Granularity in reference to spatio-temporal location and relations

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Abstract

This paper discusses the role of granularity in judgments about spatio-temporal location and relations. It distinguishes three aspects of spatio-temporal granularity: the granularity with which location is approximated, the granularity which arises as a function of the degree to which we recognize or trace over changes of location, and granularity as it affects our apprehension of spatio-temporal relations. The paper shows that there is a hierarchical structure of spatio-temporal granularity that is closely related the hierarchical structure of of places in our environment and to the hierarchical structure of temporal orders such as embodied in calendars and agendas.

Introduction

This paper analyzes how reference to spatio-temporal relations is made in natural language. Following (Smith & Brogaard 2001), (Smith & Brogaard to appear), and (Bittner 2002) it argues that to provide a satisfactory analysis of the ways such reference is effected sentences need to be considered not in the abstract but rather as *judgments*. A judgment is a sentence uttered by a judging subject in a specific situation. In order to talk in an abstract manner about a situation in which a sentence is uttered we introduce the notion of *context* and define a judgment, J, as a pair, (S, C), consisting of a sentence, S, and a context C. The context determines the way a judging subject projects onto reality (Smith & Brogaard 2001; to appear; Bittner & Smith 2001). Projection hereby can be imagined in analogy with a battery of flashlights which serve to bring certain objects in a dark room into the light while leaving others invisible. Consequently, projection determines which objects, properties, and relations are in the foreground of the attention of the judging subject when uttering a sentence S. Consider the sentence 'John and Mary were both in Hyde Park on Monday morning', as this is uttered by Larry King on Monday evening when reporting about John and Mary's whereabouts on CNN. In this case the foreground domain includes objects like John, Mary, Hyde Park, Monday morning, and the relationship same-place-same-time.

Projection not only determines which objects are in the foreground of the attention of the judging subject but also the level of granularity at which those objects are considered. For example, when reporting about John and Mary, Larry King does not bother about the individual organs, cells, or molecules John and Mary are made up of. These features are below the *level of granularity* which he employs in projecting onto reality in this specific situation. Consequently, atoms, cells, etc., are in the background of his attention. Also, Larry King does not bother about the fact that Hyde Park is in London, nor about the fact that London is in Great Britain, that Great Britain is in Europe and so forth. This is because these are facts about objects above the level of granularity in the given context. Consequently, granularity allows judging subjects (Larry King in our example) to project their attention onto objects without also projecting (i) onto their parts; and (ii) onto those larger parts of reality of which the objects under consideration are themselves parts.

Besides these rather obvious features, however, spatio-temporal granularity has more subtle aspects, which are in the center of our focus of the present paper. Consider, again, the sentence 'John and Mary were both in Hyde Park on Monday morning' uttered in the context described above. If this judgment is true, then the spatio-temporal relation *same-place-same-time* holds between John and Mary because both were in Hyde Park on Monday morning. However what does 'being in the same place at the same time' exactly mean? Moreover why does Larry King refer to 'Hyde Park' rather than to the park bench on which they sat, or to London? For all he knows he could also have reported the latitude and the longitude of the GPS-location of the cell phones of John and Mary – so why did he not do so?

Another issue is the following. To describe the very same matters of fact Larry King could also have used the relation *same-time-different-place*, thereby separately referring to John sitting on the left side of the bench and Mary sitting on the right side of the bench. He might have done this, for example, in order to emphasis that John and Mary were in the park as separate individuals; they did not go there together.

The point here is that the issues of the context and of the level of granularity in which the judgment is made are very complex and involve many factors. It is the task of this paper to characterize different aspects of granularity which are relevant for the specification of spatio-temporal relations in judgment made by judging subjects.

In general, the notion of granularity is closely related to the notion of indiscernibility, which refers to the fact that objects, properties, or relations which are beneath a certain level of resolution (not necessarily related to size) are *indiscernible*, i.e., not capable of being distinguished. Following Hobbs (Hobbs 1985) we write $x \sim y$ to mean $\forall p : p(x) \leftrightarrow p(y)$, where p ranges over predicates of some theory and $\sim$ symbolizes indiscernibility. We make the simplifying assumption that $\sim$ is an equivalence relation.

