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The qualitative structure of built environments

Thomas Bittner
Department of Computing Science
Queen's University
bittner@cs.queensu.ca

Abstract. This paper provides an ontological analysis of built environments. It shows that boundaries are ontologically salient features of built environments and that there are different kinds of boundaries that that need to be considered. It discusses in particular the important role of fiat boundaries. At the level of objects built environments are *formed* by partition forming objects and *populated* by non-partition forming objects. The underlying partition structure is the main organizational structure of a built environment. Non-partition forming objects are potentially movable and their movement is constrained by the barrier properties of the boundaries of other objects forming or populating the environment.

This paper argues that the qualitative formalization of built environments needs to take into account: (1) the fundamental role of boundaries, (2) the distinction between bona-fide and fiat boundaries and objects, (3) the different character of constraints on relations between these different kinds of boundaries and objects, (4) the distinction between partition forming and non-partition forming objects, and (5) the fundamental organizational structure of regional partitions. It discusses the notion of object-boundary sensitive rough location and shows that a formalization based on this notion takes these points into account.

1. Introduction

Imagine a computer program (I) that generates plans that look like plans of built environments [Lyn60] like shopping malls, airports, or parking lots (See, for example, the left part of Figure 1.). Program (I) generates configurations of lines of different style and width within the Euclidean plane. It is certainly not too hard to imagine the existence of such a program. Suppose our task is to design another computer program (II), which checks (1) whether or not a plan generated by (I) can possibly be a plan of a built environment and (2) in case the plan represents a built environment whether or not this environment can be apprehended easily by human beings.

In order to fulfill task (1) we need a (hopefully small) set of sufficient conditions of what it means to be a plan of a built environment. This assumes (a) that we understand the language of plans of built environments, i.e., the meaning configurations of lines of different kinds constituting a plan of a built environment and (b) that we are able to formulate our conditions, i.e., axioms, in terms of this language.

In order to fulfill task (2) we need to describe formally (a) what it means for a human being to apprehend an environment and (b) what easy means in this context. Obviously, task (2), as stated above is, much too hard to be solved. In order to simplify it we consider human navigation in built environments and ask what it means for a built environment to be easy navigationable, i.e., what it means for a built environment to be easy to apprehend with respect to the fulfillment of a navigation task. We call this task (2'). Built environments that are easy to navigate are of great practical importance, for example, if we have to deal with emergency situations. Ease of navigation is important in the design of airports. A priori knowledge of where people go in a given situation is important for the product placement in shopping malls. Obviously, program (II), even in its simplified version, performing task (1) and (2'), would be of great practical value.

Human navigation and wayfinding in general and in built environments in particular has been studied extensively in the past in architectural design, e.g., [GLM83, GBL86], in Artificial Intelligence, e.g., [Kui78, MD84, LZ89, Eps97] and in Cognitive Science, e.g., [SW75, Gol92, HH93]. Notice that all those people deal with navigation in real, physically existing environments.

The question addressed in this paper is different. The built environments this paper is dealing with do not (physically) exist yet. Consequently, our approach cannot rely on observations in reality. The only 'physical' thing we have is a plan, P, generated by program (I). Only if program (II) decides (a) that P represents a built environment and (b) it is easy to navigate then the environment is being built according to P. Whatever it means to be a built environment and whatever it means to be easy to navigate, it must be definable in the language of P and it must be decidable given P. Consequently, task (1) and (2') rest upon the same formal foundations and are, in this respect, closely related to each other. It is the aim of this paper to investigate those formal foundations.

When deriving conclusions from plans of not yet existing environments about future 'real' environments, assumptions about the relationships between physically existing built environments, human cognition, and plans of built environments are made implicitly: (i) Ontologically salient features of the environment are reflected in human cognition [Smi95a]. (ii) These features play an important role in the way humans apprehend built environments and the way they navigate within them. (iii) This is reflected by the way human beings or programs designed by human beings draw those plans, i.e., it is reflected by the language used to draw a plan of an environment. Consider plans of built environments. They are mainly made up of lines of different kind. Lines correspond to ontologically salient features in reality, i.e., boundaries. Different kinds of lines correspond to different kinds of boundaries. Consequently, it is possible to extract ontologically salient features of a not yet existing built environment from its plan. In this paper ontologically different kinds of boundaries and their role in the ontological makeup of built environments are discussed. It is shown that configurations of boundaries in an environment afford (in the sense of [Nor88]) networks of paths along which objects can be moved within built environments and that those paths can be deduced from knowledge about boundary configurations. In this context this paper goes beyond [Gib79] since it takes also fiat boundaries into account.

Program (I) produces a quantitative representation of built environments based on computational geometry (e.g., [Rou94]). The analysis of plans representing built environments, performed by program (II) will focus on qualitative aspects [FR93, Fre91, Coh97], i.e., different kinds of things [SM98], qualitative relations between lines [All83] and qualitative relations between regions [CBGG97]. At the formal level a language based on the qualitative notion of boundary sensitive rough approximations [BS98] is used to describe built environments. This notion provides the basis for the formal description of built environments and for the evaluation of the complexity of navigation.

This paper is structured as follows. It starts with an informal analysis of the ontological makeup of built environments. In Section 3 a review of the notion of rough approximations is given. In Section 4 this notion is extended in order to describe the location of spatial objects in built environments qualitatively. In Section 5 built environments are formalized based on these notions. The conclusions are given in Section 6.

2. Built environments

In this paper parking lots are used as a running example for built environments. The parking lot domain is relatively simple but its structure is rich enough to study the *ontological* makeup of built environments, which is critical for a *qualitative* formalization. An example of a parking lot in a bird's eye view is given in Fig. 1.

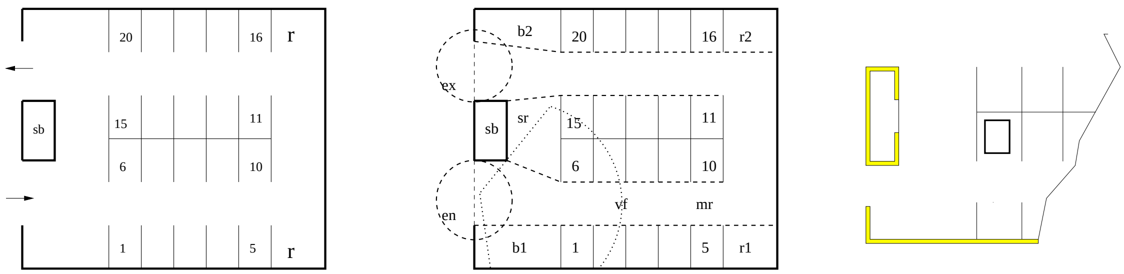


Figure 1 An empty parking lot (left). The parking lot with marked not directly observable fiat boundaries (middle). A zoom of the entrance area (right).

2.1. Boundaries

2.1.1. Bona-fide and fiat boundaries

Following [Smi95b] we distinguish bona-fide and fiat boundaries. Bona fide boundaries are boundaries *in the things themselves*. Bona fide boundaries exist independently of all human cognitive acts. They are a matter of qualitative differentiations or discontinuities of the underlying reality. Examples are surfaces of extended objects like cars, walls, the floor of a parking lot. Bona-fide boundaries are marked by bold solid lines in Figure 1.

[Smi95b] describes fiat boundaries as boundaries which exist only in virtue of different sorts of demarcation effected cognitively by human beings. Such boundaries may lie skew to boundaries of *bona-fide* sort as in the case of the boundaries of a parking spot in the center of a parking lot, e.g. spots 6-15 in Figure 1. They may also, however as in the case of a parking spot at the outer wall of the parking lot, involve a combination of fiat and *bona-fide* portions such as a wall at its back side, e.g., spots 1-5 and 16-20 in Figure 1.

The classification of boundaries generalizes to a classification of objects. Bona-fide objects have a single topologically closed bona-fide boundary (e.g., the building sb in Fig. 1). Fiat objects have fiat boundary parts (e.g., parking spot 1).

2.1.2. Observability of fiat boundaries

Fiat boundaries like the front boundaries of parking spots or the boundaries of entrance area (lower dashed ellipse) and the exit area (upper dashed ellipse) are not directly observable in reality. They are invisible but do nevertheless exist. Since fiat boundaries are not perceivable by the senses they need to be made visible or the environment must force all people to perceive them in places the designer wanted them to be.

Consider parking spot 15. It is located in the middle of the parking lot where no bona-fide boundaries are around (except the floor). When designing the parking lot the designer divided the space into parking spots. Parking spots are fiat objects. In order to be perceivable by other people the back, left, and right boundaries are marked by (usually white) paint (the thin solid lines in Figure 1). The front boundary of the parking spot is not marked but every human being knows that it is located at the straight line connecting the ends of the left and right boundaries. Non marked and hence invisible fiat boundaries are marked by dashed lines in the middle part of Figure 1. Consider the upper boundary of the side road, sr. When you cross this boundary then you are leaving the main road, mr. The lower boundary of the side road is more significant. Traffic rules force you to give way to cars on the main road when you are leaving the side road, i.e., when crossing the boundary between side and main road.

Plans of built environments to be analyzed by programs rather by human beings must contain *all* boundaries even if they are not directly observable in reality. Consequently, plans of built environments generated by program (I) and analyzed by (II) must look like the middle part of Figure 1 rather than like the left part.

2.1.3. Co-location of boundaries

Co-location of spatial objects means that they are located at exactly the same three (two)-dimensional region of three (two)-dimensional space at the same moment in time [CV95]. Co-location of boundary parts means that two (one)-dimensional boundary parts of spatial objects are located at exactly the same two (one)-dimensional region of space.

Distinct bona-fide objects cannot be co-located since they cannot overlap[1] [CV95]. [SV99a] argue that due to their physical structure not even their boundaries be co-located. Due to their atomic structure the surfaces of bona-fide objects can be brought in contact, i.e., the atoms forming their surface can come close but the atoms forming the surface of one object remain distinct from the atoms of the other. They do not mix and do not form a shared boundary. This has the consequence that between bona-fide objects the relation EC^2 (externally connected) as defined by [RCC92] can never hold [SV99a]. This fact might seem to be only of theoretical relevance but it has important impact on the semantics of lines used in plans of built environments. Here the co-location of lines representing boundaries of objects is an important feature.

Distinct fiat objects of the same ontological kind cannot overlap. Fiat objects of the same ontological kind are, for example, land properties, parking spots, and objects forming political subdivisions like countries. As a more complex example for the non-overlap of fiat objects of the same kind consider the

[1] This is consistent with the observation that bona-fide objects as wholes overlap themselves and all of their parts.

[2] See the right part of Figure 3.3.1 for a geometric example.

intersection of 5th Avenue and 110th Street in Manhattan, New York. 5th avenue and 110th Street are certainly examples of fiat objects[3] of the same kind and at first sight it might seem that both objects do overlap. So what happens at the intersection of 5th Avenue and 110th Street? There are traffic lights! These traffic lights determine when 'the intersection of 5th Avenue and 110th Street' belongs to 5th Avenue and when it belongs to 110th Street. Consequently both objects do not overlap, i.e., there are no parts of them located at the same region region of space at the *same* moment in time[4]. At street intersections without traffic lights there are complex traffic rules that determine when the intersection belongs to which street.

The situation is different in the domain of fiat objects of *different* kind and fiat boundaries. Fiat objects of ontologically different kind can be co-located [CV95]. For example, the 'City of Vienna' is co-located with the 'Federal State Vienna' of the Republic Austria. Fiat boundaries of neighboring fiat objects of the same and of different kind can be co-located [SV99a]. Consider the line separating the objects 'Main Road' (mr) and the object 'Parking Spot 1' in the middle part of Figure 1. This line marks the one-dimensional space, which is occupied by a part of the boundary of the 'Main Road' and a part of the boundary of 'Parking Spot 1'. Between the regions of space occupied by the objects 'Main Road' and 'Parking Spot 1' the relation EC hold. (See [SV99a] for an extended discussion.)

Boundary parts of bona-fide and fiat objects can be co-located. Consider the right part of Figure 1, which shows a zoom of the entrance area of our parking lot. The bona-fide boundaries of the walls of the security building and the outer wall of the parking lot are co-located with boundaries of fiat objects: The inner boundary of the wall of the security building is co-located with the boundary of the fiat object 'room for the security people' (the region enclosed by the walls of the building). The back-boundary of the fiat object 'Parking Spot 1' is co-located with a part of the bona-fide boundary of the outer wall of the parking lot, which faces the interior of the parking lot. The fiat boundaries follow all the non-regularities of the surfaces of the walls.

