Endurants and perdurants in directly depicting ontologies

Thomas Bittner [a] Maureen Donnelly [a]
Barry Smith [a,b]

[a] *Institute for Formal Ontology and Medical Information Science (IFOMIS), Saarland University*
[b] *Department of Philosophy, University at Buffalo*

We propose an ontological theory that is powerful enough to describe both complex spatio-temporal processes and the enduring entities that participate therein. For this purpose we introduce the notion of a directly depicting ontology.

Directly depicting ontologies are based on relatively simple languages and fall into two major categories: ontologies of type SPAN and ontologies of type SNAP. These represent two complementary perspectives on reality and employ distinct though compatible systems of categories. A SNAP (snapshot) ontology comprehends enduring entities such as organisms, geographic features, or qualities as they exist at some given moment of time. A SPAN ontology comprehends perduring entities such as processes and their parts and aggregates as they unfold themselves through some temporal interval. We give an axiomatic account of the theory of directly depicting ontologies and of the core parts of the meta-ontological fragment within which they are embedded.

Keywords: ontology, spacetime, endurants, perdurants, temporal mereology, substances, processes

1. Introduction

Ontologies are recognized as being of importance in almost all parts of computer science and engineering. They are critical at least for the Semantic Web [6], for data exchange among information systems [10], and for communication between software agents [11]. An important aspect of such applications is that ontologies need to be represented by means of formalisms that guarantee certain desired computational properties. Usually Description Logics (DLs) are employed for this purpose, since they are held to provide the optimal compromise between expressive power and efficiency of the underlying reasoning [1].

T-boxes of Description Logics, i.e., the part of the DL machinery which exclusively deals with classes, when used to represent subsumption relations among universals or classes (such as *living-being*, *vertebrate*, and *human-being*) are analogous to maps in geography. Maps embody a specific type of simplified and therefore highly efficient representation of a certain part of geographic reality. They represent individuals of geographic scale in their spatial relations to each other. T-boxes, similarly, represent universals or classes in their subsumption relations. In both cases the entities represented are considered as they exist at a certain moment in time.

To conceive of a map, or a T-box, in these terms is to conceive it as an inventory of things that exist at a certain moment in time and of some of the properties and relations that obtain between them. From a logical perspective a map can be considered as a vast conjunction of sentences in a language that gives us the facility to express propositions about individuals of certain simple sorts. A T-box, similarly, can be seen as a vast conjunction of sentences expressing propositions about subsumption relations among universals and classes.

Such languages are *directly depicting* in the sense of Wittgenstein's *Tractatus* [14,13]. This means that all the terms in such a language refer to entities in reality and the corresponding sentences consist, in effect, of depictions of the arrangements of such entities: the spatial arrangement of individuals in the case of maps and the hierarchical arrangement of classes and universals in the case of T-boxes.

Directly depicting languages are, given their limited expressiveness, not suitable for the expression of complex statements about ontological structures. A map itself does not contain statements about how the map itself relates to reality. We argue here, drawing on ideas set out in [8], that we need to distinguish:

- directly depicting ontologies (DDOs), which are formulated in conformity with the principles of a directly depicting language, and
- meta-ontology, which draws on greater expressive resources and expresses (*inter alia*) properties of and relationships between the different (directly depicting) ontologies.

Consider again the case of a map. Here some meta-level information is attached to the map in the form of scale, legend, and perhaps a label specifying the method of projection. If you use the map then you pay attention to these matters (which might be rather complex) only peripherally. The focus of your attention is rather on what is depicted on the map itself, on estimating distances, making decisions as to which route to take, and so on.

Similarly in the case of T-boxes. Here we have statements like:

$$Pneumonia \equiv Disease \sqcap \exists hasLocation.Lung,$$

which describes pneumonia as a subclass of diseases located in the lung. What cannot be described in a T-box framework are the properties of the individuals which instantiate these classes or the properties the *hasLocation* relation itself. As in the case of the map, such information needs to be provided on a meta-level in a language of higher expressive power.

Another problem is the static character of directly depicting ontologies such as maps. Many phenomena in reality are in one or other respect dynamic in nature. Most phenomena in reality fall into one or other of two disjoint classes of *endurants*, on the one hand, and *perdurants*, on the other. Endurants are entities which exist in full in every instant at which they exist at all. Perdurants are entities which unfold themselves over time in successive temporal parts or phases. This dichotomy needs to be reflected in our treatment of directly depicting ontologies.

We thus distinguish directly depicting ontologies of two sorts:

- SNAP (for snapshot) ontologies, which represent enduring entities (such as organisms, functions, qualities, dispositions) as they exist at a certain moment of time;
- SPAN ontologies, which represent perduring entities (such as processes, actions, events) from what we can think of as a god's eye (or atemporal) perspective.

In both cases, we have ontologies employing very simple directly depicting languages. It is the purpose of this paper to define a formal framework of meta-ontology that will allow us to show how these two types of ontologies are related together.

2. Entities and their location in spacetime

We first sketch that part of meta-ontology which deals with spatial and spatio-temporal entities, their locations in spacetime, and their different modes of persistence. We focus here on individuals. For a formal treatment of universals and classes see for example [3]. For extended treatments of endurants and perdurants see [2], [7] and [9].

We use a sorted first-order predicate logic with identity, and we assume that the domains of our models are divided into three disjoint sorts: *regions*, *material entities*, and *abstract entities*. Regions are parts of four-dimensional space and can be of any dimension (less than five), shape, and size. We use $u, v,$ and w as variables for regions. Spatio-temporal entities, i.e., endurants, perdurants, and stages, are located in spacetime. We use x, y, z as variables for spatio-temporal entities. As variables for abstract entities, i.e., entities that are not located in spacetime, we use $\alpha, \beta, \gamma, \omega_1, \omega_2,$ and ω_3. Abstract entities include directly depicting ontologies and their constituents.

All quantification is restricted to a single sort. Restrictions on quantification will be understood from conventions on variable use. Leading universal quantifiers are generally omitted.

2.1. Endurants, perdurants, and stages

Besides persisting entities such as endurants and perdurants which exist at multiple moments in time, we assume also *stages* which are instantaneous parts of perdurants [12]. Particularly important are stages which are instantaneous parts of the lives of endurants. At every moment an endurant exists, there is a stage which is the slice of the endurant's life that is limited to this moment in time.

As an example consider Figure 1. Instead of considering a four-dimensional model of spacetime, we use the subset of points of the plane which is specified by the coordinates t and s that satisfy the constraint $0 \leq t \leq t_4$ & $0 \leq s \leq 4$. In set-theoretic terms we write $\mathbf{ST} = \{(s, t) \mid 0 \leq t \leq t_4$ & $0 \leq s \leq 4\}$. The horizontal dimension in the figure is interpreted as temporal and the vertical dimension is interpreted as spatial.

