A qualitative formalization of built environments*

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Abstract. In this paper I argue that a qualitative formalization of built environments needs to take into account: (1) the ontological distinction between bona-fide and fiat boundaries and objects, (2) the different character of constraints on relations involving these different kinds of boundaries and objects, (3) the distinction between partition forming and non-partition forming objects, and (4) the fundamental organizational structure of regional partitions. I discuss the notion of boundary sensitive rough location and show that a formalization based on this notion takes all these points into account.

1 Introduction

Imagine a computer program (I) that generates configurations of lines in the Euclidean plane that look like plans of built environments [Lyn60]. Examples of built environments are shopping malls, airports, or parking lots (See, for example, the left part of Fig. 1.). Program (I) generates configurations of lines of different style and width. Suppose our task is to design another computer program (II), which checks (1) whether or not a plan generated by (I) can possibly be a plan of a built environment and, in case the plan represents a built environment, (2) whether or not this environment can be navigated easily by human beings.

Human navigation and wayfinding in general and in built environments in particular has been studied extensively in the past in architectural design, e.g., [GLM83], in Artificial Intelligence, e.g., [Kui78] and in Cognitive Science, e.g., [SW75]. Notice that all those people deal with navigation in real, physically existing environments. The question I am addressing is different. The built environments this paper is dealing with do not (physically) exist yet. Consequently, the approach I am proposing cannot rely on observations in reality. The only 'physical' thing we have is a plan, P, generated by program (I). Only if program (II) decides (a) that P represents a built environment and (b) it is easy to navigate then the environment is being built according to P. Whatever it means to be a built environment and whatever it means to be easy to navigate, it must be definable in the language of P and it must be decidable given P. Consequently, task (1) and (2) rest upon the same formal foundations and are, in this respect, closely related to each other. It is the aim of this paper to investigate those formal foundations.

Program (I) produces a quantitative representation of built environments based on computational geometry (e.g., [Rou94]). The analysis of plans representing built environments, performed by program (II) will focus on qualitative aspects [CBGG97],

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i.e., different kinds of things, qualitative relations between lines [All83] and qualitative relations between regions [CBGG97]. At the formal level I will use a language based on the qualitative notion of boundary sensitive rough location [BS98] to describe built environments. I am going to show that this notion provides the basis for the formal description of built environments and for the evaluation of the complexity of navigation.

This paper is structured as follows. I start with an informal analysis of the ontological makeup of built environments. In Section 3 I shortly review the notion of rough approximations regions and rough location of spatial objects. These notions provide the basis for the formalization. In Section 4 I give a formalization of built environments. The conclusions are given in Section 5.

2 Built environments

In this paper I use parking lots as a running example for built environments. The parking lot domain is relatively simple but its structure is rich enough to study the *ontological* makeup of built environments, which is critical for a *qualitative* formalization. An example of a parking lot in a bird's eye view is given in Fig. 1.

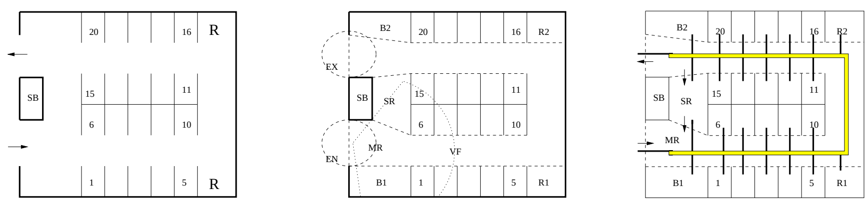


Fig. 1. An empty parking lot (left). The parking lot with invisible fiat boundaries marked (middle). The path system of the parking lot (right).

2.1 Boundaries

Following [Smi95] I distinguish bona-fide and fiat boundaries. Bona fide boundaries are boundaries *in the things themselves*. Bona fide boundaries exist independently of all human cognitive acts. They are a matter of qualitative differentiations or discontinuities of the underlying reality. Examples are surfaces of extended objects like cars, walls, the floor of a parking lot. Bona-fide boundaries are marked by bold solid lines in the left and middle parts of Fig. 1.

