A classification of spatio-temporal entities based on their location in space-time

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Abstract. We present an axiomatic theory of spatio-temporal entities based on the primitives *spatial-region*, *part-of*, and *is-an-instance-of*. We provide a classification of spatio-temporal entities according to the number and kinds of regions at which they are located in spacetime and according to whether they instantiate or are instantiated at those regions. The focus on location and instantiation at a location as the central notions of this theory makes it particularly appropriate for serving as a foundational ontology for geography and geographic information science.

1 Introduction

In geographic information science, there is a need for formal ontologies which provide semantic foundations for the terminology used in scientific theories as well as in data standards, data sets, and geographic information systems [2, 1, 7, 4]. These formal ontologies should specify the semantics of for terminology that enables the user to describe how geographic objects persist through time, change over time, and instantiate geographic categories at certain locations in space and time.

In this paper, we present an axiomatic theory which is based upon a mereology [11] of spatio-temporal regions, a distinction between (3D) spatial regions and (4D) temporal regions, and an instantiation relation holding between a particular entity, a category (or *universal*), and a spatio-temporal region where the particular instantiates the universal. We distinguish spatio-temporal entities that instantiate at the regions at which they are located (particulars) from entities that are instantiated at the regions at which they are located (universals). For example, I am a particular, an instance of the universal human being wherever I am located. My life is a particular which instantiates the universal human life at the spacetime region it occupies.

Particulars are further distinguished according to the number (a single region vs. multiple regions) and the kinds of regions (spatial regions vs. temporal regions) at which they are located. *Endurants* (objects like you, your car, planet Earth, etc.) are located at multiple spatial regions (different 3D regions at different times). *Perdurants* (processes like your life, global warming, the blood flow in my body, etc.) are located at unique temporal (4D) regions. *Stages* are located at unique spatial regions and are instantaneous parts of perdurants [10].

Universals are distinguished into universals that are instantiated by endurants, universals that are instantiated by perdurants, and universals that are instantiated by stages. An overview of the basic categories is given in Figure 1. (See also [11] and [9].)

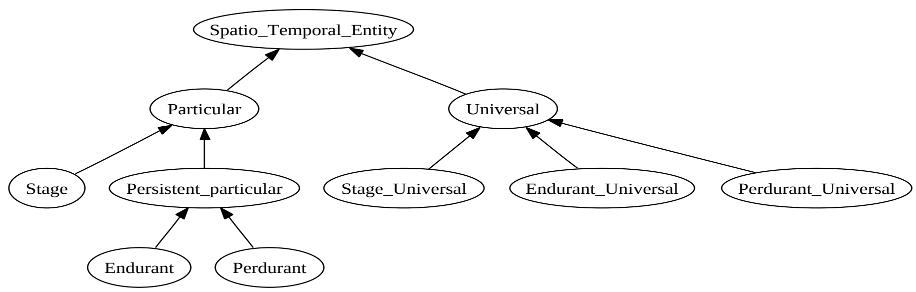

Fig. 1. A classification of spatio-temporal entities with respect to their location in space-time.

An important feature of our theory is that it describes time-dependent properties and relations (e.g. instantiation of a given universal) in terms of location in spacetime. This makes our theory particularly appropriate as an ontological foundation for geography, geographic information science, and spatio-temporal information processing. (See [5] who discuss why a formal ontology for geography and GIScience should be based on the notion of location.) The theory presented in this paper is an extension of the theory of endurants and perdurants developed in our [3].

2 Example

Consider Figure 2. Instead of considering a four-dimensional model of spacetime, we use the subset of points of the plane which is specified by the coordinates t and s that satisfy the constraint $0 \leq t \leq t_4$ & $0 \leq s \leq 4$. In set-theoretic terms we write $\mathbf{ST} = \{(s,t) \mid 0 \leq t \leq t_4 \; \& \; 0 \leq s \leq 4\}$. The horizontal dimension in the figure is interpreted as temporal and the vertical dimension is interpreted as spatial.

