Reasoning about qualitative spatio-temporal relations at multiple levels of granularity

Thomas Bittner[1]

Abstract. This paper discusses aspects of qualitative reasoning about approximate spatio-temporal location at multiple levels of granularity. We start by defining systems of granularities which are tree-like hierarchical structures and which are used as frames of reference in order to specify approximate location of objects. We then define levels of granularities within those tree structures and stratified map spaces over such granularity structures. Stratified maps are descriptions of objects in a certain domain at different levels of granularity. The structure of stratified map spaces allows us to perform reasoning about location which is specified at different levels of granularity like: Assume that John is in the same place in which Mary is (in Hyde Park) and that Mary is also in the same place in which Paula is (in London). It is then our aim to derive that John and Paula are in the same place (in London).

1 INTRODUCTION

Consider a chain of reasoning like: From (a) John and Mary are in the same place at the same time, and (b) Paula and Mary are in the same place at the same time, it follows that (c) John and Paula are in the same place at the same time. This seems to be a valid way of reasoning based on the transitivity property of the relation same-place-same-time.

However, things are more complicated since being at the same place at the same time does not mean that John, Mary, and Paula take up the same region of space at the same time and therefore overlap spatially. We do not refer to the exact location of those people but to their *approximate location*. Being in the same place at the same time means to be in the same room, or in the same building, or in the same city, etc. This shows that there are *multiple ways* of approximating spatio-temporal location. Identical things can be approximated at different levels of granularity depending on the context.

Imagine that you are an FBI agent and that it is your task to confirm whether or not John and Paula could possibly have met. In order to do so you try do derive *John same-time-same-place Paula* from the data you have. Assume that one source confirms that John and Mary were in the same place at the same time (sitting on a bench in Hyde Park on Monday morning). Another source confirms that that Mary as well as Paula were in the same place at the same time (meaning in London on Monday morning). In this case we have *John same-time-same-place Mary* and *Mary same-time-same-place Paula*. The problem is that the relations refer to approximate location at *different* levels of granularity. The question now is under which circumstances we can derive *John same-time-same-place Paula* with the interpretation that both, John and Paula, are in London. The answer of this question lies at the heart of reasoning about approximations at different levels of granularity and it is the purpose of this paper to answer this question.

Qualitative spatial, temporal, and spatio-temporal relations and reasoning have been discussed widely in the literature, for example in [1, 12, 8, 7, 10, 2]. There are fewer attempts to consider qualitative spatial and temporal relations and reasoning at different levels of granularity. Examples are [9, 3, 11]. This paper is a contribution to this line of research, however the objective here is not the reasoning about hierarchically organized sets of relations but to take a simple but relevant set of relations such as *same-time-same-place* etc. and to study their composition in the context of approximation of spatio-temporal location at different levels of granularity.

The paper is structured as follows. We start by laying out some ontological notions fundamental for this paper. We then discuss stratified map spaces and their application to systems of spatial and temporal granularities. We then define relations between elements in those spaces and the composition of those relations at the same and at different levels of granularity.

2 A SPATIO-TEMPORAL ONTOLOGY

We distinguish the domain of objects, O, and the domain of regions, R. The domain of regions is constituted by regions of different dimensionality: four-dimensional spatio-temporal regions $x \in X$, three-dimensional spatial regions, $x^s \in X^s$, and one-dimensional temporal regions, $x^t \in X^t$. Individual objects stand to individual regions in the relation of location.

Every object, $o \in O$, is exactly located at a single three-dimensional spatial region, x^s, at every instant of time, τ [6]: $\forall o \in O : \exists x^s \in R : L_\tau(o, x^s)$. The region x^s is the exact or precise spatial location of o at the time instant τ. We say that the object o is located *at* the region x in order to stress the exact fit of object and region (the object matches the region). Exact location is a functional relation.

Most objects have different exact spatial locations at different times. We say that these objects change their spatial location. If we consider a temporal region (a period of time) during which the object o existed, then o may be either (i) *at rest*, i.e., it may be located in the same region of space, x^s over the given period of time x^t; or (ii) its spatial location may change, as a result of being located in different regions of space at different time-instants during this period. In this case we consider x^s to be the mereological sum of all locations visited over the period x^t.

Often, however, it is not very interesting to know that John is exactly located at that region of space from which the air is displaced by his body at a particular instant or period of time. It is much more

[1] Institute for Formal Ontology and Medical Information Science at the University of Leipzig

interesting to know, for example, that John is in London or in Paris. Here we are interested in sentences like 'John was in Hyde Park on Monday morning', where Monday morning and Hyde Park specify the *approximate* rather than the exact temporal and spatial location of the object John over a certain period of time. (Imagine that John entered the park at 9.45 a.m., went directly to his favorite bench, rested there for 30 minutes and then left the park at 10.05 a.m.)

