S. and international) generates over 120% of the total downside risk (U.S. Equity generates about 94% of the total downside risk and Non-U.S. Equity generates about 33% of the total downside risk). It requires the diversifying effect of other asset classes as well as tactical and active decisions to reduce risk to a total of 100%.

The next largest contributor to returns is the allocation to U.S. Fixed Income, which generates on average 15.96% of the average endowment's return. In contrast to the U.S. Equity asset class, U.S. Fixed Income acts as a diversifier of downside risk, *reducing* the overall risk of the portfolio by 15.11%.

The Hedge Funds asset class is the third largest contributor to overall portfolio returns, generating 14.89%. Similar to U.S. Fixed Income, the presence of Hedge Funds in the asset allocation serves to reduce portfolio risk. In this case Hedge Funds decrease total risk by 8.19%. This is in contrast with the commonly held belief that hedge funds are a high-risk asset class. In our data, an allocation to hedge funds diversifies away downside risk and has a positive contribution to returns, being an asset class that *helps* endowments to meet their payout obligations.

Another surprising finding comes from the Venture Capital asset class. On average, target allocations to this asset class generate 5.87% of the total portfolio returns but contribute 7.49% to total downside risk. Such a small contribution to returns appears surprising given the results of Lerner et al. (2007), who document that the venture capital portfolio of university endowments has performed well when compared to venture capital portfolios of other types of investors. If this is the case, then the natural question to ask is why university endowments have not capitalized more on their ability to select venture capital investments. To reconcile our results with those of Lerner et al. (2007), we observe that the 75th percentile of the contribution of Venture Capital to total portfolio returns is high at 11.39%, indicative of the fact that there are endowments in which Venture Capital makes a large contribution to returns. However, the 75th percentile of risk contribution is also large at 13.68% (contrary to the case of Hedge Funds where the 75th percentile of risk contributions is close to zero). That is, Venture Capital is a significant generator of returns for certain endowments, as well as a major generator of risk. While the raw returns of investments in Venture Capital may be outstanding for a certain set of university endowment portfolios, our results do not support a similar role in portfolio risk reduction.

Tactical decisions do not appear to contribute significantly to either return or overall risk. One interesting result is that the tactical allocation to Venture Capital *reduces* returns (however, it also decreases downside risk). As a possible explanation, we note that the typical endowment fund is under-allocated to Venture Capital (as apparent from Table 4, where the target Venture Capital weight is on average 2.32%, while the actual weight is 1.26% on average). Among the causes of this chronic under-allocation to Venture Capital we cite slow deployment of committed capital, as well as the return of capital already allocated. Unfortunately, data on actual capital commitments of endowment funds - as opposed to target asset allocation - are not available, limiting our ability to investigate any of the potential causes. The under-allocation to Venture Capital, however, exists and it is costly to endowments in terms of returns, while its absence makes the portfolio less risky.

One last result is that the active decisions made by endowments *increase* returns by 7.20% on average, while *reducing* the downside risk by 14.62%. The cause of this finding lies in the fact that the sum of the returns generated by the internal benchmark selection decisions and security selection is negatively correlated with the actual passive portfolios held by endowments. This finding supports the literature such as Cremers and Petajisto (2007) documenting that active investing creates value.

4.3.2 Sensitivity of risk contributions with respect to payout levels

In this section we seek to answer the following question: what are the risk contributions of various asset classes and investment decisions to the overall risk of the average endowment fund if payout obligations change? This question is relevant, especially since universities require smooth payouts from their endowment fund. Typically, as the NACUBO studies state, the payout made by an endowment fund is a fixed average of the value of an endowment over a few years back. In particular, when the value of the endowment declines suddenly, the payout ratio as a fraction of the current value of the endowment increases, while if the endowment experiences a sudden positive return the same ratio decreases. It is therefore natural to analyze risk contributions when the payout levels change.

