Optimal placement of warehouse cross aisles in a picker-to-part warehouse with class-based storage

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Revised: May 2010

Abstract

Given a picker-to-part warehouse having a simple rectilinear aisle arrangement with north-south storage aisles and east-west travel aisles (or "cross aisles"), this paper investigates the optimal placement of the cross aisles as a consequence of the probability density function of the order pick locations, as determined by the storage policy. That is, for a given storage policy, what placement of the cross aisles will result in a minimal expected path length for the picker? An analytical solution procedure is developed for the optimal placement of a single middle cross aisle given for a given storage policy. A simplifying assumption is made as regards picker routing, but arbitrary non-random storage policies are considered. The solution procedure is generalized to a method for multiple cross aisles. Some example problems are solved and a simulation study is used to measure the impact of our simplifying assumptions.

Keywords: Warehousing, aisle design, order-picking, material handling

1 Introduction

A significant component in the operating cost of picker-to-part warehouses is picking time, which has been estimated to contribute up to 55% of total operating costs (see for example Tompkins et al. [2003]). In order to minimize this cost, a number of approaches are possible. Efficient picker routing algorithms reduce the distance traveled to pick a given pick list. Class-based storage policies reduce travel distance by concentrating most frequently picked items close to the I/O point. These and other factors such as order batching, warehouse shape and so on have been studied extensively, both in isolation and in combination with other factors.

One factor which has been studied less extensively is layout design. The picker uses aisles to travel through the warehouse: storage aisles in which parts are picked and cross aisles which are used to travel from storage aisle to storage aisle. Once the picker has made all of the picks in a given storage aisle, he must continue to travel through the storage aisle until he reaches the cross aisle via which he will travel to the next storage aisle. This is wasted travel in the sense that it adds no value; the less such travel, the shorter the picker's expected path will be. If it can be reduced, then picker travel will be reduced without losing value. As efficiently placed cross aisles are added, the expected distance from a pick point to the closest cross aisle will be reduced, and the picker's total expected travel distance will decrease. However, as the number of cross aisles increases, the storage density of the warehouse decreases. Eventually, returns diminish to the point where adding cross aisles decreases picking efficiency.

It should be noted that in practice cross aisle configurations may be changed without incurring prohibitive costs. Product is often stored on shelving that consists of a number of modular units bolted together. Such shelving may be reconfigured fairly easily so as to add a cross aisle or change the position of one. The greatest cost will be the effort of unloading the shelves and then reloading them once the shelves have been re-assembled in their new configuration. Therefore the optimum cross aisle positions is potentially valuable information, due to the practical possibility of acting on it.

The optimal positioning of aisles is conceptually simple: a cross aisle will provide a greater benefit if it is close to those locations where the most picks are made; to add a cross aisle in a seldom-visited area of a warehouse would be to trade storage capacity for only a small benefit in picking efficiency. If our objective is to locate cross aisles so as to shorten the expected distance from a pick point to the nearest cross aisle, we will prefer to locate cross aisles in areas where picking concentrations are high. Therefore a proper analysis of optimal aisle placement should take into account pick densities (storage policies). This paper therefore presents a solution method for the problem of where cross aisles should best be positioned to optimize picker travel distance for a given storage policy.

2 Literature Review

Substantial work has been done on picking efficiency in picker-to-part warehouses, considering different combinations of factors such as pick list size, routing policies, order batching, storage policies and so on. A good survey of the work which has been done in this area may be found in de Koster et al. [2007]. A number of simulation studies have considered the efficiency of different combinations of factors. Petersen [1999] studies the combined effects of routing policies, pick list sizes and storage policies. Petersen and Aase [2004] does a similar analysis for an extensive set of combinations of order batching policies, storage policies and routing policies. Petersen [1997] considers the effects of pick list size, warehouse shape, routing policy and I/O point location, while assuming a random storage policy. Petersen and Schmenner [1999] studies different patterns of class-based storage in combination with different routing policies, pick list sizes and I/O point locations.

