On Ontology and Epistemology of Rough Location

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Abstract. Spatial objects are located in regions of space. In this paper the notions of exact, part, and rough location are discussed. Exact location is the relation between an object and the region of space it occupies. The notion of part location characterizes relations between parts of objects and parts of regions of space. The notion of rough location characterizes the location of a spatial object within a set of regions which form a regional partition of space. It links parts of spatial objects to parts of partition elements. The relationships between rough location, vague defined spatial objects, and indeterminacy of location are discussed.
Knowledge about location of spatial objects in physical reality is based on observation and measurement. This paper argues that the the observations and measurement of location in physical reality yield knowledge about rough location rather than knowledge about exact location. The underlying regional partitions are created by the observation and measurement processes.

KEYWORDS: Location, Ontology, Epistemology, Vagueness, Approximation, Spatial Information Systems

1 Introduction

A clear understanding of the ontological and epistemological status of location is a basic precondition for representing and processing spatial data in geographic information systems (GIS). An understanding of the ontological status of location, i.e., what location is, provides the basis for the sharing of geospatial information (Frank 1997). Understanding the epistemological status of location, i.e., what people do know about the location of objects, is the basis for representation of geospatial data, which is gained by means of observation and measurement.

This paper presents a discussion of the ontological and epistemological status of location. Ontological analysis of spatial location illustrates that distinguishing between exact, part, and rough location is necessary. Epistemological analysis of spatial location illustrates that for large classes of geographic objects only part and rough location can be known and represented.

This paper is structured as follows: It starts with a discussion of three aspects, which characterize spatial objects ontologically (section 2). In section 3 the notions of exact, part, and rough location are discussed. The epistemological status of location is discussed in section 4. In section 5 a summary and the conclusions are given.

2 Spatial Objects

In this paper we assume that the ontological characterization of geographic reality requires considering three basic aspects (Smith & Mark 1998):

1. the aspects of *what* spatial objects are,
2. the aspects of *where* spatial objects are, i.e., how spatial objects are located in geographic space, and
3. aspects of scale.

These aspects reflect the special character of the geographic domain. Geographic objects are not merely physical objects, but to a significant degree there are non-physical objects created by the human mind. Furthermore geographic objects objects are intrinsically interrelated together within a single domain (called space) (Smith & Mark 1998, Mark & Frank 1995). Consequently, the compositional, topological and geometrical organization of space has deep implications for the structure of geographic categories.

In this paper we concentrate on aspects of *where* objects are and how they are embedded into geographic space. We only shortly review important notions characterizing *what* spatial objects are and how they are characterized by aspects of scale. We refer to the relevant literature for an extended discussion.

2.1 The 'What'

An important aspect characterizing *what* objects are is their compositional structure. The compositional structure of an object is characterized by the relationships between the *whole* object and the different *parts* comprising the object. Formally the compositional structure of spatial objects is characterized by constructing a relation $P(x, y)$ that connects parts of an object to the whole object. This relation is termed the *part-whole* relation (Lesniewski 1929), where P is the relation, x is the part, and y is the whole. Mereology (Leonard & Goodman 1940, Simons 1987) is the formal theory used in this context.

An extensive discussion of *what* spatial objects are and how they are made up of parts can be found in (Casati & Varzi 1994, Casati & Varzi 1997). In this paper we assume spatial wholes of homogeneous compositional structure, e.g., (Gerstl & Pribbenow 1995). Those wholes may be decomposed into parts arbitrarily. Geographic fields, forests, oceans are examples of wholes with homogeneous compositional structure from the geographic domain.

2.2 Aspects of Scale

Aspects of scale refer to the classification of spatial objects with respect to size relative to observability and modes of observation by human beings. There is a whole class of literature that deals with different classifications in this respect. An overview can be found in a paper by Freundschuh & Egenhofer (1997). Spatial objects of geographic scale, considered in this paper, are larger than the human body and cannot be perceived within a single perceptual act.