We define the notion of approximate spatio-temporal location by demanding that the exact spatial location of the object o over a certain period of time is a part of the approximating spatial region (e.g., Hyde Park) and that the time-period in question is a part of the approximating temporal regions (e.g., Monday morning):

$$L(o, x^t, x^s) \equiv \exists x' : x' \leq x^t \wedge \forall \tau \in x' : \exists x : L_\tau(o, x) \wedge x \leq x^s$$

Here $\leq$ denotes the part-of relation which holds among regions of space and among regions of time and $\tau \in x$ is an abbreviation for 'the instant τ is within the boundaries of the time-period x'. In the approximate context we trace over between the distinction between rest and change by assuming the object in question does not leave the approximating region we are referring to.

3 SYSTEMS OF GRANULARITY

Granularities are the results of the way we humans structure our surrounding world and provide the foundation for the notion of approximation and for reasoning about approximations [3, 14, 5]. In the context of approximation of spatial and temporal location the (singular) notion of granularity refers to the size of the approximating region. The plural notion of granularities then refers to systems of regions that can potentially serve as approximating regions. In our example above the regions referred to by the names 'Hyde Park' and 'London' belong to such a system of granularities. (We use names of objects occupying a certain region in order to refer to this region, e.g., 'Hyde Park', 'January 13th 2002', etc.)

Formally, a system of granularities is a pair, $G = (R, \subseteq)$, where R is a set of regions with a binary relation $\subseteq$. Following [14] we call those regions cells and the relation $\subseteq$ the subcell relation. (We use $x \subset y$ in order to denote $x \subseteq y$ and $x \neq y$.) Systems of granularities form finite tree structures induced by the subcell relation . Here we obviously assume that systems of granularity include only those places and temporal intervals which are disjoint or contain each other in such a way that no partial overlap occurs. For reasons why this is a sensible assumption and for further details see [5]. Consider the following examples: (E1) A spatial system of granularities is formed by the cells Hyde Park, Soho, Buckingham Palace, Downtown, London, York, Edinburgh, Glasgow, England, Scotland, Great Britain, Germany, Europe and the corresponding nesting of those cells (Figure 1); (E2) The political subdivision of the United States forms a (flat) system of granularities with the US as root-cell and minimal cells like Wyoming and Montana; (E3) A temporal system of granularities is formed by the subdivision of Saturday, January 13th 2002 into forenoon, afternoon, hours, half-hours, quarters, and five-minute slots.

Let $G = (R_G, \subseteq)$ be a system of granularities and let $\mathcal{G}$ the corresponding tree representation. A *level of granularity* in G then is a cut in the tree-structure in the sense of [13]: (1) Let X be the root of $\mathcal{G}$, then $\{X\}$ is a cut; (2) *sons*(X) is a cut, where *sons*(a) is the set of immediate descendants of a; (3) Let C be a cut and $v \in C$ such that *sons*$(v) \neq \emptyset$ then $C' = (C - v) \cup$ *sons*(v) is a cut. This definition ensures that (i) the elements forming a level of granularity are pair-wise disjoint, i.e., $\neg\exists v_1, v_2 \in C : v_1 \subset v_2 \vee v_2 \subset v_1$;

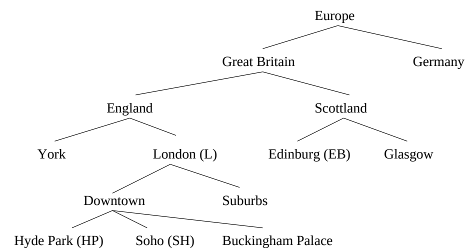

Figure 1. A system of granularities

(ii) levels of granularity are exhaustive in the sense that $\forall v \in R_G$: if $v \notin C$ then $\exists v' \in C : v \subseteq v' \vee v' \subseteq v$.

Consider Figure 1. Levels of granularity, for example, are:

$$\begin{array}{ll} g_0 & \{\textit{Europe}\} \\ g_1 & \{\textit{Great Britain}, \textit{Germany}\} \\ g_3 & \{\textit{York}, \textit{London}, \textit{Scotland}, \textit{Germany}\} \\ g_4 & \{\textit{York}, \textit{Hyde Park}, \textit{Soho}, \textit{Buckingham Palace}, \\ & \quad \textit{Suburbs}, \textit{Edinburgh}, \textit{Glasgow}, \textit{Germany}\} \end{array} \tag{1}$$

Following [13] we define an a partial order on cuts C and C' of a given tree as: $C \ll C'$ if and only if $\forall y \in C' : \exists x \in C$ such that $x \subseteq y$. The corresponding lattice is called the *granularity lattice*, **G** of G. This lattice has the root cell as maximal element with the coarsest level of granularity and the set of leaf-cells as the finest level of granularity. In Equation 1 we have the ordering: $g_0 \ll g_1 \ll g_3 \ll g_4$.

4 STRATIFIED MAP SPACES

This section outlines in a rough and informal manner the main features of the notion of stratified map spaces (Figure 2) proposed in [15].