The left part of Figure 2 shows an endurant, the line-shaped entity A, at times t_1, t_2, and t_3. The life of the endurant A is visualized as the solid two-dimensional region, *LifeOf_A*, depicted in the right part of the figure. It shows that A comes into existence at t_1 and that it continues to exist until t_4. The lives of C, B and D are proper parts of the life of A and are respectively located at the spacetime regions $loc\_lf\_C = \{(s,t) \mid t_1 \leq t \leq t_4 \; \& \; 1 \leq s \leq 2\}$, $loc\_lf\_B =$

$\{(s,t) \mid t_1 \leq t \leq t_5 \ \& \ 2 \leq s \leq 3\}$, and *loc_lf_D* $= \{(s,t) \mid t_6 \leq t_4 \ \& \ 2 \leq s \leq 3\}$ shown in the right part of Figure 2. The life of A, *LifeOf_A*, is located at the region *loc_lf_A*, which is the union of the regions *loc_lf_B*, *loc_lf_C*, and *loc_lf_D*. We also include in our model the following stages of the lives of the endurants A, C, B and D: A^{t_1}, A^{t_2}, A^{t_3}, C^{t_1}, C^{t_2}, C^{t_3}, B^{t_1}, and D^{t_3}. For example, A^{t_1} is the instantaneous slice of A's life at t_1, A^{t_2} is the instantaneous slice of A's life at t_2, and so on.

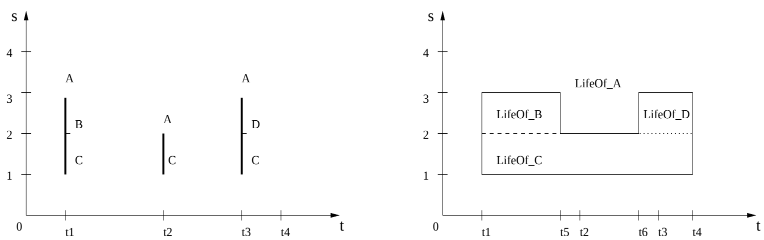

Fig. 2. The endurant A in different time-slices (left) and the life of A (right).

At a given moment during its life an endurant is exactly co-located with the stage of its life at that moment. For example, the location of A at t_1 is the location of the stage A^{t_1}: the region *loc_A_t*$_1 = \{(s,t) \mid t = t_1 \ \& \ 1 \leq s \leq 3\}$. The stages C^{t_1} and B^{t_1} are located at the regions *loc_C_t*$_1 = \{(s,t) \mid t = t_1 \ \& \ 1 \leq s \leq 2\}$ and *loc_B_t*$_1 = \{(s,t) \mid t = t_1 \ \& \ 2 \leq s \leq 3\}$. The stages A^{t_2} and C^{t_2} are both located at the region *loc_A_t*$_2 =$ *loc_C_t*$_2 = \{(s,t) \mid t = t_2 \ \& \ 1 \leq s \leq 2)$. And so on.

At every region at which endurant A is located, A instantiates the universal LINE(-segment). For example A instantiates the universal LINE at *loc_A_t*$_1$, *loc_A_t*$_2$, etc. Similarly, endurant C instantiates LINE at *loc_C_t*$_1$, *loc_C_t*$_2$, etc. The universal LINE is located wherever it is instantiated by one of its instances. Thus LINE is located at *loc_A_t*$_1$, *loc_A_t*$_2$, *loc_C_t*$_1$, *loc_C_t*$_2$, etc. Similarly, the universal LIFE-OF-A-LINE is instantiated at regions *loc_lf_A*, *loc_lf_B*, *loc_lf_C*, and *loc_lf_D*by the perdurants *LifeOf_A*, *LifeOf_B*, *LifeOf_C*, and *LifeOf_D* respectively. Hence, LIFE-OF-A-LINE is located at all those regions. Consider the universal (momentary) STAGE-IN-THE-LIFE-OF-A-LINE. This universal is instantiated for example by A^{t_1}, A^{t_2}, A^{t_3}, C^{t_1}, C^{t_2}, C^{t_3}, B^{t_1}, and D^{t_3} at their respective locations.