2.3 The 'Where'

Aspects of *where* objects are, and relations between *what* and *where* they are, have been investigated in spatial sciences for centuries. Surveyors dealt with the question of where spatial objects are located on Earth (Moffitt & Bouchard 1987). Geography dealt with the relationships between *what* objects are and *where* they are located, and with the distribution of spatial objects on Earth (Abler, Marcus & Olson 1992). Different notions were used to characterize *where* objects are. Besides the notion of location, for example, the notion place plays an important role in geography (Couclelis 1992) and architecture (Lynch 1960).

In the remainder of this paper we define different notion of location to characterize *where* objects are. We define location on an abstract and formal level in terms of relations between spatial objects and regions of space. The following assumptions about the structure of geographic reality are made (Smith & Mark 1998, Casati & Varzi 1995):

- The geographic world is populated by spatial objects of geographic scale.
- Spatial objects are things that are located in some region of space[1].
- Location can be characterized by relations between things that exist and one or more regions in which they are located.
- Spatial objects and regions of space have a homogeneous compositional structure.

3 Location

In this section we discuss relations between geographic objects and regions of space. We start reviewing the notions of exact and part location introduced by Casati & Varzi (1995). Those notions relate the compositional structure of spatial objects to the compositional structure of spatial regions. In this paper the notion of part location is refined and extended, and a classification of the relations characterizing part location is proposed. Based on the notion of part location, the notion of rough location is introduced.

As a running example the location of National Parks in partitions formed by the Federal States of the United States[2] is used to illustrate the abstract notions of part and rough location.

[1] Non-spatial kinds are not located in some region. Examples are numbers.

[2] In the context of this paper the Federal States of the United States of America are assumed to form a regional partition of the US.

3.1 Exact Location

Exact location is a binary relation between spatial objects and regions of space. Exact location, $L(x, y)$, "is a relation whose second term, y, is always a region in space ... the first term of the location relation, x, can be whatever sort of entity you have in your spatial ontology - spatial regions included ..." (Casati & Varzi 1995, p.208). Exact location of a spatial object is the region of space taken up by the object. For example, "John ... is exactly located in the space 'carved out' of the air, or of whatever medium he might be in (water if he is swimming ...)" (Casati & Varzi 1995, p. 280).

A single object cannot be exactly located in different regions in the same point in time. In the remainder of this paper the phrase 'the exact region of x' is used to refer in natural language to the region in which the spatial object x is exactly located. On the formal level we use the notion $r(x)$ in order to refer to the exact region of x. Spatial objects may be exactly located in different regions at different points in time, e.g., (Medak 1999).

The exact region of a spatial object may be a simple region of three dimensional space, for example, think of your body and the region of space it carves out of the air. The exact region of a spatial object may be a complex region, consisting of multiple regions of three dimensional space, as in the case of the exact region of the Hawaiian islands. The exact region may be a complex region, consisting of multiple regions of two dimensional space, as in the case of the representation of the Hawaiian islands on a paper map.

3.2 Part Location

Above the notion of exact location was characterized by the relation between spatial objects and the region of space in which the *whole* object is exactly located. In this section we will discuss the notion of part location and sets of relations between *parts* of spatial objects and *parts* of regions of space.

Part Location Relations. The notion of part location links parts of spatial objects to parts of spatial regions. There is a single relation characterizing exact location, i.e., there is a single region of space in which an object can be exactly located at each moment in time. There are multiple relations characterizing part location. This means that there are multiple ways in which parts of objects relate to parts of regions of space.

Consider figure 1. Grand Teton National Park, GTP, is exactly located in the region it carves out of the surface of Earth. Parts of Yellowstone National Park, YNP, are located in parts of the region of Wyoming, WY. Yellowstone National Park is partly located in Wyoming, Montana, MO, and Idaho, ID.

