A logistics model for delivery of prioritized items: Application to a disaster relief effort†

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Abstract

In this paper, a new logistics model is proposed for delivery of prioritized items in disaster relief operations. It considers multi-items, multi-vehicles, multi-periods, soft time windows, and a split delivery strategy scenario, and is formulated as a multi-objective integer programming model. To effectively solve this model we need to limit the number of available tours. Two heuristic approaches are introduced for this purpose. The first approach is based on a genetic algorithm, while the second approach is developed by decomposing the original problem. We compare these two approaches via a computational study. The multi-objective problem is converted to a single-objective problem by the weighted sum method. A case study is presented to illustrate the potential applicability of our model. Also, presented is a comparison of our model with that proposed in a recent paper by Balcik et al.[5].

keywords: vehicle routing problem, logistics, prioritizing delivery, split delivery

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1 Introduction

In recent years, much human life has been lost due to natural disasters. For example, at least 1,836 people lost their lives in Hurricane Katrina in 2005, and 86,000 died in the Kashmir Earthquake in Pakistan in 2005[36, 37]. To mitigate damage and loss in disasters, studies in pre-disaster, during disaster, and post-disaster issues have been widely conducted in the past few years. A critical challenge is to transport sufficient essential supplies to affected areas in order to support basic living needs for those trapped in disaster-affected areas. Supplies that are essential for human survival include water, food (e.g., ready-to-eat meals), and prescription medications. In general, prescription medication (e.g., diabetic supplies) are needed most urgently, followed by water and food, respectively. The requirement for delivering essential supplies that have different priorities to a disaster-affected area provides the motivation for this work.

Logistics problems are often modeled as variants of the vehicle routing problem (VRP). The classical VRP consists of determining optimal routes for vehicles that originate and terminate at a single depot, to deliver items to a set of nodes geographically scattered while minimizing the total travel distance/time/total delivery cost. The VRP has been extended to include the capacitated VRP (CVRP)[8, 18, 28], the vehicle routing problem with backhauls (VRPB)[19, 33, 34, 41], the vehicle routing problem with pickup and delivery (VRPPD)[16, 30, 38, 39], and the vehicle routing problem with time windows (VRPTW)[10, 12, 26].

We now summarize some related literature that contains similar characteristics as ours. Three particular characteristics, appearing in the vehicle routing problem literature that are related to in our new logistics model are: soft time windows, multi-period routing, and split delivery strategy. A vehicle routing problem with soft time windows (VRPSTW) is a special case of the VRPTW and has been discussed in the literature, though not often. Contrary to the general time windows constraints, usually indicating the earliest and latest allowable service time of a node, the soft time windows denote that the both upper and lower bound
of the time window can be violated with a suitable penalty. Taillard et al. [32] proposed a tabu search heuristic for VRPSTW. They employed a penalty cost that was added to the objective value when lateness at node locations occurred. Ioannou et al. [23] considered the VRPSTW for fleet planning to determine minimal fleet sizes and they proposed a nearest-neighbour method to solve the problem. Furthermore, Haghani and Jung [20] presented the formulation for the dynamic vehicle routing problem with time-dependent travel times, soft time windows, multiple vehicles with different capacities, and real-time service requests.

Multi-period vehicle routing problems are not common in the literature due to their difficulties. In 2002, Angelelli and Speranza [1] proposed a tabu search algorithm for the periodic vehicle routing problem with intermediate facilities, where vehicles can renew their capacity. The same problem was investigated by Morugaya and Vanderbeck [29], who considered the tactical planning model to scheduling visits to clients and assigning them to vehicles over a given time horizon to satisfy service level, while optimizing routes in each time period. Ozdamar et al. [31] proposed a planning model integrated into a natural disaster logistics decision support system and they required that the time-dependent transportation problem be solved repetitively in given prolonged time periods.

A split delivery vehicle routing problem (SDVRP) is defined as follows: the demand of a node can be satisfied by more than one-time delivery instead of only one-time delivery allowed in the general VRPs. Dror and Trudeau [14] first showed that benefits could be expected through split deliveries both on total travel distance and the number of vehicles required. Belenguer et al. [6] proposed a lower bound for the SDVRP according to a polyhedral study of the problem. They used a cutting-plane algorithm for small size instances. Bompadre et al. [7] presented the lower bound for the VRP with and without split deliveries. Based on the lower bound they presented, they developed the quadratic iterated tour partitioning and the quadratic unequal iterated tour partitioning heuristics for the SDVRP. Archetti et al. [2] performed a worst-case performance analysis of the SDVRP. They concluded that the maximum cost savings that can be realized is at most 50%. Recently, Archetti et al. [3] used
an empirical study and again showed that the largest benefits are obtained when average node demand is just over 50% of the vehicle capacity and when the variance of node demand is relatively small. In addition, algorithms developed for SDVRP can be found in [4, 21, 24, 25].

The main contributions of this paper are as follows:

- Soft time windows, multi-period routing, and split delivery strategy are considered simultaneously in the new model.

- Two approaches of tour determination for the model are introduced and analysis of performance for both approaches is provided.

- The consequences of not considering prioritizing delivery are highlighted with a disaster relief logistics case study

The rest of this paper is organized as follows. Section 2 presents a tour-based mathematical formulation for the prioritized items delivery problem. The overall solution strategy is discussed in Section 3. Two heuristic approaches are introduced in Section 4 to determine a desirable set of tours. Some computational examples are provided in Section 5 to demonstrate the performance of the proposed approach. In Section 6, a case study of a disaster relief logistics effort is conducted to show the benefit of our new logistics model. Finally, Section 7 contains conclusions and suggestions for future work.

2 Tour-Based Formulation

2.1 Description and Assumption

We assume that there are multiple nodes geographically dispersed and they are be served by a single depot to deliver goods. It is assumed that the supplies are unlimited in the depot and that the demand from various nodes over a planning horizon is known before the
beginning of the planning. For modeling simplicity, demand is assumed unchanged during all planning time periods. Different prioritized items are required to be delivered to nodes via the transportation network. Urgency levels of items are differentiated based on their importance. For each type of item, the allowable delivered time periods are predefined to specify the nodes’ expected waiting time to receive an item. If an item cannot be delivered within the allowable time periods, a penalty cost is incurred, although the item still can be delivered later. The longer the delay in delivering an item, the more severe the penalty cost.

Characteristics of the transportation mode and the delivery methods are considered. Multiple but limited numbers of identical vehicles are used to transport prioritized items. Vehicles are assumed to have limited weight and volume capacities. For each vehicle, tours are assigned in each period to deliver items to one or more nodes. Tours begin at the depot, continue to one or multiple nodes, and then return to the depot. The total working hours in a single time period for the operation are limited. Therefore, the total travel time of tours assigned to a single vehicle in a single time period cannot exceed the constrained working hours. We do not consider the time to load and unload items on or off the vehicle. Furthermore, any node can be served multiple times by a single vehicle or multiple vehicles to meet its demand, and the demand can also be satisfied fully or partially in a single delivery.