Stell and Worboys use the term *map* to denote an arbitrary finite collection of data. A *map space* then is a set of all possible maps describing a particular domain using some fixed representation vocabulary. As indicated in Figure 2 a map can be thought of as a paper map and a map space then is a collection of paper maps of the same scale. Stell and Worboys also give the following analogy: In database terms, a map corresponds to a database state whereas the map space corresponds to the set of possible database states that are instances of a fixed schema.

In the context of this paper a map space can be thought of as a subset of the powerset of a set of objects together with their (approximate) spatio-temporal location. Maps then are elements of such map spaces.

Map spaces are partially ordered by granularity. Intuitively, if m_1 and m_2 are maps in the same map space, then $m_1 \leq m_2$, means that m_1 has a finer level of granularity than m_2. Levels or granularity are 'measured' with respect to an underlying granularity lattice with the properties discussed above. The notion of a stratified map space then allows the translation between maps representing the same domain (or parts of it) at different levels of granularity (detail).

As indicated in Figure 2, a *stratified map space* consists of a granularity lattice, **G**, and for each granularity $g \in \mathbf{G}$, a map space $Maps(g)$. There are two special transfer functions: (1) Gen that transfers by some coarsening process a map from one level of granularity to a coarser one; and (2) $Lift$ that transfers a map from a coarser level of granularity to a finer one. Whenever $g_1 \ll g_2 \in \mathbf{G}$, there are functions

$$Gen[g_1, g_2] : Maps(g_1) \rightarrow Maps(g_2)$$
$$Lift[g_1, g_2] : Maps(g_2) \rightarrow Maps(g_1)$$

between the map spaces $Maps(g_1)$ and $Maps(g_2)$.

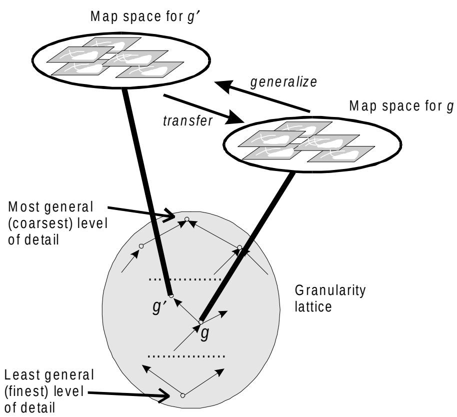

Figure 2. A stratified map space [15]

5 STRATIFIED SPATIO-TEMPORAL MAP SPACES

Let G^s be a spatial system of granularities and let $\mathbf{G}^s$ be the corresponding granularity lattice with elements $\mathbf{g}_1^s, \dots \mathbf{g}_n^s$. Let G^t be a temporal system of granularities and let $\mathbf{G}^t$ be the corresponding granularity lattice with elements $\mathbf{g}_1^t, \dots \mathbf{g}_m^t$. The granularity lattice underlying the spatio-temporal stratified map space consists of pairs $(\mathbf{g}_i^s, \mathbf{g}_j^t)$ which are ordered partially as $(\mathbf{g}_i^s, \mathbf{g}_j^t) \ll (\mathbf{g}_k^s, \mathbf{g}_l^t)$ if and only if $\mathbf{g}_i^s \ll \mathbf{g}_k^s$ and $\mathbf{g}_j^t \ll \mathbf{g}_l^t$. We call pairs of the form $(\mathbf{g}_i^s, \mathbf{g}_j^t)$ *spatio-temporal levels of granularity*.

Associated with every stratified map space there is a set of objects O such that for every $o \in O$ there are minimal cells $g^s \in G^s$ and $g^t \in G^t$ such that $L(o, g^t, g^s)$. This is not a serious limitation since we always can choose granularity-systems in such a way that they fit the associated set of objects nicely. Associated with every spatio-temporal level of granularity, $(\mathbf{g}_i^s, \mathbf{g}_j^t)$, there is a set of maps $Maps(\mathbf{g}_i^s, \mathbf{g}_j^t)$ (a map space). An element of $Maps(\mathbf{g}_i^s, \mathbf{g}_j^t)$ is a set of objects, $\mathcal{O} \subseteq O$ which is equipped with a function, Λ:

$$\Lambda : \mathcal{O} \rightarrow \mathbf{g}_j^t \times \mathbf{g}_i^s, \text{ with } \Lambda(o) = (x^t, x^s) \equiv L(o, x^t, x^s).$$

Since x^t and x^s are elements of levels of granularity and the elements of levels of granularity are mutually disjoint, the mapping Λ is well defined. In the remainder we use notations like $\Lambda_{g_k}^{\mathcal{O}_i}, \Delta_{g_k}^{\mathcal{O}_j} \in Maps(g_k)$ in order to refer to elements the map space $Maps(g_k)$.