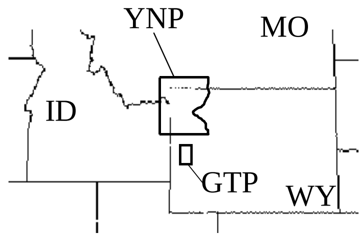

Fig. 1. Location of Yellowstone and Grand Teton National Park within the Political Subdivision of the US

Casati & Varzi (1995) introduce several predicates[3] characterizing part location. Examples are the notions of *wholly located* (WL), *partly located*[4] (PL), and *generically located* (GL):

$$WL(x,y) =_{def} \exists z(P(z,y) \wedge L(x,z))$$
$$PL(x,y) =_{def} \exists z(P(z,x) \wedge L(z,y))$$
$$GL(x,y) =_{def} \exists z \exists w(P(z,x) \wedge P(w,y) \wedge L(z,w))$$

Grand Teton National Park, GTP, is *wholly located* in the region of Wyoming, $r(WY)$, because it is exactly located in a region, which is a part of the region in which Wyoming is located, i.e., $WL(GTP, r(WY))$. Wyoming is *partly located* in the region of Grand Teton National Park because a part of Wyoming is exactly located at the region in which Grand Teton is exactly located, i.e., $PL(WY, r(GTP))$. Yellowstone National Park, YNP, is *generically located* in the region of Wyoming because there is a part of Yellowstone that is located in a part of the regions in which Wyoming is located, i.e., $GL(YNP, r(WY))$. Yellowstone is *generically located* at Montana MO, and Idaho, ID, as well, i.e., $GL(YNP, r(MO))$ and $GL(YNP, r(ID))$.

Properties of Part Location. As discussed above the notion of part location links the compositional structure of spatial objects to the compositional structure of spatial regions. The definitions and axiomatization of part location, given by Casati & Varzi (1995) make the ontological distinction between exact and part location explicit: Part location refers to *parts* of wholes, i.e.,

- *parts of* objects being located in a certain (part of a) region, or
- *parts of* regions in which (parts of) spatial objects are located.

Exact location refers to a relation between *wholes*, i.e., the sum of *all* parts of a spatial object which is exactly located in a regional whole.

[3] In this paper we use predicates and the relations they denote synonymously.

[4] The notion of part location is characterized a certain class of relations. The predicate $PL(x,y)$, 'x is partly located in y', refers to a specific relation belonging to this class of relations.

There is a *single* relation characterizing exact location. There are *multiple* relations characterizing part location relations since:

1. Spatial objects and spatial regions consist of different kinds of parts, e.g., object or parts vs. boundary parts (Smith 1997). Relations characterizing part location can be defined, for example, by taking boundary parts into account or ignoring them. This results in boundary sensitive and boundary insensitive relations. In this paper we discuss boundary insensitive relations. Boundary sensitive relations were discussed by Bittner & Stell (1998) or Bittner (1999).
2. There are multiple ways how (object) parts of spatial objects can be related to regional parts of spatial regions. Above we distinguished, for example, the relations $WL(x,y)$,$PL(x,y)$, and $GL(x,y)$.

Partitioning Sets of Part Location Relations. The definitions of the part location predicates *wholly located*, $WL(x,y)$, *partly located*, $PL(x,y)$, and *generically located*, $GL(x,y)$, by Casati & Varzi (1995) have one shortcoming: Taken as a set, they are not jointly exhaustive and pair-wise disjoint (JEPD). A set of binary predicates is JEPD (Randell, Cui & Cohn 1992) if and only if for all pairs of objects for which the predicates are defined, one and only one predicate in the set holds. Sets of jointly exhaustive and pair-wise disjoint binary predicates *partition* the domain of pairs of objects.