2.2 Multi-Objective Logistics Model

Based on the above assumptions and general description of the scenario, the new model considers a multi-item, multi-vehicle, multi-period, soft time windows, and split delivery strategy prioritized delivery problem and is constructed by a multi-objective tour-based integer programming formulation. In this section, we first summarize notation, parameters and decision variables, followed by a presentation of a mathematical formulation for the model.
2.2.1 Notation, Parameters, and Decision Variables

Sets:

\( I = \{1, 2, 3, \ldots, i, \ldots, i\} \): the set of item types, and \( i \) is the total number of item types;

\( J = \{1, 2, 3, \ldots, j, \ldots, j\} \): the set of nodes, and \( j \) is the total number of nodes;

\( K = \{1, 2, 3, \ldots, k, \ldots, k\} \): the set of tour, and \( k \) is the total number of tours;

\( L = \{1, 2, 3, \ldots, l, \ldots, l\} \): the set of vehicles, and \( l \) is the total number of vehicles;

\( T = \{1, 2, 3, \ldots, t\} \): the set of periods, and \( t \) is the total number of planning time periods.

Routing parameters:

\( v \): the index of time period when the backorder amount of demand is delivered;

\( u \): the index of severe level of delay;

\( J_k \): the set of nodes which the vehicle will visit on tour \( k \);

\( t_k \): travel time of tour \( k \);

\( t_{oi} \): allowable delivered time periods of item \( i \);

\( C_k \): travel cost of tour \( k \);

\( H \): total working time available in a single period;

\( W \): the maximum load weight of a vehicle;

\( V \): the maximum volume capacity of a vehicle;

\( M \): a big number; and

Demand parameters:

\( d_{ijt} \): demand of the item \( i \) of the node \( j \) at time \( t \);

\( p_{iu} \): penalty cost of item \( i \) if the severe level of delay is \( u \);

\( fp_i \): penalty cost of item \( i \) if there is remaining unsatisfied demand after the operation;

\( a_i \): unit weight of item \( i \);

\( b_i \): unit volume of item \( i \);

Delivery Decision Variables:

\( x_{ijkl} \): amount of item \( i \) delivered to node \( j \) on tour \( k \) by vehicle \( l \) in period \( t \);
$w_{ijklmn}$: amount of item $i$ delivered to node $j$ on tour $k$ by vehicle $l$ in period $m$ to satisfy demand in period $n$, where $m, n \in T$;

$S$: maximum difference of satisfaction rate between any two nodes;

$s_j$: satisfaction rate of node $j$.

Routing Decision Variables:

$y_{klt}$: equal to one when tour $k$ is assigned to vehicle $l$ in period $t$, and 0 otherwise.

### 2.2.2 Formulation

**Objective 1:**

\[
\min \sum_i \sum_j \sum_u \sum_{t=1}^{T-1} \sum_{u+1} \left( d_{ijt} - \sum_k \sum_l \left( x_{ijkl} + \sum_{v=t+1}^{u} w_{ijklv} \right) \right) \cdot p_{iu} + \sum_i \left( \left( \sum_j \sum_t d_{ijt} - \sum_j \sum_k \sum_l \sum_t \left( x_{ijkl} + \sum_{m>t,m \in T} w_{ijklm} \right) \right) \cdot f_{pi} \right)
\]

**Objective 2:**

\[
\min \sum_k \sum_l C_{kly_{klt}}
\]

**Objective 3:**

\[
\min \ S
\]

**Subject to:**

\[
S \geq s_p - s_q, \quad \forall p, q \in J, p \neq q
\]

\[
S \geq s_q - s_p, \quad \forall p, q \in J, p \neq q
\]

\[
s_j = \frac{\sum_i \sum_k \sum_l \sum_t \left( x_{ijkl} + \sum_{m>t,m \in T} w_{ijklm} \right)}{\sum_i \sum_t d_{ijt}}
\]
\[
\sum_k t_k y_{klt} \leq H \quad \forall l, \forall t \in T
\]  (7)

\[
x_{ijkl} \leq M y_{klt} \quad \forall i, \forall j \in J_k, \forall k, \forall l, \forall t
\]  (8)

\[
w_{ijkltn} \leq M y_{klt} \quad \forall i, \forall j \in J_k, \forall k, \forall l, \forall t, \forall n < t
\]  (9)

\[
\sum_k \sum_l \sum_t x_{ijkl} + \sum_k \sum_{m>m} \sum_t \sum w_{ijklmt} \leq \sum_t d_{ijt} \quad \forall i, \forall j
\]  (10)

\[
\sum_i \sum_j a_i \left( x_{ijkl} + \sum_{n<t, n \in T} w_{ijkltn} \right) \leq W \quad \forall k, \forall l, \forall t
\]  (11)

\[
\sum_i \sum_j b_i \left( x_{ijkl} + \sum_{n<t, n \in T} w_{ijkltn} \right) \leq V \quad \forall k, \forall l, \forall t
\]  (12)

\[
x_{ijkl} \geq 0 \quad \forall i, \forall j \in J_k, \forall k, \forall l, \forall t
\]  (13)

\[
w_{ijklmn} \geq 0 \quad \forall i, \forall j \in J_k, \forall k, \forall l, \forall m \in T, \forall n \in T
\]  (14)

\[
x_{ijkl} = 0 \quad \forall i, \forall j \notin J_k, \forall k, \forall l, \forall t
\]  (15)

\[
w_{ijklmn} = 0 \quad \forall i, j \notin J_k, \forall k, \forall l, \forall m \in T, \forall n \in T
\]  (16)

\[
y_{klt} \in \{0, 1\} \quad \forall k, \forall l, \forall t
\]  (17)

In this model, the objective function (1) is a penalty function that aims to minimize unsatisfied demand after the operation in this period. There are two parts in this objective function. The first part (before the plus sign) indicates the total accrued penalty cost, which is the sum of penalty cost for various severe delay levels during the planning periods, and the second part is the penalty cost accrued at the end of the planning periods. This objective function attempts to minimize total unsatisfied demand, especially for high priority items. The objective function (2) aims to minimize the total travel time for all tours and all vehicles. The purpose of this objective is to assign as many vehicles as possible consistent with the working hour limitation in order to deliver the largest amount of items (this assumes that the demand in a disaster scenario will be large). Objective function (3) is used to minimize the
difference in the satisfaction rate between nodes. The satisfaction rate is the ratio between the requested demand and the actual delivered amounts. The purpose of this objective is intent on balancing the service among nodes. “Fairness” is the term usually used to describe this purpose.

In addition to these objectives, equations (4-5) are used to determine the maximum satisfaction difference between two nodes, and the satisfaction rate of a node is obtained by equation (6). Equation (7) indicates that the total travel time of all tours assigned to any single vehicle in this period cannot be longer than the available working hours in a single period. Equation (8) shows that delivery units of items only can exist if corresponding tours are selected to deliver supplies. Equation (10) shows that the total delivery of items cannot exceed the demand during the planning periods. Equations (11) and (12) address the total available loading weight and the total volume limit capacity constraints of a vehicle. Equations (13 - 16) are used to ensure that vehicles can only stop and deliver to nodes on tours assigned to them. Finally, equation (17) indicates that $y_{klt}$ is a binary variable.