Consider the levels of granularity $g = (\mathbf{g}_i^s, \mathbf{g}_j^t)$ and $f, \mathbf{g}_l^t)(\mathbf{g}^s$ with $g \ll f$. Due to the tree structure of the underlying temporal and spatial systems of granularities there exists a taxonomic generalization function $GenGran : \mathbf{g}_j^t \times \mathbf{g}_i^s \rightarrow \mathbf{g}_l^t \times \mathbf{g}_k^s$. This function takes the element $(x^t, x^s) \in \mathbf{g}_j^t \times \mathbf{g}_i^s$ and returns the element $(y^t, y^s) \in \mathbf{g}_l^t \times \mathbf{g}_k^s$ with the properties $x^t \subseteq y^t$ and $x^s \subseteq y^s$. These elements exists uniquely due to the exhaustiveness and the mutual disjointness of elements of a single level of granularity. The generalization function $Gen[g, f] : Maps(g) \rightarrow Maps(f)$ assigns to the map $\Lambda : \mathcal{O} \rightarrow \mathbf{g}_j^t \times \mathbf{g}_i^s$ the map $(GenGran[g, f]\ \Lambda) : \mathcal{O} \rightarrow \mathbf{g}_l^t \times \mathbf{g}_k^s$.

Corresponding to the generalization function $GenGran : \mathbf{g}_j^t \times \mathbf{g}_i^s \rightarrow \mathbf{g}_l^t \times \mathbf{g}^s$ there exists a lifting function $LiftGran : \mathbf{g}_l^t \times \mathbf{g}^s \rightarrow \mathbf{g}_j^t \times \mathbf{g}_i^s$. The lifting function $Lift[f, g] : Maps(f) \rightarrow Maps(g)$ assigns to the map $\Lambda : \mathcal{O} \rightarrow \mathbf{g}_l^t \times \mathbf{g}_k^s$ the map $(LiftGran[f, g]\ \Lambda) : \mathcal{O} \rightarrow \mathbf{g}_j^t \times \mathbf{g}_i^s$. The lifting exists due to the constraints on O.

Consider the system of granularities G_{E1} depicted in Figure 1 with levels of granularity listed in Equation 1 and assume that our domain O contains the objects *John*, *Mary*, *Paula* and *Jack*. For simplification we discuss only the spatial component of the stratified map space. Consider the map $\Lambda_{g_4}^{\mathcal{O}_1} \in Maps(g_4)$ and let $\Lambda_{g_4}^{\mathcal{O}_1} = \{(\textit{John} \mapsto \textit{Hyde Park}), (\textit{Mary} \mapsto \textit{Hyde Park})\}$. In the same map space there might be another map $\Lambda_{g_4}^{\mathcal{O}_2} = \{(\textit{Jack} \mapsto \textit{York}), (\textit{Paula} \mapsto \textit{Soho})\}$. However there cannot be a map in this space with the tuple $(\textit{Mary} \mapsto \textit{London})$. This tuple belongs to a map in another map space, e.g., $\Delta_{g_3}^{\mathcal{O}_k} = \{(\textit{John} \mapsto \textit{London}), (\textit{Mary} \mapsto \textit{London})\} \in Maps(g_3)$.

Since we have $g_4 \ll g_3$ there is a generalization (lifting) function $Gen[g_4, g_3]\ (Lift[g_3, g_4])$ such that $(GenGran[g_4, g_3]\ \Lambda_{g_4}^{\mathcal{O}_1}) = \Lambda_{g_3}^{\mathcal{O}_1}\ ((LiftGran[g_3, g_4]\ \Delta_{g_3}^{\mathcal{O}_k}) = \Delta_{g_4}^{\mathcal{O}_k})$.

6 SPATIO-TEMPORAL RELATIONS

Consider the map space $Maps(\mathbf{g}_i)$ which is formed by maps $\Lambda^{\mathcal{O}_j} : \mathcal{O}_j \rightarrow \mathbf{g}_i$, i.e., $\Lambda^{\mathcal{O}_j} \in Maps(\mathbf{g}_i)$ or $\Lambda_{\mathbf{g}_i}^{\mathcal{O}_j}$ for short. We call pairs of the form $\gamma \mapsto g$ o with $\gamma \in \Lambda_{\mathbf{g}_i}^{\mathcal{O}_j}$ the elements of the map $\Lambda_{\mathbf{g}_i}^{\mathcal{O}_j}$. γ relates the object o to its spatio-temporal location g at the level of granularity $\mathbf{g}_i$.

Let γ_1 and γ_2 be elements of maps in the map space $Maps(\mathbf{g}_i)$ with objects o_1 and o_2 approximately located in spatial regions $x_1^s, x_2^s \in \mathbf{g}_i^s$ and approximately located in temporal regions $x_1^t, x_2^t \in \mathbf{g}_i^t$. We define a set of identity and overlap-sensitive relations between map elements γ_1 and γ_2 by distinguishing relations between the associated spatial and associated temporal regions. We distinguish the relations: identity (=), proper overlap which excludes identity but includes containment (o), and non-overlap (ø) that hold among spatial regions and among temporal regions.