Jointly exhaustive and pair-wise disjoint sets of relations are particularly important for the formalization of rough location. They provide the basis for the formalization of rough location by means of rough sets (Pawlak 1982) and location mappings (Bittner & Stell 1998, Bittner 1999). Based on the definitions of Casati & Varzi (1995) the following sets of JEPD part location predicates can be (trivially) defined:

Name	Intended Meaning	Relation Set
Contained Sensitive	the region of x is either a part of y or not	$\{WL(x,y), \neg WL(x,y)\}$
Containment Sensitive	y is either a part of the region of x or not	$\{PL(x,y), \neg PL(x,y)\}$
Overlap Sensitive	the region of x either overlaps y or not	$\{GL(x,y), \neg GL(x,y)\}$

Consider the set of part location predicates with three elements, $\{FL(x,y), OL(x,y), NL(x,y)\}$, defined as follows:

$$FL(x,y) =_{def} PL(x,y)$$
$$OL(x,y) =_{def} GL(x,y) \wedge \neg PL(x,y)$$
$$NL(x,y) =_{def} \neg GL(x,y)$$

They are obviously JEPD. The intended meaning of the predicates FL (**f***ully* **l***ocated*), OL (**o***verlap* **l***ocated*), and NL (**n***ot* **l***ocated*) is the following:

Fully located, $FL(x, y)$, means that x is completely located in y in the sense that y is a part of the region in which x is exactly located. There are *no parts* of y that are not parts of the region of x. Consider the US state of Wyoming, WY, and Grand Teton National Park, GTP, (figure 1). Wyoming is fully located on the region of Grand Teton Park, i.e., $FL(WY, r(GTP))$.

Overlap located, $OL(x, y)$, means that x is located in *parts* of y. There are parts of x that are not located in parts of y and there are parts of y in which no parts of x are located. Consider Wyoming, WY, and Yellowstone National Park, YNP, (figure 1). Wyoming is overlap located in the region of Yellowstone, i.e., $OL(WY, r(YNP))$, and Yellowstone is overlap located in the region of Wyoming, i.e., $OL(YNP, r(WY))$.

Not located, $NL(x, y)$, means that x is not located in y in the sense that there are no parts of x located at parts of y. Consider for example, Yellowstone Park, YNP and the US state of California, CA, i.e., $NL(YNP, r(CA))$ (figure 1).

The relations corresponding to the set $\{WL(x, y), PL(x, y), GL(x, y)\}$ of JEPD predicates characterize *overlap & containment sensitive* part location.

3.3 Rough Location

In this sub-section the notion of rough location is introduced. The rough location of a spatial object is characterized by the part location of spatial objects with respect to a set of regions of space that form regional partitions. Consequently, the notion rough location links parts of spatial objects to parts of partition regions.

Patterns of Part Location Relations. Consider two different kinds of geographic objects: Federal states, whose exact regions form a regional partition, and National Parks whose exact regions do not form a regional partition. National parks are located in parts of regions of Federal States. There are National Parks, which are located in parts of exact regions of multiple Federal States, e.g., Yellowstone National Park as shown in figure 1. Parts of Yellowstone are located in parts of the regions of Wyoming, Idaho, and Montana.

The rough location of the spatial object, o, within the set of regions forming the regional partition, G, is characterized by an n-tuple of relations. Those relations characterize the part location of the *single* spatial object, o, with respect to *all* elements, g, of the regional partition, $g \in G$. We say that the rough location of the object, o, is characterized by pattern of part location.

The n-tuples of relations are composed of the JEPD sets of relations discussed above. Depending on which set was chosen we distinguish containment sensitive, overlap sensitive, and overlap & containment sensitive rough location.

Rough location can be expressed in terms of logic as a conjunction of disjunctions of statements about part location (Bittner 1997). We use the notion $LOC_G(o)$ to refer to the rough location of the spatial object, o, within the regional partition, G.

Overlap Sensitive Rough Location. Let o be a spatial object, and let $\{GL, \neg GL\}$ be a set of JEPD predicates characterizing overlap sensitive part location. The overlap sensitive rough location of the object, o, within the regional partition, G, is characterized by the formula:

$$\begin{aligned} \mathrm{LOC}(o)_G =_{def} & \exists g_1, \ldots g_n \ \mathrm{isPartition}(g_1, \ldots, g_n) \\ & \bigwedge_{i=1}^{n} (GL(o, g_i) \vee \neg GL(o, g_i) \end{aligned}$$

The predicate 'isPartition' is true if the regions $g_1, \ldots, g_n$ form a regional partition[5].

