The mereology of stages and persistent entities

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Abstract. Since our world is populated by entities that persist through time and that change over time it is important to represent knowledge about those entities in a formal manner. In this paper a formal theory of the mereological structure of stages and persistent entities is presented. Stages are entities which exist only at a single moment in time. Persistent entities are entities which exist at more than one instant in time. Endurance and perdurance are identified as different modes of persistence. The underlying framework is a mereology of spacetime regions in which we can distinguish between *spatial regions* (i.e., regions of minimal temporal extend) and temporally extended regions. *Time-slices* are defined as maximal spatial regions and are used to describe the temporal properties of spacetime regions and the entities (endurants, perdurants, and stages) located at these regions.

Keywords: Formal ontology, space, time, perdurant, endurant, persistence

1 INTRODUCTION

Formal ontologies provide the semantic foundations for the use of shared terminologies which are critical for the Semantic Web [MBG$^+$04], for the use of medical terminology systems, and for a variety of other purposes.

This means that there is a need for formal ontologies which describe how things persist through time, how things change over time, how things are located in space and time, and how wholes are made up of parts. As concerns persistence, two categories of persistent entities can be distinguished: endurants and perdurants (or continuants and occurrents [Sim87]), which differ in their relation to time. Endurants are wholly present at any time at which they exist. For example, you (an endurant) are wholly present in the moment you are reading this.

Perdurants, on the other hand, are extended in time. In opposition to endurants they are only partially present at any time at which they exist. For example, at this moment only a (tiny) part of your *life* (a perdurant) is present. Larger parts of your life – such as your childhood – are not present at this moment.

Perduring and enduring entities can thus be characterized with respect to the way statements about their part-of structure can be made [Haw01]: (a) something is an endurant if and only if (i) it exists at more than one moment and (ii) statements about what parts it has are always *relative to some time or other*; on the other hand (b) something is a perdurant if and only if (i) it is extended over time and (ii) statements about what parts it has are *time-independent*.

We often make statements about the parts of endurants without *explicitly* referring to times. But when we say that Tom's little toe is part of Tom, we generally mean that Tom's little toe is part of Tom *now*. My little toe cannot be part of me in the same sense as my childhood is part of my life. This is because enduring entities can change. In particular they can gain and lose parts during their existence. For example, I might lose my toe next week. Perdurants, on the other hand, cannot change. My life cannot have my childhood as a part at one time but not at another.

Often the distinction between time-dependent parthood for endurants and time-independent parthood between perdurants is accounted for by explicitly introducing a ternary part-of relation for endurants, $P\,xyt$, which is interpreted as: x is a part of y at time t (e.g., [Tho83],[MBG$^+$04]). Among perdurants the usual time-independent binary part-of relation holds. In BFO, the Basic Formal Ontology developed at IFOMIS, the endurant/perdurant distinction is made implicitly by constructing ontological theories of two types, called SNAP and SPAN [GS04]. Ontologies of type SNAP are restricted to an instantaneous time-slice and contain statements (e.g., asserting parthood relations among endurants) which hold at that instant. Ontologies of type SPAN are not time-indexed and contain time-independent statements about perduring entities.

It is the purpose of this paper to provide a formal theory of the different kinds of part-of relations that hold among endurants and perdurants. Our theory is intended as a formal basis for the SNAP/SPAN distinction among ontologies in BFO. An advantage of the theory is that it distinguishes between time-dependent and time-independent parthood relations without referring explicitly to time. This makes our theory more flexible, since it is independent of specific assumptions about the structure of time. It also makes it simpler in that times are not included in the domain of the theory as a distinct sort, though they are represented by time-slices. If desired more temporal structure can be easily built into the theory with additional relations among time-slices.

2 ENDURANTS, PERDURANTS, AND STAGES

Besides persisting entities such as endurants and perdurants which exist at multiple moments in time, we assume also *stages* which are instantaneous parts of perdurants [Sid01]. Particularly important are stages which are instantaneous parts of the lives of endurants. At every moment an endurant exists, there is a stage which is the slice of the endurant's life that is limited to this moment in time.