In order to answer this question, we repeat the analysis performed in Subsection 4.3.1, but instead of using $MAR = \ inflation\ +\ payout$ for each endowment, we employ a set of minimal acceptable returns, $MAR = \ inflation\ +\ payout\ + x$, where x lies in the interval $[-2\%;\ 7\%]$ and describes potential variations in the endowment average payout. By changing x we develop an array of scenarios where payouts drop as low as 2.90% from their average of 4.90% reported in Table 4 and increase as high as (an admittedly extreme) 11.90%. For brevity, instead of analyzing separately the downside risk contributions of all twelve asset classes, we group them together in Equity (domestic and non-U.S.), Fixed Income (domestic and non-U.S.), Hedge Funds, Real Estate (Private and Public), Private Equity and Venture Capital (PE/VC) and Cash. As the overall risk contributions of the asset classes "Other" and "Natural Resources" are insignificant we do not present the results for these asset classes. We then increase (or decrease) the payout reported by each endowment by x and repeat the analysis of Subsection 4.3.1.

The risk contributions are presented in Figure 2. First, we observe that as we vary the minimal acceptable return (MAR), the magnitude of the risk contributions vary widely. For example, asset allocation to cash generates on average a *negative* risk contribution for low MAR levels, that is, if the required level of payout is not high, cash serves as a risk "diversifier" in the portfolio. However, as MAR increases, holding cash in the policy portfolio makes it unlikely that the payout is met; intuitively, then, an allocation to cash *increases* the risk of the portfolio. This is apparent in Figure 2: the downside risk contribution of the allocation to cash increases to about 1% if $MAR = \ inflation \ + \ payout \ + 7\%$. The same results for Fixed Income: as MAR increases, the presence of the Fixed Income asset class in the portfolio (with lower risk and lower returns) *increases* the risk that the MAR will not be met.

As for hedge funds, we observe that this asset class remains a risk diversifier even as we increase the MAR, noting that the diversification power decreases, however. The other alternative assets, private equity and venture capital, see a decrease in their risk contributions as MAR increases, although for the levels of MAR we analyze, these alternative assets continue to have a positive risk contribution.

As a summary, when MAR increases, the positive risk contributions from asset allocation shift from Equity and Private Equity/ Venture Capital to Fixed Income, Real Estate and Cash. From a normative point of view, we view this result as an indication that as payouts increase, allocations to Private Equity/ Venture Capital need to increase in order to meet the payout obligations.

Panel C of Figure 2 illustrates how as MAR increases, the risk contribution of Active Allocation or security selection changes. While, for the values of MAR currently observed in the endowments universe, security selection serves as a return enhancer and risk diversifier, we observe that as MAR increases, security selection, which currently decreases the total downside risk of the average endowment by about 14% (Panel A of Table 6) comes close to having a zero risk contribution in the average endowment's portfolio. Whereas endowments' active investing skills contribute to reducing the downside risk from the current payout levels, this may not be the case if payouts decrease. This may be due to the fact that managers are limited in their ability to add value by active management and, in the case of high levels of required payouts, those value adding skills are not sufficient. In particular, if endowments experience significant drops in returns, their active investing abilities appear insufficient to reduce the risks that payouts are not met.

Changes in Tactical Allocation contributions are presented in Panel B for completeness, but their effect is negligible and we do not discuss them.

In conclusion, risk contributions are sensitive to the minimal accepted returns, and by extension they are sensitive to the risk specification considered. Specifically, as the levels of payouts change (or equivalently, when investors employ different downside risk statistics, relative to different MARs), portfolio components may change their risk roles from becoming risk contributors to becoming risk reducers, and vice-versa. These results show the importance of measuring the risk according the risk statistic that is *relevant* to the investor, and how different the conclusions of this analysis are if a wrong risk statistic is applied.

5 Conclusions

In this study we have proposed a risk management system that decomposes the financial risk associated with a portfolio simultaneously across the management decisions used to build the portfolio as well as the individual securities held in the portfolio. This methodology of risk measurement is particularly useful to decentralized (or "top-down") investors, such as university endowments or pension plans.

The fundamental risk measure used in this study is downside risk from a prespecified minimal accepted return. This statistic is particularly useful for investors facing a liability. University endowments, which typically have a payout obligation to their beneficiary, represent such a particular type of investor.

We applied this newly developed risk measurement methodology to a sample of university endowments and analyzed the sources of returns and downside risk. Consistent with the fact that endowments invest most of their assets in U.S. Equity we found this asset class to generate most of the returns and most of the downside risk. In contrast, we found that U.S. Fixed Income, while increasing returns, appears to also decrease downside risk.