[Smi95] describes fiat boundaries as boundaries which exist only in virtue of different sorts of demarcation effected cognitively by human beings. Such boundaries may lie skew to boundaries of *bona-fide* sort as in the case of the boundaries of a parking spot in the center of a parking lot, e.g. spots 6-15 in Fig. 1. They may also, however as in the case of a parking spot at the outer wall of the parking lot, involve a combination of fiat and *bona-fide* portions such as a wall at its back side, e.g., spots 1-5 and 16-20 in Fig. 1.

Fiat boundaries like the front boundaries of parking spots are not directly observable in reality but do nevertheless exist. Since fiat boundaries are not perceivable by the senses they need to be made visible or the environment must force all people to perceive them in places the designer wanted them to be. In order to make parking spots perceivable by other people, the back, left, and right boundaries are marked by (usually white) paint (the thin solid lines in Fig. 1). The front boundary of the parking spot is not marked but every human being knows that it is located on the straight line connecting the ends of the left and right boundaries. Non marked and hence invisible fiat boundaries are marked by dashed lines in the middle part of Fig. 1. Plans of built environments to be analyzed by programs rather by human beings must contain *all* boundaries even if they are not directly observable in reality. Consequently, plans of built environments generated by program (I) and analyzed by (II) must look like the middle part of Fig. 1 rather than like the left part.

Consider the parking lot domain. Marked fiat boundaries afford people (in cars) not to cross despite the fact that there is no physical barrier. Non-marked boundaries afford crossing, e.g., the non-marked boundary of an empty parking spot 'invites' you to cross this boundary and park your car at this spot (if there is no other car parked yet). In the design of built environments fiat boundaries and their barrier properties play an important role. They provide an important organizational structure. In this paper I distinguish barrier and non-barrier boundaries. Bona-fide boundaries are barrier boundaries. Fiat boundaries may be of barrier (from one side or both sides) or non-barrier sort.

2.2 Spatial objects forming built environments

The classification of boundaries generalizes to a classification of objects. Bona-fide objects have a single topologically closed bona-fide boundary (e.g., the building SB in Fig. 1). Fiat objects have fiat boundary parts (e.g., parking spot 1). Built environments are populated by bona-fide as well as by fiat objects. There are three basic classes of axioms governing the spatial objects in built environments: $(O1)$ axioms governing spatial objects of the *same* ontological kind; $(O2)$ axioms governing spatial objects of the *different* ontological kind; $(O3)$ domain specific axioms characterizing built environments like parking lots, airports, shopping malls, or city centers. In the remainder I call the axioms $O1-O3$ ontological axioms or ontological constraints on relations that can hold between objects in built environments.

The main axiom in group $O1$ is that spatial objects of the *same* ontological kind cannot overlap [CV95]. For example, bona-fide objects like cars and walls cannot overlap. Fiat objects of the same kind like parking spots cannot overlap either. This axiom needs refinement regarding overlap of boundary parts: Boundary parts of fiat objects of the same kind can be co-located, e.g., co-located boundary parts of neighboring parking spots. Boundary parts of bona-fide objects cannot [SV99].

The main axiom in group $O2$ is that spatial objects of *different* ontological kind can overlap [CV95]. This is not a constraint. It rather says that in general there are no ontological objections against objects of different kinds to overlap. However, there are additional constraints on relations among partition forming objects (to be discussed below).

There are further *domain specific* axioms, $O3$, constraining relations that can hold between objects of ontological different kind in specific built environments. Consider the parking lot domain. There are cars and parking spots. Parking spots are such that cars can be parked in them. Parking lots are also formed by objects like blocked areas and reserved parking spots (e.g., B_1 and R_1 in Fig. 1). Examples for domain specific constraints on relations between objects in parking lots are: $(S1)$ Cars are supposed to keep blocked areas clear; $(S2)$ Regular cars should not be parked in reserved parking spots; $(S3)$ Cars should be parked within parking spots.

Regarding domain specific constraints in $O3$ it is important to notice that constraints involving objects of ontological different kind are weaker than the constraints between bona-fide objects, constraints between fiat objects of the same kind. The laws of logic prohibit objects of ontological same kind to overlap. Laws of physics prevent bona-fide objects from sharing boundary parts. Constraints involving fiat objects of ontological different kind are based on social rules and agreement and may be violated in certain situations. For example, you can die if you try to drive through a wall. You only get charged when you are parking on a reserved parking spot. The different character of constraints will play an important role in the formalization in Section 4.