Some analytical studies also exist. Le-Duc and de Koster [2005], assuming a warehouse with a single central cross aisle, a class-based storage policy and a "return" routing policy, computes the effect on efficiency of warehouse shape, pick list size and storage policy. Caron et al. [1998], assuming the same layout as Le-Duc and de Koster [2005] and class-based storage, calculates the efficiency of traversal and return routing policies. Caron et al. [2000], using the same layout, finds the optimal number of storage aisles as a consequence of pick list size and the shape of the "ABC curve" of a class-based storage policy, assuming a traversal routing strategy. Chew and Tang [1999] analyzes the effect of pick list size given class-based storage, assuming a traversal routing policy. Jarvis and McDowell [1991] calculates that for a full traversal routing policy the optimal storage policy is a "within aisle" storage policy with the fastest-moving items stored in aisles closest to the I/O point. Roodbergen and Vis [2006] finds the optimal shape of a single-block warehouse with a random storage policy, assuming either an "S-shaped" or "largest gap" routing heuristic.

Less has been written specifically on the impact of aisle layouts on picker travel distances. In fact, de Koster et al. [2007] notes that "literature on layout design for low-level manual order-picking systems is not abundant." Vaughan and Petersen [1999] uses a simulation study to calculate the optimal number of evenly-spaced cross aisles in a warehouse, assuming an "aisle-by-aisle" routing policy and a uniform (random) storage policy. Roodbergen and de Koster [2001a] extends this study by simulating a variety of routing policies in the same setting. Thalayan [2008] uses simulation to compare the effects on travel time of a number of different factors, including the number of cross aisles, storage policy and routing policy. Roodbergen and de Koster [2001b] investigates the benefit of a middle cross aisle in combination with a random storage policy and varying pick list sizes. Roodbergen et al. [2008] develops a model for calculating optimal shape and number of evenly-spaced cross aisles for a warehouse with a random storage policy, assuming an "S-shaped" routing heuristic.

There are even fewer studies which consider layouts where the travel aisles are not evenly spaced. Gue and Meller [2009] uses an analytical approach to derive an unconventional but efficient aisle configuration for a unit-load warehouse assuming a random storage policy. Pohl et al. [2009] computes the efficiency of three different aisle configurations for dual-command operation and random storage. They also prove that, for this case, the optimal position of a single east-west cross aisle in a warehouse with north-south storage aisles must be between the center of the warehouse and the top cross aisle.

This paper focuses on two factors: storage policy (represented here in the form of the distribution function of pick locations) and facility layout (more specifically the question of cross aisle position), and their effect on picker travel distances. An assumption is made that pickers will be routed by a simple heuristic that will not always generate the shortest possible route. Given this routing policy and an arbitrary storage policy, an optimal cross aisle position is calculated. Possible congestion effects are not considered.

The remainder of this paper is organized as follows. First the warehouse model is described. Then a procedure for computing the expected picker path length as a function of the position of a single interior cross aisle is presented. This solution is extended to an arbitrary number of cross aisles. Some example applications of the procedure are given. Simulations are performed to estimate the effect of the simple routing heuristic on picker path lengths and on the resulting optimal cross aisle positions.

3 Model

Consider a picker-to-part warehouse with M vertical (or "north-south") storage aisles. Each storage aisle has B discrete pick locations of uniform size (e.g. one pallet width) on each side of the aisle, numbered $1, 2, \ldots, B$ with the 1^{st} location being the "southernmost" and the B^{th} location being the "northernmost". We will assume that storage aisles are narrow enough that the lateral movement required to pick items on both sides of the aisle may be neglected. Therefore there may be more than one item which for our purposes share the same effective pick point (across the aisle from each other, or, if multiple level storage is used, above or below one another). Thus a pick list may have multiple items at what for our purposes is the same effective pick point. There are three lateral (or "east-west") cross aisles, one at $y = 0$ (i.e. "south" of all pick locations), one at $y = B$ ("north" of all pick locations) and one at $y = h$, where h is to be determined (h being the number of pick locations "south" of the middle cross aisle, where $0 < h < B$). There is a single I/O point at $y = 0$, at some x coordinate. The amount of north-south travel will be the same regardless of where the I/O point is located, therefore we may disregard the location of the I/O point when computing the optimal cross aisle position.

The picker will have to be routed through the warehouse according to some sort of heuristic, or routing policy. Many different routing policies are described in the literature, but only a few of these are appropriate for warehouses with multiple cross aisles. Three such policies are "aisle by aisle" routing, described in Vaughan and Petersen [1999], the "S-shaped" heuristic and the largest gap heuristic (as adapted for multiple cross aisles), both described in Roodbergen and de Koster [2001a].