Consider figure 1. The overlap sensitive rough location of Yellowstone Park can be represented as a conjunction of disjunctions of statements about overlap sensitive part location as discussed above. These are statements about the relations between Yellowstone Park and the regions of the Federal State partition, FSP. Due to the JEPD property of sets of those relations, between Yellowstone Park, YNP, and the exact region of every Federal State one and only one predicate can hold. Consequently, it is sufficient to list only the conjunctions of location predicates that do hold for the spatial object and each of the partition regions. This simplifies the formal representation:

$$\begin{aligned} \mathrm{LOC}(YP)_{FSP} = & GL(YNP, r(MO)) \wedge GL(YNP, r(ID)) GL(YNP, r(WY) \\ & \wedge \neg GL(YNP, r(SD)) \wedge \neg GL(YNP, r(ND)) \wedge \ldots \end{aligned}$$

Overlap & Containment Sensitive Rough Location. Let $\{NL,$
$OL, FL\}$ be a set of JEPD predicates characterizing overlap & containment sensitive part location. The overlap & containment sensitive rough location of the object, o, within the regional partition, G, is characterized by the formula:

$$\begin{aligned} \mathrm{LOC}(o)_G =_{def} & \exists g_1, \ldots g_n \ \mathrm{isPartition}(g_1, \ldots, g_n) \\ & \bigwedge_{i=1}^{n} (NL(o, g_i) \vee OL(o, g_i) \vee FL(o, g_i)) \end{aligned}$$

Consider figure 1. The overlap & containment sensitive rough location of Yellowstone is characterized by conjunctions of statements about overlap & containment sensitive part location between Yellowstone and the regions of the Federal State partition, FSP:

$$\begin{aligned} \mathrm{LOC}(YP)_{FSP} = & OL(YNP, r(MO)) \wedge OL(YNP, r(ID)) \wedge OL(YNP, r(WY) \\ & \wedge NL(YNP, r(SD)) \wedge NL(YNP, r(ND)) \wedge \ldots \end{aligned}$$

[5] For an exact definition see for example (Bittner 1999)

3.4 Rough Location, Vagueness, and Indeterminacy of Location

Spatial objects like valleys and mountains are only vaguely defined. Consider for example Mount Blanc. The boundary between rock and air is determinate and observable but where is the boundary of Mount Blanc among its foothills? (Smith & Mark 1998) The vagueness the definition of those objects causes indeterminacy of their location. Indeterminacy of location means that there are multiple candidates of exact-location-regions which are consistent with the object's definition (Cohn & Gotts 1996). Indeed the definitions are so vague that the location of the boundaries cannot be determined. Within certain limits there are arbitrary choices of boundary location (and hence exact-location-regions) possible.

The notion of rough location can be used to deal with the indeterminacy of location caused by vagueness of object definitions. Consider figure 2. A vaguely defined object, o, is located within a regional partition consisting of the three concentric regions '*core*', '*wide boundary*', and '*exterior*'. The partition is chosen such that the object's rough location is not effected by its location indeterminacy. This means for all exact-region-candidates, which are consistent with the object's vague definition, the rough location is:

$$\mathrm{LOC}(o) = FL(o, core) \wedge PL(o, wide\ boundary) \wedge NL(o, exterior)$$

In this context the notion of rough location within a partition consisting of the three concentric regions coincides with the notion of vague regions introduced by Cohn & Gotts (1996).

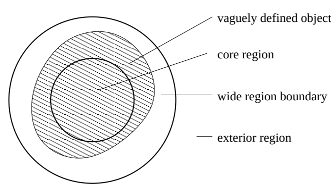

Fig. 2. Location of a vaguely defined object in a regional partition consisting of a core region, a wide boundary region, and an exterior region

4 Epistemology

The application of the notions of exact, part, and rough location to geographic information systems has another aspect: GIS represent and process *human knowledge* about spatial objects and their location resulting from observations and measurement in physical reality. This section considers the question: *What can humans know* about the location of spatial objects. This means that we discuss the Epistemology of location.