This gives raise to the nine combinatorial possible spatio-temporal relations between map elements γ_1 and γ_2 which are shown in Table 2. When considering relations between elements γ_1 and γ_2 of map spaces $Maps(\mathbf{g}_i)$, however, only the patterns 1, 3, 7, and 9 in Table 2 can occur, since the regions forming levels of granularity are pairwise disjoint and the relation *proper overlap* in the sense defined above cannot occur.

Since interpret a given element γ of a map space $Maps(\mathbf{g}_i)$ as specifying the place at which the object o is located approximately over a certain time-period, we interpret the relations 1, 3, 7, 9 in Table 2 as: (1) - same-time-same-place (stsp); (3) - same-time-different-place (stdp); (7) - different-time-same-place (dtsp); (9) - different-time-different-place (dtdp). (See [4] for further discussion of those relations.)

$R^{st}(\gamma_1, \gamma_2)$	1	2	3	4	5	6	7	8	9
$R^t(x_1^t, x_2^t)$	=	=	=	o	o	o	ø	ø	ø
$R^s(x_1^s, x_2^s)$	=	o	ø	=	o	ø	=	o	ø

(2)

7 COMPOSITION OF RELATIONS IN A MAP SPACE

We now consider the composition of spatio-temporal relations. Assume that we have γ_1 *stsp* γ_2 and γ_2 *stsp* γ_3 with γ_1, γ_2, and γ_3 being elements of maps in the map space $Maps(\mathbf{g}_i)$. We then want to perform the relation composition $\gamma_1($ *stsp* : *stsp* $)\gamma_3 = \gamma_1$ *stsp* γ_3.

In general, the composition of relation is defined as:

$$R(x,y) : S(y,z) = T_1(x,z) \dots T_n(x,z) \tag{3}$$

where R, S, and T are binary relations that hold between spatial or temporal regions. Given that R holds between x and y and S holds between y and z then between x and z one of the T_i holds. In Table 4 the composition of the relations $=$, o, and ø is given.

R \ S	=	o	ø
=	=	o	ø
o	o	=, o, ø	o, ø
ø	ø	o , ø	=, o, ø

(4)

In order to formalize the composition of spatio-temporal relations between elements γ_1 and γ_2 of maps in a map space, assume $\gamma_1 = o_1 \mapsto (x_1^t, x_1^s)$, $\gamma_2 = o_2 \mapsto (x_2^t, x_2^s)$, and $\gamma_3 = o_3 \mapsto (x_3^t, x_3^s)$ and let the relations $R(\gamma_1, \gamma_2)$ and $S(\gamma_2, \gamma_3)$ with $R, S \in \{$ *stsp* , *stdp* , *dtsp* , *dtsp* $\}$ hold. Since the definition of these relations is based on relations between regions, their composition is based on the composition of the relations between the associated regions. Since there are no assumptions about the dimension of the regions involved, Table 4 can be used in order to compute the spatial as well as the temporal component of the composition operation. Formally we write:

$$\begin{array}{rl} & R(o_1 \mapsto (x_1^t, x_1^s), o_2 \mapsto (x_2^t, x_2^s)) \\ : & S(o_2 \mapsto (x_2^t, x_2^s), o_3 \mapsto (x_3^t, x_3^s)) \\ = & T_1(o_1 \mapsto (x_1^t, x_1^s), o_3 \mapsto (x_3^t, x_3^s)) \dots \\ & T_n(o_1 \mapsto (x_1^t, x_1^s), o_3 \mapsto (x_3^t, x_3^s)) \\ & \text{with} \\ & T_i \in R^t(x_1^t, x_2^t) : S^t(x_2^t, x_3^t) \times R^s(x_1^s, x_2^s) : S^s(x_2^s, x_3^s) \end{array} \tag{5}$$

where the relation R^t (R^s) is the temporal (spatial) component of the relation R. Table 6 shows the operation table for the composition of the four relations. The composition of relations between elements of maps in a map space is closed due to the granularity structure of the underlying map space.

$R \setminus S$	*stsp*	*stdp*	*dtsp*	*dtdp*
stsp	*stsp*	*stdp*	*dtsp*	*dtdp*
stdp	*stdp*	$st\{dp, sp\}$	*dtdp*	$dt\{sp, dp\}$
dtsp	*dtsp*	*stdp*	$\{st, dt\}sp$	$\{st, dt\}dp$
dtdp	*dtdp*	$st\{sp, dp\}$	$\{st, dt\}dp$	$\{st, dt\}$ $\{sp, dp\}$

(6)