As an example consider Figure 1. Instead of considering a four-dimensional model of spacetime, we use the subset of points of the plane which is specified by the coordinates t and s that satisfy the constraint $0 \leq t \leq t_4$ & $0 \leq s \leq 4$. In set-theoretic terms we write $\mathbf{ST} = \{(s,t) \mid 0 \leq t \leq t_4 \ \& \ 0 \leq s \leq 4\}$. The horizontal dimension in the figure is interpreted as temporal and the vertical dimension is interpreted as spatial.

The left part of Figure 1 shows an endurant, the line-shaped entity A, at times t_1, t_2, and t_3. The life of the endurant A is visualized as the solid two-dimensional region, *LifeOf_A*, depicted in the right part of the figure. It shows that A comes into existence at t_1 and that it continues to exist until t_4. The lives of C, B and D are proper parts of the life of A and are respectively located at the spacetime regions *loc_lf_C* $= \{(s,t) \mid t_1 \leq t \leq t_4 \;\&\; 1 \leq s \leq 2\}$, *loc_lf_B* $= \{(s,t) \mid t_1 \leq t \leq t_5 \;\&\; 2 \leq s \leq 3\}$, and *loc_lf_D* $= \{(s,t) \mid t_6 \leq t_4 \;\&\; 2 \leq s \leq 3\}$ shown in the right part of Figure 1. The life of A, *LifeOf_A*, is located at the region *loc_lf_A*, which is the union of the regions *loc_lf_B*, *loc_lf_C*, and *loc_lf_D*.

Both figures indicate that during its life A undergoes changes in its mereological structure. We also include in our model the following stages of the lives of the endurants A, C, B and D: A^{t_1}, A^{t_2}, A^{t_3}, C^{t_1}, C^{t_2}, C^{t_3}, B^{t_1}, and D^{t_3}. For example, A^{t_1} is the instantaneous slice of A's life at t_1, A^{t_2} is the instantaneous slice of A's life at t_2, and so on.

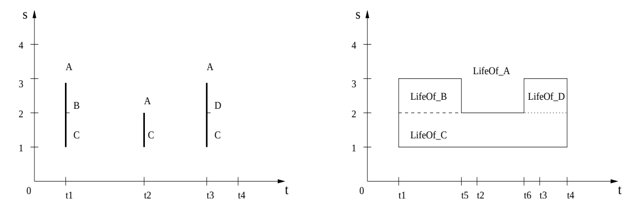

Figure 1. The endurant A in different time-slices (left) and the life of A (right).

At a given moment during its life an endurant is exactly co-located with the stage of its life at that moment. For example, the location of A at t_1 is the location of the stage A^{t_1}: the region *loc_A_t*$_1$ $= \{(s,t) \mid t = t_1 \;\&\; 1 \leq s \leq 3\}$. The stages C^{t_1} and B^{t_1} are located at the regions *loc_C_t*$_1$ $= \{(s,t) \mid t = t_1 \;\&\; 1 \leq s \leq 2\}$ and *loc_B_t*$_1$ $= \{(s,t) \mid t = t_1 \;\&\; 2 \leq s \leq 3\}$. The stages A^{t_2} and C^{t_2} are both located at the region *loc_A_t*$_2$ $=$ *loc_C_t*$_2$ $= \{(s,t) \mid t = t_2 \;\&\; 1 \leq s \leq 2)$. And so on.

We will use this example as a model for our formal theory of stages, endurants, perdurants, and their spatio-temporal locations.

3 THE MEREOLOGY OF REGIONS

We use a sorted first-order predicate logic with identity and we assume that the domains of our models are divided into two disjoint sorts: *regions* and *entities*. Regions are parts of four-dimensional space and can be of any dimension (less than five), shape, and size. We use letters u, v, and w as variables for regions. Entities are material endurants, perdurants, and stages that are located in spacetime. We use letters x, y, z as variables for entities. All quantification is restricted to a single sort. Restrictions on quantification will be understood from conventions on variable use. Leading universal quantifiers are generally omitted.