Above it was argued that fiat objects can have bona-fide boundary parts. More precisely one had to say that fiat objects can have fiat boundary parts that are *co-located* with parts of boundaries of bona-fide objects.

2.1.4. Barrier properties of boundaries

Consider the parking lot domain. Marked fiat boundaries afford people (in cars) not to cross despite the fact that there is no physical barrier. Non-marked boundaries afford crossing, e.g., the non-marked boundary of an empty parking spot 'invites' you to cross this boundary and park your car at this spot (if there is no other car parked yet). People invented signs to prevent other people from crossing non-marked fiat boundaries, think, for example, of one-way streets. In the design of parking lots and built environments in general fiat boundaries and their properties play a critical role. They provide an important organizational structure.

Fiat boundaries can be co-located with other fiat boundaries as well as with bona-fide boundaries. Fiat boundaries that are co-located with bona-fide boundaries 'inherit' some of the properties of the bona-fide boundary they are co-located with. Consider, for example, the back-boundary of 'Parking Spot

[3] They have fiat boundary parts, for example, at intersections.

[4] An interesting boundary case might be the moment the lights change in the sense of Galton's theory of domination, e.g., [Gal95, Gal00] .

1’ in Figure 1. Since it is co-located with a part of the bona-fide boundary of the outer wall of the parking lot it becomes visible and inherits the property of being a barrier for other bona-fide objects.

We distinguish four kinds of boundaries: bona-fide boundaries, non-barrier fiat boundaries, two-way barrier fiat boundaries, and one-way barrier fiat boundaries. Bona fide boundaries cannot be crossed by other bona-fide objects (think of a car and a wall). Fiat boundaries can be crossed by bona-fide objects. Two-way barrier fiat boundaries are supposed not to be crossed from both sides. One-way barrier fiat boundaries are supposed not to be crossed only from one side, from the other side they can be crossed. Non-barrier fiat boundaries can be crossed from both sides. Examples are given in Table 1. The barrier property of bona-fide boundaries is based on their physical properties. The barrier property of fiat boundaries is based on the barrier property inherited from co-located bona-fide boundaries or on social rules and agreements.

Human beings cannot see the fiat boundaries. Making fiat boundaries visible and marking them such that their barrier properties become visible is an important aspect in parking lot design and the design of built environments in general. The way fiat boundaries are marked or signs that are assigned to them makes these boundaries none, one-way, or two-way barrier fiat boundaries.

Consider a window made of glass. The boundaries of the window are a barrier for other physical objects but they are not barriers for light, i.e., we cannot walk through it but we can look through it. Consider the side boundaries of a parking spot. They are barriers for people in cars but not for people walking through a parking lot. Consequently boundaries are barriers *with respect to* movement of physical objects, or with respect to movement in a car, or with respect to human vision. In the remainder of this paper we consider bona-fide boundaries as two way barriers with respect to the movement of other bona-fide objects. We consider barrier properties of fiat boundaries with respect to the movement of bona-fide objects and in the parking lot domain we consider cars rather than people unless explicitly stated differently.

kinds of boundaries	examples
bona-fide barrier	the boundaries of the security building (sr) in Figure 1, the surface of a car
one-way barrier fiat	entrance of a one-way street, the boundaries forming the entrance and the exit of the parking lot in Figure 1, the boundaries between main road (mr) and side road (sr) in Figure 1.
two-way barrier fiat	the left/right/back boundaries of parking spots 6-15 in Figure 1.
non-barrier fiat	the front boundary of a parking spot

Table 1. Barrier properties of boundaries with respect to movement of bona-fide objects.

2.2. Spatial objects forming built environments

2.2.1. Relations between objects

Built environments are populated by bona-fide as well as by fiat objects. There are three basic classes of axioms governing the spatial objects in built environments: $(O1)$ axioms governing spatial objects of the *same* ontological kind; $(O2)$ axioms governing spatial objects of the *different* ontological kind; $(O3)$ domain specific axioms characterizing built environments like parking lots, airports, shopping malls, or city centers. In the remainder the axioms $O1 - O3$ are called ontological axioms or ontological constraints on relations that can hold between objects in built environments.

The main axiom in group $O1$ is that distinct spatial objects of the *same* ontological kind cannot overlap [CV95]. For example, bona-fide objects like cars and walls cannot overlap. Distinct fiat objects of the same kind like parking spots cannot overlap either. This axiom needs refinement regarding overlap of boundary parts: Boundary parts of fiat objects of the same kind can be co-located, e.g., co-located boundary parts of neighboring parking spots. Boundary parts of bona-fide objects cannot be co-located [SV99a].

The main axiom in group $O2$ is that distinct spatial objects of *different* ontological kind can overlap [CV95]. This is not a constraint. It rather says that in general there are no ontological objections against objects of different kinds to overlap. However, there are additional constraints on relations among partition forming objects (to be discussed below).

There are further *domain specific* axioms, $O3$, constraining relations that can hold between objects of ontological different kind in specific built environments. Consider the parking lot domain. There are cars and parking spots. Parking spots are such that cars can be parked in them. Parking lots are also formed by objects like blocked areas and reserved parking spots (e.g., b_1 and r_1 in Fig. 1). Examples for domain specific constraints on relations between objects in parking lots are: $(S1)$ Cars are supposed to keep blocked areas clear; $(S2)$ Regular cars should not be parked in reserved parking spots; $(S3)$ Cars should be parked within parking spots.

Regarding domain specific constraints in $O3$ it is important to notice that constraints involving objects of ontological different kind are weaker than the constraints between bona-fide objects, constraints between fiat objects of the same kind. Axioms deeply rooted in human intuition and the laws of logic prohibit objects of same ontological kind to overlap. Laws of physics prevent bona-fide objects from sharing boundary parts. Constraints involving fiat objects of ontological different kind are based on social rules and agreement and may be violated in certain situations. For example, you can die if you try to drive through a wall. People went to war and died for their conviction that distinct countries cannot overlap. You only get charged when you are parking on a reserved parking spot. The different character of constraints will play an important role in the formalization in Section 5.

Constraints on relations between bona-fide and fiat objects and among fiat objects of different kind have a complicated structure. Fiat objects may have boundary parts that are co-located with boundary parts of bona-fide objects. Constraints involving other bona-fide objects can only be violated at fiat boundary parts. Think of a car and a parking spot which back-boundary coincides with the wall. Laws of physics prevent the car from crossing the back boundary of the parking spot they do not prevent you from crossing the front, left and right boundary. Not to cross the left and right boundary is pure convention. Consequently, constraints involving bona-fide and fiat objects or fiat objects of different kind need to be expressed at the level of boundary segments, i.e., boundary parts that are co-located with boundary parts of other objects, rather than at the level of objects. We will discuss those issues on a formal level in Section 4.

2.2.2. Partition forming and non-partition forming sets of objects

Besides the (fundamental) distinction between bona-fide and fiat objects we distinguish two kinds of objects in built environments: partition forming objects and non-partition forming objects. *Partition forming objects* are objects that belong sets of objects that as wholes form (regional) partitions. *Non-*

partition forming objects that do not belong to such sets. A regional partition is a set of regions, which members intersect only at their boundaries $(P1)$ and, as a whole, sum up the whole space $(P2)$[5].

Consider the parking lot domain. The partition forming objects form a regional partition of the three-dimensional parking lot. Each of those objects carves out one three-dimensional region off the parking lot whole such that there is no 'no man's land' $(P2)$ and no 'double occupation' $(P1)$. Partition forming objects are, for example, parking spots, traffic lanes, sidewalks, blocked areas keeping fire exits clear, walls, pillars, and others more. Partition forming objects may be of bona-fide (pillars or walls) or of fiat sort (parking spots). Consider the middle part of Figure 1. If you ignore entrance area, en, the exit area, ex, and the visual field, vf, then you see the regional partition formed by (projections of) the partition forming objects of the parking lot.

Non-partition forming objects overlap partition forming objects of ontological different kind. Non-partition forming objects may be of bona-fide or fiat sort. Consider the parking lot domain. Examples for non-partition forming bona-fide objects are cars and people. Examples for non-partition forming fiat objects are smoking areas, the visual field (vf) in a given location or 'the entrance area', en, or the 'exit area', ex, of a parking lot (See middle part of Figure 1).

The underlying regional partition provides the main organizational structure of a built environment. The partition structure preserves in a qualitative manner: The environment's main topological structure, e.g., the adjacency of objects; Its ordering structure, e.g., relations like left and right with respect to a shared boundary segment; The relative position of the objects to each other, like 'in between parking spot 2 and 4'. In built environments it is often sufficient to describe the location of non-partition forming objects relative to the underlying regional partition, e.g., 'car1 is in parking spot 4' or 'car 2 is on the main road' or 'I am on X Avenue between Y and Z Street'. In this context we assume that 'parking spot 4', 'main road', 'X Street', $\ldots$, refer to *partition cells* rather than to isolated objects. In these examples the exact location of the object, e.g., its exact coordinates, is *approximated* with respect to the underlying regional partition. The regional partition is used as a *frame of reference*. Using the regional partition structure of an built environment as a frame of reference to describe the approximate location of non-partition forming objects within it is one of the main features of the formalism that follows below.

An important point is that there is a qualitative difference between using the regional partition formed by the partition forming objects of the environment and using an arbitrary regional partition like some raster as frame of reference. The raster structure is independent of the structure of the environment and cannot preserve its structure. The partition formed by the partition forming objects preserves the structure of the environment. Using this partition structure as frame of reference reflects the *ontological commitment* that the distinction between partition forming and non-partition forming objects exists and that these regional partitions reflect the main organizational structure of the underlying built environments.

2.2.3. Projection onto the ground

Built environments are formed by 3-dimensional objects in three dimensional space. In the remainder we consider only consider 2-dimensional objects in 2-dimensional space. Consequently, when we talk about objects in built environments then we talk about their projections onto the ground. For example the projection of a car onto the ground is a two-dimensional region. The boundaries of the projected object

[5] We will be more specific about the relations between spatial objects and the regions of space they occupy in the Section 4. In this section we use both notions synonymous.

inherit the properties (Table 1) of their originals, i.e., the boundaries of the projected region of the car are bona-fide barrier boundaries.

In essence we assume that the projection onto the ground preserves the ontological nature of boundaries and that if the axioms $O1 - O3$ and $P1$ and $P2$ are satisfied in the projected environment then so they are in the original environment. This implies that the projection needs to preserve the partition structure of the environment such that spatial location in the sense discussed above can be described equivalently.

Considering 2-dimensional projections as 'ontology preserving' is consistent with the fact that a built environment first exists as an idea in the mind of a designer. Then it exists as 2-dimensional plan, which is the major tool during the planing, design, and building process. Sometimes, in a late state of the design, a 3-dimensional model of the future environment might exists. Only the final environment is formed by three-dimensional objects with the properties discussed above.

2.3. Movement

An important aspect of the distinction between partition forming and non-partition forming objects is that the partition structure is static[6] and that non-partition forming objects can move (like cars), or shrink and grow (like the visual field). Objects move along paths. A path is a sequence of locations occupied at consecutive moments of time, which corresponds to continuous movement.

Consider the parking lot domain. It is the purpose of a parking lot to let cars park within parking spots. In order to fulfill this purpose, it must be possible to move a car from the entrance to a free parking spot. That is: (i) There must *exist* a path of movement to a free parking spot without violation of $O1 - O3$. This will be called the moveability axiom, M, of a built environment. (ii) It must be possible for a human being in a car to *find* this path. Checking the existence of paths is an instance of problem (1) discussed in the introduction. Deciding whether or not it is difficult or easy to find an existing path is an instance of problem (2). In this paper we focus on problem (1). This provides the basis for solving problem (2).

Notice that condition (i) is different from the attempt of describing the motion of objects in terms of restrictions on permitted positions. Condition (i) singles out certain paths for certain classes of objects and requires them to exist. It postulates that for an environment to *be* an environment of a certain type there must *exist* certain paths that are subject to constraints. In an empty parking lot it must be possible to drive a car to every parking spot without bumping into walls and without violating traffic rules. This does not conflict with the fact that there might be many more paths along which you can violate traffic rules or along which people can move.

Our aim is to describe and formalize the qualitative structure of *environments* rather than the behavior of objects. We do so by postulating:

1. Constraints on relations that can hold between the components of an environment, e.g., $O1 - O3$, $P1$ and $P2$;

[6] Notice that this is a simplification. For example, doors are (parts of) partition forming bona-fide objects and doors may be open or closed. Consequently, the layout of an environment is not really static. In the remainder we assume that the layout of a built environment is static in the sense that neither the location nor the type of boundary segments change. A more sophisticated approach would be to assume static boundary location but allow for change of the type of boundary segments, i.e., the type of the boundary changes from bona-fide barrier to fiat non-barrier if the door is opened.