8 COMPOSITION AT DIFFERENT LEVELS OF GRANULARITY

We now consider relations between elements of maps $(o_1 \mapsto (x_1^t, x_1^s)), (o_2 \mapsto (x_2^t, x_2^s)) \in \Lambda_{g_i}^{\mathcal{O}}$ and $(o_2 \mapsto (x_3^t, x_3^s)), (o_3 \mapsto (x_4^t, x_4^s)) \in \Lambda_{g_j}^{\mathcal{O}}$ which belong to map spaces at different levels of granularity, i.e., $Maps(g_i) \neq Maps(g_j)$. The composition of relations R and S follows the pattern:

$$\begin{array}{rl} & R(o_1 \mapsto (x_1^t, x_1^s), o_2 \mapsto (x_2^t, x_2^s)) \\ : & S(o_2 \mapsto (x_3^t, x_3^s), o_3 \mapsto (x_4^t, x_4^s)) \\ = & \dots \end{array} \tag{7}$$

In the standard sense the composition of two relations is defined only if there is a 'binding individual' (e.g., the y in Equation 3) which occurs in both relations. If we consider the composition of relations between elements of maps at different levels of granularity then we need to consider two cases: (i) $x_2^t = x_3^t$ and $x_2^s = x_3^s$ and (ii) $x_2^t \neq x_3^t$ or $x_2^s \neq x_3^s$.

In case (i) both map spaces 'overlap nicely' such that there is a 'binding individual' $(o_2 \mapsto (x_2^t, x_2^s)) = (o_2 \mapsto (x_3^t, x_3^s))$ and we can perform the composition of the relations R and S in the way defined in Equation 5.

In case (ii) there is no 'binding individual' directly connecting the relations R and S since $(o_2 \mapsto (x_2^t, x_2^s)) \neq (o_2 \mapsto (x_3^t, x_3^s))$. There is, however, a 'binding object' o_2 and it also holds that (x_2^t, x_2^s) and (x_3^t, x_3^s) are 'connected' in the underlying granularity system in the sense that $(x_2^t, x_2^s) \subset (x_3^t, x_3^s)$ or $(x_3^t, x_3^s) \subset (x_2^t, x_2^s)$. This 'connection' always exists due to the exhaustiveness-property of levels of granularities.

Consider, for example, the composition of the relations $John$ *stsp* $Mary$ with the interpretation that both are in Hyde Park on Monday morning and $Mary$ *stsp* $Paula$ with the interpretation that both are in London on Monday morning. Hyde Park and London are connected within the underlying system of granularities (Figure 1) via the chain: *Hyde Park* $\subset$ *Downtown* $\subset$ *London*. Our aim now is to derive $John$ *stsp* $Paula$ with the interpretation that both are in London on Monday morning.

One can see that in order to perform the composition of relations between map elements of distinct map spaces we need to perform generalization or lifting transformations between maps spaces. In this context it is important to know whether or not the relations *stsp* , *stdp* , *dtsp* , *dtdp* are preserved under such transformations. It will sufficient to consider the spatial components, *same-place* (sp) and *different-place* (dp), and the temporal components *same-time* (st) and *same-time* (dt) separately and to concentrate on invariance or change of the spatial component under the transformations $Gen^s[g_i, g_j](\gamma^s)$ and $Lift^s[g_j, g_i](\gamma^s)$ (the spatial components of the functions $Gen[g_i, g_j]$ and $Lift[g_j, g_i]$ applied to the spatial component $\gamma^s = o \mapsto x^s$ of $\gamma = o \mapsto (x^t, x^s)$). This is because the definitions for the temporal component follow the same pattern and are very similar.

The following holds: (1) The relation sp remains invariant under generalization, i.e., if γ_1^s sp γ_2^s then $Gen^s[g_i, g_j](\gamma_1^s)$ sp $Gen^s[g_i, g_j](\gamma_2^s)$; (2) The relation dp remains invariant under lifting, i.e., if γ_1^s dp γ_2^s then $Lift^s[g_j, g_i](\gamma_1^s)$ dp $Lift^s[g_j, g_i](\gamma_2^s)$ with $g_i \ll g_j$.

However, the relation sp is not preserved under lifting since $(John \mapsto London)$ sp_{g_i} $(Mary \mapsto London)$ in the map space $Maps(g_i)$ is perfectly consistent with $(John \mapsto$ *Hyde Park*) dp_{g_j} $(Mary \mapsto Soho)$ in the map space $Maps(g_j)$. On the other hand, the relation dp is not preserved under generalization since $(John \mapsto$ *Hyde Park*) dp_{g_j} $(Mary \mapsto Soho)$ in the map space $Maps(g_j)$ is perfectly consistent with $(John \mapsto London)$ sp_{g_i} $(Mary \mapsto London)$ in the map space $Maps(g_i)$.