Contrary to the common wisdom that alternative investments are risky investments, we found that some of these alternatives, namely hedge funds, contribute positively to returns while simultaneously decreasing downside risk. Thus hedge funds play a similar role to that traditionally attributed to U.S. Fixed Income in a university endowment portfolio. We also found that the active investment decisions made by endowments increase returns and reduce downside risk.

Although the risk management method we propose is based on downside risk, we stress that the choice of a primary risk statistic depends on each investor's unique characteristics. The risk decomposition we propose, in particular, can be generalized to risk statistics that are of particular interest to the investor.

Appendix: Downside Risk Decomposition Mathematics

In this section we prove the downside risk decomposition formula. Let the whole portfolio be denoted by $R = w_1R^1 + \ ... \ + w_NR^N$.

Let $DR\left((w_1R^1 + ... + w_NR^N), MAR\right)$ be the Downside Risk from MAR of the portfolio with weights $w_1, ..., w_N$ in asset classes 1 to N. Consider the natural extension of DR to N-uplets $(w_1, ..., w_N)$ whose sum is not necessarily one. Because

$$DR\left((\lambda w_1R^1 + ... + \lambda w_NR^N), \lambda MAR\right) = \lambda DR\left((w_1R^1 + ... + w_NR^N), MAR\right),$$

by differentiating with respect to λ then making $\lambda = 1$ we obtain the following downside risk decomposition

$$DR\left((w_1R^1 + ... + w_NR^N), MAR\right) = \sum_{i=1}^{N} w_i \frac{\partial DR}{\partial w_i} + MAR\frac{\partial DR}{\partial MAR}.$$

Recall that we denoted by $\mathcal{I}$ the subset of the probability space on which the returns of the entire portfolio $R < MAR$, and by $\mathbb{E}_{\mathcal{I}}[X] = \mathbb{E}[X\mathbf{1}_{\mathcal{I}}]$. With this notation we have that:

$$DR = \sqrt{\mathbb{E}_{\mathcal{I}}\left[(MAR - (w_1R^1 + \ ... \ + w_NR^N))^2\right]}.$$

Differentiating the above relationship with respect to MAR we obtain that:

$$\frac{\partial DR}{\partial MAR} = \frac{\mathbb{E}_{\mathcal{I}}\left[MAR - (w_1R^1 + \ ... \ + w_NR^N)\right]}{DR}$$

$$= \frac{\mathbb{E}_{\mathcal{I}}\left[(w_1 + \ ... \ + w_N)MAR - w_1R^1 - \ ... \ - w_NR^N\right]}{DR}$$

$$= \frac{\mathbb{E}_{\mathcal{I}}\left[w_1(MAR - R^1) + \ ... \ + w_N(MAR - w_N)\right]}{DR}$$

$$= w_1 \frac{\mathbb{E}_{\mathcal{I}}[MAR - R^1]}{DR} + \ ... \ + w_N \frac{\mathbb{E}_{\mathcal{I}}[MAR - R^N]}{DR}.$$

Substituting the formula for $\frac{\partial DR}{\partial MAR}$ into the decomposition of downside risk we obtain that

$$DR\left((w_1 R^1 + ... + w_N R^N), MAR\right) = \sum_{i=1}^{N} w_i \left(\frac{\partial DR}{\partial w_i} + \frac{MAR \cdot \ \mathbb{E}_{\mathcal{I}}[MAR - R^i]}{DR} \right).$$

In order to see the interpretation of our downside risk decomposition formula, note that by differentiating the definition of the downside risk with respect with w_i we obtain that:

$$\frac{\partial DR}{\partial w_i} = \frac{\partial}{\partial w_i} \ \sqrt{\mathbb{E}_{\mathcal{I}} \left[MAR - (w_1 R^1 + \ ... \ + w_N R^N) \right]}$$

$$= \frac{1}{DR} \ \mathbb{E}_{\mathcal{I}} \left[-R^i (MAR - R) \right].$$

Substituting this formula into equation (13) we obtain that:

$$w_i \left(\frac{\partial DR}{\partial w_i} + \frac{MAR \cdot \ \mathbb{E}_{\mathcal{I}}[MAR - R^i]}{DR} \right) = \frac{1}{DR} \ \mathbb{E}_{\mathcal{I}} \left[(w_i MAR - w_i R^i) \cdot (MAR - R) \right].$$

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Tables and Figures

Table 1: Portfolio structure for the XYZ fund.

