On Ontology in Image Analysis

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Object reconstruction from remote sensed images is an actual research topic in computer vision, photogrammetry, and spatial information science. One of the most interesting related questions is the modeling of the uncertainty of knowledge gained by such a process. For a better understanding of the different sources and aspects of uncertainty, indeterminacy, vagueness, and the relations between them, a better understanding of the underlying ontological and epistemological foundations of imaging is necessary.

An important aspect is to make the relationships between the objects in the world that are observed, for example, by means of remote sensing, and fiat objects created from the remote sensed data by means of spatial analysis explicit. Both kinds of objects are linked to each other by observation and analysis processes. The process of spatial analysis has three basic components which are modeled in this paper as mappings between fiat objects of different kind. We show that vagueness in the definition of fiat objects involved in this process results in indeterminacy of location of those objects. Definitorial vagueness and location indeterminacy of location result in uncertainty about the truth of conclusions about existence, properties, and location of objects in the world derived from knowledge about fiat objects created by spatial analysis.

1 Introduction

Object reconstruction from remote sensed images is an actual research topic in computer vision, photogrammetry, and spatial information science. One of the most interesting related questions is the modeling of the uncertainty of knowledge gained by such a process. Recent proposals include fuzzy sets (Molenaar & Cheng 1998) and rough sets (Worboys 1998*a*, Worboys 1998*b*).

Beyond the modeling of uncertainty for spatial analysis, we feel the need to describe ontological and epistemological aspects involved in the process of imaging. In order to get a better understanding of the different sources and aspects of uncertainty, indeterminacy, vagueness, and the relations between them, a better understanding of the underlying ontological and epistemological foundations of imaging is necessary.

We start from the distinction between bona fide and fiat objects (Smith 1995). Roughly, bona fide objects are spatial objects in physical reality which exist independently of human cognition. Examples are the planet Earth, islands, lakes. Fiat objects are created by human cognition. Examples are legal fiats like political subdivisions, land property, or — of central concern for this paper — fiat objects created by spatial analysis of remote sensed data.

We show that the process of spatial analysis has three basic components, which are modeled as mappings between fiat objects of different kinds. We relate vagueness in the definitions of fiat objects involved in this process to different kinds and sources of uncertainty. We show that vagueness in the definition of fiat objects results in indeterminacy of location of those objects.

An important aspect is to make the relationships between the objects in the world that are observed, for example, by means of remote sensing, and fiat objects created from the remote sensed data by means of spatial analysis explicit. This will provide the basis for a discussion about the truth of conclusions about existence, properties, and location of objects in the world derived from knowledge about analysis created objects.

This paper is structured as follows. We start with a short review of remote sensing and spatial analysis performed on remote sensed data (Section 2). In particular we introduce a running example which will be used to illustrate the discussion in the remainder of this paper. In Section 3 we review and introduce ontological foundations. Formal models of location in space are reviewed in Section 4. These models provide the basis of the formalization provided in this paper. In Section 5 fiat objects created during processes imaging and remote sensing are discussed and formal representations are provided. This results in the discussion of the ontological status and adequate formalization of fiat objects created by spatial analysis in Section 6. The conclusion are given in Section 7.

2 The Principle of Remote Sensing

This section gives a short wrap up of remote sensing and image processing. It provides the starting point for the ontological investigation in the later sections.

2.1 The Physical and Geometrical Model

Remote sensing is an observation technique based on sensors for spectral reflectance, registering radiation typically in optical, infrared, thermal or microwave band (Kraus 1988). The sensors are single, or ordered in lines or in rectangular arrays, however, in any case recordings are combined to two-dimensional raster images. The rectangular shape of each sensor element g_i is mapped by a central projection onto a region G_i on the earth surface, i.e., the shape of G_i is not necessarily a rectangle due to terrain elevation and image orientation. Nevertheless, the property that $\bigcup_i g_i$ forms a partition is preserved for $\bigcup_i G_i$.

It is common to call the physical sensor element as well as the raster image element a *pixel*. Physically, the sensitive area of each pixel is not identical to its rectangular shape. The area on the earth surface corresponding to the sensitive area of a pixel is called sometimes *instantaneous field of view* I (Bruegger 1995). The set $\bigcup_i I_i$ does not form a partition.

In principle, each pixel g_i registers the number of photons received in a time interval. The resulting intensity value, denoted also by g_i, represents the mean reflectance over I_i:

$$g_i = \int I_i \, \mathrm{d}t$$

For simplicity it is assumed generally that $\int I_i \stackrel{!}{=} \int G_i$. This assumption causes some (radiometric) uncertainty. Other radiometric uncertainty results from the complex and dynamic relation between surface object and reflectance, involving perspective, lighting, and surface properties, and also from the atmospherical disturbances, electronical noise and other physical limitations.

The geometry of the raster image is that of a central projection. The parameters of the mapping function between g_i and G_i are known only imprecise, so that the assignment of the value of g_i to the area of G_i is (geometrically) uncertain, too. The other source of geometric uncertainty is the regular discretization of the imaging sensor, which does not fit to partitions of space on the earth surface.

Some effects causing uncertainty are systematic and can be rectified. The remaining effects, together with the model errors in rectification, accumulate inseparably, requiring models of imaging and of image interpretation coping with uncertainty. These effects are assumed to be independent and randomly distributed. Then the observed intensity values can be interpreted as results of stochastic processes (Baarda 1967). A stochastic process is an *a-priori* indeterminate process that is characterized by a distribution. If there are many and independent elementar effects the distribution of the process is a Gaussian (central limit theorem) (Koch 1988).

The intensity value of g_i gives now some evidence to represent reflection of a class of surface objects with specific properties (color, e.g.). Evidence is expressed in terms of probability. The probabilities for all classes which are expected to be observed in the image are used to determine the object (class) at G_i. This determination is called image classification.

2.2 Classification of Remote Sensing Images

With the rectified image at hand, one is interested in information about type and properties of objects on the earth surface in the covered area. The information is extracted by visual or automatic image *interpretation* (Kraus 1988):

- Visual interpretation is made by an expert who exploits spectral features, patterns, textures, and knowledge about natural and man-made features. — Human interpretation is still too complex to be transferred to a computer.
- For the automatic image interpretation pixel-wise multispectral classification is preferred, exploiting pixel attributes only: the n intensity values at each pixel correspond to a point in an n-dimensional space. In this space, all

points are grouped in clusters. The clusters are matched with object classes, with some *a-priori* or interactively given knowledge.

The multispectral classification assumes also stochastic variations of intensity values, characterizing the assignment of a pixel to a class by a probability which is derived from the centrality in the cluster. However, as discussed before, uncertainty concerns variations of individual objects from a class prototype as well as all the geometric and radiometric random effects on the signal.

With a probability value for each class, a pixel is assigned to the class of the maximum likelihood. One can post-process a classified image to get individual objects. Image segmentation combines connected pixels of the same class label to components. Each connected component refers to an object. Boundaries of objects look crisp, but they are not, due to uncertain assignments of pixels to classes.

2.3 An Example

The running example for this paper shall be height images, as derived from laser scanning (Krzystek 1991). Such remote sensed images are specific in so far as they represent distances in their pixel values. The distances are derived by the run-time an emitted laser pulse needed to be received again. Usually the distance is rectified to the elevation of the terrain, given the height of the sensor. Our discussion for the effects causing (stochastic) uncertainty still remains valid.