3 Overall Solution Strategy

To solve the above model, we apply the well-known weighted sum method [40] to reduce the multi-objective formulation to that involving a single objective. The method transforms multiple objectives into an single objective function by multiplying each objective function by a weighting factor and summing up all weighted objective functions. This approach, well known for the fact that solutions are efficient if all weights are positive, is the most widely used for multi-objective optimization. The transformed problem is solved directly using the commercial solver CPLEX in our study. However, an important input of the model that first needs to be determined is the available tours. For a small-size delivery problem (e.g., only 3 or 5 nodes), enumerations of all possible tours is practical. However, as the problem size increases, enumerating tours is not a viable option since the resultant problem is too large
for CPLEX to solve. We propose two strategies to determine a good set of tours.

The first strategy is to use only a subset of all possible tours iteratively to solve the model. The subset of tours is replaced completely or partially by other tours based on the quality of the new solution. Tour replacement continues resulting in an efficient set of tours at the end of the iterative process. The focus of this approach is to limit the number of tours allowable at any stage while constantly seeking to improve the set of tours currently being considered.

The second strategy we propose is decomposition and parallel computation. This strategy allows the whole problem to be decomposed into several sub-problems and be solved in parallel. More specifically, for each sub-problem, we only consider a small number of nodes, for which the set of tours are more manageable. A portion of vehicles are assigned to each sub-problem to provide service between the depot and its nodes. Thus, each sub-problem can be simultaneously solved, i.e. in parallel.

4 Two Methods for Tour Determination

The two methods for tour determination outlined in Section 3 are now detailed. A GA-based approach is used for the first method, while a vehicle assignment approach is used for the second method. Prior to presentation of these details we first establish some properties from the SDVRP [14, 13, 15] that still hold in our tour-based multi-objective model. These properties prove useful for our solution methods.

4.1 Properties

If the travel cost \( \{c_{pq}\}, p, q \in J \) satisfies the triangular inequality, then the following theorem holds:

**Theorem 1.** An optimal tour collection exists such that no two tours can have more than
one split delivery node in common.

Proof. (by contradiction) Suppose that the theorem does not hold. Then in any optimal tour collection, there exists two tours that have more than one split delivery node in common. Consider such a solution $Y^*$, in which tours $r_1$ and $r_2$ are visited by the same vehicle to two split delivery nodes $p$ and $q$, $p \neq q$. In tour $r_1$, we deliver $\delta_{ip}^1$ to node $p$ and $\delta_{iq}^1$ to node $q$. In tour $r_2$ we deliver $\delta_{ip}^2$ to node $p$ and $\delta_{iq}^2$ to node $q$. Because these deliveries must satisfy the weight and volume constraints for the vehicle, we have $\sum_i \sum_{j=p,q} a_i \delta_{ij}^k \leq W$, and $\sum_i \sum_{j=p,q} b_i \delta_{ij}^k \leq V$, $k = 1, 2$. Without loss of generality, suppose $\delta_{ip}^1 = \min \{ \delta_{ip}^1, \delta_{iq}^1, \delta_{ip}^2, \delta_{iq}^2 \}, \forall i$. We now show that the same delivery amount of items in these two tours can be achieved by visiting only node $q$ in tour $r_2$. For this new setup,

$$
\hat{\delta}_{ip}^1 = 0, \forall i
$$

$$
\hat{\delta}_{iq}^1 = \delta_{iq}^1 + \delta_{ip}^1, \forall i
$$

$$
\hat{\delta}_{ip}^2 = \delta_{ip}^2 + \delta_{ip}^1, \forall i
$$

$$
\hat{\delta}_{iq}^2 = \delta_{iq}^2 - \delta_{ip}^1, \forall i.
$$

We can verify the continued satisfaction of the weight and volume constraints by noting that:

$$
\sum_i \sum_{j=p,q} a_i \hat{\delta}_{ij}^1 = \sum_i a_i \hat{\delta}_{ip}^1 + \sum_i a_i \hat{\delta}_{iq}^1
$$

$$
= 0 + \sum_i a_i \hat{\delta}_{ip}^1 + \sum_i a_i \delta_{ip}^1
$$

$$
= \sum_i \sum_{j=p,q} a_i \delta_{ij}^1 \leq W,
$$
and

\[
\sum_{i} \sum_{j=p,q} b_i \delta_{ij}^1 = \sum_{i} b_i \delta_{ip}^1 + \sum_{i} b_i \delta_{iq}^1 \\
= 0 + \sum_{i} b_i \delta_{iq}^1 + \sum_{i} b_i \delta_{ip}^1 \\
= \sum_{i} \sum_{j=p,q} b_i \delta_{ij}^1 \leq V,
\]

and

\[
\sum_{i} \sum_{j=p,q} a_i \delta_{ij}^2 = \sum_{i} a_i \delta_{ip}^2 + \sum_{i} a_i \delta_{iq}^2 \\
= \sum_{i} a_i \delta_{ip}^2 + \sum_{i} a_i \delta_{iq}^2 + \sum_{i} a_i \delta_{ip}^1 - \sum_{i} a_i \delta_{ip}^1 \\
= \sum_{i} \sum_{j=p,q} a_i \delta_{ij}^2 \leq W,
\]

and

\[
\sum_{i} \sum_{j=p,q} b_i \delta_{ij}^2 = \sum_{i} b_i \delta_{ip}^2 + \sum_{i} b_i \delta_{iq}^2 \\
= \sum_{i} b_i \delta_{ip}^2 + \sum_{i} b_i \delta_{iq}^2 + \sum_{i} b_i \delta_{ip}^1 - \sum_{i} b_i \delta_{ip}^1 \\
= \sum_{i} \sum_{j=p,q} b_i \delta_{ij}^2 \leq V.
\]

Suppose \( S^1 \) and \( S^2 \) are solutions of the original delivery strategy and the new delivery strategy, respectively, and let \( Z^1 = \{z_1^1, z_2^1, z_3^1\} \) and \( Z^2 = \{z_1^2, z_2^2, z_3^2\} \) be the corresponding objective values of the three objectives (1) - (3). Since the delivery quantities of various items to the two nodes remain the same, the penalty cost due to unsatisfied demand and the satisfactory rates in each node are unchanged. Therefore, \( z_1^1 = z_1^2 \) and \( z_2^1 = z_2^2 \). However, the total travel cost is reduced because the new tour has fewer nodes to visit and the triangular inequality holds. We can conclude that \( Z^1 \) is dominated by \( Z^2 \), contradicting the optimality of \( Y^* \). The theorem follows. \( \square \)
Definition 1. Suppose there exist $k$ distinct nodes $l_1, \ldots, l_k$ and $k$ tours in the optimal tour collection $Y^*$. Tour $r_1$ includes node $l_1$ and $l_2$, $r_2$ includes node $l_2$ and $l_3$, . . . , and $r_k$ includes node $l_k$ and $l_1$. The collection of nodes $\{l_i\}_{i=1}^k$ is called a $k$-split cycle.

Theorem 2. In the multi-objective logistics problem, there are no $k$-split cycle tours in the optimal tour collection $Y^*$, for any $k \geq 2$.