Asset Class	Benchmark	Benchmark Portfolio	Actual Portfolio
U.S. Equity	Russell 3000	$w_1^P = 50\%$	$w_1 = 40\%$
Large Cap Manager	S&P 500		$w_{1,1} = 20\%$
Small Cap Manager	Russell 2000		$w_{1,2} = 20\%$
Inflation Hedge	ML Infl. Linked	$w_2^P = 50\%$	$w_2 = 60\%$
TIPS Manager	ML Infl. Linked		$w_{2,1} = 40\%$
Commodities Manager	GSCI		$w_{2,2} = 20\%$

Table 2: The distribution assumptions on the XYZ investment horizon.

	Security Type	**Expected Annual Return**	**Standard Deviation**
Policy Benchmarks			
R3000	US Equity	8.37%	12.30%
ML Infl. Linked	Inflation Hedge	5.12%	6.60%
US Equity Benchmarks			
S&P 500	Large cap	8.28%	12.00%
R2000	Small cap	9.02%	18.00%
Inflation Hedge Benchmarks			
ML Infl. Linked	TIPS	5.12%	6.60%
GSCI	Commodities	5.00%	23.00%
Value added	Large cap	0.83%	3.70%
	Small cap	1.24%	8.50%
	TIPS	0.22%	1.50%
	Commodities	0.76%	4.40%

Table 3: Decomposition of XYZ portfolio returns and downside risk.

The Table presents the decomposition of the fictional fund XYZ returns and downside risk from a minimal accepted return (MAR). We assume that $MAR = 5\%$.

Panel A: Returns Decomposition

		Return contributions				
Asset Benchmarks	Intermediate Benchmarks	Asset Allocation	Tactical	Intermediate Benchmarking	Active	TOTAL
U.S. Equity		4.19%	-0.84%			3.88%
	Large Cap			-0.02%	0.17%	
	Small Cap			0.13%	0.25%	
Inflation Hedge		2.56%	5.12%			7.90%
	TIPS			0.00%	0.09%	
	Commodities			-0.02%	0.15%	
TOTAL					11.78%	= 11.78%

Panel B: Downside Risk Decomposition

		Downside Risk Contributions				
Asset Bechmarks	Intermediate Benchmarks	Asset Allocation	Tactical	Intermediate Benchmarking	Active	TOTAL
US Equity		2.6%	-0.5%			2.9%
	Large Cap			-0.1%	0.1%	
	Small Cap			0.6%	0.2%	
Inflation Hedge		0.9%	0.2%			2.9%
	TIPS			0.0%	0.1%	
	Commodities			1.6%	0.1%	
TOTAL					5.8%	= 5.8%

Table 4: Summary statistics of asset allocation and performance.

The Table presents summary statistics of 281 U.S. college and university endowments as of 2005. The returns are the annualized returns calculated from the entire history of the fund.

		Summary Statistics							
		Target Weights				Actual Weights			
		Mean	25th prc	Median	75th prc	Mean	25th prc	Median	75th prc
1 .	U.S. Equity	43.31%	31.45%	44.50%	55.00%	42.38%	32.32%	42.06%	53.70%
2 .	Non U.S. Equity	13.07%	10.00%	15.00%	16.00%	15.41%	11.58%	15.90%	19.70%
3 .	U.S. Fixed Income	19.95%	15.00%	20.00%	25.00%	17.44%	12.54%	16.72%	21.53%
4 .	Non U.S. Fixed Income	0.60%	0.00%	0.00%	0.00%	0.95%	0.00%	0.00%	0.00%
5 .	Public Real Estate	1.50%	0.00%	0.00%	3.00%	1.46%	0.00%	0.00%	2.50%
6 .	Private Real Estate	2.31%	0.00%	0.00%	4.81%	1.90%	0.00%	0.80%	2.80%
7 .	Hedge Funds	11.23%	2.50%	10.00%	15.00%	12.25%	4.40%	10.41%	17.54%
8 .	Venture Capital	2.32%	0.00%	0.00%	5.00%	1.26%	0.00%	0.20%	2.00%
9 .	Private Equity	3.18%	0.00%	1.50%	5.00%	2.43%	0.00%	0.80%	3.70%
10 .	Natural Resources	1.29%	0.00%	0.00%	2.00%	1.36%	0.00%	0.00%	1.94%
11 .	Other Investments	0.49%	0.00%	0.00%	0.00%	0.72%	0.00%	0.00%	0.13%
12 .	Cash	0.74%	0.00%	0.00%	0.00%	2.45%	0.10%	1.20%	3.60%
		Total Returns				Payouts			
		Mean	25th prc	Median	75th prc	Mean	25th prc	Median	75th prc
		9.82%	8.91%	9.83%	10.74%	4.90%	4.30%	4.85%	5.30%