A problem from practice is investigated by Molenaar *et al.* (Cheng, Molenaar & Bouloucos 1997, Molenaar & Cheng 1998): they deal with the classification of *foreshore*, *beach*, and *foredune* from terrestric height measurements as well as from remote sensed height images.

For laser scanning images, the error of measurement can be evaluated in a system calibration (Krzystek & Wild 1992). The result can be applied for statements about the uncertainty of the height values of each pixel, e.g., by a standard deviation σ.

The second category of uncertainty concerns the class definition. Molenaar *et al.* define the three classes by the height intervals (in m): *foreshore* (-6,-1); *beach* (-1, 2); *foredune* (2, 25). Because definitions vary in between some limits from expert to expert, Molenaar *et al.* handle the variation by a fuzzy definition of the classes, using a trapezoidal membership function (Klir & Folger 1988) for each class. A trapezoid needs four parameters: start and end point of uphill line, and start and end point of downhill line. In contrast to the crisp definition, a fuzzy definition of the classes is now: *foreshore* (-8.0, -4.0, -1.6, -0.6); *beach* (-1.6, -0.6, 1.5, 2.5); *foredune* (1.5, 2.5, 22, 28), with the cross-over points at the crisp boundaries (Fig. 1 (A-C)).

If a height value is uncertain and its classification is fuzzy, the two must be compounded. Molenaar *et al.* calculate a convolution of fuzzy membership function and likelihood function, leading to a belief function (Klir & Folger 1988) that determines the uncertainty of each pixel belonging to the different classes. The classification is visualized with its belief values in Figure 1 (D).

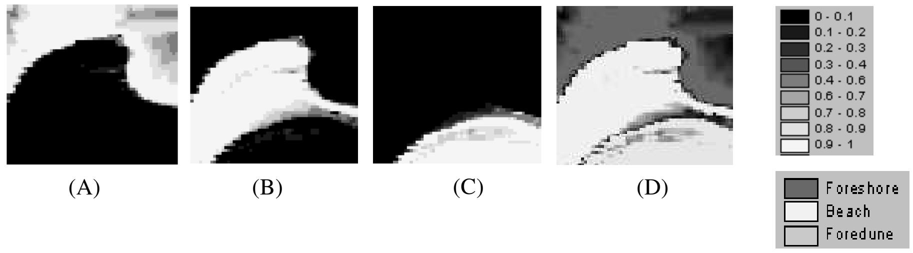

Fig. 1. The three fuzzy landscape classes (a-c), and the compound classification (d). (The figures are taken from Molenaar & Cheng 1998, with the kind permission of the authors).

3 Spatial Objects and Location

3.1 Spatial Objects

Aspects of Ontological Categorization. Following Smith & Mark (1998), we assume that in the ontological characterization of geographic reality three basic aspects are involved:

1. aspects characterizing *what* spatial objects are,
2. aspects characterizing *where* spatial objects are located, i.e., aspects of embedding of spatial objects into geographic space, and
3. aspects of scale.

In the remainder we use the notion of 'spatial object' to refer to objects characterized by those aspects.

The 'What'. *What* spatial objects are is characterized to a certain degree by their observable properties. Every object has its unique identity. During its existence every object preserves its identity even if its properties change. In order to characterize spatial objects with respect to what they are beyond observable properties we consider their identity, and:

– The compositional structure of their constituent object-parts.
– Modes of their existence within physical reality and with respect to human cognition.

The Where. Aspects characterizing *where* objects are located, and relationships between *what* objects are and *where* they are, have been investigated in spatial sciences for centuries. Surveyors deal with the question where things are located on Earth (Moffitt & Bouchard 1987). Geography deals with the relationships between what things are and where they are (Abler, Marcus & Olson 1992). In this paper aspects characterizing *where* spatial objects are, are captured by the notion of location. Location is formalized as a relation between spatial objects and regions of space.

Aspects of Scale. Aspects of scale refer to the classification of spatial objects with respect to size relative to observability and modes of observation by human beings. There is a whole class of literature that deals with different classifications in this respect. An overview can be found in (Freundschuh & Egenhofer 1997). Spatial objects of geographic scale, considered in this paper, are larger than the human body and cannot be perceived within a single perceptual act. Examples are cities, mountains, or lakes.

Bona Fide and Fiat Objects. Spatial objects do not only have constituent object-parts, they also have boundaries, which contribute much more to their ontological make-up as do object-parts (Smith & Mark 1998). One fundamental distinction characterizing *what* spatial objects are is based on the classification of boundary parts. This results in the distinction between objects with *bona fide* and objects with *fiat* boundaries.

Bona fide boundaries "... are boundaries *in the things themselves*. They would exist (and did already exist) even in the absence of all delineating or conceptualizing of our ... part. Bona fide boundaries are boundaries which exist independently of all human cognitive acts - they are a matter of qualitative differentiations or discontinuities of the underlying reality." (Smith 1995, p. 475) Examples are surfaces of extended objects like tennis balls, plants, the surface of Earth, and the shore line separating water and sand in Example 2.3.

Fiat boundaries " ... are boundaries which exist only in virtue of different sorts of demarcations effected cognitively by human beings. Such boundaries lie entirely skew to all boundaries of *bona fide* sort (as in the case of the boundaries between Utah and Wyoming). They may also, however (as in the case of Indiana and Pennsylvania), involve a combination of fiat and *bona fide* portions, or indeed they may be constructed entirely out of *bona fide* portions which however, because they are not themselves intrinsically connected, must be glued together out of heterogeneous portions in fiat fashion in order to yield a boundary that is topologically complete. " (Smith 1995, p. 476) Examples are many state and provincial borders, country lines, property lines, borders of postal districts, boundaries of soil-types.

The classification of boundaries generalizes to the objects they bound. Bona fide objects are topologically closed bounded by a fiat boundary. Examples are human beings, the planet Earth, lakes, highways. Fiat objects are bounded either by bona fide boundaries of different sorts glued together by human cognition or are at least partly bounded by boundaries of fiat sort. Examples are 'The North Sea', 'The Rocky Mountains', political subdivisions, land property.

3.2 Fiat Objects

Fiat objects can be classified with respect to the way human conceptual demarcations carve out fiat objects, i.e., create fiat boundaries, in physical reality. Those conceptual demarcations differ in the degree in which they *specify* what the objects they *create* are and where they are located. In the context of this paper we discuss three kinds of fiat objects:

1. Consensus Fiats
2. Measurement and Observation Fiats
3. Spatial Analysis Created Fiats

Consensus Fiats. "Consensus fiats may be products of informal consensus, reflecting for example usage as this evolves informally over time. Bays, peninsulas, etc., are parts of spatial reality, physical parts of the world itself. But they are parts of reality that would not be there absent corresponding linguistic and cultural practices of demarcation and categorization" (Smith & Mark 1998, p. 316). Consequently, human concepts represented and shared by means language 'carve out', i.e., create, fiat objects in physical reality. The definitions of those concepts are often subject to vagueness (Smith 1995), i.e., those definitions only vaguely specify *what* the spatial objects they create are and *where* they are located.