Proof. Omitted for the sake of brevity, since its proof follows a very similar argument to that used in the proof of Theorem 1. \hfill \Box

4.2 GA-based Approach

GA starts with an initial group of randomly selected solutions (or tour combinations) corresponding to the problem, called the population, and each solution in the population is called a chromosome, which consists of elements called genes. Chromosomes evolve iteratively, and each evolution is called a generation. As shown in Figure 1, in each generation, chromosomes in the population, called parents, are evaluated to determine the quality of each chromosome. All of them go through selection (roulette wheel method), crossover (modified Non-Wrapping Order Crossover), and mutation (insert mutation) processes to produce children. The feasibility test and enforced mutation are performed to make all children valid. In our case, a valid child indicates that all tours contained in a child are all feasible tours. Then, these children are evaluated and compared with their parents in order to eliminate some less effective children. An identical number of chromosomes as the previous population is selected among parents and children to constitute the population in the next generation. The procedure is repeated until a predefined number of iterations has run. The measure of evaluation of a chromosome is usually called the fitness value. Further discussions of some functions in the GA are now provided.
4.2.1 Initialization

The representation of the chromosome in our study is shown as in Figure 2. This chromosome is designed for a logistics problem with five nodes. The first row is the reference row, and is used to indicate the number of nodes that will be visited in the corresponding tour (i.e., the first tour is only to visit node 2). It is noted that this row will not be included in the genetic operations during the procedure. Therefore, suppose “0” indicates the depot; there are a total of six tours in this chromosome, (0-2-0), (0-1-0), (0-5-0), (0-3-4-0), (0-3-2-0), and (0-1-4-5-0).

<table>
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<tr>
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Figure 2: An example of chromosome representation

In the initial step of the GA, a set of chromosomes is generated to form a population that usually consists of 20-50 chromosomes. Suppose there are $k$ tours included in each chromosome for a $\tilde{j}$ node problem, and the number of nodes visited in each tour is $c_i, i = 1, \ldots, k$, respectively. Then there are $\sum_i c_i$ genes in a chromosome and the initial population is presented by a $N \times \sum_i c_i$ matrix, where $N$ is the size of the population. We apply
the nearest neighborhood principle to generate the initial population: Initially, a node is randomly selected to be the first gene of the chromosome; then the closest unselected node from the current one is selected to be the next gene in the chromosome, until all nodes are selected. The procedure is repeated until all genes in a chromosome are filled out. Following the same method, \( N \) chromosomes can be generated, and the initial population is obtained.

4.2.2 Evaluation

Once the initial population is available, evaluation of each chromosome takes place to explore the quality of corresponding optimal solution. The fitness function in our study is the objective function in the mathematical model. However, due to the computational effort expected to solve the whole problem (denoted by \( \psi_w \)), we instead solve the partial problem (denoted by \( \psi_p \)) and use this problem’s objective function in evaluating the fitness. The partial problem only includes one vehicle, and this vehicle has only to serve the equally divided demand in each cluster. The definition of equally divided demand is described as follows: If the demand in each cluster is \( d_i \), and total number of vehicles is \( l \), then each vehicle will take responsibility for only \( d_i/l \) in each cluster. The reason for using the partial problem as the measure to evaluate the quality of chromosomes is based on the following theorem.

**Theorem 3.** If a set of tours \( \lambda = [k_1, k_2, \ldots, k_l] \) results in the best solution in \( \psi_p \), the same set of tours can also result in the best solution in \( \psi_w \) with the same setting (e.g. demand, travel time, etc.).

**Proof.** (by contradiction) Assume \( \lambda^* \) is the best set of tours in \( \psi_p \), and the corresponding unsatisfied demand obtained by using these tours in the model is \( \tilde{d}^* \). Because all demand is equally divided and assigned to each vehicle, if every vehicle uses the same set of tours \( \lambda^* \), the unsatisfied demand resulting from solving \( \psi_p \) for each vehicle is identical, and the total unsatisfied demand will equal \( \sum_l \tilde{d}^* \). If \( \lambda \) is claimed the best set of tours in \( \psi_w \), where
\( \lambda \neq \lambda^* \), then the total unsatisfied demand from using \( \lambda \) in each vehicle is \( \sum \bar{d} \), and the unsatisfied demand from each vehicle, because they all have identical settings, is \( \bar{d} \). Because \( \sum \bar{d} < \sum \bar{d}^* \), thus \( \bar{d} < \bar{d}^* \). This contradicts the assumption. Therefore, \( \lambda^* \) is also the best set of tours in \( \psi_w \).

4.2.3 Crossover Operation

The crossover operation used in our GA is inspired by and modified from Cicirello’s Non-Wrapping Order Crossover (NWOX) [9], and we modify it based on our characteristics of the chromosome’s representation. The advantage of NWOX is that it strongly preserves relative order, but also respects the absolute positions within the parent. Our modified NWOX starts by copying parents \( P_1, P_2 \) as two children \( C_1, C_2 \). One position \( a \) is selected randomly from the interval \([1, \delta - 1]\), where \( \delta \) is the length of the chromosome and \( \delta \geq \bar{j} \), and \( b = \lfloor(\delta - a)/\bar{j}\rfloor + 1 \) is the distance of positions after position \( a \). Suppose that \( v_1(i) \) and \( v_2(i) \) represent the values of position \( i \) in the parent \( P_1 \) and \( P_2 \) respectively; hence the values \( v_2(a), v_2(a+1), \ldots, v_2(a+b) \) are searched in \( C_1 \) from the leftmost side to the rightmost side separately, and only the “first found” position which has the value we are looking for is replaced by “holes” as shown in Figure 3(b). The same procedure is repeated in \( C_2 \) to replace the holes in those first found places of corresponding values \( v_1(a) \) to \( v_1(a+b) \). Then, a sliding motion is performed to move holes into positions \( a \) to \( (a+b) \). The non-hole values are slid leftward until all of them are grouped together contiguously. All remaining non-holes in the region are slid rightward while leaving holes in the region (see Figure 3(c)). Then, values \( v_2(a), v_2(a+1), \ldots, v_2(a+b) \) of parent \( P_2 \) are placed in position \( a, a+1, \ldots, a+b \) in child \( C_1 \), and, similarly, values \( v_1(a), v_1(a+1), \ldots, v_1(a+b) \) of parent \( P_1 \) are placed in position \( a, a+1, \ldots, a+b \) in child \( C_2 \), as shown in Figure 3(d).
4.2.4 Feasibility Test and Enforced Mutation

In the GA employed in this study, one important mechanism is the feasibility test, which is used to test whether a child generated from genetic operations is feasible or not. A child is considered to be feasible if all tours contained in this chromosome are valid. In our case, a valid tour is one where any location will not be visited more than once in a tour. In addition, based on Theorems 1 and 2, we can delete those tours that have more than one node in common, or are \( k \)-split cycle tours. The benefit of doing this is that we will not waste time testing some tours that are not able to become the dominating solutions in the multi-objective optimization process.

