A Logistics Model for Emergency Supply of Critical Items in the Aftermath of a Disaster

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Abstract

In this paper, a new logistics model is proposed for the emergency supply of critical items in the aftermath of a disaster. The modeling framework considers multi-items, multi-vehicles, multi-periods, soft time windows, and a split and prioritized delivery strategy scenario, and is formulated as a multi-objective integer programming model. To effectively solve this model we limit the number of available tours for delivery vehicles; towards this end a decomposition approach is introduced that aims to decompose the original problem geographically. A computational study is conducted to evaluate the proposed approach, and a case study and related analysis are presented to illustrate the potential applicability of our model.

keywords: Vehicle routing problem, disaster relief, prioritized delivery, split delivery, soft time windows

INTRODUCTION

In recent years, much human life has been lost due to natural disasters. At least 1,836 people lost their lives in Hurricane Katrina in 2005 (*1*), and 86,000 died in the Kashmir Earthquake in Pakistan in 2005 (*2*). The most recent earthquake in Haiti caused approximately 230,000 deaths and more than 300,000 injuries (*3*). To mitigate damage and loss in disasters, studies in pre-disaster, during disaster, and post-disaster issues have been widely conducted in the past few years (*4*, *5*, *6*). A critical challenge in post-disaster is to transport sufficient essential supplies to affected areas in order to support basic living needs for those trapped in disaster-affected areas. Supplies that are essential for human survival include water, food (e.g., ready-to-eat meals), and prescription medications. In general, prescription medication (e.g., diabetic supplies) are needed most urgently, followed by water and food, respectively. The requirement for delivering essential supplies that have different priorities to a disaster-affected area provides the motivation for this work.

Logistics problems are often modeled as variants of the vehicle routing problem (VRP). The classical VRP consists of determining optimal routes for vehicles that originate and terminate at a single depot, to deliver items to a set of nodes geographically scattered while minimizing the total travel distance/time/total delivery cost. The VRP has been extended to include the capacitated VRP (CVRP), and the vehicle routing problem with time windows (VRPTW) (*7*, *8*). We now summarize related literature that contains similar characteristics to our modeling framework. Three particular characteristics, appearing in the vehicle routing problem literature that are related to our new logistics model are: soft time windows, multi-period routing, and split delivery strategy.

A vehicle routing problem with soft time windows (VRPSTW) is a special case of the VRPTW and has been discussed in the literature, though not often. Contrary to the general time windows constraints, usually indicating the earliest and latest allowable service time of a node, soft time windows denote that both the upper and lower bound of the time window can be violated with a suitable penalty. Papers in this type of problems include Taillard et al. (*9*) and Jung and Haghani (*10*). Multi-period vehicle routing problems are not common in the literature due to their difficulties. In this type of VRP, routes are constructed over a period of time. The most recent review of this kind of vehicle routing problem is seen in Francis et al. (*11*). Finally, the split delivery vehicle routing problem (SDVRP) is defined as follows: the demand of a node can be satisfied by more than one-time delivery instead of only one-time delivery allowed in the general VRPs. A detailed survey of the development of SDVRP can be found in Archetti and Speranza (*12*).

The main contributions of this paper are as follows:

- Soft time windows, multi-period routing, split delivery strategy, and prioritized delivery are considered simultaneously in the new model.
- An approach of tour determination for the model is introduced and analysis of its performance is provided.
- The consequences of not prioritizing delivery are highlighted with the help of a disaster relief humanitarian logistics case study

The rest of this paper is organized as follows. Section 2 presents a tour-based mathematical formulation for the prioritized items delivery problem. The solution methodology of solving the tour-based formulation is discussed in Section 3. Computational examples are provided in Section 4 to demonstrate the performance of the proposed approach. In Section 5, a case study of a disaster relief logistics effort is conducted to show the benefit of our new logistics model. Finally, Section 6 contains conclusions and suggestions for future work.

TOUR-BASED FORMULATION

Description and Assumption

We assume that there are multiple geographically dispersed nodes that need to be served by a single depot. It is assumed that the supplies are unlimited in the depot and that the demand from various nodes over a planning horizon is known at the beginning of the planning. For modeling simplicity, demand is assumed unchanged during all planning time periods. Different prioritized items are required to be delivered to nodes via the transportation network. Urgency levels of items are differentiated based on their importance. For each type of item, a soft time window is predefined to specify the nodes' expected waiting time to receive an item. If an item cannot be delivered within the time window, a penalty cost is incurred, although the item still can be delivered later. The longer the delay in delivering an item, the more severe the penalty cost.

