Eq. (1) can be written as
\[(e^{dx}f')' + \beta e^{dx}f = 0\]
which is a Sturm–Liouville equation with \(p(x) = e^{dx} > 0\), \(q(x) = \beta e^{dx} > 0\) and \(r(x) = 0\). The eigenfunctions defined by (1) \& (2) (i.e., the functions given by eq. (3)) satisfy the orthogonality relations
\[\int_{a}^{b} (\beta e^{dx}) \phi_n(x) \phi_m(x) \, dx = 0\]
for \(n \neq m\). Therefore, if we set
\[f(x) = \sum_{n=1}^{\infty} a_n \phi_n(x)\]
it follows that
\[a_n = \frac{\int_{a}^{b} (\beta e^{dx}) \phi_n(x) f(x) \, dx}{\int_{a}^{b} (\beta e^{dx}) \phi_n^2(x) \, dx}\]
Using eq. (1) this becomes