Besides the (fundamental) distinction between bona-fide and fiat objects I distinguish two kinds of objects in built environments: (regional) partition forming objects and non-partition forming objects. A regional partition is a set of regions, which members intersect only at their boundaries $(P1)$ and, as a whole, sum up the whole space $(P2)$. Partition forming objects are spatial objects, which form a regional partition of the underlying space.

Consider the parking lot domain. The partition forming objects form a regional partition of the three dimensional parking lot. Each of those objects carves out three dimensional regions of the parking lot but there is no 'no man's land' and no 'double occupation'. Partition forming objects are, for example, parking spots, traffic lanes, sidewalks, blocked areas keeping fire exits clear, walls, pillars, and others more. Partition forming objects may be of bona-fide (pillars or walls) or of fiat sort (parking spots).

Non-partition forming objects overlap partition forming objects of ontological different kind. Non-partition forming objects may be of bona-fide or fiat sort. Consider the parking lot domain. Examples for non-partition forming bona-fide objects are cars and people. Examples for non-partition forming fiat objects are smoking areas in public places, the visual field (VF) in a given location or 'the entrance area', EN, or the 'exit area', EX of a parking lot (See middle part of Fig. 1).

The formalization presented in this paper will be dealing with 2-dimensional objects in 2-dimensional space, i.e., with orthogonal projections of three dimensional objects onto the ground. Consequently partition forming objects form regional partitions of the plane. In this context I am assuming that the projected objects 'inherit' the ontologically significant properties of their originals as well as the barrier and non-barrier properties the boundaries of their originals.

2.3 Movement

An important aspect of the distinction between partition forming and non-partition forming objects is that the partition structure is static and that non-partition forming objects can move (like cars), or shrink and grow (like the visual field). Objects move along paths. A path is a sequence of locations occupied at consecutive moments of time, which corresponds to continuous movement.

Consider the parking lot domain. It is the purpose of a parking lot to let cars park within parking spots. In order to fulfill this purpose, it must be possible to move a car from the entrance to a free parking spot. That is: (i) There must *exist* a path of movement to a free parking spot without violation of $O1 - O3$. This will be called the moveability axiom, M, of a built environment. (ii) It must be possible for a human agent in a car to *find* this path. Checking the existence of paths is an instance of problem (1). Deciding whether or not it is possible, difficult, or easy to find an existing path is an instance of problem (2). In this paper I focus on problem (1). This provides the basis for solving problem (2).

3 Approximating regions and relations between approximations

In this section I shortly discuss the formal notions needed in the remainder of this paper. These notions were originally introduced in [BS98] and [BS00]. For an extended discussion see also [Bit99].

3.1 Approximations

Suppose we have a space R of detailed or precise regions. By imposing a partition, G, on R we can approximate elements of R by elements of ${\Omega_{bs}}^{G \times G}$. That is, we approximate regions in R by functions from $G \times G$ to the set $\Omega_{bs} = \{\mathsf{fo}, \mathsf{fbo}, \mathsf{pbo}, \mathsf{nbo}, \mathsf{no}\}$. The function which assigns to each region $r \in R$ its boundary sensitive approximation is $\alpha : R \rightarrow {\Omega_{bs}}^{G \times G}$. The value of $(\alpha r)(g_1, g_2)$ is fo if r covers all of the cell g_1, it is fbo if r covers all of the boundary segment, (g_1, g_2), shared by the cell g_1 and g_2 and some but not all of the interior of g_1, it is pbo if r covers some but not all of the boundary segment (g_1, g_2) and some but not all of the interior of g_1, it is nbo if r does not intersect with boundary segment (g_1, g_2) and some but not all of the interior of g_1, and it is no if there is no overlap between r and g_1. Consider the visual field, VF, in the parking lot in Fig. 1. The graph of the mapping $\alpha(VF)$ contains the following tuples:

(g_i, g_j)	(B_1, MR)	(B_1, PS_1)	(B_1, W)	(PS_1, B_1)	...	(PS_1, W)	...
ω	pbo	fbo	pbo	fo	...	fo	...

