Judgments about spatio-temporal relations

Thomas Bittner
Qualitative Reasoning Group, Department of Computer Science, Northwestern University
and
Institute for Formal Ontology and Medical Information Science at the University of Leipzig

Abstract

We define a family of qualitative spatio-temporal relations such as same-place-same-time and same-path-different-time, which describe the relative location of spatio-temporal objects within places or along paths. The relations in question are approximate, and this means that some of them are context-dependent. We explore the relationships between context, judgments that are made in certain contexts, and the spatio-temporal relations that do occur in those judgments. Understanding these relationships is important for understanding human judgments about spatio-temporal configurations as well as for the interaction of humans with spatio-temporal databases.

1 Introduction

Spatio-temporal databases and other spatio-temporal knowledge representation systems are of increasing importance, in the light of the fact that database technology has improved in ways which, allow databases to take into account the inherently dynamic character of our surrounding world. Modern spatio-temporal database technology is or will soon be able to represent the fact that things move, change, appear, and disappear over time.

In this paper we focus on *spatio-temporal* relations, like same-place-same-time, same-place-different-time, or different-place-different-time. Unfortunately, notions like 'same place' and 'same time' are problematic, since their meaning depends on context. For example, in order to say that two electrons are in the same place at the same time our intuition tells us that both must overlap, at least momentarily in a way which can be achieved only by using a particle accelerator. On the other hand in order to say that John and Mary are in the same place at the same time it is sufficient, depending on the context, that they are kissing each other, or that they are holding hands, or that they are in the same room. In everyday contexts, being at the same place in the same time does not require spatio-temporal objects actually to overlap.

While people seem to deal effortlessly with the different meanings of same-place-same-time in varying contexts, it is hard to formalize the context dependence that is thereby involved. Imagine a database that continuously tracks the GPS-location of the cell-phones of two terrorists, X and Y, and imagine that an FBI-investigator tries to find out whether X and Y have met by querying the database. She could start by extracting whether or not the spatial distance between X and Y dropped below a certain threshold, say 10 meters. From this, however, it does not follow that X and Y have met, since they might have been at the locations in question at different times. Adding a threshold of temporal distance does not necessarily help: For example X and Y might have been sitting for three minutes in two distinct subway trains five meters apart without having had the chance to meet. They have been in *different* places at the same time and it does not matter how close their actual distance was.

No matter how problematic the context-dependence of spatio-temporal relations like same-place-same-time and different-place-same-time might be, such relations are among the most important targets of spatio-temporal queries. This paper presents an effort towards a formalization of such qualitative spatio-temporal relations.

2 Spatio-temporal objects and their location

2.1 Objects and their location

Spatio-temporal objects fall into two major categories: continuants and occurrents (Simons 1987). Continuants change over time, for example by gaining or loosing parts, by growing older, by changing their location, but yet remain the same thing. Examples are human beings like

John and Bill Clinton, cars, substances in general. Occurrents do not change over time. They just occur. Examples are 'John's flight to New York', 'World War 2', 'The life of Bill Clinton', 'My childhood', and so on. The existence of an occurrent always depends on the existence of some continuant. The occurrent 'John's childhood' cannot exist without the continuant John. We define the relation *continuant-of*(c, o), which holds between an occurrent o and the continuant c it depends on, e.g., *continuant-of*$(John, John's\ childhood)$.

The relationship between occurrents and continuants is very complex in the sense that a single occurrent may depend on multiple continuants and in the sense that the set of continuants it depends on can change, i.e., loose and gain members. We assume for simplification: (a) that there is a single continuant for each occurrent to depend upon; and (b) that this continuant does not disappear or gets replaced during the time-intervals under consideration. Under these (very strong) assumptions the relation *continuant-of* is a functional relation, i.e., $John =$ *continuant-of*$(John's\ childhood)$.

Considering the relationship between occurrents and the continuants they depend on we can distinguish two major categories of occurrents: events and processes. Events are spatio-temporal objects that correspond the occurrence of a certain continuant in a certain state, e.g., the occurrent 'John's childhood' corresponds to the continuant John in the state of being a child. Processes, on the other hand, correspond to the occurrence of the change of some continuant. For example, the process of 'John's growing up' corresponds to certain patterns of changes of the continuant John.

We distinguish the domain of spatio-temporal objects, O, and the domain of regions, R, which both are considered as sets. The domain of regions is constituted by regions of different dimensionality which satisfy the axioms of the RCC-theory (Randall, Cui & Cohn 1992): four-dimensional spatio-temporal regions x, three-dimensional spatial regions, x^s, and one-dimensional temporal regions, x^t. The relationship between individual objects and individual regions is established by the notion of location. We write $L(o, x)$ in order to denote that the object o is located at the region x [1].

Every (four-dimensional) spatio-temporal object, $o \in O$, is exactly located at a single three-dimensional spatial region, x^s, at every instant of time, τ (Casati & Varzi 1995): $\forall o \in O : \exists x^s \in R : L^s_\tau(o, x^s)$. The region x^s is the exact or precise spatial location of o at the time instant τ. Since every spatio-temporal object is exactly located at a single region of space at a particular instant of time the relation $L^s_\tau(o, x^s)$ is a total functional relation, $\rho_\tau : O \rightarrow R$ with $x^s = \rho_\tau(o) \equiv L^s_\tau(x^s, y)$. We extend the instant-based localization function, ρ_τ, by adding an additional parameter τ ranging over instants of time. The resulting localization function is of signature: $\rho : \mathcal{T} \times O \rightarrow R$, where $\mathcal{T}$ is the time-line with all its time-instants.

Every spatio-temporal object, o, is temporally located in a unique region of time, $x^t \in R$, bounded by the beginning and end of its existence. The temporal location is a functional relation.

If we assume that space and time are distinct kinds of dimensions then spatio-temporal location is a *ternary* relation between (1) spatio-temporal objects; (2) temporal regions; and (3) regions of space, i.e., $L(o, x^t, x^s)$.

2.2 Change of spatial location

There are continuants that have different exact spatial locations at different times. We say that these objects change their spatial location. If we consider a temporal region (a period of time) during which the spatio-temporal object o existed then (i) o may be *at rest*, i.e., it may be located in the same region of space over a period of time, or (ii) its spatial location may change, by being located in different regions of space at different time-instants during this period. An example for (i) is John's location described in 'John was *at home* this morning' and an example for (ii) is John's location described in 'John was *on his way home* this morning'.

Consequently, there are two ways of defining spatio-temporal location in terms of ternary relations and we use the notations $L_R(o, x^t, x^s)$ and $L_P(o, x^t, x^s)$ in order to refer to those relations. More precisely we define:

The relation $L_R(o, x^t, x^s)$ holds if and only if the spatio-temporal object o is spatially at *rest* at x^s over x^t, i.e., its spatial location is identical for all time-instants within x^t:

$$L_R(o, x^t, x^s) \equiv \forall \tau \in x^t : \rho(\tau, o) = x^s, \qquad (1)$$

where $\tau \in x^t$ abbreviates: the time-instant τ lies within the boundaries of the temporal region x^t. We assume that there is no state of rest that lasts only for a single time-instant.