2. What kind of behavior of objects must be permitted by an environment in order to be an environment of a certain kind, e.g., in a parking lot there must exist paths along which cars can reach parking spots without violations of $O1 - O3$.

This will be discussed extensively in Section 5. The important point to keep in mind is that the purpose of what follows is to formalize the qualitative structure of environments rather than the domain of objects. Consequently we are going to describe location and change of location (e.g., movement) of objects relative to their environment, i.e., with respect to the underlying regional partition structure. Using this approximate description of location in terms of object-environment *relations* we will give axioms that constrain relations that can hold between the components of an environment and that describe behavior of objects must be permitted by an environment. Describing the structure of environments in terms of object-environment relations follows the arguments in [Gib79] and [SV99b].

3. Approximating regions of space

In the previous section we discussed important structural properties of built environments that are based on their ontological makeup. This discussion was performed in an informal way. The aim of this paper to provide a precise formalization. Based on the previous discussion the formalization needs to take two major points into account:

1. The fundamental organizational structure of the regional partition formed by the partition forming objects making up the static component of the built environment;
2. The importance of ontologically grounded constraints on (qualitative) relations that can hold between the objects forming and populating it.

To satisfy the first point the formalization uses the regional partition as a frame of reference and describes the location of all objects (partition forming and non-partition forming) with respect to this regional partition. In order to do so the notion of rough sets [Paw82] is applied to the spatial domain. The basic idea of rough set theory is to approximate the subsets of a set with respect to an underlying partition of the set. Given a set X with a partition $\{a_i \mid i \in \mathcal{I}\}$, an arbitrary subset $b \subseteq X$ can be approximated by a function $\varphi_b : \mathcal{I} \to \{\mathsf{fo}, \mathsf{po}, \mathsf{no}\}$. The value of $\varphi_b(i)$ is defined to be fo if $a_i \subseteq b$, it is no if $a_i \cap b = \varnothing$, and otherwise the value is po. In the spatial domain the (exact) location of spatial objects is approximated with respect to regional partition of space.

Every spatial object is exactly located at a single region of space at each moment in time[7] [CV95]. Consequently, we can think of the set of two-dimensional topologically regular [Req77] regions of the plane as the set of all possible locations at which two-dimensional spatial objects can be located exactly[8]. Given the set of regions of the plane, R, and a regional partition of the plane, G, regions, $r \in R$, can be approximated by describing their *spatial* relations 'full overlap' (fo), 'partial overlap' (po), and 'no overlap' (no) to the cells $g_i \in G$ of the regional partition. Corresponding to the rough set approach we get a mapping of signature $\gamma_r : G \to \{\mathsf{fo}, \mathsf{po}, \mathsf{no}\}$. In order to take the important role of boundaries into account this formalism will be extended by also representing the degree of coverage of boundary segments shared by neighboring partition cells. This will be discussed in more detail below.

[7] Your exact region at each moment in time is the region of space your body carves off the air.

[8] Spatial change makes spatial objects located at different regions of space at different moments in time.

The second point emphasizes that spatial configurations, such as built environments, can be described qualitatively in terms of spatial relations that can hold between the objects forming the configuration. These relations are subject to constraints due to the underlying ontological structure. In the remainder we concentrate on topological relations, such as disconnected, $\mathsf{DC}(x, y)$, or partial overlap, $\mathsf{PO}(x, y)$ (See Fig. 3.3.1). In the domain of objects forming built environments we assume that spatial relations between spatial objects coincide with the spatial relations that hold between their exact regions. In the remainder of this section we abstract from the distinction between spatial objects and their exact regions and use both notions synonymously.

The purpose of this section is to bring together these two points, i.e., the approximate description of the location of spatial objects with respect to an underlying regional partition and the qualitative description of configurations of spatial objects in terms of constraints on possible topological relations. In order to do so we start with the formalization of the rough approximation of spatial location within a regional partition. Given two approximations, X and Y, with respect to the same regional partition we then ask which topological relations can possibly hold between objects x and y approximated by X and Y. This will be used in the remainder of the paper in order to give a formal account of the ontological constraints $O1 - O3$ informally discussed in the previous section. The notions we are discussing in this section were originally defined in [BS98] and [BS00].

3.1. Approximations

3.1.1. Boundary insensitive approximation

Suppose a space R of spatial regions. By imposing a partition, G, on R we can approximate elements of R by elements of Ω_3^G. That is, we approximate regions in R by functions from G to the set $\Omega_3 = \{\mathsf{fo}, \mathsf{po}, \mathsf{no}\}$. The function which assigns to each region $r \in R$ its approximation will be denoted $\alpha_3 : R \to \Omega_3^G$. The value of $(\alpha_3\, r)g$ is fo if r covers all the of the cell g, it is po if r covers some but not all of the interior of g, and it is no if there is no overlap between r and g. The elements of Ω_3^G are the boundary insensitive approximations of regions $r \in R$ with respect to the underlying regional partition G.

Consider the visual field, vf, in the parking lot in Figure 1. Let b_1 be a blocked area, ps_i be parking spots, w the world outside the parking lot[9]. The graph of the mapping $VF = (\alpha_3\ vf)$[10] contains the following tuples:

$g_i \in G$	sr	mr	b_1	ps_1	ps_2	w	...
$VF\,g_i$	po	po	po	fo	po	no	...

3.1.2. Boundary sensitive approximation

We can further refine the approximation of regions $r \in R$ with respect to the partition G by taking boundary segments shared by neighboring partition cells into account. That is, we approximate regions in R by functions from $G \times G$ to the set $\Omega_{bs} = \{\mathsf{fo}, \mathsf{fbo}, \mathsf{pbo}, \mathsf{nbo}, \mathsf{no}\}$. The function which assigns to each region $r \in R$ its boundary sensitive approximation will be denoted $\alpha_5 : R \to \Omega_{bs}^{G\times G}$. The value of $(\alpha_5 r)(g_i, g_j)$ is fo if r covers all of the cell g_i, it is fbo if r covers all of the boundary segment, shared

[9] We ignore the physical extension of the wall as bona fiat object of its own and consider it as the bona-fide boundary of the exterior.

[10] We use lower case letters to denote objects and capital letters to denote approximations.

by the cell g_i and g_j[11] and some but not all of the interior of g_i, it is pbo if r covers some but not all of the boundary segment shared by g_i and g_j and some but not all of the interior of g_i, it is nbo if r does not intersect with boundary segment shared by g_i and g_j and some but not all of the interior of g_i, and it is no if there is no overlap between r and g_i.

Let bs be the boundary segment shared by the cells g_i and g_j. Approximation mappings, α_3, apply to configurations of regions in one and two-dimensional space. We define boundary sensitive approximation, α_5, in terms of pairs of approximation mappings, α_3, according to the intuitive definition above:

$(\alpha_5\, r)(g_i, g_j) =$	$(\alpha_3\ r)bs = \mathsf{fo}$	$(\alpha_3\ r)bs = \mathsf{po}$	$(\alpha_3\ r)bs = \mathsf{no}$
$(\alpha_3\ r)g_i = \mathsf{fo}$	fo	-	-
$(\alpha_3\ r)g_i = \mathsf{po}$	fbo	pbo	nbo
$(\alpha_3\ r)g_i = \mathsf{no}$	no	no	no

The pairs with $((\alpha_3\, r)\, g_i) = \mathsf{fo}$ and $((\alpha_3\, r)\, bs) \neq \mathsf{fo}$ cannot occur since $((\alpha_3\, r)\, g_i) = \mathsf{fo}$ means that r covers all of g_i including its boundary. If $((\alpha_3\, r)\, g_i) = \mathsf{no}$ then the result of $((\alpha_3\ r)\, bs)$ does not matter since for $(\alpha_5\ r)(g_i, g_j) \neq \mathsf{no}$ the region r and the cell g_i must overlap, i.e., share interior parts. The values fo, fbo, pbo, nbo, no are abbreviations for pairs $(\omega_\iota, \omega_\delta) \in \Omega_3 \times \Omega_3$. Let ω be an element of the boundary sensitive value domain Ω_{bs} with $\omega = (\omega_\iota, \omega_\delta)$. We call $\omega_\iota = (\iota\ (\omega_\iota, \omega_\delta))$ the interior component and $\omega_\delta = (\delta\ (\omega_\iota, \omega_\delta))$ the boundary component of ω.

Consider, the visual field, vf, and the entrance area, en, of the parking lot in Figure 1. The graphs of the mappings $VF = (\alpha_5\, vf)$ and $EN = (\alpha_5\, en)$ contain the following tuples:

(g_i, g_j)	(b_1, mr)	(b_1, ps_1)	(b_1, w)	(ps_1, b_1)	$\ldots$	(ps_1, w)	$\ldots$
$VF(g_i, g_j)$	pbo	fbo	pbo	fo	$\ldots$	fo	$\ldots$
$EN(g_i, g_j)$	pbo	nbo	pbo	no	$\ldots$	no	$\ldots$

3.1.3. Semantics of approximate regions

Each approximate region $X \in \Omega_3^G$ ($X \in \Omega_{bs}^{G\times G}$) stands for a set of precise regions, i.e., all those precise regions having the approximation X. This set which will be denoted $[\![X]\!]^3$ ($[\![X]\!]^5$) provides a semantics for approximate regions [BS00]:

$$[\![X]\!]^3 = \{r \in R \mid \alpha_3 r = X\}, \quad [\![X]\!]^5 = \{r \in R \mid \alpha_5 r = X\}$$

Wherever the context is clear the superscript is omitted.

3.2. Approximate operations

The domain of regions is equipped with join (union) and meet (intersection) operations, $\vee$ and $\wedge$. In this section we define join and meet operations on approximations corresponding to those on regions. In general, [BS98] showed that join meet operations on regions are approximated by pairs of greatest minimal and least maximal operations on approximations. As examples we give the definitions of the greatest minimal meet operation, $\underline{\wedge}$, and least maximal operation meet operation, $\overline{\wedge}$ on boundary insensitive approximations and the definitions of the operation $\overline{\wedge}$ on boundary sensitive approximations. A detailed discussion of these operations and their construction can be found in [BS98] and [Bit99].

[11]In case of multiple disconnected boundary segments shared by g_i and g_j we assume additional indices for distinguishing them.

3.2.1. Boundary insensitive operations

First we define operations $\underline{\wedge}$ and $\overline{\wedge}$ on the set $\Omega_3 = \{\mathsf{fo}, \mathsf{po}, \mathsf{no}\}$ as shown in the left and middle table below. These operations extend to elements of Ω_3^G (i.e. the set of functions from G to Ω_3) by defining $(X \underline{\wedge} Y)g = (Xg) \underline{\wedge} (Yg)$ and similarly for $\overline{\wedge}$.

$\underline{\wedge}$	no	po	fo
no	no	no	no
po	no	no	po
fo	no	po	fo

$\overline{\wedge}$	no	po	fo
no	no	no	no
po	no	po	po
fo	no	po	fo

$g_i \in G$	sr	mr	b_1	ps_1	w	$\ldots$
$VF\, g_i$	po	po	po	fo	no	$\ldots$
$EN\, g_i$	no	po	po	no	po	$\ldots$
$(VF \underline{\wedge} EN)\, g_i$	no	no	no	no	no	$\ldots$
$(VF \overline{\wedge} EN)\, g_i$	no	po	po	no	no	$\ldots$

Consider the boundary insensitive approximations VF of the visual field, vf, and EN of the entrance area, en, in the parking lot in Figure 1 (right table above). The result of $(VF \overline{\wedge} EN)$ approximates the intersection of vf and en correctly. On the other hand it is easy to find regions $x \in [\![VF]\!]$ and $y \in [\![EN]\!]$ with $x \wedge y = \perp$ for which the result of $(VF \underline{\wedge} EN) = \perp$ is correct[12]. In general, the outcome of the operations $\underline{\wedge}$ and $\overline{\wedge}$ on approximations X and Y constrains the possible outcome of the operation $x \wedge y$ for $x \in [\![X]\!]$ and $y \in [\![Y]\!]$ in the such that $X \underline{\wedge} Y \leq (\alpha_3\,(x \wedge y)) \leq X \overline{\wedge} Y$, where $\leq$ is a partial order between approximations defined as $X \leq Y$ if and only if for all $g \in G$ $(X\,g) \leq (Y\,g)$ with $\mathsf{no} < \mathsf{po} < \mathsf{fo}$.

3.2.2. Boundary sensitive operations

First we define the operations $\overline{\wedge}$ on the set $\Omega_{bs} = \{\mathsf{fo}, \mathsf{fbo}, \mathsf{pbo}, \mathsf{nbo}, \mathsf{no}\}$ as shown in the left table below.