If one of the tours in the chromosome is invalid, enforced mutation is applied to modify invalid tours. The idea of enforced mutation is to remove the location that is visited multiple times or causes the violation of Theorems 1 and 2, and to switch it with any other location in other tours without resulting in any invalidity. The location to be switched with is chosen randomly. This operation is performed repeatedly until all tours are valid.
4.3 Decomposition Approach

The second approach is named Vehicle Assignment Heuristic (VAH) in this paper. Assume that a set of vehicles $L = \{1, \ldots, l, \ldots, \bar{l}\}$ is used to deliver items to a set of nodes $J = \{1, \ldots, j, \ldots, \bar{j}\}$. If we use “it” to indicate the iteration number of the algorithm, $I$ as the total desired number of iterations in the algorithm, $\tau$ as the number of times that the solution is not improved after an iteration, and $TH$ as the maximum times allowed in a row that the solution is not improved, then the VAH procedures can be described as follows.

VAH algorithm:

1. Randomly assign a partial number of vehicles $\hat{L}_g$ to serve only partial nodes $\hat{J}_g$ ($\leq 3$ preferably). $G=\{1, \ldots, g, \ldots, \bar{g}\}$ is the collection of subproblems, where $\sum_g \hat{L}_g = \bar{l}$, and $\forall p \neq q, \hat{J}_p \cap \hat{J}_q = \phi$. The original problem has been decomposed into $\bar{g}$ subproblems.

2. For each subproblem $SP_g$, all feasible shortest travel time tours are enumerated.

3. For each subproblem $SP_g$, construct the mathematical model based on $\hat{L}_g, \hat{J}_g$, and the corresponding demand; solve $SP_g$ by CPLEX, and get the objective values $z_g$ and the total objective value $z_{all} = \sum_g z_g$. If it is in the initial step, set the best total objective value $z^*_all = z^0_{all}$, and $it = 1$.

4. Find a pair of groups $(p, q)$ that has the minimum and maximum objective value, respectively.

5. If $\hat{L}_g > 1$, then do steps 6 and 7:

6. Remove one vehicle from $\hat{L}_p$, and assign it to $\hat{L}_q$.

7. Go to step 3, update $z_p, z_q$, and $z_{all}$.

8. If $z_{it} < z^*_all$, update $z^*_all = z_{it}$. Go to step 13.

9. Else $z^*_{all} = z^*_all$, and $\tau = \tau + 1$. 

17
10. If $\tau < TH$, go to step 5.

11. Else Stop the algorithm.

12. Else Find the next minimum objective value group, go to step 5.

13. If $it < I$, go to step 2.

14. Else Stop the algorithm.

In each sub-problem, the tours are determined directly by enumerating all possible ones because the problem size is relatively small. Theorems 1 and 2 are employed here to construct dominating tours in each sub-problem. The difficulty of solving the model is further reduced because some tours are not valid based on Theorems 1 and 2 and the number of tours in the model for each sub-problem can be decreased as well.

5 Computational Experiments

Two parts are included in the computational experiments. In the first part, a random instance generator is designed to produce numerical instances for the following computational experiments. In the second part, a comparison of performance among different approaches of the tour determination (i.e., enumeration, GA-based approach, and VAH) is provided.

5.1 Random Instance Generator

The random instance generator has been coded in C. We assume the service area of a logistics operation is in a 50 square mile area. The location of the depot and all nodes are randomly located by the instance generator in this area. To transform Euclidean distances to road
distances, the following equation is adopted from the literature [27]:

\[
d(q, r; k, p, s) = k \left[ \sum_{i=1}^{2} |q_i - r_i|^p \right]^{1/s}
\]  

where \( q \) and \( r \) are coordinate points on the plane in the service area, and \( k, p, s \) are parameters. Suggested ranges of these parameters are \( k \in \{0.80, 2.29\} \), \( p, s \in \{0.90, 2.29\} \), respectively. In our instance generator, parameters are randomly generated from the corresponding ranges of each parameter. It is noted that only distances between the depot and each node are estimated according to (18), and distances among nodes are generated randomly based on the triangular inequality (i.e., road distances are estimated on link \( \overline{OA} \) and \( \overline{OB} \), if \( O \) is the depot and \( A, B \) are two demand clusters, but the distance \( \overline{AB} \) is generated randomly between \( (\overline{OA} + \overline{OB}) \) and \( (\overline{OA} - \overline{OB}) \) to ensure that the triangular inequality holds). After road distances of all links are obtained, the speed limit and the congestion status are given on each link randomly to represent the true scenario on roads. For simplicity, we use only three different speed limits (i.e., 30 mi/h, 45 mi/h, and 60 mi/h), and three type of congestion (i.e., 0%, 50%, and 100%). The percentage of the congestion indicates the additional travel time required to use that link (i.e., 50% congestion means an extra 50% of travel time compared with uncongested travel time). We assume here that the speed and congestion parameters are constant once they have been selected.

Three sets of problems are generated by the instance generator: small, medium, and large. The corresponding characteristics of each set of problems are provided in Table 1. Other outputs from the instance generator include the demand from each node, the number of vehicles used in each set of problems, tours in the network of each problem, and the model for each problem.
5.2 Performance Comparison

We test the performance of our approaches for tour determination on three sets of problems. The heuristic approaches proposed in this paper have been coded in C, which interfaces with the callable library in ILOG CPLEX version 11.2, which is used to solve problems after the tours have been determined. Each instance with different tour determination approaches has been run on a computer with 2.00 GB RAM, and an Intel Pentium D 3.40 GHz processor.

To compare the performance among different tour determination approaches, we present the result when the weight of the first objective function is given by 1 and others are given by 0. The reason for doing so is twofold: 1) we regard it as the most important objective among three, and 2) we desire to observe closely the impact of different tour determination approaches. We use two criteria to stop the iterative process: either the running time exceeds 3,600 seconds or the MIP gap is smaller than 0.01%. Table 2 summarizes the results from the three approaches. The first column identifies the problem IDs of each instance. For each approach, the % of demand delivered and the running time (seconds) of the corresponding approach are shown. For some instances, there is no feasible solution available within 3,600 seconds, so we designate them as n/a. Furthermore, under the GA-based approach column, the number of tours that remain in the model for small-size, medium-size, and large-size instances after this approach are 6, 7, and 8, respectively.

Table 3 summarizes the average performance of the three tour determination approaches. The GA-Based approach reduces the average running time by about 22.3%. The average percentage of delivery is 99.8%, which is almost the same as problems in which all tours are included. However, the variance of running times in the GA-based approach is still high (range is from 17.81 to 3,600 seconds), because CPLEX spends a long time in proving optimality in some cases. Figure 4 reveals this situation by showing the relationship of the running time and corresponding solutions for some instances (i.e., for these four instances, they all reach or almost reach the final optimal solution in about 600 seconds). On the other hand, the VAH approach finds a solution in a very short running time with a 4.3% reduction.
Table 1: Characteristics of problem sets

<table>
<thead>
<tr>
<th>Set</th>
<th>Problem</th>
<th>No. of clusters</th>
<th>No. of tours</th>
<th>No. of vehicles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>9</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>9</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>3</td>
<td>9</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>6</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>Medium</td>
<td>3</td>
<td>4</td>
<td>22</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>21</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>22</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
<td>31</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>33</td>
<td>18</td>
</tr>
<tr>
<td>Large</td>
<td>3</td>
<td>5</td>
<td>31</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>39</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>41</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Performance table