In this paper we consider indiscernibility of objects regarding their spatio-temporal location (or change therein), i.e., spatio-temporal granularity and indiscernibility. In particular we consider three aspects or forms of spatio-temporal indiscernibility corresponding to: (1) the granularity with which location is approximated; (2) the granularity which arises as a function of the degree to which we recognize or trace over changes of location; and (3) granularity as it affects our apprehension of spatio-temporal relations.

A spatio-temporal ontology

In order to provide a formal basis for the discussion of various forms of granularity we start by sketching an ontology of spatio-temporal objects, their locations in space and time, and relations that hold between spatio-temporal regions based on work published in (Bittner 2002).

Spatio-temporal objects fall into two major categories: continuants and occurrents (Simons 1987). Continuants are usually referred to the (atemporal) objects. This is because they may change over time – for example by gaining or losing parts, by growing older, by changing their location – but yet remain the same thing. Examples are human beings like John and Bill Clinton, cars, substances in general. Occurrents do not change over time. They just occur. One usually refers to them as events or processes rather than as objects. Examples are 'John's flight to New York', 'World War 2', 'The life of Bill Clinton', 'My childhood', and so on. The existence of an occurrent always depends on the existence of one or more continuants. The occurrent 'John's childhood' cannot exist without the continuant John.

We distinguish the domain of spatio-temporal objects, O, and the domain of regions, R. The domain of objects (regions) is the mereological sum of its constituent objects (regions). The domain of regions is constituted by regions of different dimensionality: four-dimensional spatio-temporal regions x, three-dimensional spatial regions, x^s, and one-dimensional temporal regions, x^t. Individual objects stand to individual regions in the relation of location.

Every (four-dimensional) spatio-temporal object, $o \leq O$ (o is a part of the domain of objects O), is exactly located at a single three-dimensional spatial region, x^s, at every instant of time, τ (Casati & Varzi 1995): $\forall o \leq O : \exists x^s \leq R : L^s_\tau(o, x^s)$. The region x^s is the exact or precise spatial location of o at the time instant τ. We say that the object o is located *at* the region x in order to stress the exact fit of object and region (the object matches the region).

Often, however, it is not very interesting to know that John is exactly located at that region of space from which the air is displaced by his body at a particular instant in time. It is much more interesting to know, for example, that John is in London or in Paris. We therefore define the notion of *approximate* location as a family of relations $\lambda^1_\tau, \ldots, \lambda^n_\tau$ and demand that the object o is approximately located in the region r, $\lambda^i_\tau(o, r)$ if and only if the exact location of o is a part of r: $\lambda^i_\tau(o, r) \equiv \exists y : L^s_\tau(o, y)$ *and* $y \leq r$. The λ^i_τ hereby represent approximate locations of o at τ at different levels of granularity, where level of granularity here refers to the size of the approximating regions.

Most continuants have different exact spatial locations at different times. We say that these objects change their spatial location. If we consider a temporal region (a period of time) during which the spatio-temporal object o existed, then o may be either (i) *at rest*, i.e., it may be located in the same region of space, x^s over the given period of time x^t, and write $L_R(o, x^t, x^s)$; or (ii) its spatial location may change, as a result of being located in different regions of space at different time-instants during this period. In this case we consider x^s to be the mereological sum of all locations visited over the period x^t and write $L_P(o, x^t, x^s)$. An example for (i) is the location of John on Monday morning in Hyde Park, i.e., $L_R(\textit{John}, \textit{Monday morning}, \textit{Hyde park})$, where Monday morning and Hyde Park specify approximate location. As an example for (2) imagine that John visits the Queen on Monday afternoon in Buckingham Palace and that we we intercept the communication of the Queen's Secret Services agents as they trace John's movement from the entrance through the hall to the guest-room. If we sum up all those regions, then we get John's approximate path of movement, i.e., $L_P(\textit{John}, \textit{Monday afternoon}, x^s)$, where x^s is the mereological sum of the rooms John visited in Buckingham Palace.