The left part of Figure 1 shows an endurant, the line-shaped entity A, at times t_1, t_2, and t_3. The life of the endurant A is visualized as the solid two-dimensional region, *LifeOf_A*, depicted in the right part of the figure. It shows that A comes into exis-

tence at t_1 and that it continues to exist until t_4. The lives of C, B and D are proper parts of the life of A and are respectively located at the spacetime regions $loc\_lf\_C = \{(s,t) \mid t_1 \le t \le t_4 \ \& \ 1 \le s \le 2\}$, $loc\_lf\_B = \{(s,t) \mid t_1 \le t \le t_5 \ \& \ 2 \le s \le 3\}$, and $loc\_lf\_D = \{(s,t) \mid t_6 \le t \le t_4 \ \& \ 2 \le s \le 3\}$ shown in the right part of Figure 1. The life of A, *LifeOf_A*, is located at the region *loc_lf_A*, which is the union of the regions *loc_lf_B*, *loc_lf_C*, and *loc_lf_D*.

Both figures indicate that during its life A undergoes changes in its mereological structure. We also include in our model the following stages of the lives of the endurants A, C, B and D: A^{t_1}, A^{t_2}, A^{t_3}, C^{t_1}, C^{t_2}, C^{t_3}, B^{t_1}, and D^{t_3}. For example, A^{t_1} is the instantaneous slice of A's life at t_1, A^{t_2} is the instantaneous slice of A's life at t_2, and so on.
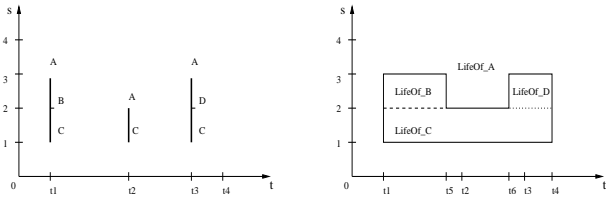

Fig. 1. The endurant A in different time-slices (left) and the life of A (right).

At a given moment during its life an endurant is exactly co-located with the stage of its life at that moment. For example, the location of A at t_1 is the location of the stage A^{t_1}: the region $loc\_A\_t_1 = \{(s,t) \mid t = t_1 \ \& \ 1 \le s \le 3\}$. The stages C^{t_1} and B^{t_1} are located at the regions $loc\_C\_t_1 = \{(s,t) \mid t = t_1 \ \& \ 1 \le s \le 2\}$ and $loc\_B\_t_1 = \{(s,t) \mid t = t_1 \ \& \ 2 \le s \le 3\}$. The stages A^{t_2} and C^{t_2} are both located at the region $loc\_A\_t_2 = loc\_C\_t_2 = \{(s,t) \mid t = t_2 \ \& \ 1 \le s \le 2\}$. And so on.

We will use this example as a model for our formal theory of stages, endurants, perdurants, and their spatio-temporal locations.

2.2. The mereology of regions

2.2.1. Regional parthood

We start by introducing the binary predicate P, where $P\,uv$ is interpreted as 'the region u is a part of the region v'. We also say that u is a *regional part* of v. We add axioms which make P reflexive (ARM1), antisymmetric (ARM2), and transitive (ARM3), i.e., a partial ordering.

$$ARM1 \ \ P\,uu \qquad \begin{array}{l} ARM2 \ \ P\,uv \wedge P\,vu \rightarrow u = v \\ ARM3 \ \ P\,uv \wedge P\,vw \rightarrow P\,uw \end{array}$$

We continue by introducing the binary predicates PP for proper parthood (D_{PP}) and O for overlap (D_O).

$$\begin{array}{l} D_{PP} \ \ PP\,uv \equiv P\,uv \wedge \neg u = v \\ D_O \ \ \ \ O\,uv \equiv (\exists w)(P\,wu \wedge P\,wv) \end{array}$$

We then add an axiom stating that if everything that overlaps u also overlaps v then u is a part of v (ARM4).

$$ARM4 \ \ (w)(O\,wu \rightarrow O\,wv) \rightarrow P\,uv$$

We then define spacetime as a predicate which holds for a region which has all regions as parts (D_{ST}). If there is a such a region then it is unique (TRM2). Finally we add an axiom stating that such a maximal region exists (ARM5) and we use the symbol $\mathcal{ST}$ to refer to it.

$$\begin{array}{ll} D_{ST} \ \ \ \ ST\,u \equiv (v)P\,vu \\ TRM2 \ \ ST\,u \wedge ST\,v \rightarrow u = v \end{array} \quad ARM5 \ (\exists u)ST\,u$$

On the intended interpretation in our example domain, spacetime is the set **ST**. Region variables range over all nonempty subsets of **ST**, and P is the subset relation, $\subseteq$.

2.2.2. Spatial regions and time-slices

We add as a new primitive the unary predicate SR. On the intended interpretation $SR\,u$ means: region u is a spatial region. Spatial regions are parts of spacetime which are either not extended at all in time or, in case of discrete time, do not extend beyond a minimal time unit. In the example model, $loc\_A\_t_1$, $loc\_B\_t_1$, and $loc\_C\_t_1$ are all spatial regions. More generally, any subset of **ST** consisting of points with a fixed time coordinate is a spatial region.

Time-slices are maximal spatial regions. In other words, a time-slice is a spatial region u such that u overlaps a spatial region v only if v is part of u (D_{TS}).

$$D_{TS} \ \ TS\,u \equiv SR\,u \wedge (v)(SR\,v \wedge O\,uv \rightarrow P\,vu)$$

In our example model, for any fixed $\mathbf{t}$ with $0 \le \mathbf{t} \le t_4$ the set $\{(s,\mathbf{t}) \mid 0 \le s \le 4\}$ is a time-slice.

We add axioms requiring that any part of a spatial region is a spatial region (AR1), every region overlaps some time-slice (AR2), and spacetime is not a spatial region (AR3).

$AR1 \;\; SR\,u \wedge P\,vu \rightarrow SR\,v$
$AR2 \;\; (\exists u)(TS\,u \wedge O\,uv)$ $AR3 \;\; \neg SR\,\mathcal{ST}$

We then can prove that there is at least one time-slice (TR1) and that any spatial region is a proper part of spacetime (TR2).

$TR1 \;\; (\exists u)TS\,u$ $TR2 \;\; SR\,u \rightarrow PP\,u\mathcal{ST}$

We can also prove that distinct time-slices do not overlap (TR3), u is a spatial region if and only if u is part of some time-slice (TR4), each region is part of at most one time-slice (TR5).