Characteristics of the transportation mode and the delivery methods are considered. Multiple but limited numbers of identical vehicles are used to transport prioritized items. Vehicles are assumed to have limited weight and volume capacities. For each vehicle, tours are assigned in each period to deliver items to one or more nodes. Tours begin at the depot, continue to one or multiple nodes, and then return to the depot. The total working hours in a single time period for the operation are limited. Therefore, the total travel time of tours assigned to a single vehicle in a single time period cannot exceed the constrained working hours. We do not consider the time to load and unload items on or off the vehicle. Furthermore, any node can be served multiple times by a single vehicle or multiple vehicles to meet its demand, and the demand can also be satisfied fully or partially in a single delivery.

Multi-Objective Logistics Model

Based on the above scenarios, the new model considers a multi-item, multi-vehicle, multi-period, soft time windows, and split delivery strategy prioritized delivery problem and is constructed by a multi-objective tour-based integer programming formulation. In this section, we first summarize notation, parameters and define decision variables, followed by a presentation of a mathematical formulation for the model.

Notation, Parameters, and Decision Variables

Sets:

$\boldsymbol{I} = \{1, 2, \ldots, i, \ldots, \bar{i}\}$: the set of item types, and $\bar{i}$ is the total number of item types;

$\boldsymbol{J} = \{1, 2, \ldots, j, \ldots, \bar{j}\}$: the set of nodes, and $\bar{j}$ is the total number of nodes;

$\boldsymbol{K} = \left\{1, 2, \ldots, k, \ldots, \bar{k}\right\}$: the set of tour, and $\bar{k}$ is the total number of tours;

$\boldsymbol{L} = \left\{1, 2, \ldots, l, \ldots, \bar{l}\right\}$: the set of vehicles, and $\bar{l}$ is the total number of vehicles;

$\boldsymbol{T} = \{1, 2, \ldots, t, \ldots, \bar{t}\}$: the set of periods, and $\bar{t}$ is the total number of planning time periods

Routing parameters:

J_k: the set of nodes which the vehicle will visit on tour k;

t_k: travel time of tour k;

$[0, to_i]$: time window for delivering item i without penalty after demand occurs, where to_i is the maximum acceptable waiting time to receive item i;

C_k: travel cost of tour k;

H: total working time available in a single period;

W: the maximum load weight of a vehicle;

V: the maximum volume capacity of a vehicle;

M: a large number;

u: the index of the length of delays after the maximum acceptable waiting time since the demand was requested, where $\bar{t} - to_i \leq u \leq \bar{t}, \forall i$;

v: the index of time periods when the backorder delivery of item i can be made after demand was requested at period t, where $t + 1 \leq v \leq t + \bar{t} - to_i$;

Demand parameters:

d_{ijt}: demand of the item i of the node j at time t;

p_{iu}: penalty cost of item i if the severe level of delay is u;

fp_i: penalty cost of item i if there is remaining unsatisfied demand after the operation;

a_i: unit weight of item i;

b_i: unit volume of item i;

Delivery Decision Variables:

x_{ijklt}: amount of item i delivered to node j on tour k by vehicle l in period t immediately after demand occurs;

w_{ijklmn}: backorder amount of item i delivered to node j on tour k by vehicle l in period m for demand in period n, where $m, n \in \boldsymbol{T}$ and $n < m$;

S: maximum difference of service level between any two nodes;

s_j: service level of node j.

Routing Decision Variables:

y_{klt}: equal to one when tour k is assigned to vehicle l in period t, and 0 otherwise.

Formulation

Objective 1:

$$\begin{aligned} min. \quad & \sum_i \sum_j \sum_{u=1}^{\bar{t}-to_i} \sum_{t=1}^{\bar{t}-to_i-u+1} \left(d_{ijt} - \sum_k \sum_l \left(x_{ijklt} + \sum_{v=t+1}^{t+u} w_{ijklvt} \right) \right) \cdot p_{iu} + \\ & \sum_i \left(\left(\sum_j \sum_t d_{ijt} - \sum_j \sum_k \sum_l \sum_t \left(x_{ijklt} + \sum_{m>t, m \in \boldsymbol{T}} w_{ijklmt} \right) \right) \cdot fp_i \right) \end{aligned} \tag{1}$$

Objective 2:

$$min. \quad \sum_k \sum_l C_k y_{klt} \tag{2}$$

Objective 3:

$$min. \quad S \tag{3}$$

Subject to:

$$S \geq s_p - s_q, \qquad \forall p, q \in \boldsymbol{J}, p \neq q \tag{4}$$

$$S \geq s_q - s_p, \qquad \forall p, q \in \boldsymbol{J}, p \neq q \tag{5}$$

$$s_j = \frac{\sum_i \sum_k \sum_l \sum_t \left(x_{ijklt} + \sum_{m>t, m \in \boldsymbol{T}} w_{ijklmt} \right)}{\sum_i \sum_t d_{ijt}}, \forall j \in J \tag{6}$$

$$\sum_k t_k y_{klt} \leq H \qquad \forall l, \forall t \in \boldsymbol{T} \tag{7}$$

$$x_{ijklt} \leq M y_{klt} \qquad \forall i, \forall j \in J_k, \forall k, \forall l, \forall t \tag{8}$$