$\overline{\wedge}$	no	nbo	pbo	fbo	fo
no	no	no	no	no	no
nbo	no	nbo	nbo	nbo	nbo
pbo	no	nbo	pbo	pbo	pbo
fbo	no	nbo	pbo	fbo	fbo
fo	no	nbo	pbo	fbo	fo

(g_i, g_j)	(b_1, mr)	(b_1, ps_1)	(b_1, w)	(ps_1, b_1)	$\ldots$
VF	pbo	fbo	pbo	fo	$\ldots$
EN	pbo	nbo	pbo	no	$\ldots$
$(VF \overline{\wedge} EN)$	pbo	nbo	pbo	no	$\ldots$

These operation extends to elements of $\Omega_{bs}^{G \times G}$ (i.e. the set of functions from $G \times G$ to Ω_{bs}) by defining $(X \overline{\wedge} Y)(g_i, g_j) = (X(g_i, g_j)) \overline{\wedge} (Y(g_i, g_j))$. The definition of the operation $\underline{\wedge}$ is similar and can be found in [BS98]. The operations $\underline{\wedge}$ and $\overline{\wedge}$ constrain the outcome of the operation $\wedge$ in the sense discussed above. Between the elements of Ω_{bs} the following order holds: $\mathsf{fo} > \mathsf{fbo} > \mathsf{pbo} > \mathsf{nbo} > \mathsf{no}$.

Consider the boundary sensitive approximations VF of the visual field, vf, and EN of the entrance area, en, in the parking lot in Figure 1. A Part of the graph of their least maximal meet, $\overline{\wedge}$, is shown in the right table above.

3.3. Approximate relations

Binary topological relations between regions (RCC relations), such as disconnected, $\mathsf{DC}(x, y)$, partial overlap, $\mathsf{PO}(x, y)$ (Figure 3.3.1) are important for the qualitative description of spatial configurations. In the context of this paper it is important to consider the relations that can hold between regions $x \in [\![X]\!]$

[12]Depending on the context the symbol $\perp$ either refers to the empty region or to the function that yields no for all $g_i \in G$.

and $y \in [\![Y]\!]$ given the approximations X and Y. Formal descriptions of qualitative relations between spatial regions have been widely studied in the literature, e.g., [EH90, CBGG97].

[BS00] propose a specific style of defining RCC relations between regions x and y, which can be generalized to define relations between approximations X and Y. They define RCC relations, $R(x, y)$, exclusively based on the outcome of the meet operation, $\wedge$, applied to regions x and y. These definitions are then generalized to approximations by syntactically replacing variables x and y ranging over regions, by variables X and Y ranging over approximations, and by replacing the meet operation $\wedge$ on regions by minimal and maximal meet operations $\underline{\wedge}$ and $\overline{\wedge}$ on approximations. This yields minimal and maximal relations $\underline{R}$ and $\overline{R}$ such that $\forall x \in [\![X]\!], \forall y \in [\![Y]\!] : \underline{R}(X,Y) \leq R(x,y) \leq \overline{R}(X,Y)$, where the ordering $\leq$ is an ordering relation between RCC-relations defined in [BS00] and discussed below. Consequently, $\underline{R}$ and $\overline{R}$ constrain possible relations, R, that can hold between $x \in [\![X]\!]$ and $y \in [\![Y]\!]$. This subsection shortly discusses those notions.

3.3.1. RCC8 relations

[BS00] describe RCC8 relations [RCC92] by defining the relationship between regions x and y using a triple, where each of the three entries may take one of three truth values F, M, T. The scheme has the form

$$(x \wedge y \not\approx \bot, x \wedge y \approx x, x \wedge y \approx y)$$

where

$$x \wedge y \not\approx \bot = \begin{cases} T & \text{if the interiors of } x \text{ and } y \text{ overlap, i.e., } x \wedge y \neq \bot \\ M & \text{if only the boundaries of } x \text{ and } y \text{ overlap, i.e., } x \wedge y = \bot \text{ and } \delta x \wedge \delta y \neq \bot \\ F & \text{if there is no overlap between } x \text{ and } y \text{, i.e., } x \wedge y = \bot \text{ and } \delta x \wedge \delta y = \bot \end{cases}$$

and where

$$x \wedge y \approx x = \begin{cases} T & \text{if } x \text{ is contained in } y \text{ and the boundaries of } x \text{ and } y \text{ are either disjoint or identical} \\ & i.e., x \wedge y = x \text{ and } (\delta x \wedge \delta y = \bot \text{ or } \delta x \wedge \delta y = \delta x) \\ M & \text{if } x \text{ is contained in } y \text{ and the boundaries are not disjoint and not identical,} \\ & i.e., x \wedge y = x \text{ and } \delta x \wedge \delta y \neq \bot \text{ and } \delta x \wedge \delta y \neq \delta x \\ F & \text{if } x \text{ is not contained within } y \text{, i.e., } x \wedge y \neq x \end{cases}$$

and where

$$x \wedge y \approx y = \begin{cases} T & x \wedge y = y \text{ and } (\delta x \wedge \delta y = \bot \text{ or } \delta x \wedge \delta y = \delta y) \\ M & x \wedge y = y \text{ and } \delta x \wedge \delta y \neq \bot \text{ and } \delta x \wedge \delta y \neq \delta y \\ F & x \wedge y \neq y \end{cases}$$

The meaning of $x \wedge y \neq \bot = T$ is that the intersection of the interior of x and y is non-empty and the meaning of $\delta x \wedge \delta y = \bot = T$ is that the meet of the boundaries of x and y is empty[13]. The correspondence between triples $(x \wedge y \not\approx \bot, x \wedge y \approx x, x \wedge y \approx y)$ and the RCC8 classification is given in the table in Figure 3.3.1. The set of triples is partially ordered by setting $(a_1, a_2, a_3) \leq (b_1, b_2, b_3)$ iff $a_i \leq b_i$ for $i = 1, 2, 3$, where the truth values are ordered by $F < M < T$. [BS00] call the

[13]Notice that, given that x and y are 2-dimensional regions, then their meet is empty, $x \wedge y = \bot$, even if their boundaries intersect since the result of this intersection is not a 2-dimensional region. The meet of 1-dimensional regions is empty if they intersect in a point.

corresponding Hasse diagram (the right part of Figure 3.3.1) the RCC8 *lattice* to distinguish it from the conceptual neighborhood graph [GC94].

Consider the definition of the relation $\mathsf{DC}(x, y)$. By Table 3.3.1 we have $x \wedge y \not\approx \bot = F$, $x \wedge y \approx x = F$, and $x \wedge y \approx y = F$. Consequently, neither the interiors nor the boundaries of x and y overlap, i.e., $x \wedge y = \bot$ and $\delta x \wedge \delta y = \bot$, and the regions x and y are disconnected. In the case of $\mathsf{EC}(x, y)$ we have $x \wedge y \not\approx \bot = M$, $x \wedge y \approx x = F$, and $x \wedge y \approx y = F$. Consequently, the interiors of x and y do not overlap but the boundaries do, i.e., $x \wedge y = \bot$ and $\delta x \wedge \delta y \neq \bot$, and the regions x and y are externally connected. In the case of $\mathsf{NTPP}(x, y)$ we have $x \wedge y \not\approx \bot = T$, $x \wedge y \approx x = T$ and $x \wedge y \approx y = F$. Consequently, x is completely contained in the interior of y: $x \wedge y \neq \bot$, $x \wedge y = x$ and since $x \wedge y \neq y$ we have $\delta x \wedge \delta y = \bot$, i.e., x is a non-tangential proper part of y. In the case of $\mathsf{EQ}(x, y)$ we have $x \wedge y \not\approx \bot = T$, $x \wedge y \approx x = T$ and $x \wedge y \approx y = T$. Both regions are identical, i.e., $x \wedge y = x$, $x \wedge y = y$, and $\delta x \wedge \delta y = \delta x = \delta y$.

$x \wedge y \not\approx \bot$	$x \wedge y \approx x$	$x \wedge y \approx y$	RCC8
F	F	F	DC
M	F	F	EC
T	F	F	PO
T	M	F	TPP
T	T	F	NTPP
T	F	M	TPPi
T	F	T	NTPPi
T	T	T	EQ

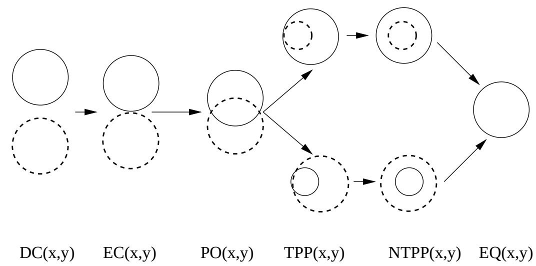


Figure 2. Definition of the RCC8 relations and the corresponding RCC8 lattice

3.3.2. Approximate RCC8 relations

Let X and Y be boundary sensitive approximations of regions x and y (i.e. functions from $G \times G$ to Ω_{bs}). [BS00] showed that based on the operations $\underline{\wedge}$ and $\overline{\wedge}$ pairs of minimal and maximal binary topological relations, $\underline{R}(X, Y)$ and $\overline{R}(X, Y)$, between the approximations X and Y can be computed. Roughly speaking, in the definitions discussed above variables x and y ranging over regions are replaced by variables X and Y ranging over approximations and the meet operation $\wedge$ is replaced by $\underline{\wedge}$ and $\overline{\wedge}$ between approximations. Consequently, relations between approximations are defined using the pair of triples:

$$((X \underline{\wedge} Y \not\approx \bot, X \underline{\wedge} Y \approx X, X \underline{\wedge} Y \approx Y), (X \overline{\wedge} Y \not\approx \bot, X \overline{\wedge} Y \approx X, X \overline{\wedge} Y \approx Y))$$

where $\underline{\wedge}$ and $\overline{\wedge}$ operations on boundary sensitive approximations, Ω_{bs}^{G}, and where

$$X \underline{\wedge} Y \not\approx \bot = \begin{cases} T & X \underline{\wedge} Y \neq \bot \\ M & X \underline{\wedge} Y = \bot \text{ and } \delta X \underline{\wedge} \delta Y \neq \bot \\ F & X \underline{\wedge} Y = \bot \text{ and } \delta X \underline{\wedge} \delta Y = \bot \end{cases} \quad X \overline{\wedge} Y \not\approx \bot = \begin{cases} T & X \overline{\wedge} Y \neq \bot \\ M & X \overline{\wedge} Y = \bot \text{ and } \delta X \overline{\wedge} \delta Y \neq \bot \\ F & X \overline{\wedge} Y = \bot \text{ and } \delta X \overline{\wedge} \delta Y = \bot \end{cases}$$

and where

$$X \underline{\wedge} Y \approx X = \begin{cases} T & X \underline{\wedge} Y = X \text{ and } (\delta X \underline{\wedge} \delta Y = \bot \text{ or } X \underline{\wedge} Y = Y) \\ M & X \underline{\wedge} Y = X \text{ and } \delta X \underline{\wedge} \delta Y \neq \bot \text{ and } X \underline{\wedge} Y \neq Y \\ F & X \underline{\wedge} Y \neq X \end{cases}$$

and similarly for $X \underline{\wedge} Y \approx Y$, $X \overline{\wedge} Y \approx X$,and $X \overline{\wedge} Y \approx Y$[14].

The meaning of $\delta X \underline{\wedge} \delta Y \neq \perp$ and $\delta X \overline{\wedge} \delta Y \neq \perp$ is the following. Assume the partial order of the RCC8-lattice. $\delta X \underline{\wedge} \delta Y \neq \perp$ is true if the least RCC8-relation that can hold between $x \in [\![X]\!]$ and $y \in [\![Y]\!]$ involves boundary intersection at a boundary segment of the underlying partition G. Consider the configurations (c) and (d) in Fig. 3. The regions x, y, and z are approximated with respect to a partition containing the cells g_i and g_j. Assume $x \in [\![X]\!]$ and $\{y, z\} \subset [\![Y]\!]$, with $(X\,(g_i, g_j)) = \mathsf{fbo}$, $(X\,(g_j, g_i)) = \mathsf{pbo}$, and with $(Y\,(g_i, g_j)) = \mathsf{no}$, $(Y\,(g_j, g_i)) = \mathsf{pbo}$. For simplification we assume that we can ignore the rest of the partition and the rest of the approximation mappings. The least relation that can hold between regions in $[\![X]\!]$ and regions in $[\![Y]\!]$ is EC and involves boundary intersection of the corresponding regions at the boundary segment shared by g_i and g_j, e.g., $\mathsf{EC}(x, y)$ in configuration (c). However there may be also regions in $[\![X]\!]$ and $[\![Y]\!]$ with relations greater than EC that may or may not involve boundary intersection at that boundary segment, e.g., $\mathsf{PO}(x, z)$ in configuration (d).