Consider Equation 5. When performing the composition of relations between map elements of different map spaces then we need to transform the parameters of one relation and leave the other relation unchanged. We focus here on the discussion of the transformation of the parameters of the first relation. When performing the transformation we need to insure that the relation still holds in the new map space. Consequently if the relation which parameters we transform is sp then we can only apply the transformation Gen^s (Equation 8). If, however, the relation which parameters we transform is dp then we can only apply the transformation $Lift^s$ (Equation 9). Of course, we can apply Gen^s ($Lift^s$) only if $x_2^s \subsetneq x_3^s$ ($x_3^s \subseteq x_2^s$) holds. (In the case of $x_2^s = x_3^s$ the transformation Gen^s ($Lift^s$) is the identity map.) If these conditions do not hold we need to transform the parameters of the second relation (Equations 10 and 11).

Let $\gamma_1 = o_1 \mapsto x_1^s$, $\gamma_2 = o_2 \mapsto x_2^s$, $\delta_1 = o_2 \mapsto x_3^s$, and $\delta_2 = o_3 \mapsto x_4^s$ be spatial projections of the map-elements $\gamma_1', \gamma_2', \delta_1'$, and δ_2' with $\gamma_1', \gamma_2' \in \Lambda^{\mathcal{O}}_{\mathbf{g}_i}$ and $\delta_1', \delta_2' \in \Lambda^{\mathcal{O}}_{\mathbf{g}_j}$. We then define:

$$\begin{array}{rl} & sp\ (\gamma_1, \gamma_2) : S(\delta_1, \delta_2) \\ = & T_1((\theta\ \gamma_1), \delta_2) \ldots T_n((\theta\ \gamma_1), \delta_2) \\ \text{iff} & x_2^s \subseteq x_3^s \wedge \delta_1 = (\theta\ \gamma_2) \wedge \theta = Gen^s[g_i, g_j] \\ \text{with } T_i \in & (\ sp\ (\theta\ \gamma_1, \delta_1) : S(\delta_1, \delta_2)), \end{array} \tag{8}$$

and

$$\begin{array}{rl} & dp\ (\gamma_1, \gamma_2) : S(\delta_1, \delta_2) \\ = & T_1((\theta\ \gamma_1), \delta_2) \ldots T_n((\theta\ \gamma_1), \delta_2) \\ \text{iff} & x_3^s \subseteq x_2^s \wedge \delta_1 = (\theta\ \gamma_2) \wedge \theta = Lift^s[g_i, g_j] \\ \text{with } T_i \in & (\ dp\ (\theta\ \gamma_1, \delta_1) : S(\delta_1, \delta_2)), \end{array} \tag{9}$$

Intuitively, Equation 8 means that if we have $x_2^s \subseteq x_3^s$ and the first relation is sp then we map all objects to the level of granularity of δ_1 (using Gen^s) and perform the relation-composition in this map space in the standard way. Equation 9 means that if we have $x_3^s \subseteq x_2^s$ and the first relation is dp then we map all objects to the level of granularity of γ_2 (using $Lift^s$) and perform the relation-composition in this map space in the standard way.

Equations 10 and 11 now cover the cases where the parameters of the second relation need to be transformed. They correspond in their structure closely to their counterparts Equation 8 and Equation 9 and a discussion is therefore omitted.

$$\begin{array}{rl} & R(\gamma_1, \gamma_2) :\ sp\ (\delta_1, \delta_2) \\ = & T_1(\gamma_1, (\theta\ \delta_2)) \ldots T_n(\gamma_1, (\theta\ \delta_2)) \\ \text{iff} & x_3^s \subseteq x_2^s \wedge \gamma_2 = (\theta\ \delta_1) \wedge \theta = Gen^s[g_j, g_i] \\ \text{with } T_i \in & (R(\gamma_1, \gamma_2) :\ sp\ (\gamma_2, \theta\ \delta_2)), \end{array} \tag{10}$$

and

$$\begin{array}{rl} & R(\gamma_1, \gamma_2) :\ dp\ (\delta_1, \delta_2) \\ = & T_1(\gamma_1, \theta\ \delta_2) \ldots T_n(\gamma_1), \theta\ \delta_2) \\ \text{iff} & x_2^s \subseteq x_3^s \wedge \gamma_2 = (\theta\ \delta_1) \wedge \theta = Lift^s[g_j, g_i] \\ \text{with } T_i \in & (R(\gamma_1, \gamma_2) :\ dp\ (\gamma_2, \theta\ \delta_2)) \end{array} \tag{11}$$

Examples are given in Table 12. (For the meaning of the abbreviations see Figure 1.) Again, for simplification we consider only the composition of the spatial component. The first column should be read as: (*John* $\mapsto$ *Hyde Park*) sp_{g_i} (*Mary* $\mapsto$ *Hyde Park*), (*Mary* $\mapsto$ *London*) sp_{g_j} (*Paula* $\mapsto$ *London*), and (*John* $\mapsto$ *London*) sp_{g_j} : sp_{g_j} (*Paula* $\mapsto$ *London*), where sp_{g_i} means that the relation *same-place* holds in the map space $Maps(g_i)$.