Fiats Created by Observation and Measurement Processes. Human beings create fiat objects during their interaction with physical reality. Visual observation of physical reality creates fiat boundaries all the time, e.g., the horizon bounding the 'visual field' (Smith 1995), my body axes create fiat boundaries separating the fiat objects 'the world in front of me', 'the world in my back', left and right of me and so on. What those objects are and where they are located is well defined in each moment of time but their location changes all the time.

Measurement is a specific and precise form of observation. Measurement processes create fiat objects, which regions form partitions[1] of space and time (Carnap 1966, Bittner 1999*a*)[2]. Consider the measurement of time. The measuring process, i.e., the use of a clock to measure temporal extension, creates fiat temporal objects, which regions partition the time line. Those temporal objects are located at time intervals bounded by 'clock-ticks'. Of central concern for this paper are fiat objects on the surface of Earth created by remote sensing processes. The boundaries of those fiat objects are created by geo-referencing of pixels of remote sensed raster images[3]. Other examples are fiat objects created by sample data and interpolation (Example 2.3).

Fiats Created by Spatial Analysis. The process of creating fiat objects by spatial analysis assumes a measuring process that creates a set of fiat objects, which form a regional partition of Earth[4]. Based on those *sets* of *simple* fiat

[1] Regional partitions are sets of regions which intersect only at their boundaries and which sum up the whole space.

[2] Smith & Mark (1998) call those fiats "mathematical fiats that are artifacts of a certain technology."

[3] In the context of this paper we see geo-referencing as an idealized process, i.e., we abstract from aspects causing radiometric and geometric uncertainty, which were discussed in Section 2.1.

[4] Prototypical examples of those processes are imaging in remote sensing and sampling of point data followed by interpolation operations (Sect. 2.3).

objects we can define, i.e., create, *complex* fiat objects as sums of primitive fiat cell objects.

For example, each fiat cell object, which was created by a measuring process, is characterized by a number of values which refer to some kind of measurement, e.g., a certain number of photons of certain wavelength measured by a particular raster sensor. We now can define sums of raster-cell-fiats which values satisfy certain conditions, for example, having received a certain number of photons with a particular wave length within a certain time interval, or, as in case of the example in Section 2.3, having registered the same run-time of single photons. Corresponding to practice in image classification (Section 2.2), the conditions often are formulated weaker, allowing a range of pixel values to be summed up. We will discuss the process of creating fiat objects by means of spatial analysis extensively in Section 6.

On Defining Fiat Objects

Definite Fiats. Consider legal fiats such as political subdivisions or land property, fiats created by measuring processes, or fiats created by point interpolation operations. The definition of those objects contains the specification of the *exact* region of space at which those objects are located[5]. In all cases the exact regions are often defined and stored in terms of semi-algebraic or semi-linear formulas (non-linear or linear polynomials) (Kanellakis, Kuper & Revesz 1990). In the case of land property those definitions are stored in property records. In the case of fiat objects created by remote sensing the exact regions are implicitly given in terms of the geo-referencing of raster images. In the case of point interpolation, the equivalence classes defined in the attribute domain induce a unique regional partition of the underlying space[6] and hence define the exact regions of the corresponding partition forming fiat objects.

Definite fiat objects, which explicitly contain the definition of their exact region of space, are the only kinds of spatial objects for which human beings can know exact location. Since the boundaries of those fiats are usually not observable in physical reality there exist rules and procedures that allow to determine the location of those boundaries and mark them such that they become observable. A very significantly marked legal boundary, for example, was the Berlin Wall separating East and West Berlin during the Cold War. May me, you marked the boundary of your property in order to separate it from your neighbor's property.

Vaguely defined fiats. Consider fiat objects like mountains, hills, valleys, ridges, or capes. "...we can all agree that they are real, and that it is obvious where the top of a mountain or the end of a cape is to be found. But where is the boundary

[5] The notion of location as a relation between spatial objects and regions of space will be discussed in the next section.

[6] Assuming continuous distribution functions describing the spatial distribution of the attribute value. See for example (Laurini & Thompson 1994).

of Cape Flattery on the island inside? Where is the boundary of Mount Blanc among its foothills?" (Smith & Mark 1998, p. 316) The human concepts and definitions that bring those fiat objects into existence do not specify their boundary locations. Consequently, we cannot apply rules and measurement in order to make those boundaries observable. Any human being can complete the definitions and draw the boundary of her or his 'Mount Blanc' or 'Cape Flattery'. The vagueness of concepts bringing fiat objects into existence cause indeterminacy of location. " ... if you point to an irregularly shaped protuberance in the sand and say 'dune', then the correlate of your expression is a fiat object whose constituent unary parts are comprehended (articulated) through the concept dune. The vagueness of the concept itself is responsible for the vagueness with which the referent of your expression is picked out. Each one of a large verity of slightly different and precisely determinate aggregates of molecules has an equal claim to being such a referent." (Smith & Mark 1998, p. 315)

Often, the vague definition might be strong enough to identify regions of space which are part of the object's exact region, which overlap the object's exact region, or where no parts of the object are located. This often allow us to draw boundaries around a 'certain' core and 'certain' exterior. Doing so we create a regional partition consisting of three concentric regions similar to those shown in Figure 2. The boundary of the vaguely defined object is located somewhere between the core and the exterior[7].

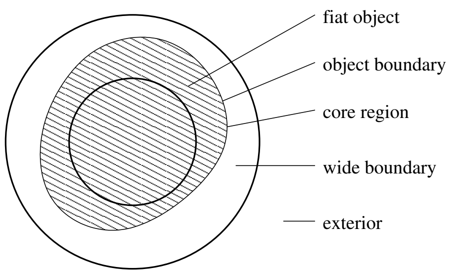

Fig. 2. A vaguely defined fiat object

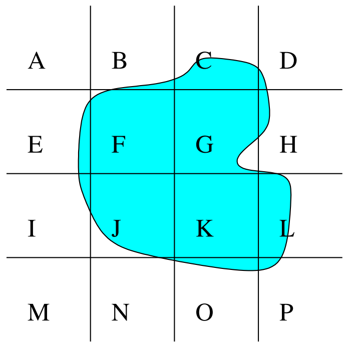

Fig. 3. Rough Location in a Regular Raster.

3.3 Location

Exact Location. Every spatial object is exactly located at a single region of space in each moment of time (Casati & Varzi 1995). This region may be a simple

[7] For an extended discussion see (Cohn & Gotts 1996).

region of three dimensional space, for example, think of your body and the region of space it carves out of the air. The exact region of a spatial object may be a complex region, consisting of multiple regions of three dimensional space, as in the case of the exact region of the Hawaiian islands. The exact region may be a complex region, consisting of multiple regions of two dimensional space, as in the case of the representation of the Hawaiian islands on a paper map. Notice that even in this case we distinguish between the spatial object 'Representation of the Hawaiian islands' consisting of several layers and blends of paint and the region of space it carves out of the map space.

Spatial change refers to the fact that spatial objects can be located at different regions of space at different points in time. This implies a concept of absolute space, i.e., regions of space do not move, shrink, or grow. Spatial objects can move, shrink, grow. This makes them being located at different regions of space at different moments of time. See for example (Borgo, Guarino & Masolo 1996).