Regional parthood. We start by introducing the binary predicate P, where $P\ uv$ is interpreted as 'the region u is a part of the region v'. We also say that u is a *regional part* of v. We add axioms which make P reflexive(ARM1), antisymmetric (ARM2), and transitive (ARM3), i.e., partial ordering.

$$ARM1 \quad P\ uu \qquad\qquad \begin{array}{ll} ARM2 & P\ uv \wedge P\ vu \rightarrow u = v \\ ARM3 & P\ uv \wedge P\ vw \rightarrow P\ uw \end{array}$$

We continue by introducing the binary predicates PP for proper parthood (D_{PP}) and O for overlap (D_O).

$$\begin{array}{ll} D_{PP} & PP\ uv \equiv P\ uv \wedge \neg u = v \\ D_O & O\ uv \equiv (\exists w)(P\ wu \wedge P\ wv) \end{array}$$

We then add an axiom stating that if everything that overlaps u also overlaps v then u is a part of v (ARM4).

$$ARM4 \quad (w)(O\ wu \rightarrow O\ wv) \rightarrow P\ uv$$

We then define spacetime as a predicate which holds for a region which has all regions as parts (D_{ST}). If there is a such a region then it is unique (TRM2). Finally we add an axiom stating that such maximal region exists (ARM5) and we use the symbol $\mathcal{ST}$ to refer to it.

$$\begin{array}{ll} D_{ST} & ST\ u \equiv (v)P\ vu \\ TRM2 & ST\ u \wedge ST\ v \rightarrow u = v \end{array} \qquad ARM5 \quad (\exists u)ST\ u$$

On the intended interpretation in our example domain, spacetime is the set **ST**. Region variables range over all subsets of **ST**, and P is the subset relation, $\subseteq$.

Spatial regions and time-slices. We add as a new primitive the unary predicate SR. On the intended interpretation $SR\ u$ means: region u is a spatial region. Spatial regions are parts of spacetime which are either not extended at all in time or, in case of discrete time, do not extend past a minimal time unit. In the example model, *loc_A_t*$_1$, *loc_B_t*$_1$, and *loc_C_t*$_1$ are all spatial regions. More generally, any subset of **ST** consisting of points with a fixed time coordinate is a spatial region.

Time-slices are maximal spatial regions. In other words, a time-slice is a spatial region u such that u overlaps a spatial region v only if v is part of u (D_{TS}).

$$D_{TS} \quad TS\ u \equiv SR\ u \wedge (v)(SR\ v \wedge O\ uv \rightarrow P\ vu)$$

In our example model, for any fixed $\mathbf{t}$ with $0 \leq \mathbf{t} \leq t_4$ the set $\{(s,\mathbf{t}) \mid 0 \leq s \leq 4\}$ is a time-slice.

We add axioms requiring that any part of a spatial region is a spatial region (AR1), every region overlaps some time-slice (AR2), and spacetime is not a spatial region (AR3).

$$AR1 \quad SR\ u \wedge P\ vu \rightarrow SR\ v \qquad \begin{array}{ll} AR2 & (\exists u)(TS\ u \wedge O\ uv) \\ AR3 & \neg SR\ \mathcal{ST} \end{array}$$

We then can prove that there is at least on time-slice (TR1) and any spatial region is a proper part of spacetime (TR2).

$$TR1 \quad (\exists u)TS\ u \qquad\qquad TR2 \quad SR\ u \rightarrow PP\ u\mathcal{ST}$$

We can also prove that distinct time-slices do not overlap (TR3), u is a spatial region if and only if u is part of some time-slice (TR4), each region is part of at most one time-slice (TR5).

$$\begin{array}{ll} TR3 & TS\ u \wedge TS\ v \wedge O\ uv \rightarrow u = v \\ TR4 & SR\ u \leftrightarrow (\exists v)(TS\ v \wedge P\ uv) \\ TR5 & P\ uv \wedge P\ uw \wedge TS\ v \wedge TS\ w \rightarrow v = w \end{array}$$

It follows from TR4 and TR5 that each spatial region is part of a unique time-slice. Finally we can prove that spacetime, $\mathcal{ST}$, is the sum of all time slices, i.e., everything overlaps $\mathcal{ST}$ if and only if overlaps some time-slice (TR6).

$$TR6 \quad O\ u\mathcal{ST} \leftrightarrow (\exists w)(\mathit{TS}\ w \wedge O\ uw)$$

We define a *temporal region* to be any region that is not a spatial region (D_{TR}).

$$D_{TR} \quad TR\,u \equiv \neg SR\,u \qquad \begin{array}{ll} TR7 & TR\,\mathcal{ST} \\ TR8 & TR\,u \wedge P\ uv \rightarrow TR\,v \end{array}$$

We can prove that spacetime is a temporal region (TR7) and that if u is a temporal region and u is a part of v then v is a temporal region (TR8). In our example model, *loc_lf_A*, *loc_lf_B*, *loc_lf_C*, *loc_lf_D*, and $\mathbf{ST}$ are all temporal regions. Note that a temporal region need not be extended in space. In the example model, $\{(1,\mathbf{t}) \mid t_1 < \mathbf{t} < t_3\}$ is a one-dimensional temporal region.