The relation $L_P(o, x^t, x^s)$ holds if and only if *o changes within x^s during x^t*. The spatial region x^s is the mereological sum of all regions at which the object, o, was exactly

[1] We say that the object o is located *at* the (spatio-temporal) region x in order to stress the exact fi t of object and region (the object matches the region). It is important to distinguish the exact match from the case of an object being located *in* a region where the region is allowed to be bigger than the object. Both are distinguished from the case of an object *covering* a region, which intuitively implies the region is smaller than the object.

spatially located over the time-period x^t:

$$L_P(o, x^t, x^s) \equiv x^s = \bigvee_{\rho(\tau,o)} \tau \in x^t. \qquad (2)$$

3 Approximate spatio-temporal location

Often, however, it is not very interesting to know that John is exactly located at that region of space from which the air is displaced by his body at a particular instant in time. It is much more interesting to know, for example, that John is in London or in Hyde Park. Hence we define a family of functions $\lambda^1_\tau, \ldots, \lambda^n_\tau : O \rightarrow R$, each of which yields a unique region of space for each spatio-temporal object $o \in O$ at instant τ. For every λ^i_τ this region is such that the exact location of the object in question is a mereological part of it, i.e., $\rho_\tau(o) \leq \lambda^i_\tau(o)$. We can think of a particular $\lambda^i_\tau(o)$ as an approximate location of o at instant τ.

The definitions of exact spatio-temporal location, $L_R(o, x^t, x^s)$ and $L_P(o, x^t, x^s)$ given above above can be generalized easily to take into account the approximative character of spatial location, temporal location, or both. In order to formalize approximate spatial location we use approximate localization functions, λ^i, rather than exact localization functions ρ. Formally we define corresponding to Definition (1): $\mathcal{L}^i_R(o, x^t, x^s) \equiv \forall\tau \in x^t : \lambda^i(\tau, o) = x^s$. We say that x^s is the place (marking the approximate location) where the event or process o occurred over the time-period x^t (or the place in which the continuant o was at rest over x^t). This does justice to the intuition behind the approximate character of John's spatial location in the sentences 'John spent Monday morning in Buckingham Palace'. The superscript i indicates the dependence of $\mathcal{L}$ on the underlying approximation function λ^i.

Imagine that we intercept the communication of the Queen's Secret Services agents as they trace John's movement from the entrance through the hall to the guest-room. If we sum up all those reported regions (entrance + hall + ... + guest-room), then we get John's approximate path of movement, x^s, during x^t, through the palace. Formally we define corresponding to Definition (2): $\mathcal{L}^i_P(o, x^t, x^s) \equiv x^s = \bigvee_{\lambda^i(\tau,o)} \tau \in x^t$.

Also, it is normally not very interesting to know that John was at a certain location for exactly 0.34768410^{10} nanoseconds. Instead of saying that John was in Hyde Park *exactly* from 10.00 a.m. to 11.00 a.m., people would often rather say that he was there on Monday morning. Even in the case where people say that John was in Hyde Park from 10.00 a.m., they do not mean that he crossed the boundary of the park at exactly 10.00 and not a second earlier or later. Consequently, if we want to do justice to ordinary reasoning, we need to define a notion of spatio-temporal location based on the approximate temporal extent of the rest or the change of spatio-temporal objects and define: [2]

$$\Lambda^i_R(o, x^t, x^s) \equiv \exists x \forall \tau \in x : \lambda^i(\tau, o) = x^s \wedge x \leq x^t \qquad (3)$$

$$\Lambda^i_P(o, x^t, x^s) \equiv \exists x : x^s = \bigvee_{\lambda^i(\tau,o)} \tau \in x \wedge x \leq x^t \qquad (4)$$

4 Context and approximate location

At each time-instant, τ, there is a single *context-free* exact localization function ρ_τ and arbitrarily many *context-dependent* approximate localization functions λ^i_τ. The exact localization function is context-free because it is same for all contexts. Approximate location is context-dependent because in a given context there is one and only one approximate localization function $\lambda^i_\tau(x)$. Those localization functions are used in order to make reference to spatio-temporal location in the given context.

The notion of context is difficult and a formalization goes beyond the scope of this paper. For us it is sufficient to assume that each context is characterized: (1) by a specific period of time over which it is active; (2) by the feature of selectivity limiting its scope to only certain objects; and (3) the feature of granularity determining the coarseness of reference to spatio-temporal location. Those aspects determine the properties of the approximate localization function associated with a given context.

4.1 Limited scope, selectivity, the notion of rest

In order to reflect the limited scope of context and associated approximation we extend instant-based approximate localization functions, λ^i_τ, by adding an additional parameter τ ranging over instants of time. In order to 'glue' approximation functions to their context we consider them to be partial functions with respect to their temporal parameter. This means that they are defined only while the corresponding context is active. *Context-dependent* approximate localization functions are of signature: $\lambda^i : T \times O \rightarrow R$ where $T \leq \mathcal{T}$ is the period of time during which the context i is active.

Another important feature of context is its selectivity. In a specific context we are not interested in the function λ^i in its range over all objects in the universe. When asking John about his whereabouts, we are not interested in the location of some statue in China. We are interested in the location

[2] One might want even weaker definitions in order to be able to capture also sentences of the form 'John was in Hyde Park from 10 to 11 a.m.', which intuitively would not be false in an everyday conversation, even if John entered the park at 9.55 or 10.05 a.m. and left it at 10.55 or 11.05.

of just some few selected objects that are in the *foreground* of our attention (John, Buckingham Palace, London, ...). Consequently we assume that in specific contexts the approximate localization functions, λ^i, are partial functions also in respect to the objects in their range.

Given the context-dependence of approximate location then the term ‘rest’ is meaningful only within a fixed context, i.e., with respect to the fixed approximate localization function associated with this context. Consider the approximate localization function λ^i which yields ‘Buckingham Palace’ for the object John and the time-period ‘Monday morning’ and the approximate localization function λ^j which yields ‘London’ regarding John’s whereabouts on Monday morning. With respect to the approximate localization function λ^i being at rest means that John does not leave the palace. With respect to the approximate localization function λ^j being at rest means that John does not leave London.

4.2 Systems and levels of granularity

Consider the sentence ‘John spent Monday at Buckingham Palace’. So far we have considered ‘Buckingham Palace’ as a the name of spatial region in which some spatio-temporal object is at rest. If we take a closer look then it turns out that ‘Buckingham Palace’ is the name of a *place*, which happens to be located at the same spatial region within which the continuant John is at rest or where the event ‘John spending this Monday morning’ occurs. In the remainder we will use the notion of place in order to refer to spatial locations where certain continuants are at rest and where certain events occur. Places are organized in a hierarchical fashion. We call those structures *systems of granularities*.

Granularities are the results of the way we humans structure our surrounding world and provide the foundation for the notion of approximation and for reasoning about approximations (Bettini, Wang & Jajodia 1998, Smith & Brogaard to appear, Bittner & Smith 2001*a*). In the context of approximation of spatial and temporal location the (singular) notion of granularity refers to the size of the approximating region. The plural notion of granularities then refers to hierarchically organized systems of regions. In our example above the regions referred to by the names ‘Hyde Park’ and ‘London’ belong to such a system of granularities.