$$w_{ijkltn} \leq M y_{klt} \qquad \forall i, \forall j \in J_k, \forall k, \forall l, \forall t, \forall n < t \tag{9}$$

$$\sum_k \sum_l \sum_t x_{ijklt} + \sum_k \sum_l \sum_{m>t, m \in \boldsymbol{T}} \sum_t w_{ijklmt} \leq \sum_t d_{ijt} \qquad \forall i, \forall j \tag{10}$$

$$\sum_i \sum_j a_i \left(x_{ijklt} + \sum_{n<t, n, t \in \boldsymbol{T}} w_{ijkltn} \right) \leq W \qquad \forall k, \forall l, \forall t \tag{11}$$

$$\sum_i \sum_j b_i \left(x_{ijklt} + \sum_{n<t, n, t \in \boldsymbol{T}} w_{ijkltn} \right) \leq V \qquad \forall k, \forall l, \forall t \tag{12}$$

$$x_{ijklt} \geq 0 \qquad \forall i, \forall j \in J_k, \forall k, \forall l, \forall t \tag{13}$$

$$w_{ijklmn} \geq 0 \qquad \forall i, \forall j \in J_k, \forall k, \forall l, \forall m \in \boldsymbol{T}, \forall n \in \boldsymbol{T} \tag{14}$$

$$x_{ijklt} = 0 \qquad \forall i, \forall j \notin J_k, \forall k, \forall l, \forall t \tag{15}$$

$$w_{ijklmn} = 0 \qquad \forall i, j \notin J_k, \forall k, \forall l, \forall m \in \boldsymbol{T}, \forall n \in \boldsymbol{T} \tag{16}$$

$$y_{klt} \in \{0, 1\} \qquad \forall k, \forall l, \forall t \tag{17}$$

In this model, the objective function (1) is a penalty function that aims to minimize total unsatisfied demand, especially for high priority items. Each item can only be backordered during to_i periods after the period t in which it is requested. If the initial demand from period t cannot be backordered during the subsequent to_i periods, a penalty is applied. There are two parts in this objective function. The first part (until p_{iu}) indicates the total accrued penalty cost during the planning periods. It is noted that $\sum_{u=1}^{\bar{t}-to_i}$ indicates the summation of the penalty due to various lengths of delays, and $\sum_{t=1}^{\bar{t}-to_i-u+1}$ indicates the summation of the penalty for demand requested at time period t with length of delay u. Moreover, $\sum_{v=t+1}^{t+u} w_{ijklvt}$ determines the total backorder amount of item i to demand requested at time period t for avoiding the penalty due to u time periods of delays. The second part (after p_{iu}) is the penalty cost accrued at the end of the planning periods for demand that cannot be delivered before the end of the operation, and it attempts to deliver higher priorities at the end of the operation.

The objective function (2) aims to minimize the total travel time for all tours and all vehicles. The purpose of this objective is to assign as many vehicles as possible consistent with the working hour limitation, in order to deliver the largest amount of items (this assumes that the demand in a disaster scenario will be large). Objective function (3) is used to minimize the difference in the satisfaction rate between nodes. The satisfaction rate is the ratio between the requested demand and the actual delivered amounts. The purpose of this objective is intent on balancing the service among nodes. “Fairness” is the term usually used to describe this purpose.

In addition to these objectives, equations (4-5) are used to determine the maximum service level between any two nodes, and it is obtained by equation (6). Equation (7) indicates that the total travel time of all tours assigned to any single vehicle in this period cannot be longer than the available working hours in a single period. Equations (8) and (9) show that deliveries can only be made if corresponding tours are selected. Equation (10) shows that the total delivery of items cannot exceed the demand during the planning periods. Equations (11) and (12) address the total available loading weight and the total volume limit capacity constraints of a vehicle. Equations (13 - 16) are used to ensure that vehicles can only stop and deliver to nodes on tours assigned to them. Finally, equation (17) indicates that y_{klt} is a binary variable.

SOLUTION METHODOLOGY

Overall Solution Process

To solve the above model, we apply the well-known weighted sum method (*13*) to aggregate the multi-objective formulations to a single objective formulation. The assumption of applying this approach is that preferences of the decision maker are known *a priori*. As a matter of fact, during the disaster relief operation, it is obvious that the most important goal is to lower situations of unsatisfied demand (objective 1), followed by balance of service levels among different nodes (objective 3), and expenditure of the operation (objective 2). We used the same consideration to determine appropriate weights of each objective in the weighted sum method.

Ideally, the transformed problem can be solved directly using the commercial solver CPLEX in our study. However, an important parameter in the aforementioned model that needs to be determined is the number of tours to be included in the model (denoted by $\bar{k}$ in the model). Theoretically, all possible tour combinations should be included in the model so that all possible solutions can be explored. For a small-size delivery problem (e.g. only 3 or 5 nodes), it is possible to enumerate all tours and include all of them in the model. However, as the problem size increases, enumerating all tours is not a viable option, because it makes the size of the resultant problem too large for CPLEX to solve (e.g. there are 4,932,055 possible tour combinations for a 10 node problem). Therefore, we propose a decomposition and assignment approach as an alternative to overcome this difficulty.