Table 5: Summary statistics for asset class benchmarks.

The Table presents summary statistics for indices used to proxy asset classes. SR is the Sharpe ratio.

			Summary statistics			
	Asset class	Benchmark index	Mean	Std	Median	SR
1.	U.S. Equity	Russell 3000	9.93	13.37	11.39	0.43
2.	Non U.S. Equity	MSCI World (Excl. US)	4.46	13.31	5.82	0.02
3.	U.S. Fixed Income	Lehman Bond Aggregate	8.04	4.41	8.64	0.88
4.	Non U.S. Fixed Income	Salomon Brothers Non US Bond Index	7.89	9.08	7.60	0.41
5.	Public Real Estate	NAREIT	13.27	12.79	9.06	0.71
6.	Private Real Estate	NCREIF	7.81	6.55	8.07	0.56
7.	Hedge Funds	HFRI-all fund Composite	14.31	7.61	13.09	1.34
8.	Venture Capital	Cambridge Associate VC index	26.59	56.20	17.42	0.40
9.	Private Equity	Cambridge Associate PE index	14.76	13.74	15.38	0.77
10.	Natural Resources	AMEX Oil (before 1992), GSCI (after 1992)	7.02	16.87	1.33	0.17
11.	Other Investments	—	—	—	—	—
12.	Cash	30-day U.S. T-Bill	4.14	1.92	4.73	—

		Correlations											
		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
1.	U.S. Equity	1.00											
2.	Non U.S. Equity	0.60	1.00										
3.	U.S. Fixed Income	-0.03	-0.57	1.00									
4.	Non U.S. Fixed Income	-0.10	-0.11	0.26	1.00								
5.	Public Real Estate	-0.02	0.20	0.08	-0.17	1.00							
6.	Private Real Estate	0.20	0.17	-0.31	-0.57	0.02	1.00						
7.	Hedge Funds	0.55	0.56	-0.05	0.07	0.03	-0.46	1.00					
8.	Venture Capital	0.37	0.45	-0.31	-0.20	-0.30	0.21	0.53	1.00				
9.	Private Equity	0.79	0.80	-0.32	-0.22	0.14	0.36	0.56	0.56	1.00			
10.	Natural Resources	0.79	0.47	-0.42	-0.09	-0.12	0.28	0.18	0.49	0.28	1.00		
11.	Other Investments	—	—	—	—	—	—	—	—	—	—	—	
12.	Cash	0.27	-0.21	0.32	-0.35	-0.40	-0.04	0.23	0.21	0.01	-0.09	—	1.00

Table 6: Returns and Risk contributions in endowments' portfolios.

The Table presents the returns and downside risk contributions of the asset allocation, tactical and active (defined as the sum of internal benchmarking decisions R^B and the security selection components R^A) parts of an endowment portfolio. The contributions are relative as follows. In order to calculate the returns contributions we perform the decomposition of returns as in formulae (4) to (8), using a $MAR = \text{payout} + \text{inflation}$. The inflation component is the annual CPI. We then divide the returns of each component of formulae (4) to (8) by the overall portfolio returns while summing R^B and R^A together. All the contributions to the portfolio returns add up to 100%. We do the same thing to downside risk, with the distinction that formula (12) is used together with formulae (4) to (8) to decompose the overall downside risk of the portfolio.

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Table 6 (cont.): Returns and Risk contributions in endowments' portfolios.