Formally we define:

$$L^\lambda_R(o, x^t, x^s) \equiv \forall \tau \in x^t : \exists x : \lambda_\tau(o, x) \textit{ and } x \leq x^s, \quad (1)$$

$$L^\lambda_P(o, x^t, x^s) \equiv x^s \geq \bigvee \{x \mid \lambda_\tau(o, x) \textit{ and } \tau \in x^t\}. \quad (2)$$

These definitions focus on approximate spatial location but 'contain' exact location as special cases. Definitions involving approximate temporal location are omitted here. See (Bittner 2002) for details.

We represent (four-dimensional) spatio-temporal regions as pairs of temporal and spatial regions, $x = (x^t, x^s)$. Consider two spatio-temporal regions $x_1 = (x^t_1, x^s_1)$ and $x_2 = (x^t_2, x^s_2)$. We define a set of identity and overlap-sensitive relations based on distinguishing identity (=), proper overlap which excludes identity but includes containment (o), and non-overlap (ø) relations among spatial regions and among temporal regions. This gives raise to nine combinatorial possible spatio-temporal relations (Bittner 2002):

$R^{st}(x_1, x_2)$	1	2	3	4	5	6	7	8	9
$R^t(x^t_1, x^t_2)$	=	=	=	o	o	o	ø	ø	ø
$R^s(x^s_1, x^s_2)$	=	o	ø	=	o	ø	=	o	ø

(3)

In order to interpret these formal definitions we need to take into account that a given pair $x = (x^t, x^s)$ can specify: (i) the location (the place) at which the continuant c is at rest ($L^\lambda_R(c, x^t, x^s)$) or the location (the place) in which the occurrent o occurs ($L_R^\lambda(o, x^t, x^s)$); and (ii) the region within which the continuant c changes location (moves), i.e., $L_P^\lambda(c, x^t, x^s)$.

It was shown in (Bittner 2002) that only a subset of the relations defined in Table 3, the relations 1, 3, 7, 9, are actually referred to in judgments occurring in naturally occurring contexts. In case (i) these relations are interpreted as: (1) - same-place-same-time; (3) - same-time-different-place; (7) - different-time-same-place; (9) - different-time-different-place. In case (ii) the relations 1, 3, 7, 9 in Table 3 are interpreted as: (1) - same-path-same-time; (3) - same-time-different-path; (7) - different-time-same-path; (9) - different-time-different-path. For a details see (Bittner 2002).

Granularity of approximation

The most obvious form of granularity occurs at the level of approximate specification of location, as in our Larry King example. Granularity here refers to the size of the region which approximates the exact location of the object in question. At each time-instant, τ, there is a single *context-free* exact location relation L_τ^s and arbitrarily many *context-dependent* approximate localization relations λ_τ^i. Approximate location is context-dependent because every context comes with its own family of approximate location relations, λ. Consequently, we assume that each context is characterized (1) by a specific period of time x^t over which it is active; and (2) by a corresponding family of approximate location relations λ which uniquely covers all instances τ in x^t.

The corresponding indiscernibility relation between objects o_1 and o_2 regarding their spatial location x^s in the context characterized by λ and x^t are is defined as:

$$o_1 \sim_{(\lambda, x^t)} o_2 \equiv L_\star^\lambda(o_1, x^t, x^s) \leftrightarrow L_\star^\lambda(o_2, x^t, x^s).$$

where $L_\star^\lambda$ expands to in L_R^λ and L_P^λ, which indicates that at this level we do not consider whether the object o is at rest at x^s over x^t or whether it moves/changes within x^s over x^t.

Examples are for reference to objects indiscernible regarding their spatio-temporal location are: 'the people in this building', 'the participants of this year's New York Marathon', 'the passengers of Flight 847', etc.