Formally, a system of granularities is a pair, $G = (R, \subseteq)$, where R is a set of regions with a binary relation $\subseteq$. Following (Smith & Brogaard to appear) we call those regions cells and the relation $\subseteq$ the subcell relation. Systems of granularities are governed by the following axioms (Bittner & Smith 2001*a*): (G1) $\subseteq$ is reflexive, transitive, and antisymmetric; (G2) Systems of granularities have unique maximal cells, called ‘the root-cell’, i.e., $\exists g \in G : \forall g_1 \in G : g_1 \subseteq g$; (G3) each cell g in a system of granularities is connected to the root cell by a finite chain: i.e., $\exists g_1, \ldots, g_n : g \subseteq g_1 \subseteq \ldots \subseteq g_n \wedge root(g_n)$; (G4) Two cells overlap each other if and only if one is a subcell of the other, i.e., $\exists z : (z = z_1 \; o \; z_2) \rightarrow (z_1 \subseteq z_2 \vee z_2 \subset z_1)$. It follows that every system of granularities can be represented as a tree structure, i.e., as rooted directed graphs without circles (Bittner & Smith 2001*a*), by taking the regions as nodes and by demanding that there is an edge from node a to node b if and only if $a \subseteq b$.

Consider the following examples: (E1) A spatial system of granularities is formed by the cells Hyde Park, Soho, Buckingham Palace, Downtown, London, York, Edinburgh, Glasgow, England, Scotland, Great Britain, Germany, Europe and the corresponding nesting of those cells (Figure 1); (E2) The political subdivision of the United States forms a (flat) system of granularities with the US as root-cell and minimal cells like Wyoming and Montana; (E3) A temporal system of granularities is formed by the subdivision of Saturday, January 13th 2002 into forenoon, afternoon, hours, half-hours, quarters, and five-minute slots.

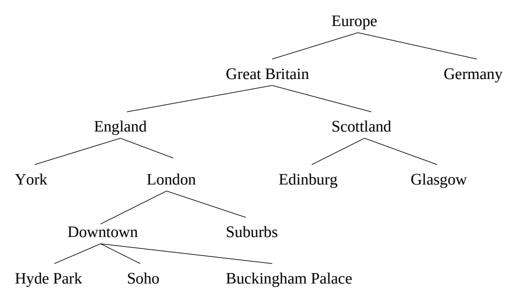

Figure 1: A system of granularities

A closer look at the examples E1-3 reveals that there are systems of granularity which are such that cells at one level sum up the next superordinate cell in the sense in which the Federal States sum up the US and in the sense in which three five-minute slots sum up quarter-of-an-hour slots etc. We call those systems of granularities *full* (Bittner & Smith 2001*b*). On the other hand there are systems of granularities like the one in E1 which do not have this property. Temporal systems of granularities are often full where systems of granularities formed by places usually lack the property of fullness.

Let $G = (R_G, \subseteq)$ be a system of granularities and let $\mathcal{G}$ the corresponding tree representation. A *level of granularity* in G then is a cut in the tree-structure in the sense of (Rigaux & Scholl 1995): (1) Let X be the root of $\mathcal{G}$, then $\{X\}$ is a cut; (2) $sons(X)$ is a cut, where $sons(a)$ is the set of immediate descendants of a; (3) Let C be a cut and $v \in C$ such that $sons(v) \neq \emptyset$ then $C' = (C - v) \cup sons(v)$ is a cut.

This definition ensures that (i) the elements forming a level of granularity are pair-wise disjoint, i.e., $\neg\exists v_1, v_2 \in C : v_1 \subset v_2 \vee v_2 \subset v_1$; (ii) levels of granularity are exhaustive in the sense that $\forall v \in R_G :$ if $v \notin C$ then $\exists v' \in C : v \subseteq v' \vee v' \subseteq v$.

Consider Figure 1. Levels of granularity, for example, are:

$$\begin{array}{ll} g_0 & \{\mathit{Europe}\} \\ g_1 & \{\mathit{Great\ Britain}, \mathit{Germany}\} \\ g_3 & \{\mathit{York}, \mathit{London}, \mathit{Scotland}, \mathit{Germany}\} \\ g_4 & \{\mathit{York}, \mathit{Hyde\ Park}, \mathit{Soho}, \mathit{Buckingham\ Palace}, \\ & \quad \mathit{Suburbs}, \mathit{Edinburgh}, \mathit{Glasgow}, \mathit{Germany}\} \\ \ldots & \end{array} \tag{5}$$

4.3 Context and granularity

Contexts have associated a unique localization functions of signature $\lambda^i : \mathcal{T} \times O \to R$ which are partial regarding their temporal parameter and partial regarding the objects in their range. We now argue that those context-dependent approximate localization functions target only spatial regions which belong to a *single* level of granularity which is fixed for the given context.

Let $\mathcal{C}$ be a context which is active over the time-period $T_\mathcal{C}$ and which has associated systems of spatial and temporal granularities $\mathcal{G}^s_\mathcal{C}$ and $\mathcal{G}^t_\mathcal{C}$, and let $\mathcal{L}_{\mathcal{G}^s_\mathcal{C}}$ and $\mathcal{L}_{\mathcal{G}^t_\mathcal{C}}$ be levels of granularities within these systems (with $\forall g \in \mathcal{L}_{\mathcal{G}^t_\mathcal{C}} : g \leq T_\mathcal{C}$ at the temporal level). A context-dependent approximation function is then of signature $\lambda^\mathcal{C} : T_\mathcal{C} \times O_\mathcal{C} \to \mathcal{L}_{\mathcal{G}^s_\mathcal{C}}$.

Context-sensitive ternary relations of approximate location are then defined as:

$$\begin{array}{l} \Lambda^\mathcal{C}_R(o, x^t, x^s) \equiv x^t \in \mathcal{L}_{\mathcal{G}^t_\mathcal{C}} \wedge \\ \qquad \exists x : \forall \tau \in x : \lambda^\mathcal{C}(\tau, o) = x^s \wedge x \leq x^t \end{array} \tag{6}$$

and

$$\begin{array}{l} \Lambda^\mathcal{C}_P(o, x^t, x^s) \equiv x^t \in \mathcal{L}_{\mathcal{G}^t_\mathcal{C}} \wedge \\ \qquad \exists x : x^s = \bigvee_{\lambda^\mathcal{C}(\tau, o)} \tau \in x \wedge x \leq x^t \end{array} \tag{7}$$

The context-dependence of the relation $\Lambda^\mathcal{C}_R$ is reflected by the fact that the spatial as well as the temporal parameters range over regions belonging to levels of granularities associated to the underlying context. The context-dependence of the relation $\Lambda^\mathcal{C}_P$ is reflected by the fact that the temporal parameter ranges over regions belonging to such a level of granularity and the fact that the spatial parameter ranges over mereological *sums* of regions belonging to such a level of granularity.

5 Spatio-temporal regions and relations

5.1 Objects and regions

We represent (four-dimensional) spatio-temporal regions as pairs of temporal and spatial regions, $x = (x^t, x^s)$. In the case of continuants the pair $x = (x^t, x^s)$ specifies the *exact* spatio-temporal region of the continuant o if and only if either $L_R(o, x^t, x^s)$ or $L_P(o, x^t, x^s)$. It specifies the *approximate* spatio-temporal region of the continuant o if and only if either $\Lambda^\mathcal{C}_R(o, x^t, x^s)$ or $\Lambda^\mathcal{C}_P(o, x^t, x^s)$.