Panel A: Mean contributions

		Return contribution			DR contribution		
		Policy	Tactical	Active	Policy	Tactical	Active
1 .	U.S. Equity	43.64%	-0.96%		94.03%	-2.13%	
2 .	Non U.S. Equity	6.00%	1.12%		33.70%	5.27%	
3 .	U.S. Fixed Income	15.96%	-2.00%		-15.11%	1.93%	
4 .	Non U.S. Fixed Income	0.48%	0.25%		-0.49%	-0.31%	
5 .	Public Real Estate	2.17%	-0.07%		-2.64%	0.01%	
6 .	Private Real Estate	1.77%	-0.27%		-1.45%	0.20%	
7 .	Hedge Funds	14.89%	1.44%		-8.19%	-0.88%	
8 .	Venture Capital	5.87%	-2.79%		7.49%	-2.50%	
9 .	Private Equity	4.51%	-1.11%		4.05%	-0.68%	
10 .	Natural Resources	0.80%	0.06%		0.99%	0.00%	
11 .	Other Investments	0.00%	0.00%		0.38%	0.29%	
12 .	Cash	0.35%	0.69%		0.37%	0.30%	
				7.20%			-14.62%
TOTAL				100 %			100 %

Panel B: The 25th percentile of contributions

		Return contribution			DR contribution		
		Policy	Tactical	Active	Policy	Tactical	Active
1 .	U.S. Equity	31.33%	-4.35%		63.39%	-9.12%	
2 .	Non U.S. Equity	4.40%	0.00%		18.13%	0.00%	
3 .	U.S. Fixed Income	10.95%	-3.40%		-19.32%	-0.09%	
4 .	Non U.S. Fixed Income	0.00%	0.00%		0.00%	0.00%	
5 .	Public Real Estate	0.00%	0.00%		-4.55%	-0.22%	
6 .	Private Real Estate	0.00%	-0.73%		-2.37%	-0.22%	
7 .	Hedge Funds	3.28%	-1.00%		-10.00%	-0.90%	
8 .	Venture Capital	0.00%	-5.17%		0.00%	-4.61%	
9 .	Private Equity	0.00%	-2.28%		0.00%	-1.18%	
10 .	Natural Resources	0.00%	0.00%		0.00%	0.00%	
11 .	Other Investments	0.00%	0.00%		0.00%	0.00%	
12 .	Cash	0.00%	0.00%		0.00%	0.00%	
				-0.12%			-35.99%
TOTAL				100 %			100 %

Panel C: The median of contributions

		Return contribution			DR contribution		
		Policy	Tactical	Active	Policy	Tactical	Active
1 .	U.S. Equity	42.03%	-0.47%		83.35%	-0.90%	
2 .	Non U.S. Equity	6.25%	0.69%		28.22%	3.12%	
3 .	U.S. Fixed Income	15.05%	-1.25%		-14.72%	1.05%	
4 .	Non U.S. Fixed Income	0.00%	0.00%		0.00%	0.00%	
5 .	Public Real Estate	0.00%	0.00%		0.00%	0.00%	
6 .	Private Real Estate	0.00%	0.00%		0.00%	0.00%	
7 .	Hedge Funds	13.88%	0.00%		-4.22%	0.00%	
8 .	Venture Capital	0.00%	0.00%		0.00%	0.00%	
9 .	Private Equity	1.86%	0.00%		0.17%	0.00%	
10 .	Natural Resources	0.00%	0.00%		0.00%	0.00%	
11 .	Other Investments	0.00%	0.00%		0.00%	0.00%	
12 .	Cash	0.00%	0.36%		0.00%	0.03%	
				7.82%			-6.61%
TOTAL				100 %			100 %

Panel D: The 75th percentile of contributions

		Return contribution			DR contribution		
		Policy	Tactical	Active	Policy	Tactical	Active
1 .	U.S. Equity	57.82%	2.43%		118.21%	4.31%	
2 .	Non U.S. Equity	7.92%	1.79%		42.55%	8.77%	
3 .	U.S. Fixed Income	20.24%	0.02%		-9.49%	3.25%	
4 .	Non U.S. Fixed Income	0.00%	0.00%		0.00%	0.00%	
5 .	Public Real Estate	3.86%	0.13%		0.00%	0.00%	
6 .	Private Real Estate	3.44%	0.23%		0.00%	0.49%	
7 .	Hedge Funds	21.38%	2.86%		0.00%	0.42%	
8 .	Venture Capital	11.39%	0.00%		13.68%	0.00%	
9 .	Private Equity	7.76%	0.12%		5.79%	0.08%	
10 .	Natural Resources	1.22%	0.00%		0.00%	0.00%	
11 .	Other Investments	0.00%	0.00%		0.00%	0.00%	
12 .	Cash	0.00%	1.00%		0.00%	0.36%	
				17.00%			24.58%
TOTAL				100 %			100 %

Figure 1: The typical investment management structure of an endowment.