Rough Location. Rough location describes the location of spatial objects within sets of regions of space that form regional partitions (Bittner 1999*b*). Rough location is characterized by sets of relationships between *parts* of the object's exact region and *parts* of partition regions. Consider Figure 3, which shows the rough location of a spatial object, o, with a non-regular shaped region, which is located within a raster-shaped regional partition. The raster shaped regional partition, G, might be created, for example, by remote sensing. Rough location can be represented by upper and lower approximation sets[8]. The lower approximation set, $\underline{o}$, of the object, o, with respect to the partition, G, is the set of all raster regions, $g \in G$, that are part of the exact region of o. The upper approximation set, $\overline{o}$, is the set of all raster regions that overlap the exact region of the object, o.

$$\underline{o} = \{K\}$$
$$\overline{o} = \{B, C, D, E, F, G, H, I, J, K, L, N, O, P\}$$

In this paper we distinguish three kinds of rough location:

1. *Overlap sensitive* rough location, which is characterized by overlap and non-overlap relations between exact regions of spatial objects and partition regions. This corresponds to upper approximation sets.
2. *Containment sensitive* rough location, which is characterized by part-of and non-part-of relations between exact regions of spatial objects and partition regions. This corresponds to lower approximation sets.
3. *Overlap & containment sensitive* rough location, which is characterized by pairs of upper and lower approximation sets.

[8] The notions of lower and upper approximation sets come from rough set theory (Pawlak 1982) and were applied to the spatial domain for example by Worboys (1998*a*) and Bittner (1999*b*).

Consider Figure 2. The overlap & containment sensitive rough location of the vaguely defined fiat object o in the regional partition consisting of three concentric regions, called core, wide boundary, and exterior can be represented the lower and upper approximation sets $\underline{o} = \{core\}$ and $\overline{o} = \{core, wide\_boundary\}$. Notice, that the rough location of the vague defined object in this regional partition is well defined and determinate. The notion of rough location allows to abstract from the indeterminacy of location caused by the vagueness of the object definition.

4 Modeling Location

4.1 Modeling Exact Location

Spatial objects and regions of space have compositional structure. We model spatial objects, regions of space, and their compositional structure using the Boolean algebra (Halmos 1963, Stell & Worboys 1997) of regular closed sets (Requicha 1977). Regular closed sets are sets which are equal to the closure of their interior. Regular closed point sets model regions of space (Gotts 1996). They are topologically well formed in that sense that they do not contain isolated points or infinitely thin 'spikes'. In a Boolean algebra join, $\vee$, and meet, $\wedge$, operations are defined, which are interpreted as (regularized[9]) union and intersection of regular closed sets.

Let O be the Boolean algebra of spatial objects and R be the Boolean algebra of regions of space. Since every spatial object is exactly located at a single region of space at each moment of time we can define a function, r, mapping spatial objects onto regions of space:

$$r : O \rightarrow R$$

The mapping r is a bijection in the domain of bona-fide objects but not in the domain of fiats. This is due to the fact that no two physical objects can be located at the same region of space in the same moment of time. It is not a bijection in the domain of fiat objects, since fiat objects of *different kind* can be located at the same region of space in the same moment of time (Casati & Varzi 1995). For example, the objects 'City of Vienna' and 'Federal State Vienna' are located at the same region of space in every moment of time.

Obviously, the mapping, r, is by no means computable. Furthermore as discussed, for example, by Bittner (1999a), for most spatial objects we cannot even know the exact region of space at which they are located. Nevertheless we know that for every spatial object the exact regions exists and hence, the mapping, r, exists. Consequently, we can use the notion $r(o)$ to refer on a formal level to the exact region of the object $o \in O$.

[9] See (Gotts 1996) or (Requicha 1977).

4.2 Approximating Regions of Space

We now review the notion of relationship mappings which was originally introduced by Bittner & Stell (1998). In this paper we use relationship mappings in order to model rough location of analysis created fiat objects within the regional partition created by the underlying measurement process.

Overlap Sensitive Approximation. Let the set $\Omega_\exists = \{\Diamond, 0^\Diamond\}$ be a set of ways elements $g \in G$ can relate to an element $r \in R$. The function $p_\exists$ is a relationship function defined as:

$$p_\exists : G \times R \to \Omega_\exists; \quad p_\exists(g, r) = \begin{cases} \Diamond & \text{if } \exists a \preceq g (a \preceq r) \\ 0^\Diamond & \text{otherwise} \end{cases}$$

The mapping $p_\exists(g, r)$ returns the value $\Diamond$ if and only if the regions g and r overlap.

The relationship function, $p_\exists$, induces an approximation function $\alpha_\exists$:

$$\alpha_\exists : R \to {\Omega_\exists}^G; \quad \alpha_\exists(r) \ =_{def} \ g \mapsto p_\exists(g, r)$$

The symbol ${\Omega_\exists}^G$ denotes a set of functions from G to $\Omega_\exists$. The mapping $\alpha_\exists$ assigns to each $r \in R$ a function $(G \to \Omega) \in {\Omega_\exists}^G$. Consider figure 3. The set of regions $G = \{A, B, C, \ldots, P\} \subset R$ forms a regional partition of the plane. The gray non-regular shaped region, $r \in R$, is located in the regional partition G. The graph[10] of the approximation function $(\alpha_\exists R) : G \to \Omega_\exists$ approximating the region, r, with respect to the partition, G is:

G	A	B	C	D	E	...
$\Omega_\exists$	$0^\Diamond$	$\Diamond$	$\Diamond$	$\Diamond$	$\Diamond$	...

Containment Sensitive Approximation. Containment sensitive approximation is represented by approximation mappings $\alpha_\forall$, which are defined based on relationship functions $p_\forall$:

$$p_\forall : G \times R \to \Omega_\forall; \;\; p_\forall(g, r) = \begin{cases} \Box & \text{if } \forall a \preceq g (a \preceq r) \\ 0^\Box & \text{otherwise} \end{cases}$$

The mapping $p_\forall(g, r)$ returns the value $\Box$ if and only if g is a subset of r. The relationship function, $p_\forall$, induces an approximation function $\alpha_\forall$:

$$\alpha_\forall : R \to {\Omega_\forall}^G; \quad \alpha_\forall(r) \ =_{def} \ g \mapsto p_\forall(g, r)$$

The approximation function $\alpha_\forall$ assigns to each $r \in R$ a function $(G \to \Omega_\forall) \in {\Omega_\forall}^G$. Consider figure 3. The graph of the approximation function $(\alpha_\forall r) : G \to \Omega_\forall$ is:

G	A	B	...	K	L	...
$\Omega_\forall$	$0^\Box$	$0^\Box$	...	$\Box$	$0^\Box$	...

[10] The graph of a mapping explicitly lists the set of tuples forming the mapping.

Overlap & Containment Sensitive Approximation. We now define the overlap & containment sensitive value domain Ω_3 as pairs of containment and overlap sensitive value domains $\Omega_\forall$ and $\Omega_\exists$:

$p_3(g,r)$	$p_\forall(g,r) = \Box$	$p_\forall(g,r) = 0^\Box$
$p_\exists(g,r) = \Diamond$	$(\Box, \Diamond)$	$(0^\Box, \Diamond)$
$p_\exists(g,r) = 0^\Diamond$	-	$(0^\Box, 0^\Diamond)$

The relationship function p_3 applied to region r and partition region g returns $(\Box, \Diamond)$ if and only if g is a part of r. It yields $(0^\Box, \Diamond)$ if g and r overlap, without g being a part of r, and $(0^\Box, 0^\Diamond)$ otherwise. Assuming $\forall g \in G(g \neq \perp)$ the case '$p_\forall(g,r) = \Box$ and $p_\exists(g,r) = 0^\Box$' cannot occur (Bittner & Stell 1998).