3 Space-time regions

In this section we briefly review our formal theory of space-time regions which was originally developed in [3]. We present the formal theory in a sorted first-order predicate logic with identity. All quantification is restricted to a single sort. Restrictions on quantification will be understood by conventions on variable usage. We use u, v, and w as variables ranging over regions and (in the second part of the paper) x, y, and z as variables ranging over entities.

Regional parthood. We start by introducing the binary predicate P, where $P\ uv$ is interpreted as 'the region u is a part of the region v'. We introduce the binary predicates PP for proper parthood (D_{PP}) and O for overlap (D_O).

$$D_{PP}\ \ PP\ uv \equiv P\ uv \wedge u \neq v \qquad D_O\ \ O\ uv \equiv (\exists w)(P\ wu \wedge P\ wv)$$

We require: P antisymmetric (AR2); P is transitive (AR3); if everything that overlaps u also overlaps v then u is a part of v (AR4); P is reflexive(AR1); and there exists a region which has all regions as parts (AR5).

$$AR2\ \ P\ uv \wedge P\ vu \rightarrow u = v$$
$$AR3\ \ P\ uv \wedge P\ vw \rightarrow P\ uw$$
$$AR4\ \ (w)(O\ wu \rightarrow O\ wv) \rightarrow P\ uv$$
$$AR1\ \ P\ uu$$
$$AR5\ \ (\exists u)(v)P\ vu$$

We then define spacetime as a predicate which holds for a region which has all regions as parts (D_{ST}). It follows that there is a unique spacetime (TR1 + AR5). We use the symbol $\mathcal{ST}$ to refer to this region.

$$D_{ST}\ ST\ u \equiv (v)P\ vu \qquad TR1\ ST\ u \wedge ST\ v \rightarrow u = v$$

On the intended interpretation in our example domain, spacetime is the set **ST**. Region variables range over all subsets of **ST**, and P is the subset relation, $\subseteq$.

Spatial regions and time-slices. We add as a new primitive the unary predicate SR. On the intended interpretation $SR\ u$ means: region u is a spatial region. Spatial regions are parts of spacetime which are either not extended at all in time or, in case of discrete time, do not extend beyond a minimal time unit. In the example model, $loc\_A\_t_1$, $loc\_B\_t_1$, and $loc\_C\_t_1$ are all spatial regions. More generally, any subset of **ST** consisting of points with a fixed time coordinate is a spatial region.

Time-slices are maximal spatial regions. In other words, a time-slice is a spatial region u such that u overlaps a spatial region v only if v is part of u (D_{TS}).

$$D_{TS}\ \ TS\ u \equiv SR\ u \wedge (v)(SR\ v \wedge O\ uv \rightarrow P\ vu)$$

We add axioms requiring that any part of a spatial region is a spatial region (AR6), every region overlaps some time-slice (AR7), and spacetime is not a spatial region (AR8).[1]

$$AR6\ \ SR\ u \wedge P\ vu \rightarrow SR\ v$$
$$AR7\ \ (\exists u)(TS\ u \wedge O\ uv)$$
$$AR8\ \ \neg SR\ \mathcal{ST}$$

[1] If desired a linear ordering on the subdomain of time-slices can be added to the theory. With such an ordering we can say that one region temporally precedes another, succeeds another, and so on.

We can prove: distinct time-slices do not overlap (i.e., each region is part of at most one time-slice) (TR2); u is a spatial region if and only if u is part of some time-slice (TR3); spacetime, $\mathcal{ST}$, is the sum of all time-slices (i.e., any region overlaps $\mathcal{ST}$ if and only if it overlaps some time-slice) (TR4).

$$TR2\ \ TS\ u \wedge TS\ v \wedge O\ uv \rightarrow u = v$$
$$TR3\ \ SR\ u \leftrightarrow (\exists v)(TS\ v \wedge P\ uv)$$
$$TR4\ \ O\ u\mathcal{ST} \leftrightarrow (\exists w)(TS\ w \wedge O\ uw)$$