<table>
<thead>
<tr>
<th>Prob. ID</th>
<th>Enumeration</th>
<th>GA-based Approach</th>
<th>VAH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% of Time</td>
<td>% of Time</td>
<td>% of Time</td>
</tr>
<tr>
<td>delivere</td>
<td>(seconds)</td>
<td>delivered (seconds)</td>
<td>delivered (seconds)</td>
</tr>
<tr>
<td>delivere</td>
<td>(seconds)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>99.98</td>
<td>178.11</td>
<td>99.96</td>
</tr>
<tr>
<td>S2</td>
<td>99.97</td>
<td>44.54</td>
<td>99.93</td>
</tr>
<tr>
<td>S3</td>
<td>98.73</td>
<td>312.16</td>
<td>98.73</td>
</tr>
<tr>
<td>S4</td>
<td>96.31</td>
<td>260.15</td>
<td>96.31</td>
</tr>
<tr>
<td>S5</td>
<td>94.74</td>
<td>17.81</td>
<td>94.74</td>
</tr>
<tr>
<td>M1</td>
<td>96.96</td>
<td>3,600</td>
<td>96.98</td>
</tr>
<tr>
<td>M2</td>
<td>n/a</td>
<td>n/a</td>
<td>98.74</td>
</tr>
<tr>
<td>M3</td>
<td>96.54</td>
<td>3,287.17</td>
<td>96.54</td>
</tr>
<tr>
<td>M4</td>
<td>n/a</td>
<td>n/a</td>
<td>92.95</td>
</tr>
<tr>
<td>M5</td>
<td>95.34</td>
<td>2,277.21</td>
<td>95.14</td>
</tr>
<tr>
<td>L1</td>
<td>n/a</td>
<td>n/a</td>
<td>96.45</td>
</tr>
<tr>
<td>L2</td>
<td>n/a</td>
<td>n/a</td>
<td>97.70</td>
</tr>
<tr>
<td>L3</td>
<td>90.94</td>
<td>3,600</td>
<td>91.00</td>
</tr>
<tr>
<td>L4</td>
<td>n/a</td>
<td>n/a</td>
<td>84.89</td>
</tr>
<tr>
<td>L5</td>
<td>79.09</td>
<td>3,600</td>
<td>73.06</td>
</tr>
</tbody>
</table>

Table 3: Average performance results

<table>
<thead>
<tr>
<th></th>
<th>Enumeration</th>
<th>GA-based Approach</th>
<th>VAH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>Average</td>
<td>Average</td>
<td></td>
</tr>
<tr>
<td>time</td>
<td>time</td>
<td>time</td>
<td></td>
</tr>
<tr>
<td>(seconds)</td>
<td>(seconds)</td>
<td>of delivery</td>
<td></td>
</tr>
<tr>
<td>2,345.14</td>
<td>1,821.54</td>
<td>99.8</td>
<td>32.33</td>
</tr>
<tr>
<td>95.5</td>
<td>32.33</td>
<td>95.5</td>
<td></td>
</tr>
</tbody>
</table>
of solution quality compared to the GA-based approach. Based on these results it is evident that only the VAH approach for tour generation is practical for large problem instances. Therefore, only this method is used in the case study presented in the next section.

![Graphs showing relationship between % optimality gap and running time](image)

Figure 4: Instances showing relationship between % optimality gap and running time

6 Case Study

To address the importance of our model for delivery items involving prioritizing delivery requirements, we applied the logistics model to the disaster relief operation in Northridge, CA, where a 6.7 moment magnitude earthquake occurred on Jan. 17, 1994. The study region we selected is Los Angeles County, CA, where the geographical size was 4,086.9 square miles, and the total population was 9,519,338 [35].
6.1 Data

The Federal Emergency Management Agency (FEMA) has developed a software named HAZUS-MH which can be used to simulate nature disasters [17]. We used HAZUS-MH to simulate the Northridge earthquake scenario. Three types of simulation output are particularly interesting for us: the number of displaced households (Figure 5), the number of severity level 1 injuries (Figure 6), and the number of households without water service (Table 4). It is noted that severity level 1 injuries are those which require medical attention, but hospitalization is not needed. These are used to generate and estimate demand for food, prescription medication, and water, respectively.

![Figure 5: Distribution of displaced households](image)

Table 4: Number of households without water service over time periods

<table>
<thead>
<tr>
<th>Total Number of Households</th>
<th>Number of Households without Service</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>At Day 1</td>
</tr>
<tr>
<td>3,133,774</td>
<td>311,689</td>
</tr>
</tbody>
</table>

Table 4 shows the number of households without water service over different time periods after the earthquake. Since the geographic distribution of households without water service
is not provided in the simulation output, we assumed that the number of households without water service in each census tract was proportional to the number of households affected by the earthquake in the track. Secondly, the number of displaced households is used to estimate the demand of food because those families, suffered damage to their houses, are required to have shelter to live temporarily, and are incapable of preparing food by themselves. The estimated number of households displaced is 25,469 according to the simulation. Finally, the number of casualties of severity level 1 is used to estimate the demand for prescription medication. Based on the simulation, the total number of injuries at this level is 1,654 in this earthquake. It is noted that we only extracted casualty data for level 1 in the residential families because the earthquake occurred at 4 AM, and we assumed the majority of people were at home during the earthquake.

In a disaster relief operation, it is common to transport supplies first from a central depot to several local distribution centers by trucks; then, supplies are distributed to individuals from local distribution centers. Therefore, we formed 9 clusters in Los Angeles County, as shown in Figure 7, simply by visualization, and one cluster center (or local distribution center) was selected in each cluster used to receive deliveries from the central depot. The demand data for a cluster can be obtained by aggregating each type of demand from all census tracts located inside the cluster. However, in the simulation output, only “one-time” demand is available for us to derive from data. Therefore, we assumed the time series data in Table 4 representing the recovery speed from the earthquake, and all supplies would follow the same pattern to recover as well. Demand data over 7 days was estimated according to an exponential fitting function by using data on number of households without water service and the direct initial data from the simulation of each item, respectively. As a result, a demand matrix containing 3 items, 9 nodes and 7 time periods was obtained. The other parameters needed for implementing the logistics model are shown in Table 5, and some of these parameters were obtained by using real data in practice [22].
Figure 6: Distribution of severe level 1 injuries

Figure 7: Clustering result in Los Angeles county
Table 5: Summary of parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vehicle:</strong></td>
<td></td>
</tr>
<tr>
<td>capacity:</td>
<td>11580 kg</td>
</tr>
<tr>
<td>volume:</td>
<td>56 m$^3$</td>
</tr>
<tr>
<td><strong>Items:</strong></td>
<td></td>
</tr>
<tr>
<td>Prescription medication</td>
<td>ship. weight: 86.5 kg; ship. volume: 0.22 m$^3$; soft time window = [0,1]</td>
</tr>
<tr>
<td>Water</td>
<td>ship. weight: 400 kg; ship. volume: 4.3 m$^3$; soft time window = [0,2]</td>
</tr>
<tr>
<td>Food</td>
<td>ship. weight: 700 kg; ship. volume: 2.3 m$^3$; soft time window = [0,3]</td>
</tr>
<tr>
<td><strong>Working hours</strong></td>
<td>12 hours</td>
</tr>
<tr>
<td><strong>Planning periods</strong></td>
<td>7 days</td>
</tr>
</tbody>
</table>