Finally we can prove that u is a temporal region if and only if it overlaps more than one time slice (TR9).

$$TR9 \quad TR\,u \leftrightarrow (\exists v)(\exists w)(\mathit{TS}\ v \wedge \mathit{TS}\ w \wedge \neg v = w \wedge O\ uv \wedge O\ uw)$$

Temporal relations. The following relations can be used to compare the temporal situations of any two regions. Two regions *temporally overlap* when they overlap some common time-slice (D_{TMPO}).

$$D_{\mathit{TMPO}} \quad \mathit{TMPO}\ uv \equiv (\exists w)(\mathit{TS}\ w \wedge O\ uw \wedge O\ vw)$$

For example the spacetime region at which my father's life is located temporally overlaps the spacetime region at which my life is located. Since my father and I are alive at some of the same times these regions cross over some common time slices.

Region v *temporally covers* region u when v overlaps any time-slice that u overlaps (D_{TCOV}).

$$D_{\mathit{TCOV}} \quad \mathit{TCOV}\ uv \equiv (w)((\mathit{TS}\ w \wedge O\ uw) \rightarrow O\ vw)$$

If my father outlives me, then the region at which his life is located will temporally cover the region at which my life is located. If I outlive him, these spacetime regions will merely temporally overlap – neither will temporally cover the other. In the example model, *loc_lf_A* temporally covers (and temporally overlaps) *loc_lf_B*, *loc_lf_C*, and *loc_lf_D*, as well as the locations of the stages of A, B, C, and D.

Regions u and v are *contemporaneous* when they overlap the same time-slices (D_{CTMP}).

$$D_{\mathit{CTMP}} \quad \mathit{CTMP}\ uv \equiv \mathit{TCOV}\ uv \wedge \mathit{TCOV}\ vu$$

In the example model the regions *loc_lf_A* and *loc_lf_C* are contemporaneous.

It is easy to see that *TMPO* *TCOV*, and *CTMP* are reflexive, that *TCOV* is transitive, and that *CTMP* is an equivalence relation. We can also prove that if two spatial regions temporally overlap then they are contemporaneous (TR10); two spatial regions are contemporaneous if and only if they are parts of the same time-slice (TR11); and that u is a spatial region if and only if there is a time-slice w such that u and w are contemporaneous (TR12).

$$\begin{array}{ll} TR10 & SR\,u \wedge SR\,v \wedge \mathit{TMPO}\ uv \rightarrow \mathit{CTMP}\ uv \\ TR11 & SR\,u \wedge SR\,v \rightarrow \\ & \quad (\mathit{CTMP}\ uv \leftrightarrow (\exists w)(\mathit{TS}\ w \wedge P\ uw \wedge P\ vw)) \\ TR12 & SR\,u \leftrightarrow (\exists w)(\mathit{TS}\ w \wedge \mathit{CTMP}\ uw) \end{array}$$

In what follows it will be useful to have a relation stating that two *spatial* regions are contemporaneous (D_{CTMP_S}).

$$D_{\mathit{CTMP}_S} \quad \mathit{CTMP}_S\ uv \equiv SR\,u \wedge SR\,v \wedge \mathit{CTMP}\ uv$$

In the example model *loc_A_*t_1, *loc_B_*t_1, and *loc_C_*t_1 are contemporaneous spatial regions.

If desired a linear ordering on the subdomain of time-slices can be added to the theory. With such an ordering we can say that one region temporally precedes another, succeeds another, and so on.