The formula $\delta X \overline{\wedge} \delta Y \neq \perp$ is true if the greatest RCC8-relation that can hold between $x \in [\![X]\!]$ and $y \in [\![Y]\!]$ involves boundary intersection at a boundary segment in G. An example is given in the configuration (a) in Fig. 3. Assume $x \in [\![X]\!]$ and $\{y, z\} \subset [\![Y]\!]$, with $(X\,(g_i, g_j)) = \mathsf{pbo}$, $(X\,(g_j, g_i)) = \mathsf{no}$, $(Y\,(g_i, g_j)) = \mathsf{no}$, $(Y\,(g_j, g_i)) = \mathsf{pbo}$. For simplification we assume that we can ignore the rest of the partition and the rest of the approximation mappings. The greatest relation that can hold between regions in $[\![X]\!]$ and $[\![Y]\!]$ is EC and involves boundary intersection of the corresponding regions at the boundary segment shared by g_i and g_j. However there may be also regions in $[\![X]\!]$ and $[\![Y]\!]$ with relations less than EC, e.g., $\mathsf{DC}(x, z)$ in configuration (b). For details see [BS00].

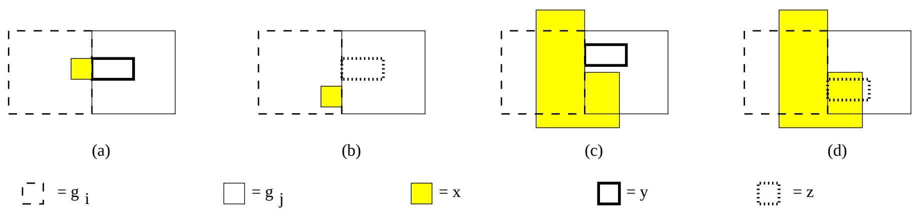


Figure 3 Example configurations for $\delta X \overline{\wedge} \delta Y \neq \perp$ (configurations a and b) and $\delta X \underline{\wedge} \delta Y \neq \perp$ (configurations c and d).

Each of the triples $(X \underline{\wedge} Y \not\approx \perp, X \underline{\wedge} Y \approx X, X \underline{\wedge} Y \approx Y)$ and $(X \overline{\wedge} Y \not\approx \perp, X \overline{\wedge} Y \approx X, X \overline{\wedge} Y \approx Y)$ defines a RCC8 relation, so the relation between X and Y is measured by a pair of RCC8 relations. These relations will be denoted by $\underline{R}\,(X, Y)$ and $\overline{R}\,(X, Y)$. Assume the ordering of the RCC8-lattice. The relation $\underline{R}\,(X, Y)$ is the *least* relation that can hold between regions $x \in [\![X]\!]$ and $y \in [\![Y]\!]$[15]. Respectively, the relation $\overline{R}\,(X, Y)$ is the *greatest* relation that can hold between those regions. Furthermore, for each $R \in RCC8$ with $\underline{R}\,(X, Y) \leq R(x, y) \leq \overline{R}\,(X, Y)$ there are $x \in [\![X]\!]$ and $y \in [\![Y]\!]$ such that $R(x, y)$ holds.

[14]In this context the bottom element, $\perp$, is either the value no or the function from $G \times G$ to Ω_{bs} which takes the value no for every element of $G \times G$.

[15]Modulo one special case which is not relevant in the context of this paper. It is discussed in detail in [BS00].

4. Boundary sensitive rough location

Every spatial object is exactly located at a single region of space in each moment of time [CV95]. This region may be a simple region of three-dimensional space, for example, think of your body and the region of space it carves out of the air. The exact region of a spatial object may be a complex region, consisting of multiple regions of three-dimensional space, as in the case of the exact region of the Hawaiian islands. It may be a complex two-dimensional region, as in the case of the exact region of a paper map representation of Hawaiian islands. For the reasons discussed in Section 2 we limit our attention to the 2D case.

Let O be the set of spatial objects and let R be the set of regions of space. Exact location is a mapping of signature $r : O \to R$, which assigns to each spatial object, $o \in O$, its exact region of space, $r \in R$, at a given moment of time. This exact location can be approximated with respect to an underlying regional partition as discussed in the previous section. We define the notion of *boundary sensitive rough location*, $loc : O \to \Omega_{bs}^{G \times G}$, as the composition of the approximation function, α_5, and the exact-location-function, r, i.e., (loc o) $\;=_{def}\;$ $(\alpha_5 \circ r)o$.

The location of a spatial object within a regional partition is not only characterized by its boundary sensitive rough location. Essentially, rough location, as introduced above, focuses on the approximation of *regions* with respect to a set of (partition forming) *regions*. But we are interested in characterizing the location of *objects* with respect to a set of (partition forming) *objects*. In this context we need to consider (Section 2): (1) There are different kinds of spatial objects and different kinds of boundaries. (2) Spatial objects have topologically closed boundaries. (3) Co-location with boundary parts of other objects or coverage by interiors of other objects carve out boundary segments off the topologically closed boundary wholes. (4) Co-located boundary segments may be of different kind and may interact with each other. The description of rough location of objects needs to take these aspects into account.

4.1. Coverage of boundary segments in regional partitions

The degree of coverage of boundary segments shared by neighboring partition cells by the approximated spatial objects plays an important role in the description of the approximate location of these objects. Given the boundary sensitive approximation X of regions $x \in [\![X]\!]$ we can easily decide for each boundary segment, (g_i, g_j)[16], of the underlying regional partition G, whether parts of the interior, the boundary, or the closure of regions $x \in [\![X]\!]$ cover this boundary segment (Figure 4). Furthermore, we can derive the degree of coverage, (fo,po,no), of (g_i, g_j) by the interior/boundary/closure of x: Let $\delta(X(g_i, g_j))$ be the boundary component of $(X(g_i, g_j))$ and $\delta(X(g_j, g_i))$ be the boundary component of $(X(g_j, g_i))$ as defined in Section 3.1.2. We define $\pi(X, (g_i, g_j))$ to be the approximation of the coverage of the boundary segment (g_i, g_j) by regions $x \in [\![X]\!]$:

$$\pi(X, (g_i, g_j)) = \max(\delta(X(g_i, g_j)), \delta(X(g_j, g_i))).$$

Due to the definition of X we have $\pi(X, (g_i, g_j)) = \mathsf{fo}$ if x completely covers the boundary segment (g_i, g_j), $\pi(X, (g_i, g_j)) = \mathsf{po}$ if x covers parts but not all of (g_i, g_j), and $\pi(X, (g_i, g_j)) = \mathsf{no}$ if x

[16] We use the notion of an ordered pair, (g_i, g_j), to refer to the boundary segment shared by the partition cells g_i and g_j. This slightly conflicts with the usage of (g_i, g_j) as argument of the approximation function α_5, e.g., in $(X\,(g_i, g_j))$, where it refers to the cells themselves. The context should make clear which interpretation is intended.

does not overlap (g_i, g_j). Consider configuration (a) in Figure 4: We have $(X(g_i, g_j)) = (\mathsf{po}, \mathsf{fo})$ and $(X(g_j, g_i)) = (\mathsf{po}, \mathsf{po})$. This yields $\pi(X, (g_i, g_j)) = \max(\mathsf{fo}, \mathsf{po}) = \mathsf{fo}$.

In order to derive the approximation of the coverage of the boundary segment (g_i, g_j) by the interior and the boundary of the region x from its approximation X, we define an operation $\ominus : \Omega_3 \times \Omega_3 \rightarrow \Omega_3$ in analogy to the subtraction of regions in the table left below[17]. Using the operation $\ominus$ we define the approximation of the coverage of the boundary segment (g_i, g_j) by boundary parts, π^δ, and by interior parts, π^ι, of the regions $x \in [\![X]\!]$ (right below).

$\ominus$	no	po	fo
no	no	po	fo
po	po	no	po
fo	fo	po	no

$$\begin{aligned} \pi^\delta(X, (g_i, g_j)) &= \delta(X, (g_i, g_j)) \ominus \delta(X, (g_j, g_i)) \\ \pi^\iota(X, (g_i, g_j)) &= \pi(X, (g_i, g_j)) \ominus \pi^\delta(X, (g_i, g_j)) \end{aligned}$$

Consider configurations (b) and (c) in Figure 4: We have $(X(g_i, g_j)) = (\mathsf{po}, \mathsf{fo})$ and $(X(g_j, g_i)) = (\mathsf{po}, \mathsf{po})$. This yields $\pi^\delta(X(g_i, g_j)) = \mathsf{fo} \ominus \mathsf{po} = \mathsf{po}$ and $\pi^\iota(X(g_i, g_j)) = \pi(X(g_i, g_j)) \ominus \pi^\delta(X(g_i, g_j)) = \mathsf{po}$.

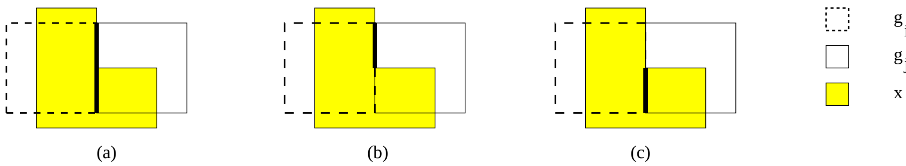


Figure 4 The coverage of the boundary segment (g_i, g_j) by the closure of x (configuration a), the boundary of x (configuration b), and the interior of x (configuration c).

4.2. Object-boundary sensitive rough location

In this subsection the notion of object-boundary sensitive rough location is introduced and used to formalize the rough location of spatial objects within a regional partition formed by a set of partition forming objects that takes the points (1)-(4) raised in the introduction of this section into account.

4.2.1. Definitions

Let the ordered pair (g_i, g_j) denote the boundary segment shared by the partition regions g_i and g_j and let the order of the pair-elements mean that we are 'looking' from the inside of g_i towards g_j. Let $BT = \{bf, fb_{IN}, fb_{OUT}, fb_2, nb\}$ be a set of symbols referring to bona-fide barrier boundaries (bf), one-way fiat barrier boundaries (fb_{IN} and fb_{OUT}), two-way fiat barrier boundaries (fb_2), and non-barrier fiat boundaries (nb) as discussed in Table 1.

In Section 2.1.4 we have seen that the barrier properties of boundaries depend on the context, e.g., glass is a barrier for the movement of bona-fide objects but not for the spread of light and for human vision. Consequently, each of the symbols bf, fb_{IN}, fb_{OUT}, fb_2 , and nb needs an additional index

[17]Notice that there is also a corresponding second operation with $\mathsf{po} \underline{\ominus} \mathsf{po} = \mathsf{po}$, which is not relevant in the context of this paper.

specifying the context, e.g., an index for 'barrier with respect to movement of bona-fide objects'. In the remainder we consider barrier properties in the context of the movement of bona-fide objects if not stated differently and omit additional indexes.

We define the set $BT^\star = BT \cup \{0\}$ by enriching the set BT by the value 0. The meaning of the symbol 0 will be discussed below. Let $(\text{loc}_{bs}\ o)$ be the mapping representation of the boundary sensitive rough location of the object, o, within the regional partition G. We define the notion of *object-boundary sensitive rough location*, $(\text{LOC}_{bs}\ o)\ :\ (G \times G) \rightarrow (\Omega_{bs} \times BT^\star \times BT^\star)$, where $(\text{loc}_{bs}\ o)$ extends in the natural way to $(\text{LOC}_{bs}\ o)$ by defining $((\text{LOC}_{bs}\ o)\ (g_i, g_j)) = (\omega, bt_\iota, bt_\delta)$ if and only if $((\text{loc}_{bs}\ o)\ (g_i, g_j)) = \omega$[18].

Assume $((\text{LOC}_{bs}\ o)(g_i, g_j)) = (\omega, bt_\iota, bt_\delta)$. The value bt_ι denotes the type of the boundary segment, of the object o, which is covered by the interior of the partition region g_i (e.g., the type of part of the boundary of x that is covered by the interior of the dashed partition cell in Figure 4 (b)). We have $bt_\iota = 0$ if $\omega \in \{\mathsf{no}, \mathsf{fo}\}$, and $bt_\iota \neq 0$ otherwise, i.e., $bt_\iota = bf$ if o is a bona-fide object, and $bt_\iota \in \{fb_{IN}, fb_{OUT}, fb_2, nb\}$ if o is a fiat object. If o is a partition forming object, i.e., $r(o) \in G$, then we have $bt_\iota = 0$ for all $(\text{LOC}_{bs}\ o)\ (g_i, g_j)$.