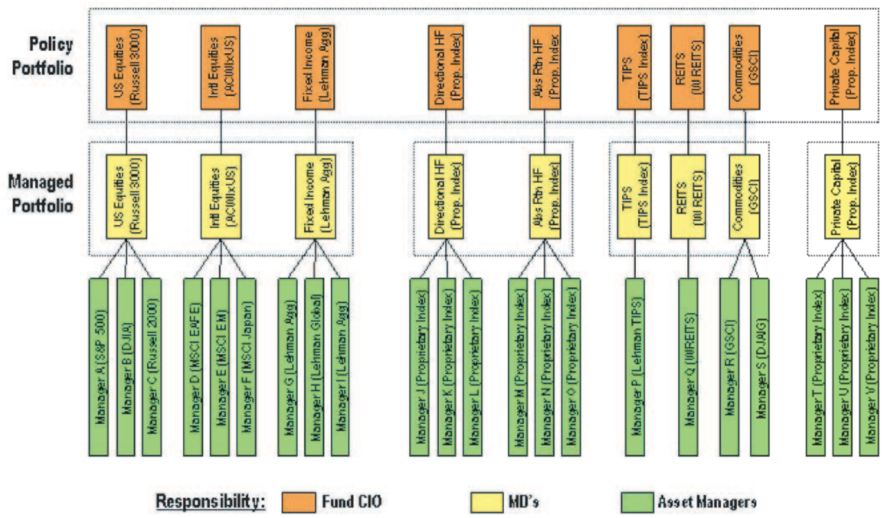

Figure 2: Sensitivity of risk contributions of asset classes and investment decisions with respect to the minimal acceptable return (MAR).

For each value of x in the interval $[-2\%;\ 7\%]$, we perform the downside risk decomposition of each endowment in our sample using $MAR = \ inflation\ +\ payout\ + x$. We then calculate the average downside risk contributions across the asset allocation, tactical and active investment decisions and across the following condensed asset classes: Equity (domestic and non-U.S.), Fixed Income (domestic and non-U.S.), Hedge Funds, Real Estate (Private and Public), Private Equity and Venture Capital (PE/VC) and Cash. The values of x (in %) are plotted on the x-axis, and the average risk contributions to total downside risk from MAR (in %) are plotted on the y-axis. Panel A presents average downside risk contributions generated by the Asset Allocation decision, Panel B presents average downside risk contributions generated by the Tactical Asset Allocation decision and Panel C presents average downside risk contributions coming from the Active (security selection) decision.