4 ENTITIES AND THEIR LOCATION

The second sort in our formal theory are material endurants, perdurants, and stages, which we call *entities*. We use the letters x, y, and z as variables for entities. We introduce the primitive binary predicate $L\ xu$ where on the intended interpretation $L\ xu$ means: entity x is exactly located at a region u [CV99]. In other words, x takes up the whole region u but does not extend beyond it. We require that every entity is exactly located at some region (AL1), and that no entity is exactly located at distinct parts of the same time-slice (AL2).

$$\begin{array}{ll} AL1 & (\exists u)(L\ xu) \\ AL2 & L\ xu \wedge L\ xv \wedge \mathit{CTMP}_S\ uv \rightarrow u = v \end{array}$$

We say that an entity is *present-at* a time-slice if and only if it is located at a region that overlaps that time-slice (D_{PrAt}).

$$D_{\mathit{PrAt}} \quad \mathit{PrAt}\ xu \equiv \mathit{TS}\ u \wedge (\exists v)(L\ xv \wedge O\ uv)$$

We can use *PrAt* to define temporal relations for entities which are analogous to the relations *TMPO*, *TCOV*, and *CTMP*. Two entities *temporally overlap* when they are present at some common time-slice (D_{TMPO_E}). Entity x *temporally covers* entity y when y is present at any time-slice at which x is present (D_{TCOV_E}). Entities x and y are *contemporaneous* when they are present the same time slices (D_{CTMP_E}).

$$\begin{array}{ll} D_{\mathit{TMPO}_E} & \mathit{TMPO}_E\ xy \equiv (\exists w)(\mathit{TS}\ w \wedge \mathit{PrAt}\ xw \wedge \mathit{PrAt}\ yw) \\ D_{\mathit{TCOV}_E} & \mathit{TCOV}_E\ xy \equiv (w)((\mathit{TS}\ w \wedge \mathit{PrAt}\ xw) \rightarrow \mathit{PrAt}\ yw) \\ D_{\mathit{CTMP}_E} & \mathit{CTMP}_E\ xy \equiv \mathit{TCOV}\ xy \wedge \mathit{TCOV}\ yx \end{array}$$

As with their regional counterparts we can prove that TMPO_E, TCOV_E, and CTMP_E are reflexive, that TCOV_E is transitive, and that CTMP_E is an equivalence relation. In the example model, both *LifeOf_A* and *LifeOf_C* temporally overlap with and cover all endurants, perdurants, and stages in the model. Also, *LifeOf_A* and *LifeOf_C* are contemporaneous.

Distinguishing endurants, perdurants, and stages. The distinct spatio-temporal character of endurants, perdurants, and stages manifests itself in the different ways they are located in spacetime. On the intended interpretation the relation $L\ xu$ holds for a perdurant x iff u is the unique *temporal* region which x exactly occupies; for a stage x, $L\ xu$ holds iff u is the unique *spatial* region which x exactly occupies; for an endurant x, $L\ xu$ holds iff u is any *spatial* region that x exactly occupies at any time during its existence. In our example model, the endurant A is exactly located at the spatial regions *loc_A_*t_1, *loc_A_*t_2, and *loc_A_*t_3 The perdurant *LifeOf_A* is exactly located at the single temporal region *loc_lf_A*. The stage A^{t_1} is exactly located at the single spatial region *loc_A_*t_1, the stage A^{t_2} is exactly located at the single spatial region *loc_A_*t_2, and so on.

We now define that an entity is a stage if and only if it is located at a single region and that region is a spatial region (D_{Stg}). Stages are instantaneous spatial entities in the sense that they are confined to a single time-slice. An entity is persistent iff it is not confined to a single time-slice ($D_{P_{st}}$).

$$D_{Stg} \quad Stg\, x \equiv (u)(v)(L\, xu \wedge L\, xv \rightarrow (SR\, u \wedge u = v))$$
$$D_{Pst} \quad Pst\, x \equiv (\exists u)(\exists v)(L\, xu \wedge L\, xv \wedge \neg CTMP_S uv)$$

Consider the left part of Figure 1. The endurant A is a persistent entity. It is located at the regions *loc_A_t*$_1$, *loc_A_t*$_2$ and *loc_A_t*$_3$ which are all parts of different time-slices and therefore not contemporaneous spatial regions. The life of A is located at the region *loc_lf_A*. Since *loc_lf_A* is a temporal region it does not stand in the $CTMP_S$ relation with itself. Consequently, the life of A is a persistent entity.