The value bt_δ denotes the type of the boundary segment of the object o, which is co-located with the boundary segment (g_i, g_j) (e.g., the type of the bold marked part of the boundary of x in Figure 4 (b)). We have $bt_\delta = 0$ if $\pi^\delta((\text{loc}\ o)\ (g_i, g_j)) = \mathsf{no}$, and $bt_\delta \neq 0$ otherwise, i.e., $bt_\delta = bf$ if o is a bona-fide object and $bt_\delta \in \{fb_{IN}, fb_{OUT}, fb_2, nb\}$ if o is a fiat object.

4.2.2. The type of co-located boundaries

Let $\{o_1, \ldots, o_n\}$ be a finite set of objects that possibly have co-located boundary parts. In the remainder capital letters are used to refer to the corresponding object-boundary sensitive rough location approximation, e.g., $O_1 = (\text{LOC}_{bs}\ o_1)$. Assume $(O_1(g_i, g_j)) = (\omega_1, bt^1_\iota, bt^1_\delta)$, ..., $(O_n(g_i, g_j)) = (\omega_n, bt^n_\iota, bt^n_\delta)$. The boundary type of the part of the boundary of o_1, which is possibly co-located with parts of the boundaries of $o_2 \ldots o_n$ at the boundary segment (g_i, g_j) is defined as: $bt^1_\delta \oplus bt^2_\delta \oplus \ldots \oplus bt^n_\delta$, where the operation $\oplus : BT \times BT \rightarrow BT$ is defined as shown in the table left below. The operation $\oplus$ extends to $BT^\star \times BT^\star \rightarrow BT^\star$ as shown right below[19].

$bt_1 \oplus bt_2$	bf	fb_2	fb_{IN}	fb_{OUT}	nb
bf	$-$	bf	bf	bf	bf
fb_2	bf	fb_2	fb_2	fb_2	fb_2
fb_{IN}	bf	fb_2	fb_{IN}	fb_2	fb_{IN}
fb_{OUT}	bf	fb_2	fb_2	fb_{OUT}	fb_{OUT}
nb	bf	fb_2	fb_{IN}	fb_{OUT}	nb

$$bt_1 \oplus bt_2 = \begin{cases} bt_1 & \text{if } bt_2 = 0 \\ bt_2 & \text{if } bt_1 = 0 \\ bt_1 \oplus bt_2 & \text{otherwise} \end{cases}$$

The operation $\oplus$ is not defined for pairs of bona-fide boundary parts since bona-fide boundaries cannot be co-located. It is commutative and associative, i.e., the order in which we compute the resulting type of multiple co-located boundary segments does not matter. Boundary types are defined in certain contexts,

[18] Notice that extending $(\text{loc}_{bs}\ r(o))$ to $(\text{LOC}_{bs}\ o)$ requires extending the operations $\underline{\wedge}$ and $\overline{\wedge}$ as well. This is omitted here due to space limitations.

[19] We are going to overload the operation $\oplus$ several times in this section but the context should make clear to which particular structures it applies.

e.g., the barrier type fb_2 with respect to movement of bona-fide objects. The operation $\oplus$ is only defined between boundary types belonging to the same context.

Consider the visual field, vf, in the parking lot in the middle of Figure 1. In this example the boundaries are barriers with respect to human vision, i.e., fb_{OUT} means that you cannot see beyond this boundary (This is another example for indirectly perceiving fiat boundaries). The graph of the mapping $VF = (\text{LOC}_{bs}\ vf)$ contains the following tuples:

(g_i, g_j)	(b_1, mr)	(b_1, ps_1)	(b_1, w)	(ps_1, b_1)	$\ldots$	(ps_1, w)	$\ldots$
ω	pbo	fbo	pbo	fo	$\ldots$	fo	$\ldots$
bt_ι	fb_{OUT}	fb_{OUT}	fb_{OUT}	0	$\ldots$	0	$\ldots$
bt_δ	0	0	bf	0	$\ldots$	bf	$\ldots$

Consider the value of bt_δ in row (b_1, mr). We have $bt_\delta = 0$ since no *segments*, i.e., one-dimensional parts, of the boundary of vf are co-located with the boundary segment (b_1, mr), i.e., $\pi^\delta(VF, (b_1, mr)) =$ no. Consider row (b_1, w). We have $bt_\delta = bf = fb_{OUT} \oplus bf$. The interpretation of the values $bt_\iota = fb_{OUT}$ in the rows $(b_1, mr) \ldots (b_1, w)$ is that the boundary segment of vf carved out by b_1 is fb_{OUT}. The interpretation of $bt_\iota = 0$ in the rows (ps_1, b_1) and (ps_1, w) is that the boundary of vf is not covered by interior parts of ps_1.

4.2.3. Sets of partition forming objects

Let $\{o_1, \ldots, o_n\}$ be a set of partition forming objects of an built environment BE, i.e., $G_{BE} = \{r(o_1), \ldots, r(o_n)\}$. There are elements of $\{o_1, \ldots, o_n\}$ that have co-located boundary segments, which types 'interact' in the sense discussed above. We define an operator Σ that computes the object-boundary sensitive rough location of the objects $\{o_1, \ldots, o_n\}$ as a (partition forming) *set*[20]. We define $\Sigma\{o_1, \ldots, o_n\} = O_1 \oplus \ldots \oplus O_n$ with

$$(O_k \oplus O_l)\,(g_i, g_j) = (\max(\omega^k, \omega^l), 0, bt_\delta^k \oplus bt_\delta^l)$$

and with $(O_k(g_i, g_j)) = (\omega^k, 0, bt_\delta^k)$ and $(O_l(g_i, g_j)) = (\omega^l, 0, bt_\delta^l)$. The interior components, bt_ι^i, are always 0 since the o_i are partition forming objects. In the end $\Sigma\{o_1, \ldots, o_n\}$ has the value $(\text{fo}, 0, bt_\delta)$ for all (g_i, g_j), where bt_δ is the boundary type resulting from the interaction of adjacent partition elements, (g_i, g_j), with $\pi^\delta(O_k, (g_i, g_j)) \,\triangle\, \pi^\delta(O_l, (g_i, g_j)) = \text{fo}$.

Consider the parking lot, PL, in the middle of Figure 1. Spatial objects have topologically closed boundaries. Co-location with parts of adjacent objects carves out boundary segments. In parking lots traffic rules assign fb_2 to side and back boundary segments of parking spots and nb to their front boundary segments. The boundaries of bona-fide objects like the outer wall and the boundaries of the security building are of type bf. Consider the side road, sr. As a fiat object it has a topologically closed fiat boundary. Co-location with the objects sb, mr, ps_6, and ps_{15} carves out boundary segments. Traffic rules assign the type fb_{IN} to the lower boundary segment $(sr, mr)^{en}$[21] near the entrance, the type fb_{OUT} to the upper boundary segment $(sr, mr)^{ex}$ near the exit and the type nb to all other boundary segments. Co-location with the bona-fide boundary of the security building, sb, gives the boundary segment

[20] We need this operator in Section 5.3 to derive paths along which non-partition forming objects can move within a built environment.

[21] The superscripts en and ex are additional indexes to distinguish the two boundary segments of sr shared with mr.

(sr, sb) the type bf. Co-location with the side boundaries of ps_6 and ps_{15} gives the boundary segments (sr, ps_6) and (sr, ps_{15}) the type fb_2. Given this we have $(SR \oplus SB)\,(sr, sb) = (\mathsf{fo}, 0, nb \oplus bf = bf)$, $(SR \oplus PS_6)\,(sr, ps_6) = (\mathsf{fo}, 0, nb \oplus fb_2 = fb_2)$, $(SR \oplus PS_{15})\,(sr, ps_{15}) = (\mathsf{fo}, 0, fb_2)$, $(SR \oplus MR)\,(sr, mr)^{en} = (\mathsf{fo}, 0, fb_{IN} \oplus nb = fb_{IN})$[22], and $(SR \oplus MR)\,(sr, mr)^{ex} = (\mathsf{fo}, 0, fb_{OUT} \oplus nb = fb_{OUT})$. Consequently we have $\Sigma\{sr, sb, ps_6, \ldots, mr\} = SR \oplus SB \oplus PS_6 \oplus \ldots \oplus MR$ with

(g_i, g_j)	(sr, sb)	(sr, ps_6)	$(sr, mr)^{en}$	$(sr, mr)^{ex}$	$\ldots$
$\Sigma_{PL}\,(g_i, g_j)$	$(\mathsf{fo}, 0, bf)$	$(\mathsf{fo}, 0, fb_2)$	$(\mathsf{fo}, 0, fb_{IN})$	$(\mathsf{fo}, 0, fb_{OUT})$	$\ldots$

5. Formalizing built environments

We are now able to formally describe the qualitative structure of built environments. In this context we distinguish three major components:

1. The (static) layout of the built environment, which is formed by the partition forming objects.
2. A system of paths along which non-partition forming objects can move within the layout.
3. A set of possible situations, where a situation is the layout of the environment and a set of non-partition forming objects populating the environment in a given moment of time.

Situations need to obey the ontological axioms, $O1-O3$ and the partition axioms, $P1-P2$. Furthermore they need to be such that the non-partition forming objects populating the environment could possibly have been moved into the location they are in this situation (axiom M). In this section we discuss formal axioms for situations in built environments. These axioms take into account:

- The ontological salience of boundaries.
- The ontological distinction between bona-fide and fiat objects and boundaries.
- The different character of constraints on relations involving bona-fide and fiat objects.
- The ontological distinction between partition forming and non-partition forming objects.
- The barrier character of boundaries.

Formally, the axioms characterizing built environments are given in terms of object-boundary sensitive rough location. In the remainder of this section the use of the notion of rough location in this context is justified and formal versions of the axioms $O1 - O3$, $P1$, $P2$, and M are given. In the end the basic components of a built environment are defined formally.

5.1. Why in terms of rough location?

In this subsection three arguments[23] in favor of the formalization of build environments based on object-boundary sensitive rough, i.e., *approximate*, location in opposition to a formalization based on *exact* location in terms of Analytical geometry[24] are given:

1. Rough location focuses on the relationships between objects and their environments and the distinction between partition forming and non-partition forming objects.

[22] One could also assume $(MR\,(mr, sr)^{en}) = (\mathsf{fo}, 0, fb_{OUT})$ and, hence, $(MR\,(sr, mr)^{en}) = (\mathsf{no}, 0, fb_{IN})$, which would yield the same result since $fb_{IN} \oplus fb_{IN} = fb_{IN} \oplus nb = fb_{IN}$.

[23] Assuming that task 1 and 2 discussed in the introduction are to be performed.

[24] Extended by a boundary calculus similar to the one discussed above.

2. The notion of rough location is qualitative in nature. It directly represents the qualitative structure of built environments. Primitives of the language directly correspond to ontologically salient features of the underlying reality. Furthermore primitives of the language correspond to features perceivable in the environment[25].
3. In Section 2 we discussed examples that showed that constraints on relations involving objects of ontological different kind are much weaker than constraints on relations between objects of the same ontological kind. In Sections 5.2 and 5.4 we will see that the notion of rough location allows to express those kinds of constraints within an algebraic framework.

5.1.1. Focus on the structure of environments

Built environments are made up of spatial objects and have to obey the axioms governing the objects forming and populating the environment. The notion of rough location rests upon a theory of objects and their exact location [CV95], [BS99]. Rough location focuses on the approximate location of objects within regional partitions. In section 2 we saw that regional partitions are formed by the partition forming objects of the environment and that they are main organizational structure of the environment. The notion of rough location implicitly takes the distinction between partition forming and and non-partition forming objects and the organizational structure of the regional partition into account.

When we describe built environments in terms of rough location then objects are second class citizens. The first class citizens are mappings representing the *rough location of objects within their environments*, i.e., object-environment relations. In fact location mappings can be interpreted as equivalence classes of objects sharing the same location: $[\![(\text{loc } o_1)]\!] = \{o \in O \mid (\text{loc } o) = (\text{loc } o_1)\}$ (Remember Section 3).

In terms of rough location we are able to talk about the potential location of objects within an environment and we can talk about the relations that can hold between objects that possibly occupy those locations. This means that we can talk about location in built environments without having knowledge of objects that actually populate it. Another important point is that no matter how big the environment might be since it is formed by finitely many partition forming objects there can be only finitely many distinct rough locations within this environment. Consequently we can *completely* analyze all possible locations even if there are (theoretically) infinitely many possible configurations in terms of objects of different kind and scale.