$TR3 \;\; TS\,u \wedge TS\,v \wedge O\,uv \rightarrow u = v$
$TR4 \;\; SR\,u \leftrightarrow (\exists v)(TS\,v \wedge P\,uv)$
$TR5 \;\; P\,uv \wedge P\,uw \wedge TS\,v \wedge TS\,w \rightarrow v = w$

It follows from TR4 and TR5 that each spatial region is part of a unique time-slice. Finally we can prove that spacetime, $\mathcal{ST}$, is the sum of all time slices, i.e., any region overlaps $\mathcal{ST}$ if and only if it overlaps some time-slice (TR6).

$TR6 \;\; O\,u\mathcal{ST} \leftrightarrow (\exists w)(TS\,w \wedge O\,uw)$

We define a *temporal region* to be any region that is not a spatial region (D_{TR}).

$D_{TR} \;\; TR\,u \equiv \neg SR\,u$ $\begin{array}{l} TR7 \;\; TR\,\mathcal{ST} \\ TR8 \;\; TR\,u \wedge P\,uv \rightarrow TR\,v \end{array}$

We can prove that spacetime is a temporal region (TR7) and that if u is a temporal region and u is a part of v then v is a temporal region (TR8). In our example model, *loc_lf_A*, *loc_lf_B*, *loc_lf_C*, *loc_lf_D*, and **ST** are all temporal regions. Note that a temporal region need not be extended in space. In the example model, $\{(1, \mathbf{t}) \mid t_1 < \mathbf{t} < t_3\}$ is a one-dimensional temporal region.

Finally we can prove that u is a temporal region if and only if it overlaps more than one time slice (TR9).

$TR9 \;\; TR\,u \leftrightarrow (\exists v)(\exists w)(TS\,v \wedge TS\,w \wedge \neg v = w \wedge$
         $O\,uv \wedge O\,uw)$

If desired a linear ordering on the subdomain of time-slices can be added to the theory. With such an ordering we can say that one region temporally precedes another, succeeds another, and so on.

2.3. Spatio-temporal entities and their location

The second sort in our formal theory are material endurants, perdurants, and stages, which we call *spatio-temporal entities*. We use x, y, and z as variables for such entities. We introduce the primitive binary predicate L where on the intended interpretation $L\,xu$ means: spatio-temporal entity x is exactly located at region u [5]. In other words, x takes up the whole region u but does not extend beyond it. We require that every spatio-temporal entity is exactly located at some region (AL1).

$AL1 \;\; (\exists u)(L\,xu)$

We define that two spatial regions u and v are simultaneous if and only if they are part of the same time-slice (D_{SIMU})

$D_{SIMU} \;\; SIMU\,uv \equiv (\exists w)(TS\,w \wedge P\,uw \wedge P\,vw)$

and require that no spatio-temporal entity is exactly located at distinct parts of the same time-slice (AL2).

$AL2 \;\; L\,xu \wedge L\,xv \wedge SIMU\,uv \rightarrow u = v$

We say that an entity is *present-at* a time-slice if and only if it is located at a region that overlaps that time-slice (D_{PrAt}).

$D_{PrAt} \;\; PrAt\,xu \equiv TS\,u \wedge (\exists v)(L\,xv \wedge O\,uv)$

The distinct spatio-temporal character of endurants, perdurants, and stages manifests itself in the different ways they are located in spacetime. On the intended interpretation the relation $L\,xu$ holds for a perdurant x iff u is the unique *temporal* region which x exactly occupies; for a stage x, $L\,xu$ holds iff u is the unique *spatial* region which x exactly occupies; for an endurant x, $L\,xu$ holds iff u is any *spatial* region that x exactly occupies at any time during its existence. In our example model, the endurant A is exactly located at the spatial regions *loc_A_t*$_1$, *loc_A_t*$_2$, and *loc_A_t*$_3$ The perdurant *LifeOf_A* is exactly located at the single temporal region *loc_lf_A*. The stage A^{t_1} is exactly located at the single spatial region *loc_A_t*$_1$, the stage A^{t_2} is exactly located at the single spatial region *loc_A_t*$_2$, and so on.

We now define that a spatio-temporal entity is a *stage* if and only if it is located at a single region and that region is a spatial region (D_{Stg}). Stages are instantaneous spatial entities in the sense that they are con-

fined to a single time-slice. A spatio-temporal entity is *persistent* iff it is not confined to a single time-slice ($D_{P_{st}}$).

D_{Stg} $Stg\,x \equiv (u)(v)(L\,xu \wedge L\,xv \rightarrow (SR\,u \wedge u = v))$
D_{Pst} $Pst\,x \equiv (\exists u)(\exists v)(L\,xu \wedge L\,xv \wedge \neg SIMUuv)$

Consider the left part of Figure 1. The endurant A is a persistent entity. It is located at the regions $loc\_A\_t_1$, $loc\_A\_t_2$ and $loc\_A\_t_3$ which are all parts of different time-slices and therefore not simultaneous spatial regions. The life of A is located at the region loc_lf_A. Since loc_lf_A is a temporal region it does not stand in the *SIMU* relation with itself. Consequently, the life of A is a persistent entity.

We can prove that no stage is persistent (TL1) and that if x is located at a temporal region then x is persistent (TL2).

$TL1$ $Stg\,x \rightarrow \neg Pst\,x$
$TL2$ $(\exists u)(TR\,u \wedge L\,xu) \rightarrow Pst\,x$

The sub-domain of persistent entities can be divided into endurants and perdurants. We define that x is an *endurant* iff x is a persistent entity which is only located at spatial regions (D_{Ed}). On the other hand, x is a *perdurant* iff it is a spatio-temporal entity which has a unique location that is a temporal region (D_{Pd}).

D_{Ed} $Ed\,x \equiv Pst\,x \wedge (u)(L\,xu \rightarrow SR\,u)$
D_{Pd} $Pd\,x \equiv (u)(v)(L\,xu \wedge L\,xv \rightarrow (TR\,u \wedge u = v))$

In our example, A is an endurant – it is located at several spatial regions in different time-slices. The life of A, on the other hand, is a perdurant – it is located at a unique temporal region.

We can prove that endurants do not have a unique location (TL3), and that nothing is both an endurant and a perdurant (TL4).

$TL3$ $Ed\,x \rightarrow (\exists u)(\exists v)(\neg u = v \wedge L\,xu \wedge L\,xv)$
$TL4$ $Ed\,x \rightarrow \neg Pd\,x$

Thus the subdomains of stages, endurants, and perdurants are pairwise disjoint. Finally we add an axiom requiring that every spatio-temporal entity is either a stage, an endurant, or a perdurant (AL3).

$AL3$ $Stg\,x \vee Ed\,x \vee Pd\,x$

It follows from AL3 that no spatio-temporal entity can be exactly located at distinct temporal regions or located both at a spatial and at a temporal region.