If we analyze the previous sections then the following observation can be made: We always referred to regions of space indirectly via the objects that are exactly located in them, e.g., the exact region of Yellowstone Park, or the exact region of Wyoming. In the context of Epistemology the following conclusions can be drawn from this observation: Humans do have knowledge about spatial objects and their identity. They assume that there *exists* a region in which a spatial object is located exactly. From this observation we can *not* conclude that humans *know* exact location beyond its existence. In the remainder of this paper we discuss what humans can know about exact location from observation and measurement.

4.1 Empirical Knowledge

Empirical knowledge is gained by the *observation* of objects in the world and *reasoning* about those observations (Carnap 1966). This paper only discusses the concepts of observation and measurement as a specific form of observation. Reasoning about empirical knowledge gained by those techniques is not discussed. For a discussion about reasoning about empirical knowledge see, for example, Carnap (1966).

Observation. For an object to be described using the notion of rough location, there must exist regional partitions of space and sets of JEPD relations linking the object and the partition regions. Knowledge about rough location is based on knowledge about those regional partitions and the observation of the relations between the object and the partition regions. People are aware of a large number of regional partitions of geographic space and know how rough location in these partitions can be observed. For example:

- Humans have knowledge about rough location in regional partitions created by human body axes. For example, an observer has knowledge about the rough location of spatial objects in the regional partition created by the half-planes in front and behind his body, or the half-planes left and right of his body. Different kinds of partitions created by human body axes were discussed, for example, by Hernandez (1994).
- Humans have knowledge about the rough location in regional partitions created by reference objects. For example, they have knowledge about rough location of spatial objects in the regional partition created by the half-plane in front of the city hall and its complement.
- Humans have knowledge about rough location in political subdivisions, e.g. knowledge about the rough location of Yellowstone Park in the political subdivision of the US (figure 1). Another example is the knowledge about rough location of cities like San Diego and Reno in the Federal States subdivision of the US as discussed by Stevens & Coupe (1978).

Knowledge about rough location in regional partitions observable in the world seems to be fundamental for human cognition, e.g., (Smith 1985, Herskovits 1986). Representation of knowledge about rough location in specific regional partitions is encoded into the core of our language system (Talmy 1983, Herskovits 1986).

In the remainder of this paper we concentrate on empirical techniques which are the source of quantitative and scientific knowledge about location of physical objects in space rather than on the sources of common-sense and qualitative knowledge.

Measurement. Measurement is a precise technique of observation. Measurement allows the transformation of observations made in nature to the abstract domain of numbers. We must have fixed rules and well defined procedures to transform the facts of nature to quantitative concepts. Indeed measurement requires sets of rules for transforming objects and relationships between them, observed in physical reality, to the domain of numbers (Carnap 1966). In the next section we discuss transformation rules specific for measurement of spatial extension and location.

4.2 Measurement of Extension

Measurement of extension has two basic components:

- A regional partition of the underlying space, and
- a rule of addition for counting the partition elements in which parts of the object to be measured are located.

The regional partition is created by the underlying measuring process. For example, in the case of time measurement, the time line is partitioned into adjacent time intervals by clock-ticks. Clock-ticks are temporal objects. Taken as a set, their exact regions form a regional partition of the time line. Geo-referencing pixels of remote sensed images creates a raster-shaped partition of the surface of Earth.

The ideal case would be to have the measured objects exactly located in a region formed by the union of a number of partition regions. Specifying the exact location of an arbitrarily shaped spatial object as the union of a number of partition elements, requires that partition elements are themselves divisible into partitioning parts. The subdivision stops if any arrangement of partition regions is identical to the exact region of the object to be measured. Thus in order to measure the exact location of arbitrarily shaped objects we need to allow for a potentially infinite number of subdivisions.

The main problem with defining measurement by exact location and the potentially infinite subdivision of partition regions is that measurement itself is based on observations. Observation is based on finite resolution. Infinite subdivisibility is an abstract concept and not observable.