Consider, for example, the movement of the continuant John from London to New York. In this case x^t is the time-period over which John flies and x^s is the path along which John moves from London to New York, i.e., $\Lambda^\mathcal{C}_P(John, x^t, x^s)$.

Occurrents (events and processes) do not change location – they just occur. Consequently for occurrents only the notions L_R and $\Lambda^\mathcal{C}_R$ are relevant. An event e is (approximately) located at a spatio-temporal region which is specified by the pair $x = (x^t, x^s)$, i.e., $\Lambda^\mathcal{C}_R(e, x^t, x^s)$, if and only if there is a continuant on which e depends such that $\Lambda^\mathcal{C}_R(\textit{continuant-of}(e), x^t, x^s)$. Consider the event of John's flight to New York. In this case x^t is the time-period over which the flight occurs and x^s is the path along which the continuant John moves from London to New York.

A process p is (approximately) located at a spatio-temporal region which is specified by the pair $x = (x^t, x^s)$, i.e., $\Lambda^\mathcal{C}_R(p, x^t, x^s)$, if and only if there is a continuant on which p depends such that $\Lambda^\mathcal{C}_P(\textit{continuant-of}(p), x^t, x^s)$. Consider the process *John's flying to New York*. In this case x^t is the time-period over which the process of flying occurs and x^s is the path along which the continuant John moves from London to New York.

That the event *John's flight* and process *John's flying* are located at the same spatio-temporal region specified by $x = (x^t, x^s)$ is due to the fact that event and process represent two different views of the same change of location of the continuant John. Those different views, however, cannot be taken simultaneously in the same context.

5.2 Relations

Given the representation of spatio-temporal regions as pairs of spatial and temporal regions the next natural thing to do is to define relations between them. The obvious way of doing this is to define relations between two spatio-temporal regions as a *pair* of temporal and spatial relations based on relations between their spatial and temporal components.

Consider two spatio-temporal regions $x_1 = (x^t_1, x^s_1)$ and $x_2 = (x^t_2, x^s_2)$. We define identity and overlap-sensitive relations based on distinguishing identity (=), proper overlap which excludes identity but includes containment (o), and non-overlap (ø) relations among spatial regions and among temporal regions. This gives raise to nine combinatorial possible spatio-temporal relations: [3]

$R^{st}(x_1, x_2)$	1	2	3	4	5	6	7	8	9
$R^t(x_1^t, x_2^t)$	$=$	$=$	$=$	o	o	o	$\emptyset$	$\emptyset$	$\emptyset$
$R^s(x_1^s, x_2^s)$	$=$	o	$\emptyset$	$=$	o	$\emptyset$	$=$	o	$\emptyset$

(8)

In order to interpret these formal definitions we need to take into account that a given pair $x = (x^t, x^s)$ can specify: (a) the location (the place) at/in which the continuant c is at rest ($L_R(c, x^t, x^s)$ or $\Lambda_R^{\mathcal{C}}(c, x^t, x^s)$); and (b) the region within (the path along) which the continuant c changes location (moves), i.e., $L_P(c, x^t, x^s)$ or $\Lambda_P^{\mathcal{C}}(c, x^t, x^s)$). This will be discussed in the Sections 6, 7, and 8.

6 Relations between occurrents and continuants at rest

6.1 Same time – same place ...

Consider two spatio-temporal regions $x_1 = (x_1^t, x_1^s)$ and $x_2 = (x_2^t, x_2^s)$ within which the continuants o_1 and o_2 are at rest or within which the occurrents o_1 and o_2 occur (or any other combination of occurrents and continuants at rest). We will use the notion of place in order to refer to spatial locations where certain continuants are at rest and where certain events occur. The relation defined in Table 8 are now interpreted as follows: (1) - same-time-same-place (stsp); (3) - same-time-different-place (stdp); (7) - different-time-same-place (dtsp); (9) - different-time-different-place (dtdp).

Consider the following examples: (a) Imagine that the continuants John and Mary are on the same airplane from London to New York. The events John's flight and Mary's flight to New York occur in the same place and at the same time. (b) The processes of John's and Mary's flying to New York occur in the same place and at the same time. (c) The philosophers Marx and Engels did much of their early work while being together in London (same-time-same-place). Later they were able to do collaborate while being in different places at the same time, with Engels in Manchester and Marx in London (same-time-different-place). (d) In order to catch the flu from another person both must be in the same place at the same time. (e) John and Mary missed their date because they were in different places at the agreed time, i.e., John was stuck in a traffic jam while Mary was waiting in the agreed place. (f) John called Mary and they agreed to meet in a different place at a different time (e.g., next day at the McDonald's by the interstate ramp). (g) When they finally met (same-place-same-time) they learned that they could not stand each other and avoided each other from this moment, thereafter they were always at different places at the same time.

[3] There have been defined more detailed sets of spatio-temporal relations, e.g., (Bennett, Cohn, Wolter & Zakharyaschev To appear) or (Hazarika & Cohn 2001). The set of relations considered here, however, proves to be particularly useful for the discussion of the reference to spatio-temporal relation in judgments.

6.2 Judgment, context, and reference to relations

Obviously, one can find plenty of examples of sentences that make use of the spatio-temporal relations corresponding to the cases 1,3,7,9 in Table 8. However these relations are only a subset of the combinatorially possible cases. There do not seem to be equally good examples of sentences that make use of spatio-temporal relations that are defined by the cases 2, 4, 5, 6, 8 in Table 8. Those cases involve spatial or temporal overlap $x_1^t \; o \; x_2^t$ or $x_1^s \; o \; x_2^s$.

From Equation 6 it follows that the regions which satisfy the relation $\Lambda_R^{\mathcal{C}}$ in a context, C, are mutually disjoint since they belong to a single level of granularity. Consequently, when considering relations between those regions (x_1^t, x_1^s) and (x_2^t, x_2^s) with $x_1^t, x_2^t \in \mathcal{L}_{\mathcal{G}_{\mathcal{C}}^t}$ and $x_1^s, x_2^s \in \mathcal{L}_{\mathcal{G}_{\mathcal{C}}^s}$ only the patterns 1, 3, 7, and 9 in Table 8 can occur.

Of course, there do exist events and processes that do overlap (without being identical). Furthermore all relations given in Table 8 do actually occur between certain occurrents in reality and, hence, can potentially be referred to in sentences regarding these matters. However it is important to distinguish between the occurrence of relationships in reality and the reference to those relations in language. Moreover it is critical to consider *judgments* rather than sentences (Smith & Brogaard 2001).

A judgment is a sentence uttered by a judging subject (the judger) in a certain context. [4] Consequently, when considering reference to spatio-temporal relations it is critical to take the context-dependence of approximate location into account. Approximate location in a given context is determined by the spatial and temporal levels of granularity and the approximate localization function associated to the context. At the formal level this is reflected by Equation 6 leading to the fact that reference to relations determined by the patterns 2, 4, 5, 6, and 8 in Table 8 does not occur in judgments that are made in naturally occurring contexts.