Recognizing or ignoring change

We argue that one can distinguish two major categories of occurrents: events and processes. Events are spatio-temporal objects whose existence depends on the occurrence of a certain continuant in a certain state, e.g., the occurrent 'John's childhood' corresponds to the continuant John persisting in the state of being a child. Processes, on the other hand, depend to the occurrence of change undergone by some continuant. For example, the process of 'John's growing up' corresponds to certain patterns of changes of the continuant John. We now shall argue that the matter of fact whether a continuant is in a certain state which does not change over a period of time or whether it undergoes a certain change during this period is a matter of granularity. It follows, therefore, that the distinction between events and processes is a matter of granularity too.

Consider the continuant John, who is flying aboard Flight 847 from London to New York. There are (at least) two co-located occurrents: the event of John's flight to New York and the process of John's flying to Now York. The event of John's flight depends on the continuant of John who is in the state of being an adult, healthy human individual who undertakes air-travel from London to New York. This state does not change while the event lasts. The process of John's flying, on the other hand depends of the change of location John's body undergoes while moving from London to New York. This change occurs as long as the process lasts.

The two occurrents (event and process) depend on the same continuant, John. Both exist over the same period of time. However the level of granularity under which a judging subject projects onto reality (the continuant John) when she is referring to the event of John's flight is distinct from the level of granularity under which she projects onto reality when she is referring to the process of John's flying.

When referring to the event John's flight the judging subject projects onto reality at a level of granularity that 'traces over' the fact that John's body is actually moving in a certain way. Under the level of granularity with which she projects onto reality the continuant John is in the state described above. For example, John's biographer could write a sentence like: 'This flight to New York marked the begin of a new chapter in John's live'.

When referring to the process of 'John's flying' then the judging subject projects onto reality at a level of granularity that focuses onto the fact that John's body is actually moving in a certain way. Referring to an occurrent as a process implies a finer level of granularity, one that does not trace over the properties of the change that occurs. For example, John's biographer could continue by writing 'The turbulences were so strong that people who had not fastened their seat belts were flying through the cabin'.

Consequently, if the occurrent to which the judging subject refers to is a process then she projects onto reality at a level of granularity that focuses on the properties of the changes that occur. This is because the process depends on the occurrence of these changes. If the judging subject refers to an event then she projects onto reality at a level of granularity that traces over the changes that occur. This is because the events depends on one or more continuants that are in a certain state.

We now can refine the indiscernibility relation between objects as regards the identification of their spatio-temporal location by taking into account whether the region x^s is: (i) a location at which the object is at rest; (ii) a region which is identical to the sum of locations visited over a period of time. More precisely in case (i) any change of location is below the level of granularity. In case (ii) objects are indiscernible only if they change in such a way that they visit the locations they visited within x^s sum up to the same region x^s. Formally we write:

$$\begin{array}{lll} \text{(i)} & o_1 \sim_R o_2 \equiv & L_R^\lambda(o_1, x^t, x^s) \leftrightarrow L_R^\lambda(o_2, x^t, x^s) \\ \text{(ii)} & o_1 \sim_P o_2 \equiv & L_P^\lambda(o_1, x^t, x^s) \leftrightarrow L_P^\lambda(o_2, x^t, x^s) \end{array}$$

Examples are (i) 'The people in this building', (ii) 'The participants of this year's New York Marathon'. Notice, however that in the second case we cannot distinguish the athlete jogging from start to finish from the drunk ‘athlete’ who gets mixed up with the race but takes the opposite direction.

Consequently, the different interpretation of the spatio-temporal relations (1,3,7,9) in Table 3 as relations between paths (e.g., 1 interpreted as same-*path*-same-time) and as relations between places (e.g., 1 interpreted as same-*place*-same-time) is a matter of spatio-temporal granularity. The different interpretations arise according to whether: (a) Judging subjects use the notion of place in order to refer to locations (spatial regions) in which objects are at rest. A judging subject considers a spatio-temporal object as being at rest if it does not change its approximation location according to level of granularity at hand. (b) Judging subjects use the notion of path in order to refer to certain spatial regions along certain kinds of change of location occurs (movement).