The relationship function, p_3, induces an approximation function α_3 as discussed above. Consider figure 3. The graph of the approximation function $(\alpha_3 r) : G \rightarrow \Omega_3$ is:

G	A	B	...	K	L	...
Ω_3	$(0^\Box, 0^\Diamond)$	$(0^\Box, \Diamond)$	...	$(\Box, \Diamond)$	$(0^\Box, \Diamond)$	...

4.3 Modeling Rough Location

In this section, so far, we discussed how spatial regions can be approximated with respect to regional partitions. Those approximations can be represented formally by means of approximation functions derived from relationship functions. The notion of location refers to relation between spatial objects, $o \in O$, and regions of space, $r \in R$. In order to model rough location using approximation functions we define location mappings as:

$$loc : O \rightarrow \Omega^G; \quad (\text{loc } o) \quad =_{def} \quad (\alpha \circ r)o$$

The function $r(o)$ refers to the exact region of the object $o \in O$. The operator $\circ$ refers to the composition of α and r defined as $(\alpha \circ r)o \ = \ \alpha(r(o))$.

Lower and upper rough approximations of the object, o, discussed in Section 3, can easily be defined in terms of the overlap & containment sensitive location mappings $loc_3 : O \rightarrow {\Omega_3}^G$ by writing $\underline{o} = \{g \in G \mid ((\text{loc}_3\ o)g) = (\Box, \Diamond)\}$ and $\overline{o} = \{g \in G \mid ((\text{loc}_3\ o)g) = (\Box, \Diamond) \text{ or } ((\text{loc}_3\ o)g) = (0^\Box, \Diamond)\}$.

5 Modeling Definite Fiat Objects

Fiat objects are created in the human mind. In the scientific realm definite fiat objects are created by exact definitions. Consequently, humans can have exact and complete knowledge about those definite fiat objects.

5.1 Defining Classes of Definite Fiats

Let A be an attribute domain ranging over classificatorial, ordering , or quantitative concepts. In abstract domains we can use concepts like real numbers in our definitions without considering issues of observability and measureability. The only assumption we have is that these definitions need to be expressible by finite means of formal language.

Let $A_1, \ldots, A_n$ be a finite number of attributes. $\phi(A_1, \ldots, A_n)$ is a class of fiat objects that are completely characterized by the attribute vector $A_1, \ldots, A_n$ and a set of formula ϕ, which characterize the class of objects $\phi(A_1, \ldots, A_n)$ in terms of attribute values $(a_1, \ldots, a_2) \in A_1 \times \ldots \times A_n$. Since we are in abstract, fiat domains it is possible to completely characterize classes of objects by finite number of attributes and finite number of formula.

Consider, for example, the class of definite fiat objects called 'land property'. Objects of this kind are classified by attributes like property Id, owner, the object's exact region, land use, etc. The region of the object might be specified by semi-algebraic formulas like linear polynomials. There might be formulas constraining possible values the attributes can take. For example the attribute domain owner should range over valid names of human beings that are alive.

5.2 Definite Fiat Objects

Let $v = (a_1, \ldots, a_n) \in A_1 \times \ldots \times A_n$, be a vector of attribute values characterizing a spatial object $o \in O$. In fiat domains we can assume that v completely characterizes the object o up to its identity. Since o is a spatial object the region $r(o)$ at which o is exactly located exists.

In definite fiat spatial domains, the exact spatial location, i.e., the exact region of the object, is often a defining property (think of land property). In this case, one of the attribute domains ranges over regions of space. The knowledge about the kind of the object, i.e., the class it belongs to, and knowledge about its exact location at a certain moment in time is sufficient to fix the object's identity. This is due to the fact that no two spatial objects of the same kind can be located at the same region of space at the same moment in time (Casati & Varzi 1995). In database systems the identity of a spatial object is often handled explicitly by identifiers and handled as attribute value.

Since the attribute value vector, v, uniquely determines the object, o, we can write $r(v)$ to refer to the region at which the object, o, is located exactly. We use the notion of a pair to refer to a definite fiat objects with its attributes and its exact location region:

$$o = (v, r(v)). \tag{1}$$

Suppose the class of objects $O' \subset O$ is characterized by attributes $A_1, \ldots, A_n$ where A_1 ranges over regions. The function $r(v)$ simply yields the attribute value $v_1 \in A_1$, i.e., $r(v) = v_1$.

A certain class of regularity shaped regions can be represented finitely by means of semi-algebraic formulas. The representation of spatial objects by a finite number of attributes $A_1, \ldots, A_n$ where A_i ranges over semi-algebraic formulas was discussed, for example, by Paredaens, Van den Bussche & Van Gucht (1994).

5.3 Definite Fiats forming Regional Partitions

Let $\phi(A_1, \ldots, A_n) = \{o_1, \ldots, o_m\}$ be a set of definite fiat objects. Every object $o_i \in \phi(A_1, \ldots, A_n)$ is characterized by a attribute n-tuple $(a_1, \ldots, a_n)$. Assume the regions of the objects $o_1, \ldots, o_m$ sum up the whole space, i.e., $r(o_1) \vee \ldots \vee r(o_m) = U$. Since the objects $o_1, \ldots, o_m$ are of the same kind, their exact-location-regions cannot overlap (Casati & Varzi 1995). Consequently, the set of regions $\{r(o_1), \ldots, r(o_m)\}$ forms a regional partition of space. Under these assumptions, the set of objects $\phi(A_1, \ldots, A_n)$ can be modeled as mapping.

$$c : \{r(o_1), \ldots, r(o_m)\} \rightarrow A_1 \times \ldots \times A_n$$

If we assume that the region of space is a defining property of objects in class $\phi(A_1, \ldots, A_n)$ and A_1 ranges over a set of regions forming a partition of space, then the mapping can be written as $c : A_1 \rightarrow A_2 \times \ldots \times A_n$. Under the additional assumption that A_1 ranges over regions representable by semi-algebraic formulas the mapping, c, representing the class of objects $\phi(A_1, \ldots, A_n)$, has the signature $c : \Gamma(x_1, \ldots, x_d) \rightarrow A_2 \times \ldots \times A_n$. In this formula $\Gamma(x_1, \ldots, x_d)$ is a set of semi-algebraic formulas with d variables representing the particular regional partition of the space of dimension d.

5.4 Definite Fiats Created by Measuring Processes

Measuring processes create regional partitions of the underlying domain. Examples are time intervals bounded by clock-ticks in the case of time measurement (Carnap 1966), or raster cells on Earth in the case of remote sensing (Section 2). Let $G = \{g_1, \ldots, g_n\}$ be a regional partition created by a remote sensing process. At each partition region attribute values $a_1, \ldots, a_k$ are observed or measured. The measuring process yields the set of fiat objects $O = \{o_1, \ldots, o_n\}$ of kind $\phi(A_1, \ldots, A_k)$ with partition $G = \{r(o_1), \ldots, r(o_n)\}$. The objects are brought into existence by applying Definition 1 in the mind of the human being performing the measurement. Each single object is created by defining $o_i =_{def} (v_i, g_i)$, where $v_i = (a_1, \ldots, a_k)$ is the vector of attribute values measured at region g_i. Due to the partition structure of the regions of the created objects, O, the outcome of a measurement process can be represented as a mapping $c : G \rightarrow A_1 \times \ldots \times A_k$.