5.1.2. Qualitativeness

The notion of rough location allows for qualitative representation and allows to abstract from any kind of measurement. Furthermore, it focuses on properties that are ontologically salient and that are perceivable in the sense discussed above. In this context we assume that topological relationships between spatial objects, boundaries and co-location of boundary segments are observable/perceivable.

Remember the thought experiment in the introduction. We assumed a program (I) that generates potential plans for built environments. It is fair to assume that (I) is based on standard algorithms of computational and Analytical geometry. The output of (I) is quantitative and focuses on metric knowledge.

[25] As discussed in Section 2, we consider fiat boundaries, like the front boundary of a parking spot, as being perceivable by human beings. Even if one cannot see them one is able to perceive them in suitably designed built environments.

The program (II) extracts qualitative knowledge and builds a corresponding object-boundary sensitive rough location representation.

One might ask 'Why do we need a qualitative description if we have a quantitative geometric model?'. The answer is threefold: (1) The basic structure of a built environment IS qualitative. Consequently, we need qualitative notions in order to decide whether the generated geometric model represents a built environment. (2) It is the purpose of (II) to evaluate the plan of the environment with respect to the degree it facilitates human way finding. Human cognition is based on processing qualitative rather than quantitative knowledge, e.g., [Fre91]. (3) Knowledge about actual situations is based on observations of reality and is qualitative in nature. The quantitative description generated by (I) represents reality as it is or it is planned to be. The question is not whether or not to use this quantitative description, but to derive qualitative spatial relations *between ontologically salient features, which correspond to relations observable in reality*. Given the assumption that topological relationships between spatial objects are observable, boundaries are observable, and co-location of boundaries is observable, we are justified to derive object-boundary sensitive rough location of spatial objects from the underlying quantitative representation and to claim that (1) this corresponds to relations observable in reality; (2) this represents ontologically salient features; (3) this reflects the qualitative structure of the environment.

5.2. Formalizing ontological constraints

In Section 2 we analyzed the kinds of objects, which form and populate built environments in general and parking lots in particular. At the conceptual level we identified a number of constraints on relations that need to hold between those spatial objects. Essentially these constraints are constraints on topological relations that need to hold among exact regions of spatial objects of different kinds. The constraints specify what kinds of objects and regions can overlap, and what kinds of boundaries can be co-located. Those relations will now be expressed formally in terms of object-boundary sensitive rough location as described in Sections 3 and 4. Using this notion we are even more specific since we take the distinctions between bona-fide and fiat objects, the distinction between partition forming and non-partition forming objects, and the distinctions between different kinds of barriers into account.

In the remainder of this subsection we are discussing five constraints, $F1 - F5$. The constraints $F1$ and $F2$ belong to the class $O1$. The constraints $F3$ and $F4$ belong to the class $O2$. Constraints $F1$-$F4$ as a set govern partition forming objects. Constraint $F5$ applies to non-partition forming bona-fide objects that are supposed to be movable within the built environment and contributes to the formal version of of the moveability axiom M.

5.2.1. Bona-fide objects

Distinct bona fide objects do not overlap and do not have co-located boundary parts. Let o_1 and o_2 be bona-fide objects, i.e., $o_1, o_2 \in BF$[26]. In terms of objects and their exact location we postulate $\forall o_1, o_2 \in BF \ : \ o_1 \neq o_2 \Rightarrow \mathsf{DC}(r(o_1), r(o_2))$. In terms of rough location we define:

$$F1(o_1, o_2) \ \equiv \ \underline{R}\,((\mathrm{loc}_{bs}\ o_1), (\mathrm{loc}_{bs}\ o_2)) = \mathsf{DC}$$

[26] BF is a finite (but may be very large) set of things that count as bona-fide objects with respect to the definitions given in [SV99a].

and postulate $\forall o_1, o_2 \in BF \;:\; o_1 \neq o_2 \Rightarrow F1(o_1, o_2)$. The location bona-fide objects, o_1 and o_2, can have in an environment is such that the minimal relation between them, consistent with their rough location $(\mathrm{loc}_{bs}\ o_1)$ and $(\mathrm{loc}_{bs}\ o_2)$, is disconnected, $\mathsf{DC}(r(o_1), r(o_2))$. There cannot exist an environment that forces bona-fide objects to be connected[27].

Bona-fide objects cannot connect or overlap even if they share the same rough location. But in terms of rough location it is impossible to postulate that bona-fide objects cannot be connected. Notice the difference: In terms of rough location we specify what an environment *cannot do* to bona-fide objects populating or forming it - it cannot make them being connected. On the other hand - objects themselves are governed by the underlying theory of objects.

Consider Figure 1. Imagine two cars on the main road. Both share the same rough location. The main road would not be a road if it were too small for both cars to fit into it without collision (connection). That is what we postulate in terms of rough location. But in terms of rough location we cannot exclude the possibility for the cars to overlap. This is the business of the theory of objects.

5.2.2. Fiat objects

Distinct fiat objects of the same kind do not overlap but may have co-located boundary parts. Let o_1 and o_2 be fiat objects of kind ϕ, i.e., $o_1, o_2 \in F^{\phi}$[28]. In terms of objects and their exact location we postulate $\forall o_1, o_2 \in F^{\phi} : o_1 \neq o_2 \Rightarrow \mathsf{DC}(r(o_1), r(o_2))$ or $\mathsf{EC}(r(o_1), r(o_2)))$. In terms of rough location we define:

$$F2(o_1, o_2) \;\equiv\; \underline{R}((\mathrm{loc}_{bs}\ o_1), (\mathrm{loc}_{bs}\ o_2)) \leq \mathsf{EC}$$

and postulate $\forall o_1, o_2 \in F^{\phi} \;:\; o_1 \neq o_2 \Rightarrow F2(o_1, o_2)$. There cannot exist a built environment that forces fiat objects of the same kind to overlap. As in the case of bona-fide objects, in terms of rough location it is impossible to postulate that distinct fiat objects of the same kind cannot overlap. This is the business of the theory of objects.

5.2.3. Partition forming objects

Let o_1 and o_2 be bona-fide partition forming objects. In terms of boundary-sensitive rough location we define:

$$F3(o_1, o_2) \;\equiv\; \overline{R}((\mathrm{loc}_{bs}\ o_1), (\mathrm{loc}_{bs}\ o_2)) = \mathsf{DC}$$

and postulate $\forall o_1, o_2 \in BF \;:\; (o_1 \neq o_2$ and $r(o_1), r(o_2) \in G) \;\Rightarrow\; F3(o_1, o_2)$. Due to the underlying partition structure we are able to postulate that partition forming bona-fide objects cannot be connected. The largest relation that can hold between two partition forming bona-fide objects is DC. Consequently, bona-fide objects cannot be located at neighboring partition cells.

Let o_1 and o_2 be partition forming objects such that o_1 is of fiat kind and o_2 is of bona-fide or of fiat kind. Boundary parts of those objects may be co-located, i.e., their exact regions may be externally connected, EC. In terms of boundary-sensitive rough location we define:

$$F4(o_1, o_2) \;\equiv\; \overline{R}((\mathrm{loc}_{bs}\ o_1), (\mathrm{loc}_{bs}\ o_2)) = \mathsf{EC}$$

---

[27] Two objects, o_1 and o_2 are connected if they are externally connected or they overlap, i.e., $(r(o_1), r(o_2)) \in \{EC, PO, TPP(i), NTPP(i), EQ\}$ in the sense of Figure 3.3.1.

[28] F^{ϕ} the set of fiat objects of kind ϕ in the sense of [SV99a].

and postulate $\forall o_1 \in F^{\phi}, \forall o_2 \in F^{\psi} \cup BF \;:\; (o_1 \neq o_2 \text{ and } r(o_1), r(o_2) \in G) \;\Rightarrow\; F4(o_1, o_2)$. Due to the underlying partition structure we are able to postulate that distinct partition forming objects, which are not both bona-fide, cannot overlap, i.e., the largest relation that can hold between them is EC.

5.2.4. Boundaries and non-partition forming objects

Let o_1 be a non-partition forming bona-fide object, let o_2 and o_3 be partition forming fiat objects, and let $(r(o_2), r(o_3))$ be a non-barrier boundary segment shared by o_2 and o_3. For example, o_1 could be a car, and the boundary segment $(r(o_2), r(o_3))$ could be the boundary segment shared by parking spot 2 and the main road, mr, or the upper boundary segment shared by the main road and the side road, sr, in Figure 1. The boundary segment $(r(o_2), r(o_3))$ must wide enough to let the car o_1 pass through. Otherwise it would be a (two-way fiat) barrier with respect to the movement of o_1. It would become a bona-fide barrier boundary if the neighboring boundary segments were of bona-fide type[29]. Assuming $((\mathrm{LOC}_{bs}\;\; o_2)\;\; (r(o_2), r(o_3))) \;=\; ((\mathsf{fo}, (0, bt_\delta))$ we have $(((\mathrm{loc}_{bs}\;\; o_1) \triangle (\mathrm{loc}_{bs}\;\; o_2))\;\; (r(o_2), r(o_3))) \geq \mathsf{fbo} \Rightarrow bt_\delta \in \{fb_2, bf\}$ $(F5)$.

5.3. Built environments

In this subsection the constraints defined above are used in order to describe the components built environments (layout, path system, situations) formally.

5.3.1. The layout

The layout of a built environment is formed by a set of partition forming objects. Formally, it is a triple $L =< G, BF_G, F_G >$, where G is a set of regions forming a regional partition, BF_G is a set of partition forming bona-fide objects, and F_G a set of partition forming fiat objects such that the following hold to be true:

1. $\forall o_1, o_2 \in BF_G : \quad o_1 \neq o_2 \Rightarrow F3(o_1, o_2)$
2. $\forall o_1 \in F_G, \forall o_2 \in BF_G \cup F_G : \quad o_1 \neq o_2 \Rightarrow F4(o_1, o_2)$
3. $G = \{r(o) \mid o \in BF_G\} \cup \{r(o) \mid o \in F_G\}$
4. $\forall g_i, g_j \in G : g_i \wedge g_j \neq \perp \Rightarrow g_i = g_j, \quad \bigvee G = \top$

The axioms 3 and 4 are formal versions of the partition axioms $P1$ and $P2$, where $\bigvee G = g_1 \vee g_2 \vee \ldots \vee g_n$, $g_i \in G$ and $\top$ is the universal region U if the exterior, EXT, of the environment belongs to the partition G otherwise $\top$ is the universal region without the exterior.

5.3.2. The path system

Let $\Gamma^G = (V, E, h)$[30] be a directed version of the dual graph of the topological graph of the regional partition[31], G. Consequently, every vertex, v_i, corresponds to a partition element g_i and $h(e) = (v_i, v_j)$

[29] In fact, the entrances of parking garages are usually designed such that they are bona-fide barriers for cars above a certain size.

[30] A directed graph $\Gamma = (V, E, h)$ is a structure that consists of a finite set of vertices, V, a finite set of edges, E, and a function $h : E \to V \times V$ which maps each edge $e \in E$ onto a tuple (p, q) of vertices $p, q \in V$. $h(e) = (p, q)$ means that the edge e joins the vertices p and q from p to q.

[31] In the dual graph of the topological graph of a regional partition the partition cells are the vertices and the boundary segments shared by neighboring partition cells are the edges. See [NC88] and [Bit99] for details. In the remainder vertexes,

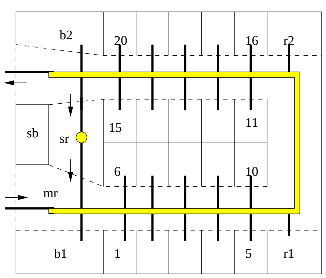
refers to the boundary segment (g_i, g_j) where the order of the tuple interpreted as described above. Let G be the regional partition formed by the partition forming objects of a built environment with the layout L. The path system of the layout, Γ^L, is a sub-graph[32] of $\Gamma^G = (V, E, h)$. The graph $\Gamma^L = (V' \subseteq V, E' \subseteq E, h')$ is defined such that for the mapping $h' : E' \to (V' \times V')$ the following holds:

$$h'(e') = (v'_i, v'_j) \iff (\Sigma(BF_G \cup F_G)\,(v'_i, v'_j)) = (\mathsf{fo}, 0, b_\delta) \text{ and } b_\delta \in \{fb_{IN}, nb\}.$$

The edges, $e' \in E'$, correspond to boundary segments of partition-forming fiat objects of non-barrier sort in direction (g_i, g_j). The vertices V' are the vertices joined by those edges. If Γ^L has disconnected components then there are places within the environment to which no path exists.