Decomposition and Assignment Approach

The proposed approach allows the whole problem to be decomposed into several sub-problems – this allows assignment of vehicles to each sub-problem. More specifically, for each sub-problem, we only consider a small number of nodes that are close to each other geographically such that the number of possible tours in each sub-problem is countable and more manageable. A portion of vehicles are assigned to each sub-problem to provide service between the depot and included nodes in the sub-problem. Thus, all sub-problems can be simultaneously solved in parallel, since the size of each sub-problem is solvable by CPLEX in reasonable time.

We have named this approach the Decomposition and Assignment Heuristic (DAH). Assume that a set of vehicles $\boldsymbol{L} = \left\{1, \ldots, l, \ldots, \bar{l}\right\}$, where $\bar{l} > 1$ is used to deliver items to a set of nodes $\boldsymbol{J} = \{1, \ldots, j, \ldots, \bar{j}\}$. We use λ to indicate the iteration number of the algorithm, $\bar{\lambda}$ the total desired number of iterations in the algorithm, τ the number of times that the solution is not improved after an iteration, and $\bar{\tau}$ the maximum consecutive times allowed in a row that the solution is not improved. If we set $\lambda = 0$ initially, then the DAH procedures can be described as follows:

DAH algorithm:

1. Randomly select a node that is not included in any group, and put it in a new group $\hat{J}_g$.
2. Find the nearest node (not currently included in any group) to the last assigned node in the group, and repeat the process until the predefined number of nodes in a group is met or there is no ungrouped node left.
3. **IF** there is a node that does not belong to any group, **Go to step 1**.

4. **ELSE** Equally assign a number of vehicles $\hat{L}_g$ to each group $g \in G$, where ***G*** = $\{1, \ldots, g, \ldots, \bar{g}\}$ is the collection of groups, $\sum_g \hat{L}_g = \bar{l}$, and $\forall p \neq q,\ \hat{J}_p \cap \hat{J}_q = \phi$. The original problem has now been decomposed into $\bar{g}$ subproblems with assigned nodes and vehicles, respectively, and each subproblem is labeled as SP_g.
5. For each subproblem SP_g, all feasible tours are enumerated and constructed using the shortest time principle.
6. For each subproblem SP_g, construct the mathematical model based on $\hat{L}_g$, $\hat{J}_g$, and the corresponding demand of nodes in the subproblem; solve SP_g by CPLEX, and get the objective values z_g and the total objective value $z_{all} = \sum_g z_g$. If $\lambda = 0$, set the best total objective value $z^*_{all} = z_{all}$.
7. **IF** $\lambda \leq \bar{\lambda}$, find a pair of groups (p, q) that has the minimum and maximum objective value, respectively.
8. **IF** $\hat{L}_p > 2$, then do **Steps 9** and **10**.
9. Remove a random number of vehicles ν from $\hat{L}_p$, where $1 \leq \nu \leq \hat{L}_p - 1$, and assign it to $\hat{L}_q$.
10. **Go to Step 6**, update z_p, z_q, and z_{all}.
11. **IF** $z^\lambda_{all} < z^*_{all}$, update $z^*_{all} = z^\lambda_{all}$. **Go to Step 8**.
12. **ELSE** Set $\tau = \tau + 1$.
13. **IF** $\tau \leq \bar{\tau}$. **Go to Step 8**.
14. **ELSE** Set $\lambda = \lambda + 1$. **Go to Step 1**.
15. **ELSEIF** There are more than one group in addition to $\hat{J}_p$ and $\hat{J}_q$; find the next maximum objective value group. **Go to Step 8**.
16. **ELSE** Set $\lambda = \lambda + 1$. **Go to Step 1**.
17. **ELSE Stop and Exit**.

Steps 1-3 of the DAH algorithm decompose the original problem into several sub-problems by forming groups with a predefined maximum number of nodes allowed in a group. In step 4, we equally assign vehicles to serve each sub-problem following the order of groups in G. It is noted that we may have to assign a fewer number of vehicles than the average integer number of vehicles to the last formed group. The next two steps are used to construct all tours, and to create corresponding mathematical formulations for each sub-problem. The objective values of sub-problems and the overall objective value under the current decomposition result are obtained in step 6. Steps 7 - 15 aim to improve the solution by adjusting vehicles assignments among groups. If the solution is not improved for $\bar{\tau}$ successive iterations, groups are reformed and the procedure is repeated until the maximum number of iterations $\bar{\lambda}$ (i.e., times of regrouping) is met.

Using this algorithm, we solve several smaller-scale problems, instead of a single large-scale problem. Tours in each sub-problem are countable and manageable, and each sub-problem is also solvable by CPLEX in reasonable computational time.