Panel A: Contributions from the Asset Allocation decision

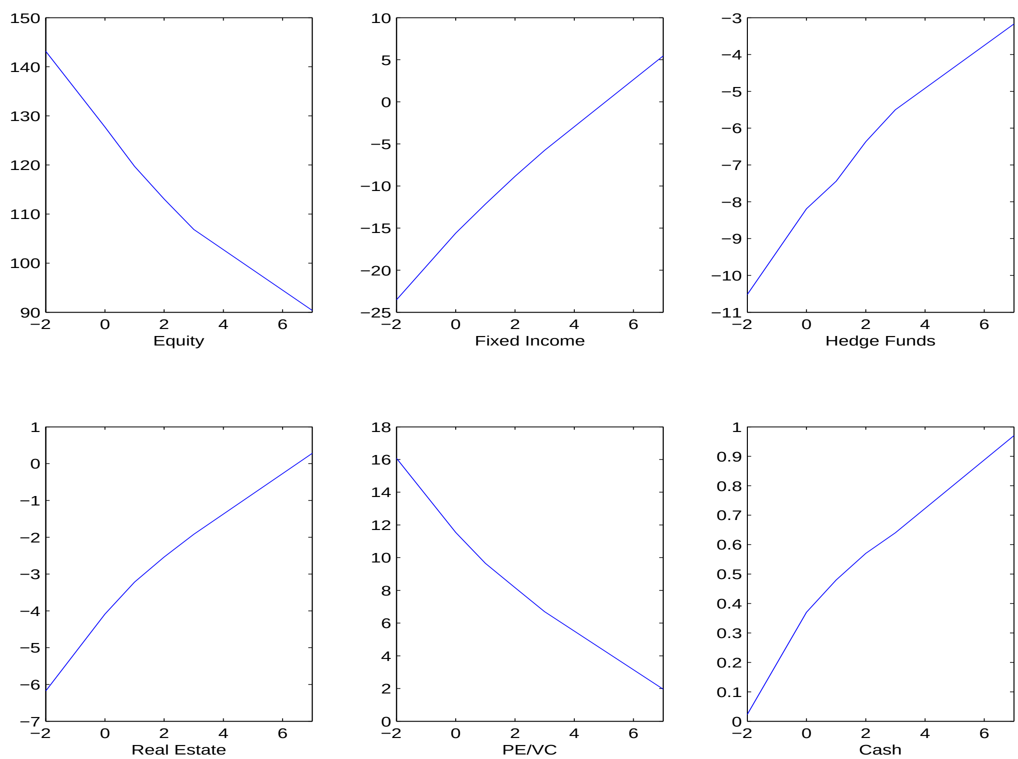

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Figure 2 (cont'd)

Panel B: Contributions from the Tactical Asset Allocation decision

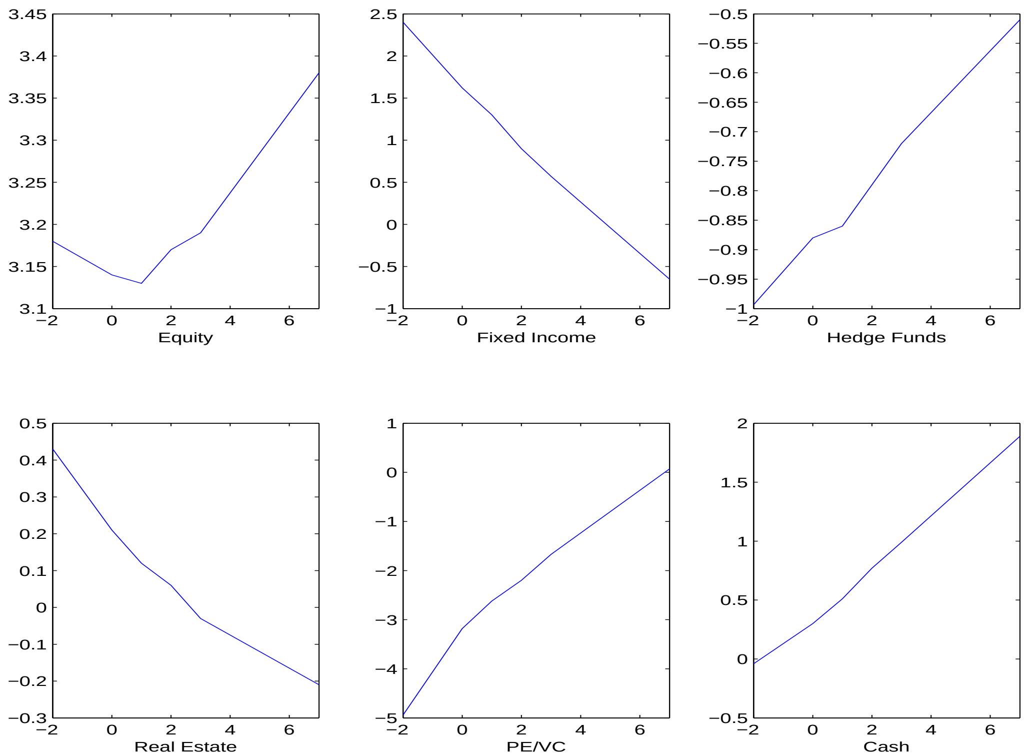

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Figure 2 (cont'd)

Panel C: Contributions from the Active Allocation decision