John	R	Mary	S	Paula	John	(R:S)	Paula
HP	sp	HP | L	sp	L	L	sp	L
L	sp	L | HP	sp	HP	L	sp	L
HP	sp	HP | HP	dp	EB	HP	dp	EB
HP	sp	HP | L	dp	EB	L	dp	EB
HP	dp	EB | EB	sp	EB	HP	dp	EB
HP	dp	SH | L	dp	EB	HP	dp	EB

(12)

As already pointed out, the treatment of the temporal component is similar. In the temporal case we need to consider the transformations $Gen^t[g_i, g_j](\gamma^t)$ and $Lift^t[g_j, g_i](\gamma^t)$, i.e., the temporal components of the transformations $Gen[g_i, g_j]$ and $Lift[g_j, g_i]$ applied to the temporal component $\gamma^t = o \mapsto x^t$ of the map element $\gamma = (o, x^t, x^s)$. The equations 8–10 then can be rephrased in the obvious manner.

9 CONCLUSIONS

In this paper we applied the notion of stratified map spaces to the composition of relations about approximate spatio-temporal location at different levels of granularity. Critical for the whole approach are the existence of systems of granularities which provide frames of reference for the specification of approximate spatio-temporal location. Future work should further study the properties of those structures, the way human beings facilitate different frames of reference in different contexts, as well as more complex sets of relations.

ACKNOWLEDGEMENTS

The support by the Alexander von Humboldt Stiftung under the auspices of its Wolfgang Paul program, DARPA under the Rapid Knowledge Formation program, and the National Science Foundation under the Research on Learning and Education program is gratefully acknowledged.

REFERENCES

[1] James F. Allen, 'Towards a general theory of action and time', *Artificial Intelligence*, **23**(2), 123–154, (1984).

[2] B. Bennett, A.G. Cohn, F. Wolter, and M. Zakharyaschev, 'Multi-dimensional modal logic as a framework for spatio-temporal reasoning', *Applied Intelligence*, (To appear).

[3] C. Bettini, X. Wang, and S. Jajodia, 'A general framework for time granularity and its application to temporal reasoning', *Annals of Mathematics and Artificial Intelligence*, **22**, 29–58, (1998).

[4] T. Bittner, 'Judgments and spatio-temporal relations', in *Proceedings of the Eighth International Conference on Principles of Knowledge Representation and Reasoning (KR2002)*. Morgan Kaufmann, (2002).

[5] T. Bittner and B. Smith, 'A taxonomy of granular partitions', in *Spatial Information Theory, COSIT '01*, ed., D.R.Montello, volume 2205 of *Lecture Notes in Computer Science*, pp. 28–43. Berlin/New York: Springer, (2001).

[6] R. Casati and A.C. Varzi, 'The structure of spatial localization', *Philosophical Studies*, **82**(2), 205–239, (1995).

[7] C. Claramunt, M. Theriault, and C. Parent, 'A qualitative representation of evolving spatial entities in two-dimensional spaces', in *Innovations in GIS V*, ed., S. Carver, 119–129, Taylor & Francis, (1997).

[8] Max J. Egenhofer and Robert D. Franzosa, 'Point-set topological spatial relations', *International Journal of Geographical Information Systems*, **5**(2), 161–174, (1991).

[9] Jerome Euzenat, 'An algebraic approach for granularity in qualitative space and time representation', in *Proceedings of the IJCAI-95*, pp. 894–900, (1995).

[10] S. M. Hazarika and A. G. Cohn, 'Qualitative spatio-temporal continuity', in *Spatial Information Theory: Foundations of Geographic Information Science; Proceedings of COSIT'01*, ed., D. R. Montello, volume 2205 of *LNCS*, pp. 92–107. Springer Verlag, (2001).

[11] D. Papadias and Egenhofer M, 'Algorithms for hierarchical spatial reasoning', *Geoinformatica*, **1**(3), (1997).

[12] D.A. Randall, Z. Cui, and A.G. Cohn, 'A spatial logic based on regions and connection', in *Principles of Knowledge Representation and Reasoning. Proceedings of the Third International Conference (KR92)*, eds., B. Nebel, C. Rich, and W. Swartout, pp. 165–176. Morgan Kaufmann, (1992).

[13] P. Rigaux and M. Scholl, 'Multi-scale partitions: Application to spatial and statistical databases', in *Advances in Spatial Databases (SSD'95)*, eds., M. Egenhofer and J. Herrings, number 951 in Lecture Notes in Computer Science. Springer-Verlag, Berlin, (1995).

[14] B. Smith and B. Brogaard, 'Quantum mereotopology', *Annals of Mathematics and Artificial Intelligence*, (to appear).

[15] J.G. Stell and M.F. Worboys, 'Stratified map spaces: A formal basis for multi-resolution spatial databases', in *SDH'98 Proceedings 8th International Symposium on Spatial Data Handling*, eds., T. K. Poiker and N. Chrisman, pp. 180–189. International Geographical Union, (1998).