COMPUTATIONAL EXPERIMENTS

Two parts are discussed in this section. In the first part, a random instance generator is designed to produce numerical instances for the following computational experiments. In the second part, a comparison of performance of our proposed approach for tour determination and other approaches is provided.

Random Instance Generator

A random instance generator has been coded in C. We assume that the service area of a logistics operation is in a 50 square miles area. The location of the depot and all nodes are randomly located by the instance generator in this area. Instead of using Euclidean distances between any two nodes, we transform Euclidean distances to estimated road distances to obtain much more realistic estimations of travel times. This can be achieved by using the following equation (*14*):

$$d\left(q,r;k,p,s\right) = k\left[\sum_{i=1}^{2}\left|q_i - r_i\right|^p\right]^{1/s} \tag{18}$$

where q and r are coordinate points on the plane in the service area, and k, p, s are parameters. Suggested ranges of these parameters are $k \in \{0.80, 2.29\}$, $p, s \in \{0.90, 2.29\}$, respectively. In our instance generator, parameter values are randomly generated from their corresponding ranges. It is noted that only distances between the depot and each node are estimated according to equation (18), and distances among nodes are generated randomly based on the triangular inequality (i.e., road distances are estimated on link $\overline{OA}$ and $\overline{OB}$, if O is the depot and A, B are two demand nodes, but the distance $\overline{AB}$ is generated randomly between $\left(\overline{OA} + \overline{OB}\right)$ and $\left(\overline{OA} - \overline{OB}\right)$ to ensure that the triangule inequality holds).

Three sets of problems are generated by the instance generator: small size, medium size, and large size. Each size of problem involves 5 random instances, respectively, where each instance has a distinct network structure, problem size and number of vehicles. Specifically, the set of small size problems consists of 3 nodes and 6-9 tours. Problems with 4 nodes and 20-30 tours are denoted as medium size. The large size problems have 5 nodes and 30-40 tours included in each instance. Regardless of problem size, 10-20 vehicles are used in each instance, selected randomly. Other outputs from the instance generator include the demand of three types of critical items in each node, and structures of tours for each instance.

Performance Comparison

The performance of the proposed approach for tour determination is evaluated on three sets of problems. The heuristic approach proposed in this paper has been coded in C, which interfaces with the callable library in ILOG CPLEX version 11.2, which is used to solve sub-problems after the tours have been determined in each sub-problem. Each instance with different tour determination approaches has been run on a computer with 2.00 GB RAM,and an Intel Pentium D 3.40 GHz processor.

We consider two other tour determination approaches to compare our proposed approach. The first approach is to enumerate all possible tours of problems and to include all of them in the model. Under this approach, we aim to find the exact optimal solutions of these problems. The second approach, proposed by Lin (*15*), is based on a Genetic Algorithm (named GA-based approach), a popular meta-heuristic method that has been applied in various applications (*16*, *17*). This approach uses a Genetic Algorithm to generate a small number of tours in each iteration, and the model with these manageable number of tours is solved by CPLEX. The idea is to improve the solution at each iteration by only retaining a reasonable number of good tours in the model until the end of the search process.

To compare the performance among different tour determination approaches, we assume that the weights for objectives 1, 2, and 3 are 0.6, 0.1, and 0.3, respectively; these selections conform to general preferences of dealing with disaster relief, where the first goal of the operation is to satisfy as many demand requests as possible, followed by the fairness issue among different demand nodes; the shipment cost of items is typically of the least concern. We use two criteria to stop the iterative process: either the running time exceeds 3,600 seconds or the MIP gap is smaller than 0.01%. In order to obtain a more accurate evaluation, each instance was solved 10 times with different randomly generated demand requests, and the average results of 10 replications for each instance is presented in the TABLE 1.

TABLE 1 summarizes the logistics performance of using three different tour determination approaches in the solution process. The first column identifies the IDs of each instance. For each approach, percentages of demand delivered and running times (seconds) of the corresponding approach are presented. For some instances, there is no feasible solution available within 3,600 seconds, so we designate them as n/a. Furthermore, under the GA-based approach column, the number of tours that remain in the model for small-size, medium-size, and large-size instances are 6, 7, and 8, respectively.

The average performance of the three tour determination approaches is summarized as follows. The GA-Based approach reduces the average running time by about 22.3%, compared to instances with all tours included. The average percentage of complete delivery is 99.8%, which is almost the same as instances in which all tours are included. However, the variance of running times in the GA-based approach is still high (range is from 17.81 to 3,600 seconds), because CPLEX spends a long time in proving optimality in some cases. On the other hand, the DAH approach finds a desirable solution in a very short running time with a 4.3% reduction of solution quality (i.e., only can achieve 95.5% of complete delivery percentage) compared to the GA-based approach. Based on these results it is evident that the DAH approach is both fast and sufficient method and we therefore use this for large-scale problems.