Granularity of sets of relations

In Table 3 we defined relations between spatio-temporal regions by considering the relations $=$, o, and $\emptyset$ among the relevant spatial and the temporal components of such regions. Since the nine relations we distinguished are jointly exhaustive and pair-wise disjoint they subdivide the domain of spatio-temporal regions into equivalence classes, i.e., they impose a granularity. Formally we define: $(x_1, x_2) \sim (y_1, y_2) \equiv \forall p \in \{=, o, \emptyset\} \times \{=, o, \emptyset\} : p(x_1, x_2) \leftrightarrow p(y_1, y_2)$. The indiscernibility relation thus defined applies to regions rather than to objects. Since we assume absolute space and time (regions do not change but objects do) the granularity imposed by $\sim$ is context independent.

The theory underlying the definitions in Table 3 is a fragment of mereology. Moreover they do not take the ordering underlying the temporal domain into account. As shown in (Bittner to appear) and (Bittner 2002) the relations in Table 3 can be refined by taking full mereology as well as by taking ordering relations into account. Consider, for example, the relation (7) in Table 3 in the interpretation of same-place-different-time. Taking the direction of the time-line into account same-place-same-time refines to same-place-before and same-place-after.

Refining the sets of spatio-temporal relations results into a finer granularity of the subdivision in the domain of pairs of objects. From the structure of the definitions of the relations provided in (Bittner to appear) it follows that the corresponding granularities are ordered hierarchically in such a way that multiple relations on a finer level of granularity are subsumed by a single relation on a coarser level.

Relations between spatio-temporal regions are context independent which means that for every pair of spatio-temporal regions there are relations that hold no matter what the context. However those relations are structured hierarchically and the judging subject has the freedom to choose the level of granularity which she considers to be most appropriate in order to make a judgment in a determinate and efficient manner. For example, overlap and identity sensitive relations like same-place-same-time and same-place-different-time are often used when a subject makes a judgment about causal relationships between spatio-temporal objects that require the objects to be in the same place at the same time. In those contexts finer distinctions at the level of relations (like same-place-before or same-place-after) are often irrelevant and therefore omitted by choosing a coarse level of granularity. On the other hand, when reporting the spatio-temporal locations of cars that were involved in a traffic accident to the jury in a curt of law the investigating officer will judge about spatio-temporal relations at the finest available level of granularity.

Ordering structure

We use the notation $\Gamma_\sim$ in order to refer to the set of equivalence classes, i.e., the granularity, generated or imposed by $\sim$. Different forms of granularity create different subdivisions of spatio-temporal objects and relations in into equivalence classes. We now define a partial ordering between the granularities Γ_i and Γ_j with indiscernibility relations $\sim_i$ and $\sim_j$:

$$\Gamma_i \preceq \Gamma_j \equiv \forall x, y : x \sim_i y \Rightarrow x \sim_j y.$$

This ordering forms a partial order with the domain itself as unique maximal element. There is a finest level of granularity if there are atoms.

As discussed above sets of jointly exhaustive and pair-wise disjoint spatio-temporal relations create hierarchically ordered granularities. It is important to remember that these granularities subdivide the domain of spatio-temporal regions (and are therefore context free). In order to link the granularity of pairs of regions to objects we need the notion of location, which is itself subject to granularity.

Given a fixed collection of objects, O, (atoms or not) and their spatial location at a certain time-instant the finest spatio-temporal granularity we can achieve is by considering the exact location of those objects. Notice, however that even in this case we cannot distinguish objects that are spatio-temporally exactly co-located. Let $\Gamma^O_{L^s_\tau}$ be the granularity imposed on O by the indiscernibility of exactly spatio-temporally co-located objects and let $\Gamma^O_{\lambda^i_\tau}$ be the granularity on O created by considering approximate location at the instant τ determined by λ^i_τ. $\Gamma^O_{L^s_\tau}$ is then of finer granularity than $\Gamma^O_{\lambda^i_\tau}$ since objects indiscernible in $\Gamma^O_{L^s_\tau}$ are also indiscernible in $\Gamma^O_{\lambda^i_\tau}$, i.e., $\Gamma^O_{L^s_\tau} \preceq \Gamma^O_{\lambda^i_\tau}$