Temporal regions. We define a *temporal region* to be any region that is not a spatial region (D_{TR}). Hence, spacetime is a temporal region. We can prove that u is a temporal region if and only if it overlaps more than one time-slice (TR5).

$$D_{TR}\ \ TR\ u \equiv \neg SR\ u$$
$$TR5\ \ TR\ u \leftrightarrow (\exists v)(\exists w)(TS\ v \wedge TS\ w \wedge v \neq w \wedge O\ uv \wedge O\ uw)$$

In our example model, *loc_lf_A*, *loc_lf_B*, *loc_lf_C*, *loc_lf_D*, and **ST** are all temporal regions. Note that a temporal region need not be extended in space. In the example model, $\{(1, \mathbf{t}) \mid t_1 < \mathbf{t} < t_3\}$ is a one-dimensional temporal region.

Simultaneous regions. Two regions are *simultaneous* if and only if they are parts of the same time-slice (D_{SIMU}).

$$D_{SIMU}\ SIMU\ uv \equiv (\exists w)(TS\ w \wedge P\ uw \wedge P\ vw)$$

It immediately follows that *SIMU* an equivalence relation (reflexive, symmetric, transitive) on the sub-domain of spatial regions. Notice that *SIMU uv* is always false if u or v is a temporal region.

4 Instantiation at regions of space-time

The second sort in our formal theory is *spatio-temporal entities* (*entities* for short). Recall that variables x, y, and z are used for entities.

We introduce a ternary relation *Inst* between two entities and a region and interpret *Inst xyu* as y is *instantiated by* x at region u (or, equivalently, x *instantiates* y at region u or x is an *instance* of y at region u). For example, I am an instance of human being wherever I am located. Consider Figure 2. Wherever the entites A, B, C, and D are located they instantiate the entity LINE (a universal). Wherever the entites *LifeOf_A*, *LifeOf_B*, *LifeOf_C*, and *LifeOf_D* are located they instantiate the entity LIFE-OF-A-LINE (a universal).

We define: entity x is *located* at region u if and only if there exists an entity y such that x instantiates y at u or x is instantiated by y at u (D_L); entity x is a *particular* if and only if x instantiates wherever x is located (D_{Part}); entity x is an *universal* if and only if x is instantiated wherever x is located (D_{Uni}); entity x is *uniquely located* if and only if x is located at a single region (D_{UL}).

$$D_L\ \ \ L\ xu \equiv (\exists y)(Inst\ xyu \vee Inst\ yxu)$$
$$D_{Part}\ \ Part\ x \equiv (u)(L\ xu \rightarrow (\exists y)(Inst\ xyu))$$
$$D_{Uni}\ \ Uni\ x \equiv (u)(L\ xu \rightarrow (\exists y)(Inst\ yxu))$$
$$D_{UL}\ \ UL\ x \equiv (u)(v)(L\ xu \wedge L\ xv \rightarrow u = v)$$

Since spatio-temporal entities and regions are disjoint sorts L is asymmetric and irreflexive. On the intended interpretation $L\ xu$ means: spatio-temporal entity x is *exactly located* at region u [6]. In other words, x takes up the whole region u but does not extend beyond u. In our example model, the entity A is exactly located at the regions $loc\_A\_t_1$, $loc\_A\_t_2$, and $loc\_A\_t_3$. The entity *LifeOf_A* is exactly located at the region *loc_lf_A*. The entities A^{t_1}, A^{t_2}, A^{t_3}, C^{t_1}, C^{t_2}, C^{t_3}, B^{t_1}, D^{t_3}, *LifeOf_A*, *LifeOf_B*, *LifeOf_C*, *LifeOf_D*, A, B, C, and D are particulars. The entities LINE and LIFE-OF-A-LINE are universals.