TABLE 1 Performance Comparison of Different Tour Determination Approaches

Instance	Enumeration		GA-based Approach		DAH	
ID	% of delivered	Time (seconds)	% of delivered	Time (seconds)	% of delivered	Time (seconds)
S1	99.98	178.11	99.96	0.99	99.86	17
S2	99.97	44.54	99.93	0.97	99.82	52
S3	98.73	312.16	98.73	271.69	96.97	18
S4	96.31	260.15	96.31	241.16	95.00	14
S5	94.74	17.81	94.74	17.94	87.10	5
M1	96.96	3,600	96.98	192.96	95.16	79
M2	n/a	n/a	98.74	1,794.91	98.05	42
M3	96.54	3,287.17	96.54	58.57	94.87	42
M4	n/a	n/a	92.95	3,600	92.60	39
M5	95.34	2277.21	95.14	3,340.12	93.33	15
L1	n/a	n/a	96.45	301.91	88.89	15
L2	n/a	n/a	97.70	3,473.27	94.57	22
L3	90.94	3,600	91.00	1278.63	77.48	45
L4	n/a	n/a	84.89	291.52	91.00	66
L5	79.09	3,600	73.06	120.37	63.34	14

CASE STUDY

To illustrate the capability of our model in the context of delivering prioritized items, we applied the logistics model to the disaster relief operation in Northridge, CA, where a 6.7 magnitude earthquake occurred on Jan. 17, 1994. The study region selected is Los Angeles County, CA, with a geographical size of 4,086.9 square miles, and a total population of 9,519,338 (*18*).

Data

Federal Emergency Management Agency (FEMA) has developed a software named HAZUS-MH which can be used to simulate nature disasters (*19*). We used HAZUS-MH to simulate the Northridge earthquake scenario. Three types of simulation output are of particular interest for us: the number of displaced households, the number of severity level 1 injuries, and the number of households without water service. These output are used to generate and estimate demand for food, prescription medication, and water, respectively. It is noted that severity level 1 injuries are those which require medical attention, but hospitalization is not needed. We now justify this method.

First of all, the number of households without water service is used to generate demand of water. Since the geographic distribution of households without water service is not provided in the simulation output, we assumed that the number of households without water service in each census tract was proportional to the number of households affected by the earthquake in the track. Second, the number of displaced households is used to estimate the demand of food. This is because these families suffered damage to their houses, require a temporary shelter, and are thus incapable of preparing food by themselves. The estimated number of households displaced is 25,469 according to the simulation. Finally, the number of casualties of severity level 1 is used to estimate the demand for prescription medication. Based on the simulation, the total number of injuries at this level is 1,654 in this earthquake. It is noted that we only extracted casualty data for level 1 in the residential families because the earthquake occurred at 4 AM, and we assumed that the majority of people were at home during the earthquake.

In a disaster relief operation, it is common to transport supplies first from a central depot to several local distribution centers by trucks; then, supplies are distributed to individuals from local distribution centers. Therefore, we actually formed 9 clusters in Los Angeles County, as shown in FIGURE 1, simply by geographic relationships, and identified a node in each cluster as the local distribution center to receive deliveries from the central depot. The demand data in each node is obtained by aggregating each type of demand from all census tracts located inside the cluster. However, in the simulation output, only "one-time" demand is available for us to derive from data, except the number of households without water service data that consists of different numbers in discrete time points (e.g. day 1 to day 90). Therefore, we assumed that the time series data in this dataset represents the recovery speed from the earthquake, and all supplies would follow the same pattern to recover as well. Demand data of all kinds of items over 7 days can be estimated, respectively, based on the initial "one-time" data from the simulation and the trend of the recovery speed, that is represented by an exponential fitting function, obtained by fitting data of the number of households without water service. As a result, a demand matrix containing 3 items, 9 nodes and 7 time periods was obtained. Based on the catalogue published in a relief organization (*20*), we defined that the truck used in the study has 11,590 kg and 56 m^3 of maximum loading and volume limits, respectively. The shipping weights and volumes of each unit of medication, water and food are 86.5 kg and 0.22 m^3, 400 kg and 4.3 m^3, and 700 kg and 2.3 m^3, respectively. We determined the time windows for medication, water, and food for delivery without penalty cost as 0-1 days, 0-2 days, and 0-3 days, respectively. Finally, the maximum working hours in each day was constrained as 15 hours.