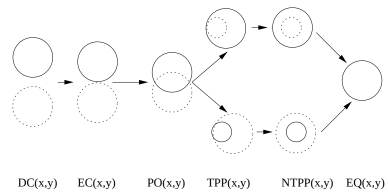
Each approximate region $X \in {\Omega_{bs}}^{G \times G}$ stands for a set of precise regions, i.e., all those precise regions having the approximation X. This set, $[\![X]\!]$, provides a semantics for approximate regions: $[\![X]\!] = \{r \in R \mid \alpha r = X\}$. In the remainder I use the notion *boundary sensitive rough location*, $(\text{loc}\ o) = (\alpha_5 \circ r)o$ in order to refer to the approximation of the (exact) region[1], $r(o)$, of the object o with respect to an underlying partition G.

[1] The region of space it exactly occupies.

3.2 Relations between approximations

In the domain of regions we distinguish a set of 8 well known binary topological relation between spatial regions, the RCC8 [CBGG97] relations.

We distinguish the relations $DC(x, y)$ (disconnected), $EC(x, y)$ (externally connected), $PO(x, y)$ (partial overlap), $TPP(x, y) = TPPi(y, x)$ (tangential proper part), $NTPP(x, y) = NTPPi(y, x)$ (non-tangential proper part), and $EQ(x, y)$.



In the remainder I use the notion $RCC8$ in order to refer to this set of relations. The elements of the set are jointly exhaustive and pairwise disjoint [CBGG97] and form a lattice with respect an ordering relation $\leq$ [BS00]. Possible geometric interpretations and the ordering relation ($R_1 \leq R_2$ is indicated by an arrow from R_1 to R_2) are shown in the figure above.

Let X and Y be boundary sensitive approximations. [BS00] showed that there exists a mapping

$$\Psi \ : \ {\Omega_{bs}}^{G \times G} \times \ {\Omega_{bs}}^{G \times G} \quad \rightarrow \quad RCC8 \times RCC8$$

such that $\Psi(X, Y) = (R^8_{min}, R^8_{max})$ if and only if (1) $R^8_{min}(x, y)$ is the least RCC8 relation that can hold between $x \in [\![X]\!]$ and $y \in [\![Y]\!]$, (2) $R^8_{max}(x, y)$ is the greatest RCC8 relation that can hold for x and y as above, and (3) for each R with $R^8_{min} \leq R \leq R^8_{max}$ there are $x \in [\![X]\!]$ and $y \in [\![Y]\!]$ such that $R(x, y)$, where $\leq$ is the ordering shown in the figure above. For details see [BS00]. In the remainder I use the notions $R^8_{min}(X, Y)$ in order to refer to the least relation and $R^8_{max}(X, Y)$ in order to refer to the greatest relation that can hold between $x \in [\![X]\!]$ and $y \in [\![Y]\!]$.

4 Formalizing built environments

Formally, built environments have three major components: The layout of the built environment, which is formed by the partition forming objects; A system of paths along which non-partition forming objects can move within the layout of the built environment; A set of possible situations, where a situation in a built environment is the layout of the environment and a set of non-partition forming objects populating it in a given moment of time.

Situations need to obey the ontological axioms, $O1 - O3$ and the partition axioms, $P1 - P2$. Furthermore they need to be such that the non-partition forming objects populating the environment could possibly have been moved into the location they are in this situation (axiom M). In this section I give axioms for situations in built environments. These axioms take into account: (1) The distinction between bona-fide and fiat objects; (2) The distinction between partition forming and non-partition forming objects; (3) The different strength of constraints on relations involving bona-fide and fiat objects. Formally, the axioms characterizing built environments are given in terms of boundary sensitive rough location.

4.1 Formalizing ontological constraints

Bona-fide objects do not overlap and do not have co-located boundary parts. Let o_1 and o_2 be bona-fide objects, i.e., $o_1, o_2 \in BF$[2]. In terms of rough location we define: $F1(o_1, o_2) \equiv R^8_{min}((\text{loc}_{bs}\ o_1), (\text{loc}_{bs}\ o_2)) = DC$ and postulate $\forall o_1, o_2 \in BF : o_1 \neq o_2 \Rightarrow F1(o_1, o_2)$. Bona-fide objects can be located in a built environment such that the minimal relation between their exact regions, which is consistent with their rough location $(\text{loc}_{bs}\ o_1)$ and $(\text{loc}_{bs}\ o_2)$, is disconnected, i.e., $DC(r(o_1), r(o_2))$. There cannot exist an environment that forces bona-fide objects to be connected[3].