6.3 Choosing levels of granularity

We now discuss how judging subjects choose levels of granularity. We concentrate hereby on the spatial case. In this context we make the following assumptions: (i) Judging subjects have a certain degree of freedom in choosing systems and levels of granularity; (ii) Judging subjects as-

[4] We limit ourselfs to judgments that can be uttered in naturally occurring contexts and admit the possibility to construct 'strange' counter examples in certain circumstances.

sociate the chosen level of granularity to the context before uttering the judgment in question; and (iii) There is no vagueness involved and the judging subjects have complete knowledge in the sense that there is no need to choose a coarse level of granularity due to vagueness of lack of knowledge.

Imagine that John and Mary's second date was arranged to take place in a certain downtown bar. Consider the time while Mary waited on her table for John to arrive and consider the following scenarios : (a) John was the whole time while Mary was waiting on her table stuck in the restroom due to his diarrhea; and (b) John was sitting on Mary's table.

Assume a system of granularities containing a cell 'the downtown bar' (B) with subcells guest room (G) and restroom (R) where the guest room has a separate subcell for the neighborhood of each table (T_i). In judgments then reference to the following spatio-temporal relations can be made:

J_1	$John$ *stsp* $Mary$	$\Lambda_R(John, B)$	$\Lambda_R(Mary, B))$
J_2	$John$ *stdp* $Mary$	$\Lambda_R(John, G)$	$\Lambda_R(Mary, R))$
J_3	$John$ *stsp* $Mary$	$\Lambda_R(John, T_i)$	$\Lambda_R(Mary, T_i))$

Assume now that the judgments, J_i, are made in a context where the judger has the intention to specify whether or not John and Mary have met. In situation (a) the judgments J_1 and J_2 can potentially uttered truly and in situation (b) the judgments J_1 and J_3 can potentially uttered truly. However in situation (a), given the intention of the judger, she will choose the level of granularity allowing her to distinguish the guest room from the restroom and she will judge J_2 rather than J_1. In situation (b) on the other hand she will judge J_3 rather than J_1.

It is important to emphasis the judger has the freedom and the obligation to choose the level of granularity in such a way that the judgment is true either in situation (a) or in situation (b) but not in both. This is because given the freedom to choose an appropriate level of granularity – why would a judger who's intension is to make a meaningful judgment (rather than to make a joke or to utter nonsense) use choose a level of granularity such that a judgment (implied by J_i) about whether or not John and Mary have met is subject to truth-value indeterminacy? Since there are no such reasons in naturally occurring contexts we hold that levels of granularity are chosen in such a way that the resulting judgments are meaningful and determinate.

7 Movement along paths

Consider the relation $\Lambda_P^{\mathcal{C}}(o, x^t, x^s)$ and assume that it holds for the object o, the temporal region x^t and the spatial region x^s. As discussed in Equation 7 the region x^t is an element of a level of granularity $\mathcal{L}_{\mathcal{G}_{\mathcal{C}}^t}$ belonging to a system of granularities $\mathcal{G}_{\mathcal{C}}^t$ and the region x^s is the mereological sum of elements of a level of granularity $\mathcal{L}_{\mathcal{G}_{\mathcal{C}}^s}$ belonging to a system of granularities $\mathcal{G}_{\mathcal{C}}^s$.

In our examples so far we (intuitively) always interpreted the region x^s as a path of movement rather than as the sum of regions occupied over a sequence of arbitrary changes of location. Let us now more carefully distinguish movement from arbitrary change of location like change of shape, growth and shrinking. Let $x_1, \dots, x_n$ be (pair-wise non-identical) regions at which the continuant o was exactly located at $\tau_1 \dots \tau_n$. We say that the change of location is movement if and only if the joint intersection of all those regions is empty. [5]

In Definition 7 we demanded that the regions summing up x^s need to belong to the same level of granularity. Since regions forming a single level of granularity are pair-wise disjoint it follows that for change of location, which is not movement in the sense defined above, the notions $\Lambda_P^{\mathcal{C}}$ and $\Lambda_R^{\mathcal{C}}$ coincide in the sense that given a fixed level of granularity we have $\Lambda_P^{\mathcal{C}}(o, x^t, x^s)$ if and only if $\Lambda_R^{\mathcal{C}}(o, x^t, x^s)$. Consequently it is sufficient to consider the approximation of location of continuants that move along paths.

7.1 Same path – same time ...

Let the spatio-temporal regions $x_1 = (x_1^t, x_1^s)$ and $x_2 = (x_2^t, x_2^s)$ be the motion-*paths* along which the continuants o_1 and o_2 change location (move). In the case of approximate location the relation $\Lambda_P^{\mathcal{C}}(o, x^t, x^s)$ holds and x^t belongs to a level of granularity and x^s is the sum of regions belonging to some level of granularity.

The relations 1, 3, 7, 9 in Table 8 are now interpreted as follows: (1) - same-time-same-path; (3) - same-time-different-path; (7) - different-time-same-path; (9) - different-time-different-path.

Consider the following examples: (a) The flying of the continuants John and Mary from Chicago to New York in the same airplane is an example for same-time-same-path. (b) If John flew on Monday and Mary flew on Tuesday then this would be an example for different-time-same-path. (c) If John flew to New York on Monday morning and Mary flew to Los Angeles the same morning then between their flying the relation same-time-different-path would hold. (d) If John flew to New York on Monday and Mary flew to Los Angeles on Tuesday, then this would be an example for different-time-different-path.

[5] This is a very rough definition and it is certainly possible to construct counter-intuitive examples (e.g., a car that moves only a few inches). However the definition does capture the intuition that if something moves then usually its end-location is disjoint from the one it started from.

Of course, for each relation in Table 8 there are continuants in reality which change in such a way that those relations hold. Consider the case (4). An example would be two continuants moving (e.g., two cars) along the same path and the time of their journey overlaps without being identical (e.g., one started earlier or one finished earlier or ...). Consider the cases (2) and (5). Examples are such that the paths of two moving occurrents do *cross* each other. For example consider the paths of airline-passengers with connecting flights who share the same airplane for a part of their journeys.

But, again, it is important to distinguish between the occurrence of relationships in reality and the reference to relations in judgments. For example, it is hard to imagine a context in which a judging subject (the judger) would use (4) in order to actually make a judgment. For example, in the case of a car-race she would rather say that one car finished earlier than the other (and thus won). In this case she uses a set of relations with a *finer granularity* at the temporal level than the relations in Table 8. She might use, for example, the Allen relations (Allen 1983), which allow her to distinguish: relations like starts, finishes, or during which are refinements of proper overlap as defined above.

The same argument holds in the case the relations (2) and (5). A judging subject would use relations of a finer level of granularity in order to make judgments. She would use spatio-temporal relations in order to distinguish continuants moving along forking paths, merging paths, or properly crossing paths. (These relations, of course, are refinements of the cases (2) and (5).)

7.2 Refined relations

Consider Figure 2 which shows a number of paths $p_1, \ldots p_4$ in a street network with block structure. Along those paths move the continuants $o_1 \ldots o_4$ over the time-period x^t. The street network forms a system of granularities with cells: 1st, 2nd, 3rd, 4th street; a, b, c avenue; block segments $(1, r_1)$, $(1, r_2)$, ..., (a, c_1), (a, c_1), etc. Those cells form levels of granularity in the obvious way. We call the level of granularity which is formed exclusively by segments *the block level* $\mathcal{L}_B$.