2.4. Lives of endurants

To tie endurants to perdurants, we define the binary relation *LifeOf* to hold between an endurant and a perdurant where, on the intended interpretation, *LifeOf xy* means: perdurant x is the life of endurant y.

D_{LifeOf} $LifeOf\,xy \equiv Pd\,x \wedge Ed\,y \wedge$
    $(v)(L\,xv \rightarrow (w)(O\,wv \leftrightarrow (\exists u)(L\,yu \wedge O\,uw)))$

D_{LifeOf} tells us that perdurant x is the life of endurant y if and only if x is exactly located at the sum of all spatial regions at which y is exactly located. In the example model, *LifeOf_A* is exactly located at loc_lf_A, which is the sum of all spatial regions at which A is located. Similarly, loc_lf_B, loc_lf_C, and loc_lf_D are the sums of all spatial regions occupied by, respectively, B, C, and D.

Axiom ATP2 requires that every endurant has a life:

$ATP2$ $Ed\,x \rightarrow (\exists y)(LifeOf\,yx)$

3. The mereology of spatio-temporal entities

3.1. Stages

We now define parthood among stages as follows: x is a stage-part of y if and only if x and y are stages and for all u and v, if x is located at u and y is located at v then u is a regional part of v ($D_{P_{st}}$).

$D_{P_{st}}$ $P_{st}\,xy \equiv Stg\,x \wedge Stg\,y \wedge$
    $(u)(v)(L\,xu \wedge L\,yv \rightarrow P\,uv)$

In other words, stage x is a stage-part of stage y if and only if the unique spatial region at which x is located is a part of the unique spatial region at which y is located. In the example model, both B^{t_1} and C^{t_1} are stage-parts of A^{t_1}. The stage of my hand at this moment is a stage-part of the stage of me at this moment.

We can prove that x is a stage if and only if x is a stage-part of itself (TST1) and that stage-parthood is transitive (TST2).

$TST1$ $Stg\,x \leftrightarrow P_{st}\,xx$
$TST2$ $P_{st}\,xy \wedge P_{st}\,yz \rightarrow P_{st}\,xz$

We cannot, however, prove that P_{st} is antisymmetric. In order to force co-located stages to be identical, we add an axiom of antisymmetry (AST1).

$$AST1 \ P_{st} \, xy \wedge P_{st} \, yx \rightarrow x = y$$

Thus, in our example model the co-located stages C^{t_2} and A^{t_2} must be identical.

3.2. Endurants

The way an endurant endures through time is characterized by its relation to stages in different time-slices. In order to capture this mode of persistence, we introduce the binary predicate *Ed-Stg*, where *Ed-Stg xy* (y is a stage of the endurant x) if and only if (i) x is an endurant and y is a stage; and (ii) x and y are both located at some spatial region ($D_{Ed\text{-}Stg}$). It follows immediately that *Ed-Stg* is irreflexive and asymmetric.

$$D_{Ed\text{-}Stg} \ Ed\text{-}Stg \, xy \equiv Ed \, x \wedge Stg \, y \wedge (\exists u)(L \, yu \wedge L \, xu)$$

Consider our example model. Here we have *Ed-Stg* AA_{t^1}, *Ed-Stg* AA_{t^2}, *Ed-Stg* AA_{t^3}, *Ed-Stg* BB_{t^1}, *Ed-Stg* CC_{t^1}, *Ed-Stg* CC_{t^2}, *Ed-Stg* CC_{t^3}, and *Ed-Stg* DD_{t^3}.

We can prove that every endurant has at most one stage per time-slice (TED1).

$$TED1 \ Ed\text{-}Stg \, xz \wedge Ed\text{-}Stg \, xy \wedge$$
$$(\exists w)(PrAt \, zw \wedge PrAt \, yw) \rightarrow y = z$$

Notice, that one single stage can be the stage of several different endurants. Consider our example model. Here A^{t_2} and C^{t_2} are identical but this stage is the stage of distinct endurants: A and C. Consider a statue and the bronze of which it is constituted. The statue and the portion of bronze are distinct endurants which have identical stages in some, but not all, time-slices.

We add an axiom stating that wherever an endurant x is located there exists a stage which is the stage of x for this time-slice (AED1).

$$AED1 \ (Ed \, x \wedge L \, xu) \rightarrow (\exists y)(Ed\text{-}Stg \, xy \wedge L \, yu)$$

Because each endurant is located at multiple regions, parthood relations among endurants are more complicated than parthood relations among stages or among perdurants. We now define a number of distinct parthood relations between endurants: The endurant x is a *temporary part* of the endurant y *at time-slice* u iff there exists a stage of x which is present at u and which is part of a stage of y ($D_{P_{Ed}^t}$).

$$D_{P_{Ed}^t} \ P_{Ed}^t \, xyu \equiv Ed \, x \wedge Ed \, y \wedge (\exists z_x)(\exists z_y)$$
$$(Ed\text{-}Stg \, xz_x \wedge Ed\text{-}Stg \, yz_y \wedge PrAt \, uz_x \wedge P_{st} \, z_x z_y)$$

The endurant x is a *temporary part* of the endurant y iff there exists a time-slice at which x is a temporary part of y ($D_{P_{Ed}^t}$).

$$D_{P_{Ed}^t} \ P_{Ed}^t \, xy \equiv (\exists u)P_{Ed}^t \, xyu$$

In our example model A, B, C, are all temporary parts of A. My blood cells, my wisdom teeth, and my beard are temporary parts of me.

The endurant x is a *permanent part* of the endurant y iff every stage of x is a part of a stage of y ($D_{P_{Ed}^p}$).

$$D_{P_{Ed}^p} \ P_{Ed}^p \, xy \equiv Ed \, x \wedge Ed \, y \wedge (z_x)(Ed\text{-}Stg \, xz_x \rightarrow$$
$$(\exists z_y)(Ed\text{-}Stg \, yz_y \wedge P_{st} \, z_x z_y))$$

In our example, the endurants A, C, B and D are temporary parts of A as well as permanent parts of A. Most of my blood cells and my heart are permanent parts of me. My wisdom teeth (which still exist but have been removed) are not permanent parts of me.

The endurant x is a *lifelong part* of the endurant y iff x is a permanent part of y and every stage of y has a stage of x as part ($D_{P_{Ed}^l}$).

$$D_{P_{Ed}^l} \ P_{Ed}^l \, xy \equiv P_{Ed}^p \, xy \wedge (z_y)(Ed\text{-}Stg \, yz_y \rightarrow$$
$$(\exists z_x)(Ed\text{-}Stg \, xz_x \wedge P_{st} \, z_x z_y))$$

In our example the endurant C is the only lifelong part of the endurant A besides A itself.