We require: if x instantiates y at u then no z is an instance of x at some u (AE1); every spatio-temporal entity is located at some region (AE2); every entity is located only at spatial regions or only at temporal regions (AE3); if x instantiates y at a temporal region u then x uniquely located (AE4); if x is a particular and x is located at simultaneous regions u and v then u and v are identical (AE5); if y is instantiated by a uniquely located entity then each entity that instantiates y is uniquely located (AE6); if x instantiates y at u then there exist an entity z and a region v such that z instantiates y at v and x is distinct from z and u is distinct from v (AE7).

$$AE1\ Inst\ xyu \rightarrow \neg(\exists z)(\exists v)(Inst\ zxv)$$
$$AE2\ (\exists u)(L\ xu)$$
$$AE3\ [(u)(L\ xu \rightarrow SR\ u) \vee (u)(L\ xu \rightarrow TR\ u)]$$
$$AE4\ Inst\ xyu \wedge TR\ u \rightarrow UL\ x$$
$$AE5\ Part\ x \rightarrow (u)(v)(L\ xu \wedge L\ xv \wedge SIMU\ uv \rightarrow u = v)$$
$$AE6\ Inst\ xyu \wedge UL\ x \rightarrow (z)(v)(Inst\ zyv \rightarrow UL\ z)$$
$$AE7\ Inst\ xyu \rightarrow (\exists z)(\exists v)(Inst\ zyv \wedge z \neq x \wedge u \neq v)$$

Axiom (AE1) guarantees that there is a distinction between entities that instantiate and entities that are instantiated. We can prove that if x instantiates y at u then x is a particular and y is a universal (TE1). From (AE1) and (AE2) it follows: if x is a particular then x is not a universal (TE2) and every entity is either a particular or a universal (TE3); x is a particular if and only if x instantiates somewhere (TE4) and that x is a universal if an only if x is instantiated somewhere (TE5).

$$TE1\ Inst\ xyu \rightarrow (Part\ x \wedge Uni\ y)$$
$$TE2\ (Part\ x \rightarrow \neg Uni\ x)$$
$$TE3\ (Part\ x \vee Uni\ x)$$
$$TE4\ Part\ x \leftrightarrow (\exists y)(\exists u)(Inst\ xyu)$$
$$TE5\ Uni\ x \leftrightarrow (\exists y)(\exists u)(Inst\ yxu)$$

Theorems TE2 and TE3 tell us that the sub-domains of universals and particulars partition the domain of entities.

We can also prove: wherever a particular x is located there is a co-located universal that is instantiated by x (TE6); wherever a universal y is located there is a co-located particular x which instantiates y (TE7).

$$TE6\ Part\ x \wedge L\ xu \rightarrow (\exists y)(Uni\ y \wedge Inst\ xyu)$$
$$TE7\ Uni\ x \wedge L\ xu \rightarrow (\exists y)(Part\ y \wedge Inst\ yxu)$$

Notice that particulars can instantiate multiple universals at the same region. For example, at my current location I instantiate (among others) the universals human being and animal. Moreover, an individual x may instantiate the universal y at region u and fail to instantiate y at region v. For example, at my current location I instantiate the universal adult. There are, however, regions at which I instantiated the universal child.

From (AE2) and (D_L) it follows that on the sub-domain of uniquely located entities the location relation L is a function. It also follows that if x instantiates y at u then x and y are located at u. Note, that the converse does not hold: there are situations in which two entities are located at the same region without one being an instance of the other. For example the City of Vienna and the Austrian Federal State of Vienna are located at the same spatial region in many time-slices, but neither is an instance of the other at any region.

Axiom (AE3) requires that entities cannot be located at different kinds of regions. The implications for instantiation are made explicit in the following theorems: if x instantiates y at a spatial region then everything that has x as an instance or instantiates y is located only at spatial regions (TE8). Similarly, if x instantiates y at a temporal region then everything that has x as an instance or instantiates y is located only at temporal regions (TE9).

$$TE8\ \mathit{Inst}\ xyu \wedge SR\ u \rightarrow (z)(v)[(\mathit{Inst}\ xzv \vee \mathit{Inst}\ zyv) \rightarrow (w)(L\ zw \rightarrow SR\ w)]$$
$$TE9\ \mathit{Inst}\ xyu \wedge TR\ u \rightarrow (z)(v)[(\mathit{Inst}\ xzv \vee \mathit{Inst}\ zyv) \rightarrow (w)(L\ zw \rightarrow TR\ w)]$$

Axioms (AE4) and (AE5) provide additional constraints on how particulars can be located in space time. Particulars are located at no more than one temporal region, i.e., particulars that are located at temporal regions are uniquely located at those regions (TE10).

$$TE10\ \mathit{Part}\ x \wedge L\ xu \wedge TR\ u \rightarrow (v)(L\ xv \rightarrow u = v)$$

Moreover particulars are located at no more than one spatial region per time-slice, i.e., particulars are not located at distinct simultaneous regions.