We can prove that no stage is persistent (TL1) and that if x is located at a temporal region then x is persistent (TL2).

$$TL1 \quad Stg\, x \rightarrow \neg Pst\, x \qquad TL2 \quad (\exists u)(TR\, u \wedge L\, xu) \rightarrow Pst\, x$$

The sub-domain of persistent entities can be divided into endurants and perdurants. We define that x is an endurant iff x is a persistent entity which is only located at spatial regions (D_{Ed}). On the other hand, x is a perdurant iff it is an entity which has a fixed location that is a temporal region (D_{Pd}).

$$D_{Ed} \quad Ed\, x \equiv Pst\, x \wedge (u)(L\, xu \rightarrow SR\, u)$$
$$D_{Pd} \quad Pd\, x \equiv (u)(v)(L\, xu \wedge L\, xv \rightarrow (TR\, u \wedge u = v))$$

In our example, A is an endurant – it is located at several spatial regions in different time-slices. The life of A, on the other hand, is a perdurant – it is located at a unique temporal region.

We can prove that endurants do not have a fixed location (TL3), and that nothing is both an endurant and a perdurant (TL4).

$$TL3 \quad Ed\, x \rightarrow (\exists u)(\exists v)(\neg u = v \wedge L\, xu \wedge L\, xv)$$
$$TL4 \quad Ed\, x \rightarrow \neg Pd\, x$$

Thus the subdomains of stages, endurants, and perdurants are pairwise disjoint. Finally we add an axiom requiring that every entity is either a stage, an endurant, or a perdurant (AL3).

$$AL3 \qquad Stg\, x \vee Ed\, x \vee Pd\, x$$

It follows from AL3 that no entity can be exactly located at distinct temporal regions or located both at a spatial and at a temporal region.

5 STAGES

We now define parthood among stages as follows: x is a stage-part of y if and only if x and y are stages and for all u and v, if x is located at u and y is located at v then u is a regional part of v ($D_{P_{st}}$).

$$D_{P_{st}} \quad P_{st}\, xy \equiv Stg\, x \wedge Stg\, y \wedge (u)(v)(L\, xu \wedge L\, yv \rightarrow P\, uv)$$

In other words, stage x is a stage-part of stage y if and only if the unique spatial region at which x is located is a part of the unique spatial region at which y is located. In the example model, both B^{t_1} and C^{t_1} are stage-parts of A^{t_1}. The stage of my hand at this moment is a stage-part of the stage of me at this moment.

We can prove that x is a stage if and only if it is a stage-part of itself (TST1) and that stage-parthood is transitive (TST2).

$$TST1 \quad Stg\, x \leftrightarrow P_{st}\, xx \qquad TST2 \quad P_{st}\, xy \wedge P_{st}\, yz \rightarrow P_{st}\, xz$$

We cannot, however, prove that P_{st} is antisymmetric. In order to force co-located stages to be identical we add an axiom of antisymmetry (AST1).

$$AST1 \quad P_{st}\, xy \wedge P_{st}\, yx \rightarrow x = y$$

Thus, in our example model the co-located stages C^{t_2} and A^{t_2} must be identical.

We can prove: if x is a stage part of y then x and y are contemporaneous (TST3); for stages temporal overlap, temporal covering, and contemporaneousness are equivalent (TST4); and every stage is present at a unique time-slice (TST5).

$$TST3 \quad P_{st}\, xy \rightarrow CTMP_E\, xy$$
$$TST4 \quad Stg\, x \wedge Stg\, y \rightarrow (TMPO_E\, xy \leftrightarrow TCOV_E\, xy \leftrightarrow CTMP_E\, xy)$$
$$TST5 \quad Stg\, x \rightarrow (\exists w)(PrAt\, xw \wedge (v)(PrAt\, xv \rightarrow v = w))$$

6 ENDURANCE

The way an endurant endures through time is characterized by its relation to stages in different time-slices. In order to capture this mode of persistence we introduce binary predicate *Ed-Stg* xy (y is a stage of the endurant x) if and only if (i) x is an endurant and y is a stage; and (ii) x and y are both located at some spatial region ($D_{Ed\text{-}Stg}$). It follows immediately that *Ed-Stg* is irreflexive and asymmetric.

$$D_{Ed\text{-}Stg} \quad Ed\text{-}Stg\, xy \equiv Ed\, x \wedge Stg\, y \wedge (\exists u)(L\, yu \wedge L\, xu)$$

Consider our example model. Here we have *Ed-Stg* AA_{t^1}, *Ed-Stg* AA_{t^2}, *Ed-Stg* AA_{t^3}, *Ed-Stg* BB_{t^1}, *Ed-Stg* CC_{t^1}, *Ed-Stg* CC_{t^2}, *Ed-Stg* CC_{t^3}, and *Ed-Stg* DD_{t^3}.