We can prove that P_{Ed}^t, P_{Ed}^p and P_{Ed}^l are reflexive on the sub-domain of endurants and that P_{Ed}^p and P_{Ed}^l are transitive. But we cannot prove that P_{Ed}^l (lifelong parthood) is antisymmetric. In other words we cannot prove that if x and y are lifelong parts of each other then they are identical. If desired, this can be required with an additional axiom (AED2):

$$AED2 \ P_{Ed}^l \, xy \wedge P_{Ed}^l \, yx \rightarrow x = y$$

3.3. Perdurants

Whereas endurants have only endurants as parts and stages have only stages as parts, perdurants can have either perdurants or stages as parts. We define a binary predicate P_{Pd} where $P_{Pd} \, xy$ means: x is a part of the perdurant y. $P_{Pd} \, xy$ holds if and only (i) x is a stage or a perdurant and y is a perdurant; and (ii) for all u and v: if x is located at u and y is located at v then u is a regional part of v ($D_{P_{Pd}}$).

$$D_{P_{Pd}} \quad P_{Pd}\, xy \equiv (Stg\ x \vee Pd\ x) \wedge Pd\ y \wedge$$
$$(u)(v)(L\ xu \wedge L\ yv \rightarrow P\ uv)$$

In the example model, the perdurants *LifeOf_B*, *LifeOf_C*, and *LifeOf_D*, as well as the stages A^{t_1}, B^{t_1}, C^{t_1}, and so on, are all parts of the perdurant *LifeOf_A*.

P_{Pd} is reflexive on the subdomain of perdurants and transitive. We can also prove: if stage x is part of stage y and y is part of the perdurant z then x is a part of the perdurant z (TTP1); any stage of an endurant is part of its life (TTP2); if endurant x is a permanent part of perdurant y then x's life is part of y's life (TTP3).

$TTP1 \quad P_{st}\, xy \wedge P_{Pd}\, yz \rightarrow P_{Pd}\, xz$
$TTP2 \quad Ed\text{-}Stg\, xy \wedge LifeOf\, zx \rightarrow P_{Pd}\, yz$
$TTP3 \quad P_{Ed}^{p}\, xy \wedge LifeOf\, z_x x \wedge LifeOf\, z_y y \rightarrow P_{Pd}\, z_x z_y$

We cannot however prove that P_{Pd} is antisymmetric. To require this we add the following axiom:

$$ATP1 \quad P_{Pd}\, xy \wedge P_{Pd}\, yx \rightarrow x = y$$

We then can prove that no endurant has more than one life (TTP4).

$$TTP4 \quad LifeOf\, yx \wedge LifeOf\, zx \rightarrow y = z$$

Notice that (ATP1) rules out the possibility of co-located but distinct processes, such as the simultaneous heating and rotation of a metal rod. (According to ATP1, the heating and the rotating would be different aspects of the same process.) Thus ATP1, though not untenable, is somewhat controversial and may be not appropriate in every context. If desired it can be weakened or eliminated (in this case, however, Theorems TTP4 and TOB1 need to be replaced by axioms).

4. Directly depicting ontologies (DDOs)

4.1. DDOs, constituents, and projection

We now consider directly depicting ontologies (DDOs) as subjects of study from a meta-ontological perspective. Specific representations of such ontologies include maps, figures, lists of names, factory inventories, biological taxonomies, engineering partonomies, component catalogues, and so on. In this section we focus on the relationships between directly depicting ontologies and their constituents. For example, if a DDO is represented as a map, then the constituents of the ontology are represented as symbols/features on the map. Between a constituent represented by a symbol for a church, for example, and the church itself there holds what we shall call a relation of ontological projection.

Intuitively, we can compare a directly depicting ontology, α, with a rig of spotlights projecting down onto an orchestra during the performance of a symphony. Each constituent of α corresponds to some spotlight in the rig. Some constituents (spotlights) will project upon single players, others onto whole sections of the orchestra (string, wind, percussion, and so forth). One constituent (spotlight) will project upon the orchestra as a whole. Note that the spotlights do not hereby create the objects which they cast into relief. When once the rig has been set, and the members of the orchestra have taken their places, then it will be an entirely objective matter which objects (individuals and groups of individuals) are located in which illuminated constituents.

Consider the map in Figure 2. Constituents of this ontology are abstract entities represented by regions that have the shape of corresponding federal states. The relation of ontological projection then holds between the constituent represented Montana-shaped region (additionally labeled 'Montana') and the federal state Montana, between the constituent represented by the Idaho-shaped region and the the federal stated Idaho, and so on. This relation of ontological projection is a generalization of the more familiar relation between a name such as 'Mount Everest' and the corresponding mountain.

Fig. 2. A map of parts of the United States.

Consider the case in which the DDO is a taxonomic structure of the sort that can be represented within a DL T-box:

Fruit $\sqsubseteq$ *Food*
Vegetable $\sqsubseteq$ *Food*
Fruit $\sqcap$ *Vegetable* $\sqsubseteq$ $\perp$

Here the constituents of the underlying ontology are represented symbolically by means of either primitive

symbols or complex descriptions. Targets of their projection are universals or classes of entities in reality. Projection is the semantic relation of assignment which underlies the intended interpretation.

Having given these examples we will present a general framework for classifying directly depicting ontologies and for describing the relationships among them and their constituents.

4.2. Directly depicting ontologies and their constituents

Directly depicting ontologies and their constituents are abstract entities and we use the Greek letters $\alpha, \beta, \gamma, \omega$ as variables for abstract entities. We write $\mathcal{O}\,\alpha$ in order to signify that α is a directly depicting ontology. We introduce the primitive binary relation, CO, which in our intended interpretation holds between directly depicting ontologies and their constituents, i.e., $CO\,\omega\alpha$ is interpreted as 'ω is a constituent of the directly depicting ontology α'.