Two bona-fide objects cannot be connected even if they share the same rough location. In terms of rough location it is impossible to postulate that bona-fide objects cannot be connected. Consider Fig. 1 and imagine two cars on the main road. Both share the same rough location and we have $R^8_{max} = EQ$. In terms of rough location we cannot exclude the possibility for the cars to be connected. Notice the important point: In terms of rough location we specify what an environment *cannot do* to bona-fide objects populating or forming it - it cannot make them being connected. The objects themselves are governed by the underlying theory of objects.

Fiat objects of the same kind do not overlap but may have co-located boundary parts. Let o_1 and o_2 be fiat objects of kind ϕ, i.e., $o_1, o_2 \in F^{\phi}$[4]. In terms of rough location we define: $F2(o_1, o_2) \equiv R^8_{min}((\text{loc}_{bs}\ o_1), (\text{loc}_{bs}\ o_2)) \leq EC$ and postulate $\forall o_1, o_2 \in F^{\phi} : o_1 \neq o_2 \Rightarrow F2(o_1, o_2)$. There cannot exist a built environment that forces fiat objects of the same kind to overlap. In terms of rough location it is impossible to postulate that fiat objects of the same kind cannot overlap. This is the business of the theory of objects.

Partition forming objects. Let o_1 and o_2 be bona-fide partition forming objects. In terms of boundary-sensitive rough location we define: $F3(o_1, o_2) \equiv R^8_{max}((\text{loc}_{bs}\ o_1), (\text{loc}_{bs}\ o_2)) = DC$ and postulate $\forall o_1, o_2 \in BF : (o_1 \neq o_2 \text{ and } r(o_1), r(o_2) \in G) \Rightarrow F3(o_1, o_2)$. Due to the underlying partition structure we are able to postulate that partition forming bona-fide objects cannot be connected. The largest relation that can hold between two partition forming bona-fide objects is DC. We have $R^8_{min} = R^8_{max} = DC$. Consequently, bona-fide objects cannot be located at neighboring partition regions.

Let o_1 and o_2 be partition forming objects such that o_1 is of fiat kind and o_2 is of bona-fide or of fiat kind. Boundary parts of those objects may be co-located, i.e., their exact regions may be externally connected, EC. In terms of boundary-sensitive rough location we define: $F4(o_1, o_2) \equiv R^8_{max}((\text{loc}_{bs}\ o_1), (\text{loc}_{bs}\ o_2)) \leq EC$ and postulate $\forall o_1 \in F^{\phi}, \forall o_2 \in (F^{\psi} \cup BF) : (o_1 \neq o_2 \text{ and } r(o_1), r(o_2) \in G) \Rightarrow F4(o_1, o_2)$. Due to the underlying partition structure we are able to postulate that partition forming fiat objects cannot overlap, i.e., the largest relation that can hold between two partition forming bona-fide objects is EC.

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[2] BF is a finite (but may be very large) set of things that count as bona-fide objects with respect to the definitions given by [SV99].

[3] Two objects, o_1 and o_2 are connected if they are not disconnected i.e., $(r(o_1), r(o_2)) \notin DC$.

[4] F^{ϕ} the set of fiat objects of kind ϕ in the sense of [SV99].

4.2 Built environments

In this subsection I use the constraints defined above in order to describe the components built environments (layout, path system, situations) formally.

The layout of a built environment is formed by a set of partition forming objects. Formally, it is a triple $L =< G, BF_G, F_G >$, where G a set of regions forming a regional partition, BF_G is a set of partition forming bona-fiat objects, and F_G a set of partition forming fiat objects such that the following holds to be true: (1) $\forall o_1, o_2 \in BF_G : \quad o_1 \neq o_2 \Rightarrow F3(o_1, o_2)$; (2) $\forall o_1 \in F_G, \forall o_2 \in BF_G \cup F_G : \quad o_1 \neq o_2 \Rightarrow F4(o_1, o_2)$; (3) $G = \{r(o) \mid o \in BF_G\} \cup \{r(o) \mid o \in F_G\}$; (4) $\bigvee G = \top$. These are formal versions of the partition axioms $P1$ and $P2$, where $\bigvee G = g_1 \vee g_2 \vee \ldots \vee g_n$, $g_i \in G$ and $\top$ is the universal region, U, without the exterior, EXT, of the environment.