---

Figure 8: Transportation network and sub-networks
6.2 Results

Since this case study has nine demand nodes and one depot, we applied the VAH algorithm to decompose the original network into three subnetworks that are depicted by different type of lines in Figure 8, which shows the transportation network among the central depot and nine local distribution centers in Los Angeles County, CA. The route between any two nodes was first determined by the online map provider, and consisted of detoured partial segments if the best route involved some damaged roadways due to the earthquake. The detours were made according to the government report [11]. Then the travel time between any two cluster centers or between a cluster center and the depot was estimated by taking the average of the travel time in the traffic and in congested traffic, because it is reasonable to assume that some congestion occurred around the area after the earthquake. In addition, we assume the weights for objectives 1 to 3 are 0.6, 0.1, and 0.3, respectively. This reflects the ordinary scenario in a disaster relief operation that the first goal of this operation is to satisfy as many demand requests as possible; this is followed by the fairness issue among different locations, and the travel cost to ship supplies is the least concern for authority officers.

It is common to involve resources from a variety of organization in a disaster relief operation. The total number of vehicles was assumed to be 400 from 20 different organizations (e.g. American Red Cross, military units, etc.). Each organization is responsible for servicing the delivery tasks that it gets assigned from the disaster operations manager, and each organization is also responsible for managing its own fleet to accomplish the relief effort. As a result, during seven days of operations based on the data described above and the proposed new model, a total of 621 assignments of delivery tasks had been made to different organizations, and this was equivalent to 12,420 deliveries by trucks. Table 6 presents the overall performance of the humanitarian relief effort in each time period. For each item, two kinds of information are shown in the table. The first row shows the total percentage of demand being satisfied in each time period by either immediate or backorder deliveries. The second row indicates the percentage of demand being delivered immediately during the time
period when the demand arose. Furthermore, the last column indicates the best scenario for the satisfaction levels, subject to time and resource restrictions in the case study. Among the three item types, an average of 90.1% of prescription medication demand was satisfied in all nine nodes, while only about 70% and 16.4% of water demand and food demand, respectively, were delivered on average. We note that only 30.8%, 32.5%, and 7.5% of demand of prescription medication, water, and food, respectively, were satisfied during the time period when the demand arose — these low numbers are due to the soft time window feature in the model which allowed delayed satisfaction of demand. The amount of delay can be changed and offers great flexibility to the operations manager. It is noted that our proposed model has significant ability to prioritize delivery, and achieves high satisfaction rates for prescription medication. In addition, note that the fluctuation for immediate delivery of medication is significant. The reason for this is revealed from the loadings of vehicles in the solution. It is interesting to see that the full truck load of a single type item is a preferred method of delivery in our application, when the overall vehicle capacity is not capable of satisfying all demand, i.e., in time period three, because much medication demand in time period two is not satisfied after time period two, more vehicles are used to deliver demand for time period two; therefore, if extra capacity is available on vehicles to the same demand locations, more medication for fulfilling demand in time period three is carried on vehicles to deliver.

Table 6: Overall performance of the humanitarian relief effort

<table>
<thead>
<tr>
<th></th>
<th>medication</th>
<th>water</th>
<th>food</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 1 satisfaction</td>
<td>100.0%</td>
<td>100.0%</td>
<td>27.2%</td>
</tr>
<tr>
<td>Immediate delivery</td>
<td>60.9%</td>
<td>48.4%</td>
<td>14.4%</td>
</tr>
<tr>
<td>t = 2 satisfaction</td>
<td>100.0%</td>
<td>97.0%</td>
<td>13.7%</td>
</tr>
<tr>
<td>Immediate delivery</td>
<td>7.4%</td>
<td>27.9%</td>
<td>2.0%</td>
</tr>
<tr>
<td>t = 3 satisfaction</td>
<td>100.0%</td>
<td>62.1%</td>
<td>13.9%</td>
</tr>
<tr>
<td>Immediate delivery</td>
<td>23.6%</td>
<td>23.3%</td>
<td>5.7%</td>
</tr>
<tr>
<td>t = 4 satisfaction</td>
<td>95.3%</td>
<td>55.1%</td>
<td>14.2%</td>
</tr>
<tr>
<td>Immediate delivery</td>
<td>18.9%</td>
<td>16.8%</td>
<td>0.6%</td>
</tr>
<tr>
<td>t = 5 satisfaction</td>
<td>76.6%</td>
<td>54.8%</td>
<td>14.3%</td>
</tr>
<tr>
<td>Immediate delivery</td>
<td>2.9%</td>
<td>22.6%</td>
<td>5.9%</td>
</tr>
<tr>
<td>t = 6 satisfaction</td>
<td>76.6%</td>
<td>55.0%</td>
<td>15.0%</td>
</tr>
<tr>
<td>Immediate delivery</td>
<td>25.4%</td>
<td>32.7%</td>
<td>9.3%</td>
</tr>
<tr>
<td>t = 7 satisfaction</td>
<td>76.6%</td>
<td>55.7%</td>
<td>14.5%</td>
</tr>
<tr>
<td>Immediate delivery</td>
<td>76.6%</td>
<td>55.7%</td>
<td>14.5%</td>
</tr>
<tr>
<td>Avg. of satisfaction</td>
<td>90.1%</td>
<td>70.0%</td>
<td>16.4%</td>
</tr>
<tr>
<td>Best scenario</td>
<td>100.0%</td>
<td>78.9%</td>
<td>42.6%</td>
</tr>
</tbody>
</table>

Table 28
6.3 Discussion

A major feature of our proposed model is the ability to prioritize delivery. In this section, we conducted further analysis and discussed some insights of the model.

We first compared our model to other similar models in the literature. Balcik et al.[5] proposed a model to deliver relief supplies in disasters. Although our model and theirs have the same purpose, they are different for two reasons. First, they considered two types of supplies. The first type are critical items for which demand occurs once at the beginning of the planning horizon (e.g. shelters, blankets) and has to be fully satisfied during the planning horizon (i.e., a hard constraint in the model). The second type items are those that are consumed regularly and whose demand occurs periodically over the planning horizon (e.g. food, prescription medication, and water), and cannot be backordered if it is not satisfied on time. In ours, we are only interested in the second type of items and backorders are permitted. Second, their model did not consider the soft time window, which means that the demand must be served immediately when it occurs.

We assumed that the demand of shelters after the earthquake is zero since our interest is not in this type of item. Therefore, the total available capacity of vehicles would not be affected because of shipping this type of item. To conform to their model, we collapsed our model to one in which backorders are not permitted and the soft time window is not considered. In addition, we adjusted the weights of three objectives from the original (0.6,0.1,0.3) to (0.5,0.5,0) since they only considered the penalty cost due to unsatisfied demand and the travel cost in their model. We first applied their model to obtain the solution, and used their travel cost as the budget limit in our model in order to get comparable results. The comparison is shown in Tables 7 and 8. Table 7, indicating the percentage of demand being satisfied in each time period, shows that though their model reveals a prioritized delivery pattern similar as ours, their model was unable to concentrate on delivering the item with the highest priority (e.g. prescription medication). On the contrary, our collapsed model showed significant ability to deliver the highest priority item. The improved performance
of our model is due to the penalty cost function structure as shown in Equation 1. Table 8 shows the number of injuries or households suffering shortage of supplies. It is apparent that our collapsed model can avoid more injuries or households without sufficient supplies, under the same budget as theirs.