We then add axioms stating that: if ω is a constituent of α then α is a DDO and ω is not a DDO (AO1). It immediately follows that CO is irreflexive and asymmetric. We also require that no ontology is empty (AO2), i.e., that every ontology has at least one constituent, and that there exists at least one DDO (AO3).

$$(AO1)\quad CO\,\omega\alpha \to \mathcal{O}\,\alpha \wedge \neg\mathcal{O}\,\omega$$
$$(AO2)\quad \mathcal{O}\,\alpha \to (\exists\omega)CO\,\omega\alpha$$
$$(AO3)\quad (\exists\alpha)\mathcal{O}\alpha$$

We define that an abstract entity is a constituent if and only if it is a constituent of some ontology (D_{Cst}).

$$D_{Cst}\quad Cst\,\omega \equiv (\exists\alpha)CO\,\omega\alpha$$

We then add an axiom stating that every abstract entity is is either a DDO or a constituent (AO4).

$$(AO4)\quad \mathcal{O}\,\alpha \vee Cst\,\alpha$$

We continue by introducing the binary predicate Π, where $\Pi\,\omega x$ signifies that the constituent ω projects onto the spatio-temporal entity x. We then demand that something is a constituent if and only if it projects upon (refers to) some spatio-temporal entity, i.e., constituents project to entities and ontologies do not project to entities (AO5). Finally we add an axiom ensuring that every constituent projects onto at most one entity (AO6).

$$(AO5)\quad Cst\,\omega \leftrightarrow (\exists x)(\Pi\,\omega x)$$
$$(AO6)\quad \Pi\,\omega x \wedge \Pi\,\omega y \to x = y$$

It follows that Π is a function from the sub-domain of constituents into the sub-domain of spatio-temporal entities.

An ontology α *acknowledges* a spatio-temporal entity x if and only if there is some constituent ω of α which projects onto x (D_{Ackn}).

$$(D_{Ackn})\qquad Ackn\,\alpha x \equiv (\exists\omega)(CO\,\omega\alpha \wedge \Pi\,\omega x)$$

Consider again the map in Figure 2. The ontology represented by this map acknowledges the federal states Montana, Idaho, Wyoming, Washington, North Dakota, South Dakota, and so on.

4.3. Directly depicting ontologies of spatio-temporal entities

Corresponding to the two different modes of persistence – endurance and perdurance – we now introduce two kinds of directly depicting ontologies: DDOs of type SNAP, acknowledging enduring entities that are all present at a certain time-slice; and DDOs of type SPAN, acknowledging perdurants and stages, i.e., entities that are *uniquely* located.

Formally we introduce a binary predicate SNAP and a unary predicate SPAN. SNAP αu is defined to hold between a directly depicting ontology α and time-slice u if and only if every entity acknowledged by α is an endurant that is present at u (D_{SNAP}).

$$D_{\text{SNAP}}\quad \text{SNAP}\,\alpha u \equiv \mathcal{O}\,\alpha \wedge TS\,u \wedge$$
$$(x)(Ackn\,\alpha x \to (Ed\,x \wedge PrAt\,xu))$$

The unary predicate SPAN is defined to hold for directly depicting ontologies which acknowledge only perdurants or stages (D_{SPAN}).

$$D_{\text{SPAN}}\quad \text{SPAN}\,\alpha \equiv \mathcal{O}\,\alpha \wedge$$
$$(\forall x)(Ackn\,\alpha x \to (Pd\,x \vee Stg\,x))$$

We then can prove that no directly depicting ontology can be both of type SNAP and of type SPAN (TSS1).

$$TSS1\qquad \neg(\text{SNAP}\,\alpha u \wedge \text{SPAN}\,\alpha)$$

Finally we demand that all directly depicting ontologies are either of type SNAP or SPAN (ASNSP1).

$$ASNSP1\quad \mathcal{O}\,\alpha \to ((\exists u)(\text{SNAP}\,\alpha u) \vee \text{SPAN}\,\alpha)$$

ASNSP1 reflects the basic ontological distinction between perduring and enduring entities at the level of DDOs.

5. Parthood in directly depicting ontologies

In the previous section we characterized constituents of a directly depicting ontology as abstract entities which project onto real entities outside the ontology. DDOs, however, are not just collections of constituents. They also have a structure, i.e., there are relations that hold among constituents. Relations among the constituents of a DDO are included in that DDO. In this section we will show how relations between the constituents of a DDO mirror the relations between the entities to which they project.

DDOs are specific granular partitions in the sense of [4]. The theory of granular partitions has two main components:

– theory A concerns the way constituents of ontologies are organized into hierarchical structures (the nested boxes in Figure 3).
– theory B concerns the way these constituent-structures project onto reality (in Figure 3 indicated by the arrows connecting constituents to portions of reality).
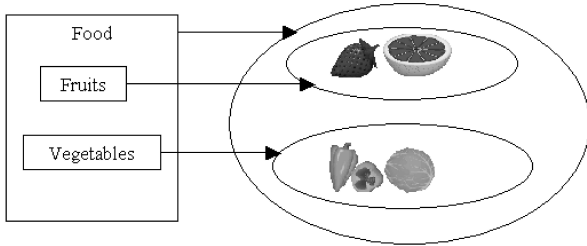

Fig. 3. Relationships between constituents and entities.

Consider a directly depicting ontology of type SNAP whose constituents target parts of the body of some human being named Tom. The DDO may present Tom's body as subdivided into head, torso, and limbs, and Tom's limbs as subdivided into left leg, left arm, right leg, and right arm. The representation of such a DDO as a tree is given in the left part of Figure 4.

In this paper, we make the simplifying assumption that all relations among the constituents of a DDO accurately reflect relations among entities in the spatio-temporal world to which the constituents project. Also,
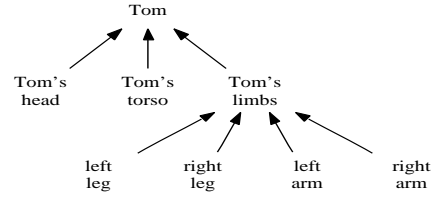

Fig. 4. Hierarchical subdivision of the body of Tom.

we will only consider representations of parthood relations. Notice, however, that other relations can be represented in DDOs. For example DDOs portrayed as maps represent also topological relations like connection, and tangential and interior parthood.

The underlying reason for subdividing DDOs into SNAP and SPAN ontologies has to do with the difference between parthood relations among *uniquely* located entities (stages and perdurants) and parthood relations among multiply located entities (endurants). The SPAN ontologies represent time-independent views of the time-independent parthood relations P_{st} and P_{Pd} among stages and perdurants. The SNAP ontologies give us time-restricted views of the time-dependent parthood relations among endurants. When α is a SNAP ontology for time-slice u, then α represents parthood relations that hold among the endurants acknowledged by α at u.

5.1. The sub-constituent relation

We introduce the primitive sub-constituent relation $\preceq$, which holds between two constituents and an ontology. On the intended interpretation, $\omega_1 \preceq_\alpha \omega_2$ means that ω_1 is a sub-constituent of ω_2 in ontology α. We demand that if ω_1 is a sub-constituent of ω_2 in ontology α then both, ω_1 and ω_2 are constituents of α (AOA0).

$$(AOA0) \ \ \omega_1 \preceq_\alpha \omega_2 \rightarrow CO \ \omega_1\alpha \wedge CO \ \omega_2\alpha$$

If $\omega_1 \preceq_\alpha \omega_2$ holds then we also say that ω_1 is an α-sub-constituent of ω_2.