The path system. Let $\Gamma^G = (V, E, h)$ be a directed version of the dual graph of the topological graph[5] of the regional partition, G[6]. Consequently, every vertex, v_i, corresponds to a partition element g_i and $h(e) = (v_i, v_j)$ refers to the boundary segment (g_i, g_j) 'looking' from g_i to g_j[7,8]. The path system of the layout, Γ^L, is a sub-graph [NC88] of $\Gamma^G = (V, E, h)$. The graph $\Gamma^L = (V' \subseteq V, E' \subseteq E, h')$ is defined such that the edges, $e' \in E'$, correspond to boundary segments of partition-forming fiat objects of non-barrier sort in direction (g_i, g_j). The vertexes V' are the vertexes joined by those edges. For details see [Bit99]. Consider the right part of Fig. 1. It shows the path system of the parking lot discussed in Section 2. The long grey bar on the main road is the stretched vertex corresponding to the partition region occupied by the main road. The bold solid lines represent edges corresponding to non-barrier boundary segments. The arrows along the edges show their direction. If there are edges for each direction then the arrows are omitted[9].

Path system and movement. Let $r_t(o)$ be the exact region of the object o at moment t, let $r_T(o)$ be the set of all regions at which o was exactly located within the time interval $T = (t_1, t_2)$, i.e., $r_T(o) = \{r_t(o) \mid t_1 \leq t \leq t_2\}$, and let $\bigvee r_T(o)$ be the sum of all those regions. Let $\Gamma^L = (V', E', h')$ be the path system of the layout L. A path within the path system from v_1 to v_2, $\Gamma^L_{v_1,v_2} = (V'', E'', h'')$, is a connected subgraph of Γ^L beginning at v_1 and ending at v_2. This path is a path for the object o, $\Gamma^L_{v_1,v_2}(o)$, if and only if: (1) $R^8_{min}((\alpha\,(\bigvee r_T(o))), (\alpha\,\bigvee\{v_i \mid h''(e'') = (v_i, v_j)\})) = NTPP$[10]; (2) $h''(e'') = (v_i, v_j) \Rightarrow R^8_{min}((\alpha\,(\bigvee r_T(o))), (\alpha\, v_i)) = PO$. This says that $(\bigvee r_T(o))$ overlaps all regions along its path (2) and that $(\bigvee r_T(o))$ is a non-tangential proper part of the sum of all partition regions along its path (1).

Situations. A situation in a built environment is a triple $S =< L, BF_S, F_S >$, where L is the layout of the environment, BF_S is a set of non-partition forming bona-

[5] See [NC88] and [Bit99] for details.

[6] Boundary sensitive approximations, $(\mathrm{loc}\, o)$, correspond to labeled versions of this graph [Bit99].

[7] Multiple, disconnected boundary segments are distinguished by additional indices.

[8] In the remainder I use v_i and g_i synonymously.

[9] In this paper I only consider partition forming objects as *wholes*. In fact partition forming objects have parts which are caved out by fiat boundaries. A path system taking parts of partition forming objects into account is much better structured.

[10] Since the v_i refer to partition regions we have $R^8_{min} = R^8_{max}$.

fide objects and F_S is a set of non-partition forming fiat objects. The members of both sets are populating L in situation S. In a situation S the following holds: (1) $\forall o_1, o_2 \in BF_S \cup BF_G : o_1 \neq o_2 \Rightarrow F1(o_1, o_2)$; (2) $\forall o_1, o_2 \in F_S$: $(\phi o_1$ and ϕo_2 and $o_1 \neq o_2) \Rightarrow F2(o_1, o_2)$; (3) $\forall o \in BF_S : \Gamma^{L \cup \{EXT\}}_{r(EXT), r(o)}(o)$. Axioms (1) and (2) govern the non-partition forming objects as discussed in the sections 2 and 4.1. Consider axiom (3). The symbol EXT denotes the 'The world exterior to the environment L' and $\Gamma^{L \cup EXT}$ is the graph representing the path system of the environment L with its exterior EXT. Consequently, $\Gamma^{L \cup \{EXT\}}_{r(EXT), r(o)}(o)$ is a path for the object, o, from the exterior to its current location. Axiom (3) ensures that for each non-partition forming bona-fide object within the environment there exists a path along which this object could have been moved from the entrance to its current position. This is a formal version of the axiom M discussed in Section 2.