Consider the example in Section 2.3. At each pixel, i, exactly one attribute a_1^i was measured over the area g_i on Earth: the run-time of a photon on the distance sensor-Earth-sensor. Consequently, the objects, O, are of kind 'photon run-time measured to a region on Earth'. These objects are characterized by a single attribute value run_time and a region of space g_i. The definite fiat object, o_i, is created by applying Definition 1, i.e., $o_i =_{def} (run\_time_i, g_i)$. The set of fiat objects, O, created by the whole measurement process is represented by the mapping $c : G \rightarrow Photon\_Run\_Time$. The attribute $Photon\_Run\_Time$ ranges over values run_time. For example, the beginning of the graph of the mapping c is:

G	g_{11}	g_{12}	...
$Photon\_Run\_Time[ns]$	4822	4826	...

6 Modeling Fiats Created by Spatial Analysis

A special kind of fiat spatial objects are objects created by spatial analysis. Assume a set of fiat objects $O = \{o_1, \ldots, o_n\}$ of kind $\phi(A_1, \ldots, A_k)$ represented by the mapping $c : G \to A_1 \times \ldots \times A_k$ with $G = \{r(o_1), \ldots, r(o_n)\}$. This set might be created, for example, by a measuring process such as remote sensing as discussed in the previous section. In spatial analysis fiat objects are created in a process that has three basic components, which are modeled as mappings between different kinds of fiat objects:

1. Measuring processes yield sets of definite fiat objects forming regional partitions carrying certain attribute values representing the outcome of the measuring process. Examples are attribute values representing measured heights, strength of radiation, wavelength of measured radiation, or run time length of photons. The mapping $transform\_attributes$ transforms those attribute values into attribute values characterizing classes of fiat objects. For example, the measured run time of photons can be transformed into the attribute domain 'elevation height' which can be used to define classes of fiat objects like foreshore, shore, or foredune.
   In general, mappings $transform\_attributes$ are of signature $(G \to A_1 \times \ldots \times A_k) \to (G \to B_1 \times \ldots \times B_l)$ transforming tuples $((g_i \mapsto (a_1^i, \ldots, a_k^i))$ into tuples $(g_i \mapsto (b_1^i, \ldots, b_l^i))$.
2. There are mappings $classify\_location$, which yield values $\omega \in \Omega$ characterizing the mode of location of an object, o, at a particular partition region, g_i, depending on a given input, $(b_1^i, \ldots, b_l^i)$, related to this region. Possible modes of location are overlap sensitive, containment sensitive, or overlap & containment sensitive location.
   Mappings $classify\_location$ are of signature $(G \to B_1 \times \ldots \times B_l) \to (G \to \Omega)$, i.e., they return the value ω depending on the input vector $(b_1, \ldots, b_l)$ for each $g_i \in G$.
3. There are mappings $create\_object\_attributes$, which compute attribute values for the created fiat object from the attribute values of the measurement created objects, for example, the average elevation hight for the new object from the 'elevation height' of the raster cells at which it is located.
   In general, mappings $create\_object\_attributes$ computing the attribute values of the new object are of signature $(G \to B_1 \times \ldots \times B_l) \to B_1 \times \ldots \times B_m$.

The whole creative human process of creating fiat objects of kind $\pi(B_1, \ldots, B_m)$ by spatial analysis is modeled as a mapping, $analyze$, transforming a mapping representation of a measurement process into the new, analysis created, fiat object: $analyze : (G \to A_1 \times \ldots \times A_k) \to ((B_1 \times \ldots \times B_m), (G \to \Omega))$. The fiat object is modeled as a pair consisting of an n-tuple of attribute values and a location mapping representing rough location in the underlying regional partition, i.e., $o = ((b_1, \ldots, b_m), (\text{loc } o))$. We are now discussing those components in detail and give more examples.

6.1 Transforming Measured Attributes

Creating fiat objects by means of spatial analysis assumes the definability of those fiat objects of kind $\pi(B_1, \ldots, B_m)$ in terms of sets of fiat objects of kind $\psi(B_1, \ldots, B_l)$. Consider, for example, remote sensing. The measuring process creates fiat objects of kind $\phi(A_1, \ldots, A_k)$, which attributes range, for example, over the intensity of radiation of a certain wavelength emitted by the surface of Earth, or the run time of photons emitted by a laser and reflected from the surface of Earth. Assume we are interested in finding places on Earth belonging to the class foreshore as discussed in the example in Section 2.3. Molenaar *et al.* defined the class of fiat objects *foreshore* of kind $\pi(terrain\_elevation)$ in terms of terrain elevations values within the interval of $-6m$ to $-1.1m$, i.e., $(-6m \leq b \leq -1.1m) \in \pi(B)$. The observations from remote sensed images are photon run times; they must be transformed into relative heights with respect to sea level on Earth. This is a two step process. Firstly: At each pixel exactly one attribute a_1 was measured over the area g_i, the run-time of a photon. Multiplied with the speed of light, the kind of the attribute is (double) *distance*. Secondly: Relative height with respect to sea level on Earth is computed by subtracting the distance from (given) sensor height.

We define the mapping $transform\_attributes\ :\ (G \rightarrow Photon\_Run\_Time) \rightarrow (G \rightarrow terrain\_elevation)$. It transforms the attribute run_time into the attribute $terrain\_elevation$ for every partition element $g \in G$. For example, the photon run-time $a_1 = 4822ns$ measured at location g_{11} is transformed into the attribute value $b_1 = -5.49m$ of kind *terrain elevation* which remains assigned to location g_{11}.

The mapping $transform\_attributes$ creates a set of fiat objects of kind $\psi(terrain\_elevation)$ which members are co-located with fiat objects of kind $\phi(Photon\_Run\_Length)$. Sets of fiat objects of kind $\psi(terrain\_elevation)$ will be used to create objects like foreshores of kind $\pi(terrain\_elevation)$ by spatial analysis. This is discussed in the next section.

6.2 Creating Fiat Objects

Defining Classes of Analysis Created Objects. Measurement processes, such as remote sensing or point sampling, create sets of definite fiat objects of kind $\phi(A_1, \ldots, A_k)$. Those are transformed into objects of kind $\psi(B_1, \ldots, B_l)$. The attribute domains, $B_1, \ldots, B_l$, create the attribute space $B_1 \times \ldots \times B_l$. Every possible instance, $(b_1, \ldots, b_l)$, is interpreted as the coordinate of a point in this attribute space. In the context of concrete measurement those attribute points are not arbitrarily distributed. They form clusters (Section 2.2). The underlying assumption of spatial analysis is that those clusters in the attribute space correspond to classes of objects in reality (Kraus 1990). Classes of fiat objects created by spatial analysis are defined such that they correspond to clusters in the attribute space.

Consider the example of a terrain elevation image (Section 2.3). We profit from the natural shape of foreshore, beach and dune, which can be separated in the height profile (Fig. 4 left). With that property elevation yields a good selectivity of height clusters. The histogram of an elevation image, i.e. the function accumulating the frequency the occurrences of elevation values, shows a three-modal distribution (Fig. 4 right). Each mode[11] corresponds to one object class.