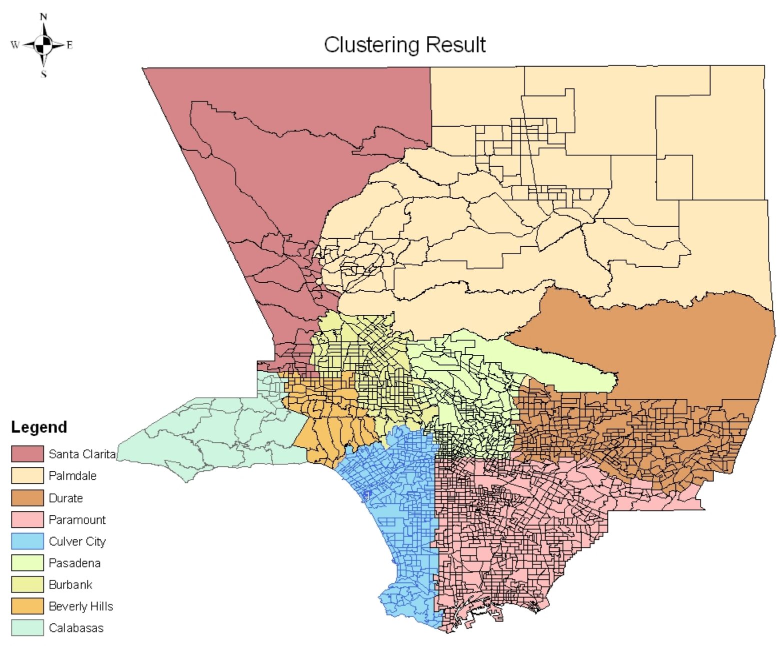

FIGURE 1 Clustering Result in Los Angeles County

Results

Since this case study has nine demand nodes and one depot, the total number of tours is too large to be included in the model, as discussed earlier. We therefore applied the DAH algorithm to decompose the original network into three subnetworks that are depicted by different type of lines in FIGURE 2, which shows the transportation network among the central depot and nine nodes in Los Angeles County, CA. The route between any two nodes was first determined by the online map provider, and consisted of detoured partial segments if the best route involves some damaged roadways due to the earthquake. The detours were established according to a government report (*21*). Then the travel time between any two nodes or between a node and the depot was estimated by taking the average of the travel time in the traffic and in congested traffic, because it is reasonable to assume that congestion usually occurs around the area after the earthquake. In addition, we again assume the weights for objectives 1, 2, and 3 are 0.6, 0.1, and 0.3, respectively.

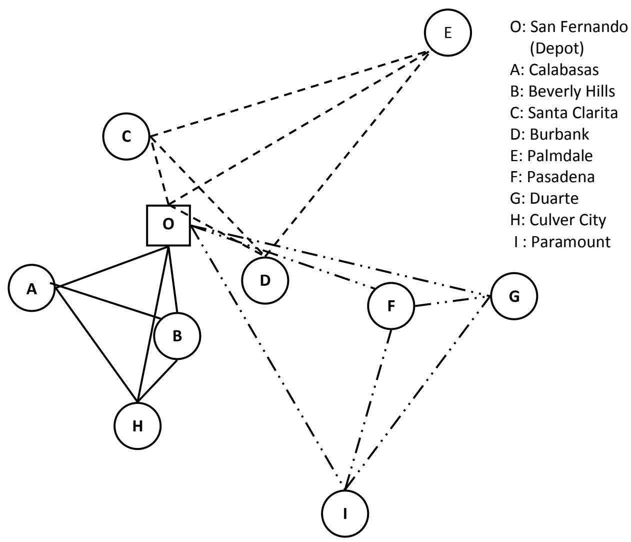

FIGURE 2 Transportation Network and Sub-networks

It is common to involve resources from a variety of organizations in a disaster relief operation. The total number of vehicles was assumed to be 400 from 20 different organizations (e.g. American Red Cross, military units, etc.). Each organization is responsible for servicing the delivery tasks assigned from the disaster operations manager, and each organization is also responsible for managing its own fleet to accomplish the relief effort. As a result, during seven days of operations based on the data described above and the proposed new model, a total of 621 assignments of delivery tasks had been made to different organizations, and this was equivalent to 12,420 deliveries by trucks.

Indeed, among the three types of items, an average of 90.1% of prescription medication was delivered in all nine nodes, while only about 70% and 16.4% of water and food, respectively, were delivered on average. In fact, these numbers reveal the compromise among objectives, compared with 100.0% delivery of medication, 78.9% delivery of water, and 42.6% delivery of food in the best scenario, which is best performance we can achieve if all resources are used to deliver one type of items. We note that only 30.8%, 32.5%, and 7.5% of demand of prescription medication, water, and food, respectively, were satisfied during the time period when the demand arose — these low numbers are due to the soft time window feature in the model which allowed delayed satisfaction of demand. In addition, it is interesting to see that the full truck load of a single type item is a preferred method of delivery in our application, when the overall vehicle capacity is not capable of satisfying all demand; therefore, if extra capacity is available on vehicles to the same demand locations, more of the same type of items are carried on vehicles.