4.3 Specific built environments

In Section 2 we discussed that domain specific constraints on relations involving objects of different kind are weaker than constraints involving objects of the same kind. They can be violated without violating the laws of logic or physics, i.e., *it is possible to violate those constraints*. On the other hand the built environment *must permit* the satisfaction of those constraints in order *to be* an environment of a given kind.

Consider a parking lot with layout $L = (G, BF_G, F_G)$ and the informal axioms $S1$ and $S3$ as discussed in Section 2. Let $PS \subset F_G$ be the set of parking spots and let $BA \subset F_G$ the set of blocked areas of the parking lot. Let $CARS \subset BF_S$ be the set of cars populating the parking lot. We postulate: (1) $\forall o_1 \in CARS, \forall o_2 \in PS$: $\max\{R^8_{max}((\text{loc}_{bs}\ o_1), (\text{loc}_{bs}\ o_2)) \mid R^8_{max} \in RCC8\} = NTPP$[11]; (2) For each parking spot, $o_2 \in PS$ there must exist a path for a car $o_1 \in CARS$, i.e., $\Gamma^{L \cup \{EXT\}}_{r(EXT), r(o_2)}(o_1) = (V, E, h)$, which keeps blocked areas clear, i.e., $\forall e \in E$: $h(e) = (v_i, v_j) \Rightarrow \neg \exists ba \in BA : r(ba) = v_i$.

Axiom (1) states that cars need to fit into parking spots. Two remarks. Firstly, (1) is consistent with $\exists o \in CARS, \exists o_2 \in PS : PO(r(o_1), r(o_2))$, i.e., when we postulate that a parking lot *must be* such that cars do *fit* into parking spots we do *not* rule out the possibility that there are cars parked across boundaries of parking spots. Axiom (1) ensures the possibility for cars to be parked in parking spots. Secondly, stating axiom (1) in terms of rough location rather in terms of exact location has the advantage that we can effectively check its satisfaction since there are only finitely many different rough location in a built environment and we have the calculus proposed by [BS00] to compute the possible relationships.

Axiom (2) states that it must be possible for cars to avoid blocked areas. Again postulating this for an environment does not conflict with the fact that there cars that drive through or park at blocked areas.

[11] Since $r(PS_i) \in G$ we have $R^8_{max} = R^8_{min}$ and hence $\max\{rcc8(o_1, o_2) \mid rcc8 \in RCC8\} = NTPP$.

5 Conclusions

Given that task 1 and 2 are to be performed by program (II), there are three main arguments in favor of the formalization of build environments based on *rough location within environments* in opposition to the formalization based on *exact location of objects*: Rough location focuses on the relationships between objects and their environments; Concentrating on properties of the environment allows to abstract the different character of constraints on relations between the different kinds of objects forming and populating it; The notion of rough location is qualitative in nature.

Firstly. Rough location focuses on the approximate location of objects within regional partitions. In built environments the regional partitions formed by the partition forming objects are the main organizational structure. They provide a frame of reference within which non-partition forming objects are located. The notion of rough location implicitly takes the distinction between partition forming and and non-partition forming objects and the organizational structure of the regional partition into account.

When we describe built environments in terms of rough location then objects are second class citizens. The first class citizens are mappings representing the *rough location of objects within their environments*. These mappings can be interpreted as equivalence classes of objects sharing the same rough location. Since built environments are formed by finitely many partition forming objects there are only finitely many different rough locations within an environment. Given the calculus presented in [BS00] the satisfaction of the axioms presented in this paper can be checked effectively.

Secondly. Concentrating on properties that need to satisfied by the (built) *environment* allows to abstract from the different character of constraints on relations between spatial *objects*. The different character of the constraints on relations between objects is due to the fact that there are constraints that are enforced by the laws of logic, there are constraints that are enforced by the laws of physics, and there are constraints that are enforced by human conventions. The laws of logic prohibit objects of ontological same kind and partition forming objects to overlap. Laws of physics prevent bona-fide objects from being connected. Constraints involving fiat objects of ontological different kind are based on social rules and agreement and may be violated in certain situations. An environment *must permit* the satisfaction of *all* constraints in order *to be* an environment of a given kind *independently* of the character of the constraints between the objects forming or populating it.