In the context $\mathcal{C}$ we specify approximate spatial location at block level, e.g., $\Lambda_P^{\mathcal{C}}(o_1, x^t, p_1)$, ..., $\Lambda_P^{\mathcal{C}}(o_4, x^t, p_4)$ with $p_1 = (a, c_2) + (2, r_2) + (b, c_3) + (3, r_3)$ and $p_2 = (b, c_2) + (b, c_3) + (b, c_4)$.

In order to understand why in the case of judgments about spatio-temporal relations between continuants moving along paths a finer set of relations is used than the one given in Table 8 we need to take three properties of paths into account: (i) Paths like $p_1 = g_1 + \ldots + g_n$ and $p_2 = h_1 + \ldots + h_m$ (with $g_i, h_j \in \mathcal{L}_B$) are formed by sets of

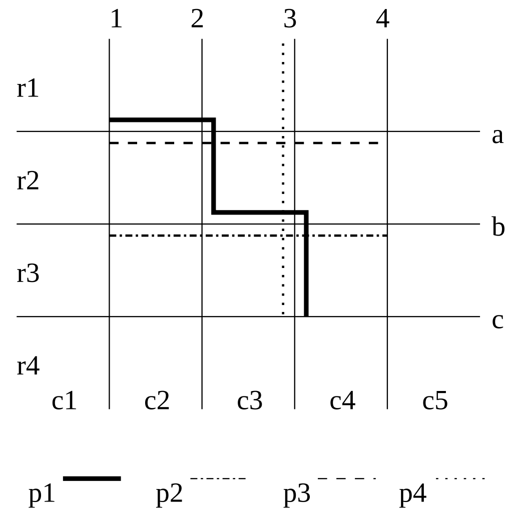

Figure 2: The paths $p_1 \ldots p_4$ in a city block network formed by 1st, 2nd, 3rd, 4th street and a, b, c avenue.

cells which elements can potentially overlap; (ii) Paths like p_1 and p_2 have a mereo-*topological* structure in the sense that some cells are externally connected; (iii) Paths like p_1 and p_2 have an internal *ordering* structure in that sense that connected cells form chains with beginnings and endings.

Mereological overlap. Consider the spatio-temporal regions (x^t, p_1) and (x^t, p_2). The spatial regions p_1 and p_2 are mereological sums of cells of a level of granularity rather than single cells in that level, e.g., $p_1 = (a, c_2) + (2, r_2) + (b, c_3) + (3, r_3)$ and $p_2 = (b, c_2) + (b, c_3) + (b, c_4)$. Those sums of cells can potentially overlap in the sense that, when considered as sets (e.g., $\{(a, c_2), (2, r_2), (b, c_3), (3, r_3)\}$), they share constituting cells. For example, in Figure 2 we have $(p_1 \; o \; p_2)$, $(p_1 \; o \; p_3)$, and $(p_1 \; o \; p_4)$. [6]

However, as argued above, in judgments about relations between paths of movement humans usually would make finer distinctions by saying that the paths p_1 and p_2 cross each other, while the paths p_1 and p_3 fork and the paths p_1 and p_4 merge. These distinctions are possible at the level of approximations because in most contexts, given paths of the form $p_1 = (a, c_2) + (2, r_2) + (b, c_3) + (3, r_3)$, we can define an ordering $(a, c_2) \prec (2, r_2) \prec (b, c_3) \prec (3, r_3)$. In order to see this we need to take topological and ordering structure into account.

Topological structure. Levels of granularities that are used to specify approximate location of moving continu-

[6] Notice that the overlap of the paths p_2 and p_4 as well as the overlap of the paths p_3 and p_4 cannot be referred to at the block level of granularity since there are no shared cells.

ants are *full* in the sense that their mereological sum is identical to the next superordinate unit in the underlying system of granularity (Section 4.2). If we assume that cells forming systems of granularity are topologically simple then it follows that cells forming a level of granularity are either externally connected or they are disconnected in the sense that they either do share boundary parts or they do not. If we assume continuous movement then it follows that paths of movement are formed by chains of externally connected cells.

Ordering structure. We now assume that paths of movement correspond to those chains which are not self-intersecting, i.e., we omit cases where the continuant under consideration drives around the block in search for a parking spot. The order in which the cells appear in chains such as $(a, c_2) \prec (2, r_2) \prec (b, c_3) \prec (3, r_3)$ corresponds to the order in which the continuant in question was located at them at subsequent periods of time. We now can distinguish the begin and the end of a chain of cells as the minimal and the maximal element of a chain with respect to the ordering $\prec$.

Let p_i and p_j be two paths of movement such that $\Lambda_P^{\mathcal{C}}(o_i, x^t, p_i)$ and $\Lambda_P^{\mathcal{C}}(o_j, x^t, p_j)$. We say that (a) p_i and p_j fork if and only if $p_i \neq p_j$ and $\min(p_i) = \min(p_j)$; (b) they merge if and only if $p_i \neq p_j$ and $\max(p_i) = \max(p_j)$; and (c) they cross if and only if $p_i \cap p_j \neq \emptyset$ and $\{\min(p_i), \min(p_j), \max(p_i), \max(p_j)\} \cap (p_i \cap p_j) = \emptyset$.

Obviously, those definitions only account for simple situations, however we suggest that due to the feature of granularity it is those simple situations that are referred to in judgments. Also, if $p_i \subset p_j$ then we can distinguish the Allen-relations relations p_i *starts* p_j, p_i *finishes* p_j, and p_i *during* p_j in a similar fashion.

8 Relations between places and paths

Consider two spatio-temporal regions $x_1 = (x_1^t, x_1^s)$ and $x_2 = (x_2^t, x_2^s)$ representing spatio-temporal location. Let x_1 be the location (place) of rest of a continuant or the place at which an event or process occurs, i.e., $\Lambda_R^{\mathcal{C}}(o_1, x_1^t, x_1^s)$, and let x_2 be the motion-path of the continuant o_2, i.e., $\Lambda_P^{\mathcal{C}}(o_2, x_2^t, x_2^s)$. The relations in Table 8 are now interpreted as antisymmetric relations with places as first and paths as second argument. The patterns are interpreted as follows: (2,5) - place-on-path(-at-the-same-time), (8) place-on-path-at-different-time, (3,6,9) - place-not-on-path. Hereby (2,5) abbreviates that either pattern 2 or 5 holds.

Consider the following examples. *place-on-path*: The oil-slick on the path of the nuclear waste transport; the speed trap on the path of your journey home. *place-on-path-at-different-time*: the oil-slick that was removed from the road before the nuclear waste transport took this path. *place-not-on-path*: The traffic jam on route I95 while you are traveling home on route I90; or The North Pole and your journey to the South Pole.

In order to justify this grouping and this interpretation of the patterns in Table 8 we need to discuss the spatial and the temporal dimension separately.