Consider Figure 5. The left part shows the path system of the parking lot discussed in Section 2. The long grey bar on the main road is the stretched vertex corresponding to the partition cell occupied by the main road. The bold solid lines represent edges corresponding to non-barrier boundary segments. The arrows along the edges show their direction. If there are edges for each direction then the arrows are omitted.

Consider the right part of Figure 5. So far we considered partition forming objects as *wholes*. In fact partition forming objects have parts which are caved out by fiat boundaries. Consider, for example, the part of the main road sharing boundary parts with the parking spots 1 and 6. Remember that the main road is a one-way street. Consequently, cars parking in parking spots 1 and 6 cannot leave the parking spot towards the entrance. There is a one-way fiat barrier boundary connecting the left side boundary of ps_1 and the opposite side boundary of ps_6. The same kind of one-way fiat barrier boundary exists for parking spots ps_2 and ps_7, ps_3 and ps_8 and so on. Taking (partition forming) parts of partition elements into account refines the underlying regional partition and the corresponding path system and results in a path system shown in the right part of Figure 5.



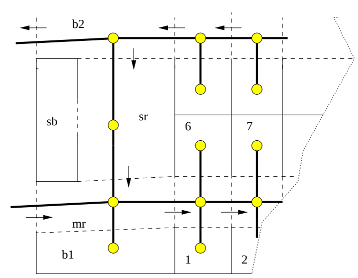


Figure 5. Path systems of a parking lot

5.3.3. Path system and movement

Let $r_t(o)$ be the exact region of the object o at a particular moment in time, t, let $r_T(o)$ be the set of all regions at which o was exactly located within the time interval $T = (t_1, t_2)$, i.e., $r_T(o) = \{r_t(o) \mid t_1 \leq$

$v_i \in V$, and partition cells, $g_i \in G$, will sometimes be used synonymously. Multiple disconnected boundary segments are represented by multiple separate edges.

[32]A subgraph of $\Gamma = (V, E, h)$ is a directed graph Γ' with $(V' \subseteq V, E' \subseteq E, h' : E' \to V' \times V')$.

$t \leq t_2\}$, and let $\bigvee r_T(o)$ be the sum of all those regions. Let $\Gamma^L = (V', E', h')$ be the path system of the layout L. A path within the path system from v_1 to v_2, $\Gamma^L_{v_1,v_2} = (V'', E'', h'')$, is a connected subgraph[33] of Γ^L beginning at v_1 and ending at v_2. This path is a path for the object o, $\Gamma^L_{v_1,v_2}(o)$, if and only if

1. $\underline{R}((\alpha_5 (\bigvee r_T(o))), (\alpha_5 \bigvee\{v_i \mid h''(e'') = (v_i, v_j), e'' \in E''\})) = NTPP$[34]
2. $h''(e'') = (v_i, v_j) \Rightarrow \underline{R}((\alpha_5 (\bigvee r_T(o))), (\alpha_5\, v_i)) = PO$

This ensures that $(\bigvee r_T(o))$ overlaps all partition cells along its path (2) and that it is a non-tangential proper part of the sum of those cells (1). This implies that constraint $F5$ is satisfied for o and the boundary segments shared by partition cells along this path.

5.3.4. Situations

A situation in a built environment is a triple $S =< L, BF_S, F_S >$, where L is the layout of the environment, BF_S is a set of non-partition forming bona-fide objects and F_S is a set of non-partition forming fiat objects. The members of both sets are populating L in situation S. In a situation S the following holds:

1. $\forall o_1, o_2 \in BF_S \cup BF_G :\ o_1 \neq o_2 \Rightarrow F1(o_1, o_2)$;
2. $\forall o_1, o_2 \in F_S :\ (\phi o_1$ and ϕo_2 and $o_1 \neq o_2) \Rightarrow\ F2(o_1, o_2)$;
3. $\forall o \in BF_S\ :\exists \gamma\ :\ \gamma = \Gamma^{L\cup\{EXT\}}_{r(EXT),r(o)}(o)$.

Axioms (1) and (2) govern the non-partition forming objects as discussed in Sections 2 and 5.2.

Consider axiom (3). The symbol EXT denotes the ‘The world exterior to the environment L’ and $\Gamma^{L\cup EXT}$ is the graph representing the path system of the environment L with its exterior EXT. Consequently, $\Gamma^{L\cup\{EXT\}}_{r(EXT),r(o)}(o)$ is a path for the object, o, from the exterior to its current location. Axiom (3) ensures that for each non-partition forming bona-fide object within the environment there exists a path along which this object could have been moved from the entrance to its current position without violation of $O1 - O3$. This is a formal version of the axiom M discussed in Section 2. Axiom (3) also excludes cases where big objects are being assembled within an environment.

5.4. Specific built environments

In Section 2 we discussed that domain specific constraints on relations involving objects of different kind are weaker than constraints involving objects of the same kind. They can be violated without violating the laws of logic or physics, i.e., *it is possible to violate those constraints*. On the other hand the built environment *must permit* the satisfaction of those constraints in order *to be* an environment of a given kind.

Consider a parking lot with layout $L = (G, BF_G, F_G)$ and the informal axiom $S3$, ‘cars need to fit into parking spots’, discussed in Section 2.2.1. Let $PSP \subset F_G$ be the set of parking spots and let $CARS \subset BF_S$ be the set of cars populating the parking lot. We postulate[35]:

$$\forall o_1 \in CARS, \forall o_2 \in PSP\ :\ \max\{\,\overline{R}((\text{loc}_{bs}\ o_1), (\text{loc}_{bs}\ o_2)) \mid \overline{R} \in RCC8\} = NTPP$$

[33]Every vertex $v' \in V'$ can be reached along directed edges. Consequently the graph $(\{sr, mr, ps_1\}, \{e'_1, e'_2, e'_3\}, h')$ with $h'(e'_1) = (sr, mr)$, $h'(e'_2) = (mr, ps_1)$, $h'(e'_3) = (ps_1, mr)$ is not a directed subgraph of Γ^L.

[34]Since the v_i refer to partition cells we have $\underline{R} = \overline{R}$. Consequently, we could also write $NTPP((\bigvee r_T(o)), (\bigvee\{v_i \mid h''(e'') = (v_i, v_j)\}))$.

[35]Since $r(ps_i) \in G$ we have $\overline{R} = \underline{R}$ and hence $\max\{R(o_1, o_2) \mid R \in RCC8\} = NTPP$.

This ensures that cars need to fit into parking spots. This is consistent with $\exists o \in CARS, \exists o_2 \in PSP : PO(r(o_1), r(o_2))$, i.e., when we postulate that a parking lot *must be* such that cars do *fit* into parking spots we do *not* rule out the possibility that there are cars parked across boundaries of parking spots. It ensures the possibility for cars to be parked in parking spots.

In axiom $S1$ in Section 2.2.1 we demanded that it must be possible for cars to avoid blocked areas. This is ensured by axiom 3 in Section 5.3.4. Let $BA \subset F_G$ the set of blocked areas of the parking lot. Given the barrier property of the boundary of blocked areas axiom 3 ensures that for each parking spot, $o_2 \in PSP$ there must exist a path for a car $o_1 \in CARS$, i.e., $\Gamma^{L \cup \{EXT\}}_{r(EXT), r(o_2)}(o_1) = (V, E, h)$, which keeps blocked areas clear, i.e., $\forall e \in E : h(e) = (v_i, v_j) \Rightarrow \neg \exists ba \in BA : r(ba) = v_i$. Again postulating $S1$-like axioms for an environment does not conflict with the fact that there may be cars that drive through or park in blocked areas.

6. Conclusions

This paper started with an ontological analysis of built environments. It was shown that boundaries are ontologically salient features of built environments and that there are bona-fide boundaries that correspond to discontinuities in the underlying reality and fiat boundaries that are the result of human demarcation. The distinction of bona-fide vs. fiat boundaries generalizes to the distinction of bona-fide and fiat objects. Another important characterization of boundaries is their barrier character. In this context bona-fide barrier, two-way fiat barrier, one-way fiat barrier and non-barrier boundaries were distinguished.

Built environments are *formed* by partition forming objects and *populated* by non-partition forming objects. The partition forming objects form the static layout of the environment. There is no no-man's land in the layout of a built environment and no double occupation. Partition forming objects may be of bona-fide and fiat sort. The partition structure is the basic organizational structure of a built environment. Within the layout formed by the partition forming objects non-partition forming objects are located. They may be of bona fide and fiat sort too.

Non-partition forming objects are potentially movable within the environment. Their movement is constrained by the barrier properties of the boundaries of other objects forming or populating the environment. This paper concentrated on the way partition forming objects constrain the movement of non-partition forming objects. It was shown that boundaries of partition forming objects induce paths along which non-partition forming objects can be moved. The paths are induced by the barrier and non-barrier properties of the boundaries of the partition forming objects. The barrier and non-barrier properties of boundaries are caused by constraints on relations that can hold among bona-fide objects, among fiat objects, and between bona-fide and fiat objects.

The notion of object-boundary sensitive rough location was proposed for the formalization of the ontological, that is the qualitative, structure of built environments. Rough location is based on the approximation of the exact location of spatial objects with respect to a regional partition in terms of relations between objects and partition cells. Remember the thought experiment in the introduction. Given that task 1 and 2' are to be performed by program (II), then there are three main arguments in favor of the formalization of build environments based on *approximate location within environments* in opposition to the formalization based on *exact location of objects*: Rough location focuses on the relationships between objects and their environments; Concentrating on properties of the environment and on object-environment-relations allows to abstract the different character of constraints on relations between the different kinds of objects forming and populating it; The notion of rough location is qualitative in nature.

Firstly. Rough location focuses on the approximate location of objects within regional partitions and implicitly takes the distinction between partition forming and and non-partition forming objects and the organizational structure of the regional partition into account. When we describe built environments in terms of rough location then objects are second class citizens. The first class citizens are mappings representing the rough location of objects within their environments, i.e., *object-environment-relations*. These mappings can be interpreted as equivalence classes of objects sharing the same rough location, i.e., classes of objects having the same relations to the cells of the underlying partition. Possible relations between objects can be computed given their rough location. Constraints on relations that can hold between objects can be expressed indirectly in terms of constraints on their rough locations. Since built environments are formed by finitely many partition forming objects there are only finitely many different rough locations within an environment.

Secondly. Concentrating on properties that need to satisfied by an environment and on constraints on object-environment-relations, allows to abstract from the different character of constraints on relations between spatial *objects*. The different character of the constraints on relations between objects is due to the fact that there are constraints that are based axioms rooted in human intuition and the laws of logic, there are constraints that are based on the laws of physics, and there are constraints that are based by human conventions. Axioms deeply rooted in human intuition and the laws of logic prohibit objects of ontological same kind and partition forming objects to overlap. Laws of physics prevent bona-fide objects from being connected. Constraints involving fiat objects of ontological different kind are based on social rules and agreement and may be violated in certain situations. An environment *must permit* the satisfaction of *all* constraints in order *to be* an environment of a given kind *independently* of the character of the constraints between the objects forming or populating it. This can be formulated quite naturally in terms of rough location.

Thirdly. We assumed a program (I) that generates potential plans for built environments based on standard algorithms of computational geometry. The output of (I) is quantitative and focuses on metric knowledge. The program (II) extracts the qualitative structure of the environment and builds a corresponding boundary sensitive rough location representation. We argued that the important question is to derive qualitative spatial relations *between ontologically salient features, which correspond to relations observable in reality* from this description and showed that this is exactly what happens when we describe built environments in terms of boundary sensitive rough locations of objects forming and populating them. The qualitative description allows us to capture the essence of what a built environment is. We showed that it is justified to claim that this representation (1) corresponds to relations observable in reality; (2) represents ontologically salient features; (3) this reflects the qualitative structure of the environment.

To summarize: The proposed formalization of built environments takes into account: (1) the fundamental role of boundaries, (2) the distinction between bona-fide and fiat boundaries and objects, (3) the different character of constraints on relations involving these different kinds of boundaries and objects, (4) the distinction between partition forming and non-partition forming objects within built environments, (5) the fundamental organizational structure of the regional partition formed by the partition forming objects, and (6) the importance of paths along which non-partition forming objects can move within a built environment.

It was shown that based on the notion of boundary sensitive rough location task (1) of program (II) can be performed, i.e., it is possible to decide whether or not a configuration of lines in the plane represents a built environment using the axioms given in Section 5. It was also shown how to derive paths within a build environments along which non-partition forming objects can move. This provides the formal foundations for task (2'), i.e., to evaluate those paths with respect to the complexity of the way finding task to be solved in order to navigate along them. Subject of ongoing research in this context is to apply the model for the evaluation of the complexity of wayfinding tasks proposed by [RE98].

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