Thirdly. We assumed a program (I) that generates potential plans for built environments. It is fair to assume that (I) is based on standard algorithms of computational geometry. The output of (I) is quantitative and focuses on metric knowledge. The program (II) extracts qualitative knowledge and builds a corresponding boundary sensitive rough location representation.

One might ask ‘Why do we need a qualitative description if we have a quantitative geometric model?’. The answer is that it is the purpose of (II) to evaluate the plan of the environment with respect to axioms specifying what a plan of a built environment is AND with respect to the degree it facilitates human way finding. (i) In order to capture the essence of what a built environment is one needs to abstract from metric properties of particular instances. What a built environment is can be described in terms of (qualitative) relationships between ontologically salient features of the environment.

(ii) Human cognition is based on processing qualitative rather than quantitative knowledge. Qualitative knowledge about actual situations is based on observations of reality. Consequently, the question is not whether or not to use the quantitative description generated by (I), but to derive qualitative spatial relations *between ontologically salient features, which correspond to relations observable in reality* from this description. This is exactly what happens when we describe built environments in terms of boundary sensitive rough locations of objects forming and populating them.

In this paper I have shown that based on the notion of boundary sensitive rough location task (1) of program (II) can be performed, i.e., it is possible to decide whether or not a configuration of lines in the plane represents a built environment using the axioms given in Section 4. I, furthermore, showed how to derive paths within a build environments along which non-partition forming objects can move. This provides the formal foundations for task (2), i.e., to evaluate those paths with respect to the complexity of the way finding task to be solved in order to navigate along them. Subject of ongoing research in this context is to apply the model for the evaluation of the complexity of wayfinding tasks proposed by [RE98].

References

[All83] J.F. Allen. Maintaining knowledge about temporal intervals. *Communications of the ACM*, 26(11):832–843, 1983.

[Bit99] T. Bittner. The qualitative structure of built environments. Technical report, Department of Computer Science, Queen's University, 1999.

[BS98] T. Bittner and J. G. Stell. A boundary-sensitive approach to qualitative location. *Annals of Mathematics and Artificial Intelligence*, 24:93–114, 1998.

[BS00] T. Bittner and J. Stell. Approximate qualitative spatial reasoning. Technical report, Department of Computing and Information Science, Queen's University, 2000.

[CBGG97] A.G. Cohn, B. Bennett, J. Goodday, and N. Gotts. Qualitative spatial representation and reasoning with the region connection calculus. *geoinformatica*, 1(3):1–44, 1997.

[CV95] R. Casati and A. Varzi. The structure of spatial localization. *Philosophical Studies*, 82(2):205–239, 1995.

[GLM83] T. Gaerling, E. Lindberg, and Maentylae. Orientation in buildings: Effects of familarity, visual access, and orientation aids. *Applied Psychology*, 68:177–186, 1983.

[Kui78] B. Kuipers. Modeling spatial knowledge. *Cognitive Science*, 2:129–154, 1978.

[Lyn60] Kevin Lynch. *The Image of the City*. MIT Press, Cambridge, 1960.

[NC88] T. Nishizeki and N Chiba. *Planar Graphs: Theory and Applications*. North Holland, Amsterdam, 1988.

[RE98] M. Raubal and M. Egenhofer. Comparing the complexity of wayfinding tasks in built environments. *Environment & Planning B*, 25(6):895–913, 1998.

[Rou94] Joseph O' Rourke. *Computational Geometry in C*. Cambridge University Press, 1994.

[Smi95] B. Smith. On drawing lines on a map. In A.U. Frank and W. Kuhn, editors, *Conference on Spatial Information Theory, COSIT*, volume 988. Springer, Semmering, Austria, 1995.

[SV99] B. Smith and A.C. Varzi. Fiat and bona fide boundaries. *Philosophy and Phenomenological Research*, 1999.

[SW75] A. Siegel and S. White. *The development of spatial representations of large-scale environments*, volume 10, pages 9–55. Academic Press, 1975.