8.1 The spatial dimension

Consider the relations in Table 8. The use of spatio-temporal relations in judgments about spatio-temporal relationships between places and paths[7] seems to be limited to the mentioned prototypical cases. The cases 1, 4, and 7 defined in Table 8 do not occur in judgments about relationships between places and path. This is because the spatial extension of the places and the paths, that are involved in those relations, must be (significantly) distinct in size. To see this consider the following:

The special case of spatial co-location of place and path occurs in cases similar to the following. Consider the spatial location of the event 'John's flight to New York', the process 'John's flying to New York', and the movement of the continuant 'John' from London to New York. Here pattern 1 in Table 8 does potentially hold. However the event 'John's flight', the process 'John's flying', and the movement of the continuant'John' represent distinct views onto the same continuant (John) within the same time-period. However only *one* of those views can be taken in a single context. Consequently patterns 1, 4, and 7 in Table 8 cannot be used to specify a relation in a judgment which is bound to a single context.

In general the spatial extension of the place is (much) smaller than the extension of the path, i.e., the maximal diameter of the place is (much) smaller than the length of the path. Given the definitions of $\Lambda_R^{\mathcal{C}}$ and $\Lambda_P^{\mathcal{C}}$ (Equations 6 and 7) this can be understood as follows. A judgment is made in a specific context and within that context we have a *single* level of granularity for spatial reference. Cells forming places as well as the cells summing up paths belong to this level. Consequently, paths formed by a *multitude* of cells therefore must be (significantly) larger than the place formed by a single cell. (We assume that cells at same level of granularity are of roughly the same size.)

Notice that natural language seems to make additional distinctions regarding the scale of the difference of the size

[7] We use the phrase 'relation between place and path' as an abbreviation for 'relation between the place x_1^s at which the event o_1 occurs (the continuant o_1 is at rest) over the interval x_1^t and the path of movement, x_2^s, of a continuant o_2 over the period of time x_2^t'.

between places and path by distinguishing relationships like place-*on*-path ('There was an oil-slick on his path') or place-*along*-path ('There was a fire along his path') in the sense that the underlying level of granularity in the case of place-*on*-path is much finer than in the case of place-*along*-path.

There are cases where the spatial extension of the place is (much) larger than the extension of the path. In those cases the path is for example *in* the place as in 'When inside the Federal Building take the elevator to the 13th floor'. In this case, however, the place is used in order to specify the approximate location of the path in the sense of $\Lambda_R^{\mathcal{C}}$ and we are dealing with a different set of relations.

8.2 The temporal dimension

Consider the definitions of place-on-path(-at-the-same-time) and place-not-on-path. At the temporal level we allow for $x_1^t = x_2^t \vee x_1^t \ o \ x_2^t$ in the case of place-on-path and for $x_1^t = x_2^t \vee x_1^t \ o \ x_2^t \vee x_1^t \ \emptyset \ x_2^t$ in the case of place-not-on-path. This indicates that the relations in Table 8 are *too fine* in granularity in order to be used directly in judgments about relations between paths and places.

Consider the judgment 'the oil-slick is on the path of the nuclear waste transport'. For the binary relation 'oil-slick' place-on-path 'path of the nuclear waste transport' to hold the existence of the oil-slick must overlap temporally the existence of the spatio-temporal object 'path-of the nuclear waste transport'. Overlap here is meant in a sense that includes identity. The oil-slick might have existed before and might continue to exist afterwards. It might be placed on the road after the transport has begun its journey. It might even have been removed by a security team before the transport has actually arrived at this particular place. Similar examples can be found for the relation place-not-on-path.

9 Discussion

In an attempt to understand the way people make judgments about relations between spatio-temporal objects it is important to understand the role of systems and levels of granularity utilized in the process of judging.

Judgments about spatio-temporal relations are based on the specification of approximate spatio-temporal location which is based on reference to hierarchically organized systems of cells forming levels of granularity. When making a certain judgment the judging subject has a certain degree of freedom to chose a system and a particular level of granularity which she considers appropriate in order a judgment in a *determinate* fashion and in order to convey useful information. However the *degree* of freedom of choice is different in the case of (a) specifying location events and processes or continuants at rest using the relation $\Lambda_R^{\mathcal{C}}$ and in the case (b) of specification of change of location of a continuant along a path by means of the relation $\Lambda_P^{\mathcal{C}}$.

If we assume that judgments are made in such a way that they convey *determinate and useful information* then we can state the thesis that the more freedom the judging subject has in choosing an appropriate level of granularity the less relations are needed in order to specify spatio-temporal relations in a determinate manner. On the other hand the more restrictive the choice of level of granularity the richer the vocabulary that is needed in order to refer to spatio-temporal relations in a determinate fashion.

In other words there are two different strategies that judging subjects employ in order to make judgments that convey determinate and useful information: The adjustment of system and level of granularity of approximation of location in case (a) versus the refinement of the granularity of the relations in case (b). The remaining question now is why judging subjects have different degree of freedom of choosing system and level of granularity of approximation.

The answer to this question can be found by analyzing the properties of the systems and levels of granularities underlying the judgments in question. Consider systems of granularities formed by hierarchically organized places that underlie judgments of type (a). Those systems of granularity have only a weak structure in the sense that they usually lack the property of fullness (Section 4.2) and, hence, also lack topological and ordering structure in the sense discussed in Section 7.2.

Due to the lack of structural properties the judging subject has the freedom to choose subsystems (like the one in Figure 1) and levels of granularity within them in ways which she considers to be appropriate without having to worry about preserving structural properties such as fullness, topology, and ordering. The advantage of those systems of granularities is that (1) they are easy to construct, and (2) they contain only cells which are relevant in the context at hand. This flexibility allows the judging subject to choose a level of granularity in such a way that only a few relations (a small vocabulary) such as same-time-same-place etc. (the relations corresponding to the pattern 1,3,7,9 in Table 8) are needed in order to make judgments in the required determinate fashion as discussed in Section 6.3.

Systems of granularities that underly judgments of type (b) are usually full and have sophisticated topological and ordering structure as discussed in Section 7.2. Those systems of granularity are in their structure firmly grounded in reality and therefore are less flexible in the sense that they are either used in judgments as they are or not used at all. On the other hand, their sophisticated structure allows for a rich vocabulary for specifying spatio-temporal relations as demonstrated in Section 7.2.

10 Related work

Qualitative spatio-temporal relations have been discussed widely in the literature, e.g., (Claramunt, Theriault & Parent 1997, Hazarika & Cohn 2001, Bennett et al. To appear). (Hazarika & Cohn 2001) propose a spatio-temporal logic between four-dimensional regions based on the notions of spatial, temporal, and spatio-temporal connectedness. (Bennett et al. To appear) proposed a multi-dimensional modal logic for spatio-temporal reasoning as a temporalized version of the region connection calculus (RCC). Consequently, at the spatial level they distinguish the eight RCC relations and at the temporal level they distinguish the thirteen Allen relations (Allen 1983, Randall et al. 1992). A similar set of relations was defined in (Claramunt et al. 1997) in a point-set theoretical framework employing Egenhofer's intersection model (Egenhofer & Franzosa 1991). Within those approaches spatio-temporal relations between *occurrents* based on their *exact* location can be represented.