Note that there are no such constraints for universals in our theory: universals may be located at multiple spatial regions per time-slice or at multiple temporal regions. For example, in this time-slice (at this moment in time) the universal building is located at every spatial region exactly occupied by a building. The universal erosion process is located at every temporal region where an erosion process is located.

Axioms (AE6 and AE7) enforce additional constraints on how universals are instantiated: no universal is instantiated by uniquely located and non-uniquely located particulars; every universal has at least two instances that are located at distinct regions.

5 Basic categories of particulars

As specified so far, location is a relation which can hold between a single entity and multiple regions. In our example model, particular A is exactly located at multiple spatial regions including *loc_A_t*$_1$, *loc_A_t*$_2$, and *loc_A_t*$_3$. On the other hand, the particular *LifeOf_A* is located at the single temporal region *loc_lf_A*. In this section we discuss ways of distinguishing particulars according to the number and the kinds of regions at which they can be located.

Stages and persistent particulars. A particular is a *stage* if and only if it is located at a single region and that region is a spatial region (D$_{Stg}$). Consequently, stages are instantaneous spatial particulars in the sense that they are confined to a single time-slice.

$$D_{Stg}\ Stg\ x \equiv Part\ x \wedge (u)(v)(L\ xu \wedge L\ xv \rightarrow (SR\ u \wedge u = v))$$

A particular is *persistent* iff it is not confined to a single time-slice (D$_{Pst}$).

$$D_{Pst}\ Pst\ x \equiv Part\ x \wedge (\exists u)(\exists v)(L\ xu \wedge L\ xv \wedge \neg SIMUuv)$$

In our example model the entities A^{t_1}, A^{t_2}, A^{t_3}, C^{t_1}, C^{t_2}, C^{t_3}, B^{t_1}, and D^{t_3} are stages and the entities A, B, C, D, *LifeOf_A*, *LifeOf_B*, *LifeOf_C*, *LifeOf_D* are persistent particulars. Other examples of stages include: every momentary slice of my life, every momentary stage of an erosion process, every momentary stage of the development of a geo-political entity (including the time-slice at which the geo-political entity was brought into existence by some administrative act). Persistent entities include myself, my life, planet Earth, the process of global warming on Earth, etc.

We can prove: stages are uniquely located (TI1); no stage is persistent (TI2); every particular is either a stage or a persistent entity (TI3). It follows from (TI2 and TI3) that the subdomains of stages and persistent entities partition the sub-domain of particulars.

$$TI1\ Stg\ x \rightarrow UL\ x$$
$$TI2\ Stg\ x \rightarrow \neg Pst\ x \qquad TI3\ Part\ x \leftrightarrow (Stg\ x \vee Pst\ x)$$

Kinds of persistent particulars. Persistent particulars are distinguished into endurants and perdurants. Entity x is an *endurant* iff x is a persistent particular and x is located at some spatial region (D_{Ed}). On the other hand, x is a *perdurant* iff it is a particular and x is located at some temporal region (D_{Pd}).

$$D_{Ed}\ \ Ed\ x \equiv Pst\ x \wedge (\exists u)(L\ xu \wedge SR\ u)$$
$$D_{Pd}\ \ Pd\ x \equiv Part\ x \wedge (\exists u)(L\ xu \wedge TR\ u)$$

In our example model the entities *LifeOf_A*, *LifeOf_B*, *LifeOf_C*, and *LifeOf_D* are perdurants and the entities A, B, C, and D are endurants.