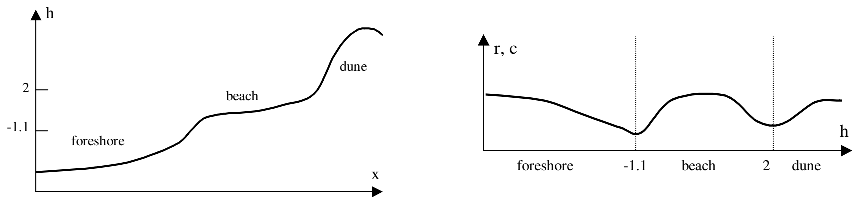

Fig. 4. A height profile through typical beach terrain (*left*), with h: terrain elevation, and x: direction of profile. — The histogram of the remote sensed image (*right*), with h: terrain elevation, and r, c: the image indices.

Creating Fiat Object Instances. Spatial objects are characterized by aspects specifying *what* they are and *where* they are located (Section 3). What fiat objects are, is characterized by their attribute values, $(b_1, \ldots, b_l)$. For fiat objects created by spatial analysis these attribute values are derived from *sets* of attribute values $(b_1, \ldots, b_k)$ belonging to a cluster in the attribute space $B_1 \times \ldots \times B_l$. The location of fiat objects created by spatial analysis, is characterized by their rough location within the regional partition, G, created by the underlying measurement process.

We define spatial analysis created objects of kind $\pi(B_1, \ldots, B_m)$, which is based on a measuring process that yields the set O of kind $\phi(A_1, \ldots, A_k)$ as pair containing an attribute vector $(b_1, \ldots, b_m) \in B_1 \times \ldots \times B_m$ and a location mapping of signature $(\mathrm{loc}\ o) : G \rightarrow \Omega^{12}$,

$$o =_{def} ((b_1, \ldots, b_m), (\mathrm{loc}\ o)).$$

Consequently, the act of creating a fiat object, o, consists of two components:

1. The computation of the attribute vector $(b_1, \ldots, b_l)$ from the attributes $(a_1^i, \ldots, a_k^i)$ of objects $o_i \in O$ created by the measuring process.
2. The computation of the rough location representation, $(\mathrm{loc}\ o)$, of o

[11] We speak of three *modes* meaning a robust concept of local frequency maxima.
[12] Ω can be either $\Omega_\exists$ or $\Omega_\forall$

Both components perform operations on the mapping representation of the underlying measuring process, $c : G \rightarrow A_1 \times \ldots \times A_k$. The process of creating fiat object by spatial analysis is modeled by the mapping *analyze*:

$$\begin{aligned} analyze \quad &: \quad (G \rightarrow A_1 \times \ldots \times A_k) \rightarrow ((B_1 \times \ldots \times B_m), (G \rightarrow \Omega)) \\ analyze\ c \ &=_{def} ((create\_object\_attributes \circ transform\_attributes)\ c), \\ &\quad\ ((classify\_location \circ transform\_attributes)\ c) \end{aligned}$$

Attributes. The mapping $create\_object\_attributes : (G \rightarrow B_1 \times \ldots \times B_l) \rightarrow B_1 \times \ldots \times B_m$ computes a single attribute tuple for the analysis created fiat object of kind $\pi(B_1, \ldots, B_m)$, by describing the cluster in the attribute domain created by the attributes of the set of measurement created objects $O = \{o_1, \ldots, o_n\}$ by parameters, like mean and other moments of the distribution.

Consider, for example, all objects, O, of a class, e.g., *beach*. The attributes values, *terrain_elevation*, characterizing objects of this class vary in between some limits, and they form a distribution. In Figure 4 (right), this distribution is characterized by the second mode of the histogram (neglecting the overlaps with continuing distributions of *foreshore* and *dune*). A multi-modal distribution is separated ideally at (robust) local minima, so in this case we decide to separate at $-1.1m$ and $2m$, forming a beach-cluster of $[-1.1m \ldots 2m]$. The histogram function in this interval can be used now to describe the distribution of beach objects, e.g., by a mean and a standard deviation.

Rough location. The rough location representation of a fiat spatial object, o', of kind $\psi(B_1, \ldots, B_l)$ within a regional partition, G, created by a set of objects, O, of kind $\phi(A_1, \ldots, A_k)$, is defined as the application the composition of the mappings $transform\_attributes$ and $classify\_location$ to the mapping c:

$$\begin{aligned} (\text{loc } o') \quad &: \quad G \rightarrow \Omega \\ (\text{loc } o') &=_{def} (classify\_location \circ transform\_attributes)\ c. \end{aligned}$$

We already discussed the mapping c, representing the outcome of the measuring process, and the mapping $transform\_attributes$ transforming those results into attribute domains that can be used to define fiat objects in terms of clusters within these attribute domains. The mapping $classify\_location$ finally creates the location mapping representation of the analysis created object within the underlying regional partition.

6.3 Classifying Location

We now discuss the mapping $classify\_location$. It performs a classification of partition elements, $g_i \in G$, characterized by an attribute n-tuple $(b_1^i, \ldots, b_m^i)$, with respect to the class definition, $\pi(B_1, \ldots, B_m)$, of the fiat object to be constructed. The classification is based on the class definition of the fiat object in terms of clusters in the attribute space $B_1 \times \ldots \times B_m$. Clusters in the attribute space do not necessarily have sharp and well defined boundaries (See Section

2.3). This has the consequence that we need to distinguish crisp and vague class definitions.

Crisp class definitions correspond to clusters on the attribute domain with well defined boundaries. Vague class definitions correspond to clusters with ill defined boundaries. In the remainder of this section we show that

- crisp class definitions cause the classification process of the partition elements to yield containment sensitive rough location, and
- vague class definitions cause the classification process of the partition elements to yield overlap & containment sensitive rough location.

Crisp class definitions. Let $classify\_location$ be the mapping

$$classify\_location : (G \to B_1 \times \ldots \times B_l) \to (G \to \Omega_\forall)$$

$$classify\_location(g_i \mapsto (b_1, \ldots, b_l)) = \begin{cases} (g_i \mapsto \Box) \text{ if} \\ \quad containment\_condition(b_1, \ldots, b_l) \\ (g_i \mapsto 0^\Box) \text{ otherwise} \end{cases}$$

The condition *containment_condition* needs to be expressed formally in terms of constraints on the values of attribute l-tuples, $(b_1, \ldots, b_l)$. Those constraints describe the clusters in the attribute domain, which define the class of the object to be constructed.

Regarding to the spatial extent, i.e., the *where*, these constraints define conditions that are assumed to be sufficient to postulate that:

- *given* an input value $(b_1, \ldots, b_l)$ related to region g_i
- *then* we postulate that there exists an analysis created fiat object, o, of kind $\psi(B_1, \ldots, B_l)$ such that the region g_i is part of the exact region of the object o, i.e., $((\mathrm{loc}_\forall\ o)\ g_i) = \Box$.

With crisp class definitions, only containment and non-containment classifications occur. Consequently, the mapping $classify\_location$ yields containment sensitive rough location. The exact region of the analysis created fiat object $r(o)$ coincides with the sum of partition regions for which the 'containment condition' holds.