Discussion

A major feature of our proposed model is its ability to prioritize delivery. To test this feature of our model, we attempted a comparison with a similar model in the literature. Balcik et al. (*22*) proposed a model to deliver relief supplies in disasters. Although our model and theirs have the same purpose, they are different for two reasons. The first difference is that Balcik et al.(*22*) consider two types of supplies. The first type are critical items for which demand occurs once at the beginning of the planning horizon (e.g. shelters, blankets) and has to be fully satisfied during the planning horizon (i.e., a hard constraint in the model). The second type of items are those that are consumed regularly and whose demand occurs periodically over the planning horizon (e.g. food, prescription medication, and water), and cannot be backordered if it is not satisfied on time. In our model, we are only interested in the second type of items and backorders are permitted. The second difference is that Balcik et al. (*22*) do not consider soft time windows, which means that demand must be served immediately when it occurs.

To facilitate the comparison of our model with that of Balcik et al. (*22*) we assumed that the demand of shelters after the earthquake is zero since our interest is not in this type of item. Therefore, the total available capacity of vehicles would not be affected because of shipment of this type of item. To conform to the Balcik et al. model, we collapsed our model to one in which backorder is not permitted and soft time windows are not considered. In addition, we adjusted the weights of the three objectives from the original (0.6,0.1,0.3) to (0.5,0.5,0) since they only considered the penalty cost due to unsatisfied demand and travel cost in their model. We first applied their model to obtain the solution, and then used their travel cost as the budget limit in our model in order to get comparable results. The comparison is shown in TABLE 2, in two parts. Part A in TABLE 2 indicates the percentage of demand being delivered in each time period and Part B indicates the number of injuries or households suffering a shortage of items. It shows that though their model reveals a prioritized delivery pattern similar to ours, their model was unable to concentrate on delivering the item with the highest priority (e.g. prescription medication). In contrast, our collapsed model showed a significant ability to deliver the highest priority item. The improved performance of our model is due to the penalty cost function structure as shown in Equation 1. Furthermore, it is apparent that our collapsed model provides superior results to that of Balcik et al. under the same budget.

TABLE 2 Comparison of Logistics Performance between Models

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	
A. Percentage of Satisfied Demand								**Average**
Medication:								
Balcik's model	85.4%	84.3%	86.7%	91.9%	87.4%	91.9%	76.8%	86.4%
Proposed model (collapsed)	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	95.3%	99.4%
Water:								
Balcik's model	54.2%	58.1%	59.1%	61.7%	56.2%	61.6%	58.1%	58.3%
Proposed model (collapsed)	76.0%	77.8%	78.4%	78.9%	79.5%	38.5%	12.8%	64.6%
Food:								
Balcik's model	12.2%	12.7%	13.8%	10.9%	12.2%	12.8%	10.1%	15.6%
Proposed model (collapsed)	12.1%	12.2%	13.0%	14.0%	15.0%	8.6%	8.7%	12.0%
B. Number of Injuries or Households without Sufficient Supply								**Total**
Medication:								
Balcik's model	0	216	223	181	106	159	98	982
Proposed model (collapsed)	0	0	0	0	0	0	0	0
Water:								
Balcik's model	0	0	14,282	12,081	11,356	10,230	11,252	59,201
Proposed model (collapsed)	0	0	7,482	6,405	5,998	5,625	5,261	30,771
Food:								
Balcik's model	0	0	0	23,263	21,411	20,328	20,227	85,230
Proposed model (collapsed)	0	0	0	23,285	21,546	20,529	19,524	84,885

Conclusions and Future Work

This paper proposes a new logistics model for the emergency supply of critical items in the aftermath of a disaster. Our model considers multi-items, multi-vehicles, multi-periods, soft time windows, and a split delivery strategy, and is formulated as a multi-objective integer programming model. The distinguishing feature of our work is the consideration of delivery priorities of different items. In addition, a heuristic approach is developed, named the Decomposition and Assignment heuristic (DAH), to overcome the computational challenge of enumerating all possible tours. The performance of the approach is analyzed and its efficiency is investigated. We found that, in general, the DAH approach provides solutions in reasonable computational time while it has about a 4.3% reduction in solution quality compared to the other two tour determination approaches.

To verify the importance of the new model for delivery tasks containing prioritized delivery requests, we conducted a case study of an earthquake scenario in Los Angeles County. For this case study, our model is capable of satisfying 90.1% of prescription medication demand, 70% of water demand, and 16.4% of food demand, under the limited delivery capacity of vehicles and limited working time in each time period. The results from the case study showed that the model performed well in this disaster relief operation. Further analysis showed that our model performed better than a model in the literature by Balcik et al. (*22*) which has a similar purpose to ours.

We suggest three directions for future work. The first is to investigate the robustness of our model with respect to uncertainty in demand values, congestion levels, network accessibility, and correlations of node locations with respect to the highway systems. The second is to develop more efficient multi-objective optimization methodologies for this type of problem and to perform a trade-off analysis to understand the compromises among different goals. The third is to consider a distributed scenario in which several temporary depots are required to be located and serve as "bridges" between the major depot and nodes.

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