Table 7: Percentage of satisfied demand between models

<table>
<thead>
<tr>
<th></th>
<th>$t = 1$</th>
<th>$t = 2$</th>
<th>$t = 3$</th>
<th>$t = 4$</th>
<th>$t = 5$</th>
<th>$t = 6$</th>
<th>$t = 7$</th>
<th>Avg. % of satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Medication:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balcik's model</td>
<td>85.4%</td>
<td>84.3%</td>
<td>86.7%</td>
<td>91.9%</td>
<td>87.4%</td>
<td>91.9%</td>
<td>76.8%</td>
<td>86.4%</td>
</tr>
<tr>
<td>Proposed model (collapsed)</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>95.3%</td>
<td>99.4%</td>
<td></td>
</tr>
<tr>
<td><strong>Water:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balcik's model</td>
<td>54.2%</td>
<td>58.1%</td>
<td>59.1%</td>
<td>61.7%</td>
<td>56.2%</td>
<td>61.6%</td>
<td>58.1%</td>
<td>58.3%</td>
</tr>
<tr>
<td>Proposed model (collapsed)</td>
<td>76.0%</td>
<td>77.8%</td>
<td>78.4%</td>
<td>78.9%</td>
<td>79.5%</td>
<td>38.5%</td>
<td>12.8%</td>
<td>64.6%</td>
</tr>
<tr>
<td><strong>Food:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balcik's model</td>
<td>12.2%</td>
<td>12.7%</td>
<td>13.8%</td>
<td>10.9%</td>
<td>12.2%</td>
<td>12.8%</td>
<td>10.1%</td>
<td>15.6%</td>
</tr>
<tr>
<td>Proposed model (collapsed)</td>
<td>12.1%</td>
<td>12.2%</td>
<td>13.0%</td>
<td>14.0%</td>
<td>15.0%</td>
<td>8.6%</td>
<td>8.7%</td>
<td>12.0%</td>
</tr>
</tbody>
</table>

Table 8: Number of injuries or households without sufficient supply

<table>
<thead>
<tr>
<th></th>
<th>$t = 1$</th>
<th>$t = 2$</th>
<th>$t = 3$</th>
<th>$t = 4$</th>
<th>$t = 5$</th>
<th>$t = 6$</th>
<th>$t = 7$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Medication:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balcik's model</td>
<td>0</td>
<td>216</td>
<td>223</td>
<td>181</td>
<td>106</td>
<td>159</td>
<td>98</td>
<td>982</td>
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<tr>
<td>Proposed model (collapsed)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Water:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balcik's model</td>
<td>0</td>
<td>0</td>
<td>14,282</td>
<td>12,081</td>
<td>11,356</td>
<td>10,230</td>
<td>11,252</td>
<td>59,201</td>
</tr>
<tr>
<td>Proposed model (collapsed)</td>
<td>0</td>
<td>0</td>
<td>7,482</td>
<td>6,405</td>
<td>5,998</td>
<td>5,625</td>
<td>5,261</td>
<td>30,771</td>
</tr>
<tr>
<td><strong>Food:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balcik's model</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>23,263</td>
<td>21,411</td>
<td>20,328</td>
<td>20,227</td>
<td>85,230</td>
</tr>
<tr>
<td>Proposed model (collapsed)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>23,285</td>
<td>21,546</td>
<td>20,529</td>
<td>19,524</td>
<td>84,885</td>
</tr>
</tbody>
</table>

Furthermore, we are also interested in the impact of the penalty cost of corresponding items to the prioritizing delivery strategy. An analysis of comparing the original proposed model with modified models with different penalty cost settings was conducted. We considered two situations. The first situation is when all items have the exactly same penalty cost, and we denoted it as model-EP. The second situation is one where we employed the reverse penalty cost setting, denoted as model-RP, which means that the food is the highest priority item, followed by water and prescription medication. The result is shown in Table 9. The percentage of demand being delivered for different items is significantly affected by the
penalty setting. If equal penalties are applied, decisions regarding which type of items should be delivered become more complicated (e.g., shipping size and weight, travel distance, etc.) than with the model with the original setting. The model is also very sensitive to changes of the penalty cost settings from the original one to the reverse one. The percentage of demand of prescription medication being satisfied drops by 10% when we shifted the highest priority item from prescription medication to food, while the percentage of demand for food being satisfied increases around 10%, though the delivered percentage of food remains low because of its heaviest shipping weight.

<table>
<thead>
<tr>
<th>Table 9: Comparison between different penalty cost design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Prescription medication</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>Model</td>
</tr>
<tr>
<td>Model-EP</td>
</tr>
<tr>
<td>Model-RP</td>
</tr>
</tbody>
</table>

7 Conclusions and Future Work

This paper proposes a new logistics model for delivery of prioritized items for logistics operations that is applicable to a disaster relief effort. Our model considers multi-items, multi-vehicles, multi-periods, soft time windows, and split delivery strategy scenario, and is formulated as a multi-objective integer programming model. The distinguishing feature of our work is to consider the delivery priorities of different items and to encompass this idea as an objective. In addition, two heuristic approaches are developed, the GA-based approach and the Vehicle Assignment heuristic (VAH), to limit the number of available tours in the model in order to solve it effectively. The performance of these two approaches is analyzed and their efficiency is investigated. We found that, in general, the GA-based approach can resolve infeasible situations effectively, and provide good solutions. On the other hand, the VAH approach provides solutions in a short computational time while they have about a 4.3% reduction in solution quality compared to the GA-based approach.
To verify the importance of the new model for delivery tasks containing prioritized delivery requests, we conducted a case study in the disaster relief operation effort in the earthquake scenario. Overall, our model is capable of satisfying 90.1% of prescription medication demand, 70% of water demand, and 16.4% of food demand, under the limited delivery capacity of vehicles and limited working time in each time period. The result showed that the model performed well in the disaster relief operation, where prioritizing delivery is important because only correct deliveries of supplies can make sure to keep isolated people alive as long as possible. Further analysis showed that our model performed better than the model in the literature which has a similar purpose as ours. In addition, our model is sensitive to the penalty cost settings as seen through the case study that suggested that appropriate determinations of the penalty cost would make our model more efficient.

We suggest three directions for future work. The first is to investigate the robustness of our model with respect to uncertainty in demand values, congestion levels, network accessibility, and correlations of cluster locations with respect to the highway roadways. The second is to develop more efficient multi-objective optimization methodologies for this type of problem, and tradeoff analysis can be analyzed to understand the compromises among different goals. The third is to consider a distributed scenario in which several temporary depots are required to be located and serve as “bridges” between the major depot and nodes.

References