We can prove that an endurant temporally covers all of its stages (TED1) and that every endurant has at most one stage in a time-slice (TED2).

$$TED1 \quad Ed\text{-}Stg\, xy \rightarrow TCOV_E\, xy$$
$$TED2 \quad Ed\text{-}Stg\, xz \wedge Ed\text{-}Stg\, xy \wedge CTMP_E\, yz \rightarrow y = z$$

Notice, that a single stage can be the stage of different endurants. Consider our example model. Here the stages A^{t_2} and C^{t_2} are identical but this stage is the stage of distinct endurants: A and C. Consider a statue and the bronze of which it is constituted. The statue and the portion of bronze, are distinct endurants which have identical stages in some, but not all time-slices.

We add an axiom stating that wherever an endurant x is located there exists a stage which is the stage of x in this time-slice (AED1).

$$AED1 \quad (Ed\, x \wedge L\, xu) \rightarrow (\exists y)(Ed\text{-}Stg\, xy \wedge L\, yu)$$

Because each endurant is located at multiple regions, parthood relations among endurants are more complicated than parthood relations among stages or among perdurants. We now define a number of distinct parthood relations between endurants: The endurant x is a *temporary part* of the endurant y iff there exists a stage of x which is part of a stage of y ($D_{P^t_{Ed}}$).

$$D_{P^t_{Ed}} \quad P^t_{Ed}\, xy \equiv Ed\, x \wedge Ed\, y \wedge (\exists z_x)(\exists z_y)(Ed\text{-}Stg\, xz_x \wedge Ed\text{-}Stg\, yz_y \wedge P_{st}\, z_x z_y)$$

In our example model A, B, C, are all temporary parts of A. All of my blood cells, my wisdom teeth, and my heard are temporary parts of me.

The endurant x is a *permanent part* of the endurant y iff every stage of x is a part of a stage of y ($D_{P^p_{Ed}}$).

$$D_{P^p_{Ed}} \quad P^p_{Ed}\, xy \equiv Ed\, x \wedge Ed\, y \wedge (z_x)(\textit{Ed-Stg}\, xz_x \rightarrow (\exists z_y)(\textit{Ed-Stg}\, yz_y \wedge P_{st}\ z_x z_y))$$

In our example the endurants A, C, B and D are enduring temporary parts of A as well as permanent parts of A. Most of my blood cells and my heart are permanent parts of me. My wisdom teeth (which were removed in fact) are not permanent parts of me.

The endurant x is a *livelong part* of the endurant y iff x is a permanent part of y and every stage of y has a stage of x as part ($D_{P^l_{Ed}}$).

$$D_{P^l_{Ed}} \quad P^l_{Ed}\, xy \equiv P^p_{Ed}\, xy \wedge (z_y)(\textit{Ed-Stg}\, yz_y \rightarrow (\exists z_x)(\textit{Ed-Stg}\, xz_x \wedge P_{st}\ z_x z_y))$$

In our example the endurant C is the only lifelong part of the endurant A besides A itself.

We can prove that P^t_{Ed}, P^p_{Ed} and P^l_{Ed} are reflexive on the subdomain of endurants and that P^p_{Ed} and P^l_{Ed} are transitive. But we cannot prove that P^l_{Ed} (livelong parthood) is antisymmetric. In other words we cannot prove that if x and y are lifelong parts of each other then they are identical. If desired, this can be required with an additional axiom (AED2):

$$AED2 \quad P^l_{Ed}\, xy \wedge P^l_{Ed}\, yx \rightarrow x = y$$

Finally we prove: if x is a temporary part of y then x and y temporally overlap (TED3); if x is a permanent part of y then y temporally covers x (TED4); and if x is a lifelong part of y then x and y are contemporaneous (TED5).

$$TED3 \quad P^t_{Ed}\, xy \rightarrow \textit{TMPO}_E\ xy$$
$$TED4 \quad P^p_{Ed}\, xy \rightarrow \textit{TCOV}_E\ xy$$
$$TED5 \quad P^l_{Ed}\, xy \rightarrow \textit{CTMP}_E\ xy$$