We then add axioms ensuring that the sub-constituent relation, when restricted to a single DDO, is reflexive (AOA1), antisymmetric (AOA2), and transitive (AOA3), i.e., that is a partial ordering.

(AOA1) $CO \ \omega\alpha \rightarrow \omega \preceq_\alpha \omega$
(AOA2) $(\omega_1 \preceq_\alpha \omega_2 \wedge \omega_2 \preceq_\alpha \omega_1) \rightarrow \omega_1 = \omega_2$
(AOA3) $(\omega_1 \preceq_\alpha \omega_2 \wedge \omega_2 \preceq_\alpha \omega_3) \rightarrow \omega_1 \preceq_\alpha \omega_3$

Using the sub-constituent relation $\preceq$ we define the relations proper sub-constituent ($D_\prec$), constituent-overlap ($D_{O_\preceq}$), and introduce binary predicates which distinguish the root constituent (D_R) and the atoms (D_{At}) of a DDO.

$D_\prec$ $\omega_1 \prec_\alpha \omega_2 \equiv \omega_1 \preceq_\alpha \omega_2 \wedge \neg(\omega_1 = \omega_2)$
$D_{O_\preceq}$ $O_\preceq \omega_1 \omega_2 \alpha \equiv (\exists \omega)(\omega \preceq_\alpha \omega_1 \wedge \omega \preceq_\alpha \omega_2)$
D_R $R\, \omega \alpha \equiv CO\, \omega \alpha \wedge (\forall \omega_1)(CO\, \omega_1 \alpha \rightarrow \omega_1 \preceq_\alpha \omega)$
D_{At} $At\, \omega \alpha \equiv CO\, \omega \alpha \wedge \neg(\exists \omega_1)(\omega_1 \prec_\alpha \omega)$

The proper α-sub-constituent relation $\omega_1 \prec_\alpha \omega_2$ holds if ω_1 is a α-sub-constituent of ω_2 but ω_1 and ω_2 are distinct. $O_\preceq \omega_1 \omega_2 \alpha$ is the relation of α-overlap between constituents ω_1 and ω_2 of the DDO α. The relation $R\, \omega \alpha$ holds if and only if ω is a root in the DDO α. The relation $At\, \omega \alpha$ (ω is an atom within α) holds for constituents of α without proper sub-constituents in α.

In AOA4, we demand that every directly depicting ontology α has a root constituent. AOA5 ensures that if everything that α-overlaps ω_1 also α-overlaps ω_2 then ω_1 is a α-sub-constituent of ω_2. AOA6 ensures that every constituent of a given DDO α has at least one atom as α-sub-constituent.

(AOA4) $(\exists \omega) R\, \omega \alpha$
(AOA5) $(\omega)(O_\preceq \omega \omega_1 \alpha \rightarrow O_\preceq \omega \omega_2 \alpha) \rightarrow \omega_1 \preceq_\alpha \omega_2$
(AOA6) $CO\, \alpha \omega \rightarrow (\exists \omega_1)(At\, \omega_1 \alpha \wedge \omega_1 \preceq_\alpha \omega)$

We then can prove the following theorems. There exists exactly one root in every ontology (TOA1); if ω_1 is a constituent of α and all atomic α-sub-constituents of ω_1 are α-sub-constituents of ω_2 then ω_1 is an α-sub-constituent of ω_2 (TOA2). TAO2 is a specific version of what is called the principle of atomic essentialism [15]. It implies that two constituents of a DDO α are identical if and only if they have the same atoms as α-sub-constituents (TAO3).

$TOA1$ $R\, \omega_1 \alpha \wedge R\, \omega_2 \alpha \rightarrow \omega_1 = \omega_2$
$TOA2$ $[CO\, \omega_1 \alpha \wedge (\omega)(At\, \omega \alpha \rightarrow$
         $(\omega \preceq_\alpha \omega_1 \rightarrow \omega \preceq_\alpha \omega_2))] \rightarrow \omega_1 \preceq_\alpha \omega_2$
$TOA3$ $CO\, \alpha \omega_1 \rightarrow (\omega_1 = \omega_2 \leftrightarrow$
         $(\omega)(At\, \omega \alpha \rightarrow (\omega \preceq_\alpha \omega_1 \leftrightarrow \omega \preceq_\alpha \omega_2)))$

5.2. Projection onto reality

The projective relationship between constituents of a directly depicting ontology and the entities in its target domain is complex. In the context of this paper we focus on ontologies with particularly well-defined projection relations. In a more general framework one could, for example, give up axiom AO6, i.e., allow for constituents that project onto more than one spatio-temporal entity. One could also give up axiom AO5, i.e., allow for poorly defined constituents which do not project at all. For details see [4].

Given axioms AO5 and AO6, it follows that every constituent of a directly depicting ontology projects onto a unique spatio-temporal entity. We now need to ensure that the projection function Π preserves the sub-constituent structure of each DDO. Intuitively: if the constituent ω_1 is a sub-constituent of ω_2 within the DDO α and ω_1 and ω_2 project onto the spatio-temporal entities x_1 and x_2 respectively, then x_1 should be part of x_2. Conversely, if the spatio-temporal entity x_1 is a part of x_2 and x_1 and x_2 are projected onto respectively by the constituents ω_1 and ω_2 of the ontology α, then ω_1 should be an α-sub-constituent of ω_2.

However, we need to spell out these restrictions differently for SNAP and SPAN ontologies. We start with SPAN ontologies. Recall that all constituents of a SPAN ontology project to either stages or perdurants. We demand that if (i) α is a SPAN ontology, (ii) ω_1 is a α-sub-constituent of ω_2, (iii) ω_1 projects onto x_1, and (iv) ω_2 projects onto x_2, then x_1 is a part of x_2 in the sense of either P_{st} or P_{Pd} (AOB1).

$$AOB1 \;\; \text{SPAN } \alpha \wedge \omega_1 \preceq_\alpha \omega_2 \wedge \Pi \omega_1 x_1 \wedge \Pi \omega_2 x_2$$
$$\rightarrow (P_{st}\, x_1 x_2 \vee P_{Pd}\, x_1 x_2)$$

Conversely we demand that if (i) α is a SPAN ontology, (ii) ω_1 and ω_2 are constituents of α, (iii) ω_1 projects onto x_1, (iv) ω_2 projects onto x_2, and (v) x_1 is a part of x_2 in the sense of either P_{st} or P_{Pd}, then ω_1 is an α-sub-constituent of ω_2 (AOB2).