(Stell 2001) takes the approximate nature of spatio-temporal location into account and defines qualitative spatio-temporal extents like somewhere-sometime, always-somewhere, etc. which are based on specifications of location with respect to full levels of granularity.

The issue of granularity was discussed in spatial and temporal contexts for example in (Bettini et al. 1998, Euzenat 1995, Stell & Worboys 1998), however mostly in full systems of granularity and not in the context of reference to relations in judgments. Spatial and temporal relations in the context of approximate spatial and temporal location were discussed in (Bittner & Stell 2000), (Bittner to appear). The present paper extends this work to the spatio-temporal domain, to non-full systems of granularities, and by explicitly taking the notion of context into account.

11 Conclusions

We discussed qualitative relations between spatio-temporal objects based on their location in space and time. The location of a spatio-temporal object in a region of space or time may be exact or approximate. Exact location emphasizes the exact fit between object and region. A spatio-temporal object is located approximately in a region of space or time if its exact region is a part of this region. There are multiple ways of approximating exact location at different levels of granularity.

Critical for the understanding of qualitative relations between spatio-temporal objects is the distinction between continuants and occurrents and the closely related distinction between rest and change. Only continuants are capable of change and hence of rest. Occurrents do not change. They just occur and characterize modes of rest or change of occurrents. The spatio-temporal occurrence of events and processes is closely related to the notion of place in the sense that events and processes occur in places. The change of location of continuants takes place along paths.

We defined and discussed relations between various kinds of spatio-temporal objects. This discussion showed that the formalization of spatio-temporal relations needs to take into account the notion of context. While people seem to deal effortlessly with the different meanings of relations like same-place-same-time in varying contexts, it is hard to formalize the context dependence that is thereby involved. We addressed the problem of context by distinguishing relationships that hold between spatio-temporal objects from the reference to spatio-temporal relations in judgments made in specific contexts.

A judgment in a specific context is characterized by: (a) a statement about spatio-temporal relation between two objects; (b) the types of those objects (continuants or occurrents); (c) a certain level of granularity at which the approximate location of the two objects is specified; (d) a certain level of granularity of the particular spatio-temporal relations that are used in this judgment. The levels of granularity of approximate location and spatio-temporal relations are chosen by the judging subject in such a way that the resulting judgment provides determinate and useful information (is not subject to truth-value indeterminacy).

The relationship between the level of granularity of approximate location and the level of granularity of the spatio-temporal relations at hand is quite complex. We showed that: (1) In contexts where a judgment was made about relationships between (among) occurrents or continuants at rest, the level of granularity of approximate location is chosen in such a way that a small number of relations at an intermediate level of granularity is used (same-place-same-time, and so on). (2) In contexts where a judgment is made about relationships between changing continuants, the level of granularity of approximate location is relatively fixed and cannot be chosen by the judging subject as freely as in the previous case. However in those contexts a finer set of spatio-temporal relationships is used in order to make the distinctions that are necessary in order to make a determinate judgment.

Ongoing research is directed towards giving a more formal account of the context-dependence involved in judgments about spatio-temporal relations. This work is based on the theory of granular partitions proposed in (Bittner & Smith 2001*a*) and (Bittner & Smith 2001*b*).

Acknowledgements

Thanks are due to Barry Smith, Stephan Winter, and one of the anonymous reviewers for their helpful comments. The support by the Alexander von Humboldt Stiftung under the auspices of its Wolfgang Paul Programme, DARPA under the Rapid Knowledge Formation program, and the National Science Foundation under the Research on Learning and Education program is gratefully acknowledged.

References

Allen, J. (1983), 'Maintaining knowledge about temporal intervals', *Communications of the ACM* **26**(11), 832–843.

Bennett, B., Cohn, A., Wolter, F. & Zakharyaschev, M. (To appear), 'Multi-dimensional modal logic as a framework for spatio-temporal reasoning', *Applied Intelligence*.

Bettini, C., Wang, X. & Jajodia, S. (1998), 'A general framework for time granularity and its application to temporal reasoning', *Annals of Mathematics and Artificial Intelligence* **22**, 29–58.

Bittner, T. (to appear), 'Approximate temporal reasoning', *Annals of Mathematics and Artificial Intelligence*.

Bittner, T. & Smith, B. (2001*a*), A taxonomy of granular partitions, *in* D.R.Montello, ed., 'Spatial Information Theory, COSIT '01', Vol. 2205 of *Lecture Notes in Computer Science*, Berlin/New York: Springer, pp. 28–43.

Bittner, T. & Smith, B. (2001*b*), 'A theory of granular partitions', *Department of Computer Science, Northwestern University*.

Bittner, T. & Stell, J. (2000), Rough sets in approximate spatial reasoning, *in* 'Proceedings of the Second International Conference on Rough Sets and Current Trends in Computing (RSCTC'2000)', Lecture Notes in Computer Science (LNCS), Springer-Verlag.

Casati, R. & Varzi, A. (1995), 'The structure of spatial localization', *Philosophical Studies* **82**(2), 205–239.

Claramunt, C., Theriault, M. & Parent, C. (1997), A qualitative representation of evolving spatial entities in two-dimensional spaces, *in* S. Carver, ed., 'Innovations in GIS V', Taylor & Francis, pp. 119–129.

Egenhofer, M. J. & Franzosa, R. D. (1991), 'Point-set topological spatial relations', *International Journal of Geographical Information Systems* **5**(2), 161–174.

Euzenat, J. (1995), An algebraic approach for granularity in qualitative space and time representation, *in* 'Proceedings of the IJCAI-95', pp. 894–900.

Hazarika, S. M. & Cohn, A. G. (2001), Qualitative spatio-temporal continuity, *in* D. R. Montello, ed., 'Spatial Information Theory: Foundations of Geographic Information Science; Proceedings of COSIT'01', Vol. 2205 of *LNCS*, Springer Verlag, pp. 92–107.

Randall, D., Cui, Z. & Cohn, A. (1992), A spatial logic based on regions and connection, *in* B. Nebel, C. Rich & W. Swartout, eds, 'Principles of Knowledge Representation and Reasoning. Proceedings of the Third International Conference (KR92)', Morgan Kaufmann, pp. 165–176.

Rigaux, P. & Scholl, M. (1995), Multi-scale partitions: Application to spatial and statistical databases, *in* M. Egenhofer & J. Herrings, eds, 'Advances in Spatial Databases (SSD'95)', number 951 *in* 'Lecture Notes in Computer Science', Springer-Verlag, Berlin.

Simons, P. (1987), *Parts, A Study in Ontology*, Clarendon Press, Oxford.

Smith, B. & Brogaard, B. (2001), 'A unified theory of truth and reference', *Logique et Analyse*.

Smith, B. & Brogaard, B. (to appear), 'Quantum mereotopology', *Annals of Mathematics and Artificial Intelligence*.

Stell, J. & Worboys, M. (1998), Stratified map spaces: A formal basis for multi-resolution spatial databases, *in* T. K. Poiker & N. Chrisman, eds, 'SDH'98 Proceedings 8th International Symposium on Spatial Data Handling', International Geographical Union, pp. 180–189.

Stell, J. G. (2001), Qualitative extents for spatio-temporal granularity, *in* 'Workshop on Spatial Vagueness, Uncertainty and Granularity'.