7 PERDURANCE

Whereas endurants have only endurants as parts and stages have only stages as parts, perdurants can have either perdurants or stages as parts. We define a binary predicate P_{Pd} where $P_{Pd}\ xy$ means: x is a part of the perdurant y. $P_{Pd}\ xy$ holds if and only (i) x is a stage or a perdurant and y is a perdurant; and (ii) for all u and v: if x is located at u and y is located at v then u is a regional part of v ($D_{P_{Pd}}$).

$$D_{P_{Pd}} \quad P_{Pd}\, xy \equiv (\textit{Stg}\ x \vee Pd\, x) \wedge Pd\, y \wedge (u)(v)(L\ xu \wedge L\ yv \rightarrow P\ uv)$$

In the example model, the perdurants *LifeOf_B*, *LifeOf_C*, and *LifeOf_D*, as well as the stages A^{t_1}, B^{t_1}, C^{t_1}, and so on, are all parts of the perdurant *LifeOf_A*.

P_{Pd} is reflexive on the subdomain of perdurants and transitive. We can also prove that if stage x is part of stage y and y is part of the perdurant z then x is a part of the perdurant z (TTP1).

$$TTP1 \quad P_{st}\ xy \wedge P_{Pd}\, yz \rightarrow P_{Pd}\ xz$$

We cannot however prove that P_{Pd} is antisymmetric. To require this we add the following axiom:

$$ATP1 \quad P_{Pd}\ xy \wedge P_{Pd}\ yx \rightarrow x = y$$

Notice that (ATP1) rules out the possibility of co-located but distinct processes, such as the simultaneous heating and rotation of a metal rod. (According to ATP1, the heating and the rotating would be different aspects of the same process.) Thus ATP1 though not untenable, is somewhat controversial and may be not appropriate in every context. If desired it can be weakened or eliminated.

To tie endurants to perdurants, we define the binary relation *LifeOf* to hold between an endurant and a perdurant where, on the intended interpretation, *LifeOf* xy means: perdurant x is the life of endurant y.

$$D_{\textit{LifeOf}} \quad \textit{LifeOf}\, xy \equiv Pd\, x \wedge Ed\, y \wedge (v)(L\ xv \rightarrow (w)(O\ wv \leftrightarrow (\exists u)(L\ yu \wedge O\ uw)))$$

$D_{\textit{LifeOf}}$ tells us that perdurant x is the life of endurant y if and only if x is exactly located at the sum of all spatial regions at which y is exactly located. In the example model, *LifeOf_A* is exactly located at *loc_lf_A*, which is the sum of all spatial regions at which A is located. Similarly, *loc_lf_B*, *loc_lf_C*, and *loc_lf_D* are the sums of all spatial regions occupied by, respectively B, C, and D.

Axiom ATP2 requires that every endurant has a life:

$$ATP2 \quad Ed\, x \rightarrow (\exists y)(\textit{LifeOf}\, yx)$$

We then can prove: no endurant has more than one life (TTP2); any stage of an endurant is part of its life (TTP3); if endurant x is a permanent part of perdurant y then x's life is part of y's life (TTP4); and if x is y's life, then x and y are contemporaneous (TTP5).

$$TTP2 \quad \textit{LifeOf}\, yx \wedge \textit{LifeOf}\, zx \rightarrow y = z$$
$$TTP3 \quad \textit{Ed-Stg}\, xy \wedge \textit{LifeOf}\, zx \rightarrow P_{Pd}\ yz$$
$$TTP4 \quad P^p_{Ed}\ xy \wedge \textit{LifeOf}\, z_x x \wedge \textit{LifeOf}\, z_y y \rightarrow P_{Pd}\ z_x z_y$$
$$TTP5 \quad \textit{LifeOf}\, xy \rightarrow \textit{CTMP}_E\ xy$$

8 CONCLUSIONS

The theory presented in this paper describes time-dependent properties and relations among entities without making explicit reference to time. Consequently, one is not forced to make any commitments about the specific structure of time. For example, nothing prevents one from requiring that time is discrete and interpreting time-slices as having a positive but minimal temporal extension. With this interpretation, time-slices could be spatially maximal slices of spacetime during which nothing changes. Stages would have a minimal temporal extension, but would still represent fixed configurations of the material world.

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