Consider the example in Section 2.3. The attribute value $b_1 = -5.49m$ of the pixel g_{11} (remember the discussion in section 6.1) is contained in the interval, $[-6m, -1.1m]$, characterizing objects of class foreshore crisply. In terms of the mapping $classify\_location$ this is expressed as:

$$classify\_location : (G \to terrain\_elevation) \to (G \to \Omega_\forall)$$

$$classify\_location(g_i \mapsto b) = \begin{cases} (g_i \mapsto \Box) \text{ if } -6m \le b \le -1.1m \\ (g_i \mapsto 0^\Box) \text{ otherwise} \end{cases}$$

Vague class definitions. We now consider vague definitions of classes of fiat objects. Vague definition means that there are no sharp boundaries of clusters in the attribute space $B_1 \times \ldots \times B_m$. Vague class definitions result in overlap & containment sensitive rough location of the corresponding object instances in regional partitions created by the the underlying measurement processes. Let $classify\_location$ be the mapping

$$classify\_location \ : \ (G \to B_1 \times \ldots \times B_l) \to (G \to \Omega_3)$$

$$classify\_location(g_i \mapsto (b_1, \ldots, b_l)) = \begin{cases} (g_i \mapsto (\Box, \Diamond)) \text{ if} \\ \quad containment\_condition(b_1, \ldots, b_l) \\ (g_i \mapsto (0^{\Box}, \Diamond)) \text{ if} \\ \quad overlap\_condition(b_1, \ldots, b_l) \\ (g_i \mapsto (0^{\Box}, 0^{\Diamond})) \text{ otherwise} \end{cases}$$

The conditions *containment_condition* and *overlap_condition* need to be expressed formally in terms of constraints on the values of attribute l-tuples, $(b_1, \ldots, b_l)$. We already discussed the containment condition in the context of crisp class definitions above. The *overlap_condition* defines conditions that are sufficient to postulate that:

- given an input value $(b_1, \ldots, b_l)$ related to region g_i
- then we postulate that there exists an object o of kind $\psi(B_1, \ldots, B_l)$ such that the region g_i and the exact region of the object o overlap but g_i, is not a part of the object's exact region. i.e., $((\mathrm{loc}_3\ o)\ g_i) = (0^{\Box}, \Diamond)$.

The distinction of three modes of location rather than two reflects the vagueness of the class definition and corresponds to the location of vague defined fiat objects in concentric regional partitions consisting of a core region an exterior region and a wide boundary between them (Fig. 2)[13]. In the case of rough location in regional partitions the *core* region is formed by those partition elements which are classified as $(\Box, \Diamond)$, the *exterior* is formed by partition regions classified as $(0^{\Box}, 0^{\Diamond})$, and the *wide_boundary* is formed by partition elements classified as $(0^{\Box}, \Diamond)$.

Consider the example in Section 2.3. The membership in a the class *foreshore* was defined by a trapezoidal function with breakpoints at $(-8m, -4m, -1.6m, -0.6m)$. In terms of the mapping $classify\_location$ this is expressed as follows:

$$classify\_location \ : \ (G \to terrain\_elevation) \to (G \to \Omega_3)$$

$$classify\_location(g_i \mapsto b) = \begin{cases} (g_i \mapsto (\Box, \Diamond)) \text{ if } -4m \leq b \leq -1.6m \\ (g_i \mapsto (0^{\Box}, \Diamond)) \text{ if } -8m \leq b < -4m \text{ or} \\ \quad -1.6m < b \leq 0.6m \\ (g_i \mapsto (0^{\Box}, 0^{\Diamond})) \text{ otherwise} \end{cases}$$

In the vague definition of the class *foreshore*, the pixel with an attribute $b_1 = -5.49m$ satisfies the 'overlap condition', but not the 'containment condition' of the class *foreshore*: $classify\_location(g_{11} \mapsto -5.49m) = (g_{11} \mapsto (0^{\Box}, \Diamond))$.

[13] Notice that Fig. 2 is an idealization. In general, the partition regions may be complex regions consisting of multiple disconnected parts.

7 Conclusions

The purpose of spatial analysis performed on data resulting from measurement in physical reality is to identify objects, o', in physical reality that satisfy certain conditions. The process of spatial analysis yields objects, o, that are created by classification and other analysis operations. They are of fiat sort since they are the result of a human classification and analysis process. They are roughly located within the underlying regional partition, G, created by the measuring process. The exact location of the fiat spatial object o is unknown if o is subject to definitorial vagueness.

In the process of spatial analysis fiat objects are created in a process that has three basic components, which were modeled as mappings.

1. The mapping $transform\_attributes : (G \to A_1 \times \ldots \times A_k) \to (G \to B_1 \times \ldots \times B_l)$ transforming sets of fiat objects created by measurement or observation processes and represented by mappings of signature $c : (G \to A_1 \times \ldots \times A_k)$, into a set of co-located fiat objects represented by mappings of signature $d : (G \to B_1 \times \ldots \times B_l)$. Their attribute values form cluster in the attribute space $B_1 \times \ldots \times B_l$. Those cluster provide the basis for the definition of the final, analysis created object of kind $\pi(B_1, \ldots, B_m)$.
2. The mapping $classify\_location : (G \to B_1 \times \ldots \times B_l) \to (G \to \Omega)$ which yields the mapping representation, $(\mathrm{loc}\ o) : (G \to \Omega)$, of the rough location of the analysis created fiat object within the regional partition, G, created by the underlying measurement process.
3. The mapping $create\_object\_attributes : (G \to B_1 \times \ldots \times B_l) \to B_1 \times \ldots \times B_m$ computing the attribute values of the new object.

The whole creative human process of bringing fiat objects of kind $\pi(B_1, \ldots, B_m)$ into existence by means of spatial analysis, is modeled as a mapping, $analyze : (G \to A_1 \times \ldots \times A_k) \to ((B_1 \times \ldots \times B_m), (G \to \Omega))$. This mapping transforms a mapping representation, c, of a measurement process into the new, analysis created, fiat object, i.e., $analyze\ c = (b_1, \ldots, b_m, (\mathrm{loc}\ o))$.

It is important to distinguish fiat objects created by spatial analysis and the objects in the world they are supposed to correspond to. Both objects are related to each other in that sense that the radiation emitted by the measured object 'object in the world' caused sensor response during the remote sensing process which was recorded and caused fiat objects created by the remote sensing process to have certain attribute values. The fiat object 'object created by spatial analysis on remote sensed data' was created by performing spatial analysis on those objects. Obviously both objects, the measured object 'object in the world' and the fiat object 'object created by spatial analysis on remote sensed data', are different in nature and related to each other in the sense that the fiat object 'object created by spatial analysis on remote sensed data' *owes its existence* to:

- the existence of the object 'object in the world',
- the existence of a measuring act such as remote sensing, and
- the existence of a creative human act of spatial analysis.

Subject of ongoing research are the following questions:

- What is the justification to derive conclusions about bona fide and fiat objects in reality from knowledge about fiat objects created by spatial analysis?
- How does the *vagueness* of definitions of analysis created objects and the *indeterminacy* of their location relate to *uncertainty* about the truth of conclusions about the corresponding bona fide and fiat objects in physical reality?

Acknowledgements

The research of the first author was financed by the GEOIDE network. This support is gratefully acknowledged.

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