The physical act of measurement is always governed by the limits of the observations. Consequently, the act of measuring something always results in finite resolution. The physical act of measurement is based on operations on units of maximal resolution. The act measurement of extension is determined by counting the partition elements

– in which parts of the measured object are located, or
– which are parts of the measured object's exact region.

Consider the left part of figure 3. The regional partition, G, created by a measuring process (e.g., remote sensing), is a regular raster. Partition regions at the maximal level of resolution might have an assumed[6] extension of $1m$ in each dimension. Partition regions are labeled by strings of capital letters. Assume overlap & containment sensitive rough location. Let the upper approximation set, $U(o)_G$, be the set of all partition elements, $g \in G$, in which parts of the spatial object, o, are located and let the lower approximation set, $L(o)_G$, be the set of all partition elements, which are parts of the exact region of the object o[7]:

$$
\begin{aligned}
L(o)_G &=_{def} \{g \in G \mid FL(o,g)\} \\
&= \{CH, CI, CK, DH, DK, EK\} \\
U(o)_G &=_{def} \{g \in G \mid OL(o,g) \vee FL(o,g)\} \\
&= \{CH, CI, CK, DH, DK, EK, BG, CG, DG, EG, EH, DI, DJ, \\
&\quad EJ, FJ, FK, FL, EL, DL, CL, BL, AK, BK, BJ, CJ, AJ, BI, BH\}
\end{aligned}
$$

The cardinality of the lower approximation set is 6, i.e., $\#L(o) = 6$. The cardinality of the upper approximation set is 28, i.e., $\#L(o) = 28$. The spatial extension of the spatial object satisfies the constraint: $6m^2 < ext(o) < 28m^2$, i.e., is between $6m^2$ and $28m^2$.

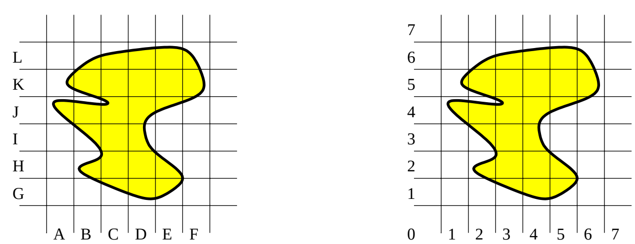

Fig. 3. Measurement of Extension (left) and Location (right)

4.3 Measurement of Location

For the measurement of extension relative magnitudes, based on *counting* partition elements, are sufficient. In order to measure location we have to extend the rules for measuring extension by a rule for *labeling* partition elements by tuples of natural numbers. This results in absolute magnitudes if the following conditions are satisfied:

[6] See Carnap (1966) for a discussion about the justification of those assumptions.

[7] The notions of lower and upper approximation sets come from rough set theory (Pawlak 1982) and were applied to the spatial domain for example by Worboys (1998) and Bittner (1999).

- Let m be the dimension of the embedding space and let CV_i be coordinate values ranging over natural numbers. Partition elements, $g \in G$, can be uniquely labeled by m-tuples of coordinate values using the labeling mapping, λ, of signature:

$$\lambda : G \to (CV_1 \times \ldots \times CV_m).$$

- There is a distance function (a binary function that satisfies the distance axioms, e.g., (Jaenich 1994)), with the signature

$$dist : (CV_1 \times \ldots \times CV_m) \times (CV_1 \times \ldots \times CV_m) \to CV,$$

  defined for pairs of coordinates. The labeling function, λ, and the distance function, $dist$, need to be defined such that the metric in the created coordinate space preserves the neighborhood of partition elements. This means that coordinates of neighboring partition elements need to have minimal distance.
  Let the set $NR_G \subseteq G \times G$ be set the pairs of distinct elements of the underlying regional partition, G, which share a boundary segment[8]. The labeling mapping λ needs to satisfy the implication:

$$\forall (g_i, g_j) \in NR_G \Rightarrow \quad (dist(\lambda(g_i), \lambda(g_j)) = \min\{dist(\lambda(g_k), \lambda(g_l)) \mid g_k, g_l \in G,\ k \neq l\})$$