$$AOB2 \;\; \text{SPAN } \alpha \wedge CO\, \omega_1 \alpha \wedge CO\, \omega_2 \alpha \wedge \Pi \omega_1 x_1 \wedge$$
$$\Pi \omega_2 x_2 \wedge (P_{st}\, x_1 x_2 \vee P_{Pd}\, x_1 x_2) \rightarrow \omega_1 \preceq_\alpha \omega_2$$

For SNAP-ontologies we demand that if (i) α is a SNAP-ontology for the time-slice u, (ii) ω_1 is a α-sub-constituent of ω_2, (iii) ω_1 projects onto x_1 and (iv) ω_2 projects onto x_2, then x_1 is a temporary part of x_2 at u (AOB3).

$$AOB3 \;\; \text{SNAP } \alpha u \wedge \omega_1 \preceq_\alpha \omega_2 \wedge$$
$$\Pi \omega_1 x_1 \wedge \Pi \omega_2 x_2 \rightarrow P_{Ed}^t\, x_1 x_2 u$$

Conversely we demand that if (i) α is a SNAP-ontology for the time-slice u, (ii) ω_1 and ω_2 are constituents of α, (iii) ω_1 projects onto x_1, (iv) ω_2 projects onto x_2,

and (v) x_1 is a temporary part of x_2 at time-slice u, then ω_1 is an α-sub-constituent of ω_2 (AOB4).

$$AOB4 \quad SNAP \; \alpha u \wedge CO \; \omega_1 \alpha \wedge CO \; \omega_2 \alpha \; \wedge$$
$$\Pi \omega_1 x_1 \wedge \Pi \omega_2 x_2 \wedge P^t_{Ed} \; x_1 x_2 u \rightarrow \omega_1 \preceq_\alpha \omega_2$$

We then can prove that if x is projected onto by the constituents ω_1 and ω_2 of the ontology α, then ω_1 and ω_2 are identical

$$TOB3 \quad CO \; \omega_1 \alpha \wedge CO \; \omega_2 \alpha \wedge \Pi \; \omega_1 x \; \wedge$$
$$\Pi \; \omega_2 x \rightarrow \omega_1 = \omega_2$$

It follows that the restriction of Π to the constituents of a fixed DDO α is a one-one mapping. In fact, it is an order isomorphism to α's target domain.

6. Conclusions

The theory outlined above contains the resources for describing both complex spatio-temporal processes and the enduring entities which participate therein. We argued that to deal with such phenomena we need a plurality of directly depicting ontologies together with a meta-ontological framework to deal with the relations between them.

Directly depicting ontologies are granular in nature and based on directly depicting languages. From the constraints imposed on the structure of such ontologies, it follows that computation about parthood relations within ontologies can be reduced to computation in atomic lattice structures. A special class of atomic lattices are finite lattices for which there exists a wide variety of efficient algorithms for performing operations on such structures [1].

Meta-ontology, i.e., the theory presented in order to specify what directly depicting ontologies are, how they are related to each other, and how they relate to reality, is more complex and requires the expressive power of full first order logic. The complexity of the meta-ontology, however, does not add to the complexity at the level of DDOs.

We distinguished two major categories of directly depicting ontologies: SPAN and SNAP. These DDOs represent orthogonal inventories of reality – one (SNAP) acknowledging enduring entities, and the other (SPAN) acknowledging perduring entities and stages. We showed that the distinction between perduring and enduring entities itself needs to be established on the meta-level since it is outside the scope of directly depicting ontologies themselves.

Distinguishing between ontologies of types SNAP and SPAN is also designed to take into account that perduring and enduring entities behave differently with respect to the type of part-whole relations that can be represented within them: for endurants parthood relations must be represented relative to some time (-slice); for perdurants parthood relations can be represented in a time independent manner.

Due to their granular nature and and the separation into DDOs of type SNAP and SPAN, the mereological structure represented within a a given DDO is simpler than the mereological structure on the side of entities. However we showed that despite the inherent simplicity on the side of the DDO, important aspects of the mereological structure of the represented domain can still be captured.

References

[1] Franz Baader, Ian Horrocks, and Ulrike Sattler. Description logics as ontology languages for the semantic web. In Dieter Hutter and Werner Stephan, editors, *Festschrift in honor of Jörg Siekmann*, Lecture Notes in Artificial Intelligence. Springer, 2003.

[2] T. Bittner and M. Donnelly. The mereology of stages and persistent entities. In *Proceedings of the 16th European Conference on Artificial Intelligence*, 2004.

[3] T. Bittner, M. Donnelly, and B. Smith. Individuals, universals, collections: On the foundational relations of ontology. In *Proceedings of the International Conference on Formal Ontology in Information Systems, FOIS04*, 2004.

[4] T. Bittner and B. Smith. A theory of granular partitions. In M. Duckham, M. F. Goodchild, and M. F. Worboys, editors, *Foundations of Geographic Information Science*, pages 117–151. London: Taylor & Francis, 2003.

[5] R. Casati and A. C. Varzi. *Parts and Places*. Cambridge, MA: MIT Press., 1999.

[6] D. Fensel, F. van Harmelen, I. Horrocks, D. McGuinness, and P. Patel-Schneider. Oil: An ontology infrastructure for the semantic web. *IEEE Intelligent Systems*, 16(2), 2000.

[7] A. Gangemi, N. Guarino, C. Masolo, A. Oltramari, and L. Schneider. Sweetening ontologies with DOLCE. In *Proceedings of EKAW*, Siguenza, Spain, 2002.

[8] P. Grenon. The spatio-temporal ontology of reality and its formalization. In H. Guesgen, D. Mitra, and J. Renz, editors, *AAAI Spring Symposium on Foundations and Applications of Spatio-Temporal Reasoning (FASTR)*, pages 27–34. AAAI Press, 2003.

[9] P. Grenon and B. Smith. SNAP and SPAN: Prolegomenon to geodynamic ontology. *Spatial Cognition and Computation*, 2004.

[10] N. Guarino. Formal ontology and information systems. In Nicola Guarino, editor, *Formal Ontology and Information Systems, (FOIS'98)*. IOS Press, 1998.

[11] Eduardo Mena, Arantza Illarramendi, and Alfredo Goni. Automatic ontology construction for a multiagent-based software gathering service. In *Cooperative Information Agents*, pages 232–243, 2000.

[12] T. Sider. *Four–Dimensionalism*. Clarendon Press, Oxford, 2001.

[13] Barry Smith. Characteristica universalis. In Mulligan. K, editor, *Language, Truth and Ontology*. Kluwer, 1992.

[14] Barry Smith and Kevin Mulligan. Pieces of a theory. In *Parts and Moments. Studies in Logic and Formal Ontology*. Munich: Philosophia, 1982.

[15] A. Varzi. Mereology. In Edward N. Zalta, editor, *Stanford Encyclopedia of Philosophy*. Stanford: CSLI (internet publication), 2003.