We can prove: perdurants are uniquely located (TI4); perdurants are persistent entities (TI5); endurants are not uniquely located (TI6); nothing is both an endurant and a perdurant (TI7) ; x is a particular if and only if x is an endurant or a perdurant or a stage (TI8).

$TI4\ Pd\ x \rightarrow UL\ x$
$TI5\ Pd\ x \rightarrow Pst\ x$
$TI6\ Ed\ x \rightarrow \neg UL\ x$
$TI7\ Ed\ x \rightarrow \neg Pd\ x$
$TI8\ Part\ x \leftrightarrow (Ed\ x \vee Pd\ x \vee Stg\ x)$

It follows from TI2, TI3, TI7, and TI8 that the subdomains of stages, endurants, and perdurants partition the sub-domain of particulars.

6 Kinds of universals

Corresponding to the three kinds of particulars we distinguish universals whose instances are endurants (*universals of endurants*), universals whose instances are perdurants (*universals of perdurants*), and universals whose instances are stages (*universals of stages*).

$$D_{UniEd}\ \ UniEd\ x \equiv Uni\ x \wedge (y)(u)(Inst\ yxu \rightarrow Ed\ y)$$
$$D_{UniPd}\ \ UniPd\ x \equiv Uni\ x \wedge (y)(u)(Inst\ yxu \rightarrow Pd\ y)$$
$$D_{UniStg}\ \ UniStg\ x \equiv Uni\ x \wedge (y)(u)(Inst\ yxu \rightarrow Stg\ y)$$

For example: Human being, city, lake, building, etc. are universals of endurants; Human life, erosion process, administrative process, etc. are universals of perdurants; Momentary stages of human lives are instances of the universal human-life-stage. Other examples of universals of stages include classes of momentary events such as the becoming effective of a law, the establishment of political subdivisions, etc. In our example model, the entity LINE is a universal of endurants, LIFE-OF-A-LINE is a universal of perdurants, and STAGE-IN-THE-LIFE-OF-A-LINE is a universal of stages.

We then can prove that universals of endurants, universals of perdurants, and universals of stages are disjoint kinds of universals (TU1-TU3). We can also prove that x is a universal if and only if x is either a universal of endurants, a universal of perurants, or a universal of stages (TU4).

$$TU1\ \ UniEd\ x \rightarrow \neg(UniPd\ x \vee UniStg)$$
$$TU2\ \ UniPd\ x \rightarrow \neg(UniEd\ x \vee UniStg)$$
$$TU3\ \ UniStg\ x \rightarrow \neg(UniEd\ x \vee UniPd)$$
$$TU4\ \ Uni\ x \leftrightarrow (UniEd\ x \vee UniPd\ x \vee UniStg\ x)$$

7 Conclusions

We developed an axiomatic theory of spatio-temporal entities based on the primitives *spatial-region*, *part-of*, and *is-an-instance-of* and provided a classification of those entities according to the number and kinds of regions at which they are located in spacetime and according to whether they instantiate or are instantiated at those regions. The various categories and their implication hierarchy are depicted in Figure 1. The arrows correspond to axioms and theorems of our theory. The categorization is exhaustive in the sense that every spatio-temporal entity falls in *exactly one* category at the level of leaf nodes of the depicted tree.

There are a number of top-level ontologies that distinguish spatio-temporal entities into universals and particulars and particulars into edurants, perdurants and stages [11, 8, 9]. None of these ontologies, however, develop those distinction based on the relation of location. The focus on location as one of the central notions of this theory makes it particularly appropriate for serving as a foundational ontology for geography and geographic information science. Also, in our theory we treat time in a way that is analogous to the way we treat space.

Another important feature of our theory is that it describes time-dependent properties and relations (e.g. instantiation of a given universal) without making assumptions about the structure of time. Thus we are not forced to make commitments on the specific structure of time (e.g., assume that time is continuous, rather than discrete). For example, our theory can have coarse grained models in which the minimal time unit is a calendar year and each stage in the development of a geographic process (e.g., climate change or the development of a nation) corresponds to a different year.

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