Consider now the spatial location of the objects in O over the interval x^t. Since objects in O potentially change their location we consider the spatio-temporal location $L^\lambda_P(o, x^t, x^s)$ and assume that x^s is the mereological sum of the regions at which o was located exactly at some instant in x^t (the limit case in Equation 2)). This results in the granularity $\Gamma^O_{L_P}$. If we consider exact spatial location over time intervals x^t rather than at time instants τ then we gain a finer granularity, since now we can also distinguish objects that are only temporarily exactly co-located. Since we assume that x^s is the mereological sum of the regions at which o was located exactly at some instant in x^t, we can distinguish objects whose change of location does not sum up to the same region (motion-path) x^s. Consequently, we have $\Gamma^O_{L^s_\tau} \preceq \Gamma^O_{L_P}$ with $\tau \in x^t$.

Of course, there are multiple ways of imposing a granularity on a collection of objects by considering their approximate location. Given two of those granularities they may or may not be refinements of each other. In order to determine an ordering structure here we need to consider how granularity is imposed in specific judgments. Here the judging subject actively reaches out and in principle can choose arbitrary regions that satisfy definitions 1 and 2. If, however, we analyze language expressions that specify the approximate spatial location of some given object, then we see that the approximate location is specified with respect to some place, such as 'Hyde Park', 'Soho', 'The theater district', 'London', etc. Those places have the following properties: (a) they are well known and (b) they do themselves not change. Place with those properties serve as objects of reference for the description of approximate spatial location.

The situation is similar for approximate temporal location. In our example the approximate temporal location is given with respect to a calendar unit such as Monday morning. Calendar units such as minutes, hours, days, weeks, etc. partition the time line into jointly exhaustive and pairwise disjoint temporal regions at different levels of granularity and provide frames of reference. Again, an important aspects of the temporal frames of reference are that they are well known and do themselves not change. For details see (Bittner to appear).

It follows from the above that: (a) the hierarchical structure of spatial granularities of approximate specifications of spatio-temporal location coincides with the hierarchical structure of places in our environments; (b) the hierarchical structure temporal granularities of approximate specifications of spatio-temporal location coincides with the hierarchical structure embodied in our calendars; and (c) the hierarchical structure of spatio-temporal granularity is the cross-product of the hierarchical structures of the spatial and temporal components involved.

When making a judgment about spatio-temporal relations judging subjects traverse those ordering structures in order to find an appropriate level of granularity for specifying approximate location. Larry King chose 'Hyde Park' in order to specify the approximate spatial location of John and Mary and 'Monday morning' in order to specify their approximate temporal location. The choice of these levels of granularity allowed him to specify the relation same-place-same-time in order to describe the spatio-temporal relation between John and Mary. 'Hyde Park' is the most specific place he can expect his audience to know. Not many people in the United States are likely to know the placement of every bench in Hyde Park. Therefore referring to a specific bench would result in a true but useless judgment since it does not convey the intended information to the intended audience.

Conclusions

In this paper we analyzed three aspects of spatio-temporal granularity: (i) the granularity of our specifications of approximate location; (ii) the ways we recognize or trace over changes of location; (iii) granularity of spatio-temporal relations. Aspects (i) and (ii) determine the relationships between spatio-temporal objects and corresponding regions of space and time and involve the factor of context-dependence. Aspect (iii) relates to the hierarchical organization of spatio-temporal relations and so does not involve this factor.

The most important aspect of spatio-temporal granularity is the granularity in which approximate spatio-temporal location is specified. The hierarchical structure of spatio-temporal granularity is a function of the hierarchical structures of the underlying spatial and temporal components. The nature of temporal granularities is relatively well understood (Bettini *et al.* 1998). In order to gain a better understanding of levels of granularity in spatial approximations it is important to gain a better understanding of the way places and hierarchies of places are formed. A promising approach towards an understanding of this subject seems to be the theory of granular partitions proposed in (Bittner & Smith 2001).

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