Consider the right part of figure 3. The mapping λ is given by assigning pairs of numbers to partition regions (rather than strings of capital letters as in the extension case). The distance function is $dist((x_1, y_1), (x_2, y_2)) \;=_{def}\; |x_1 - x_2| + |y_1 - y_2|$. The resulting coordinate representation of the lower approximation set of the object o is:

$$L(o)_G = \{(3,2), (3,3), (3,5), (4,2), (4,5), (5,5)\}$$

Consequently, coordinate representations of rough location are based on lower and upper approximation sets containing absolute magnitudes. Absolute magnitudes are created by the labeling mapping, λ, which transforms partition regions to the domain of m-tuples of numbers.

4.4 Knowledge from Measurement

We cannot measure nor observe exact location. We observe objects and relations between them. We assume that spatial objects are exactly located in a region of space at each moment of time. The rules of measurement are such that the exact-regions of unit-objects, like clock-ticks or geo-referenced pixels of remote sensed images, create a regional partition of the underlying temporal or spatial

[8] A boundary segment is a one-dimensional part of the boundary of a two-dimensional region of space.

domain. During the measurement process spatial relations between these partition forming unit-objects and the object being measured are being observed. These observations allow us to count unit objects for which certain relationships hold. Counting of unit-objects transforms to counting of partition elements and provides the transformation to the domain of relative magnitudes. Transforming partition elements to a coordinate space, which metric preserves the partition structure, provides the mapping into the domain of absolute magnitudes.

Exact measurement provides:

- a regular partition of space,
- means of observation of rough location within this partition, and
- a transformation to the domain of numbers.

Exact Measurement of location yields *quantitative representations of rough location* of spatial objects within regional partitions created by the measuring process. Often only the upper or the lower approximation or a mixture of both are represented.

Epistemologically, rough location resulting from observation and measurement can be seen as an *approximation* of exact location in terms of partition regions. The finer the underlying partition the better the approximation. This approximation view of rough location was discussed by Worboys (1998). It needs to be separated from the discussion about the relationships between rough location and the indeterminacy of location caused by vagueness of object definitions.

5 Conclusions

The geographic world is populated by spatial objects that are located in regions of space. In this paper several notions of location were discussed which characterize the relationships between spatial objects and regions of space. Notions of locations describe the relationships between the compositional structure of spatial objects and the compositional structure of regions of space.

Exact location is characterized by a unique relation between spatial wholes and regional wholes. Part location links parts of objects to parts of regions of space. Multiple jointly exhaustive and pair-wide disjoint sets relations characterizing different types of part location were discussed.

Rough location refers to the location of spatial objects within sets of regions that form regional partitions of space. Rough location links parts of spatial objects to parts of partition regions. The rough location of a spatial object within a regional partition is characterized by an n-tuple of relations. Each of those relations characterizes the part location between the spatial object and a single element of the underlying regional partition.

This paper discussed empirical methods of observation and measurement of location in physical reality. We argued that the observation and measurement of location in physical reality yields knowledge about rough location.

Measurement as a method of gaining quantitative knowledge about empirical facts about the world is based on the observation of rough location in the regional partition created by the underlying measuring process. The transformation to the domain of numbers, i.e., the quantitative representation of empirical facts of the world, is based on two concepts: Counting of partition elements with certain part location relations, and representation of rough location in coordinate spaces of numbers. Counting yields relative magnitudes transformation into coordinate representation yields absolute magnitudes.

So far, the notion of rough location in regional partitions was discussed in the context of representation of spatial reality in geographic information systems. The notion of rough location in regional partitions seems to play an important role in human cognition in general and should be investigated from a much broader point of view.

From a cognitive point of view, rough location can be seen as a way of understanding objects as wholes by considering how their parts relate to a frame of reference with a known structure. The frame of reference is given by the underlying regional partition. Spatial objects which structure and geometry as wholes people do not understand can at least understood partly. In this way rough location seems to be used by geographers often to understand spatial phenomena continuously distributed on Earth, i.e., geographic fields, or vaguely defined objects.

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