We will begin by assuming an "aisle by aisle" routing model as used in Vaughan and Petersen [1999]: the picker begins at the leftmost storage aisle from which items must be picked and picks all items in that aisle, then proceeds to the nearest aisle to the right that has any items to be picked, picks all items in that aisle, and so on until all items have been picked. (Pickers are able to turn around in storage aisles and to traverse them in either direction, but always move west-to-east in travel aisles, except before making the first pick or after making the last.) The shortest path using this routing may be calculated by dynamic programming, however the necessary computations are still complex, and an analytical solution will be correspondingly difficult to obtain.

In order to simplify this computation sufficiently and allow us to develop an analytical solution, we will make the additional simplifying "naïve routing assumption" that after making the final pick in a given storage aisle, the picker then departs *via the closest cross aisle to his current location*, without considering the picking locations to be visited in subsequent aisles. If equidistant from two cross aisles, the picker will choose the cross aisle closest to the I/O point. Note that this will not always result in the picker choosing the optimal route (see figure 1). See appendix A for a more detailed discussion of the effects of the naïve routing assumption.

The optimal placement of a single cross aisle will result in minimal expected "north-south" travel (that is, travel in storage aisles), given the assumption that pickers will employ our simplified routing strategy. Note that use of aisle by aisle routing ensures that for a given storage policy the expected amount of lateral or "east-west" travel (that is, travel in the cross aisles) will be the same regardless of the value of h. Therefore we may disregard this quantity when computing the optimal cross aisle position for a given storage policy. However it will be of interest when we compare the efficiency of one storage policy to that of another.

We will use the following terms (after Vaughan and Petersen [1999]):

N is the pick list size

M is the number of vertical (storage) aisles

K_m is the number of pick locations to be visited in storage aisle m

$X_m(t)$ is the t^{th} pick location in aisle m

X_m^+ is the largest ("northernmost") pick location in aisle m

X_m^- is the smallest ("southernmost") pick location in aisle m

Assume that pick locations are independent random variables, and that each pick location will be distributed among the different storage aisles according to some arbitrary probability mass function $g_M(m)$, and within each storage aisle $m = 1, 2, \dots, M$ according to some set of M arbitrary probability mass functions $f_{X_m}(x)$. Thus a given pick will be at storage location x in storage aisle m with probability $g_M(m) f_{X_m}(x)$.

Note that X_m^+ and X_m^- are respectively the K_m^{th} and 1^{st} order statistics for a discrete sample of K_m items, and will thus have probability mass functions, given by Siotani [1956], of the form:

$$f_{X_m^+}(x) = (F_{X_m}(x))^{K_m} - (F_{X_m}(x-1))^{K_m}$$

and

$$f_{X_m^-}(x) = (1 - F_{X_m}(x-1))^{K_m} - (1 - F_{X_m}(x))^{K_m}$$

and their joint pmf is

$$\begin{aligned} f_{X_m^+, X_m^-}(x,y) &= [F_{X_m}(x) - F_{X_m}(y-1)]^{K_m} - [F_{X_m}(x) - F_{X_m}(y)]^{K_m} \\ &\quad - [F_{X_m}(x-1) - F_{X_m}(y-1)]^{K_m} + [F_{X_m}(x-1) - F_{X_m}(y)]^{K_m} \end{aligned}$$

As the formulas for these pmfs include the value K_m they clearly depend on knowing the number of picks made in storage aisle m. There are therefore $N \times M$ instances each of $f_{X_m^+}(x)$, $f_{X_m^-}(x)$ and $f_{X_m^+, X_m^-}(x,y)$. The algorithm presented below is such that we will always know the appropriate values of K_m and will therefore know which pmf to use at what time. For the sake of simplicity of presentation this detail will be omitted, but we should make clear that the pmf being used must in each case be the appropriate one given the number of picks in the storage aisle under consideration.

4 Algorithm

Define P_m as the north-south travel distance in aisle m, and P as the total north-south travel distance. In order to find the value of h which minimizes $E[P]$, we must find a way to compute $E[P]$ for a given value of h. We do this by conditioning over the ways the picks are distributed among the storage aisles. The N picks must be distributed among the M storage aisles in some way. That is, we have some ordered set $\mathcal{K} = \{K_1, K_2, \ldots, K_M\}$ such that $\sum_{m=1}^{M} K_m = N$. If we know all the different possible patterns of picks among our aisles, and the probability of each pattern, then we can calculate $E[P]$ as follows:

$$E[P] = \sum_{\mathcal{K} \in \mathcal{K}^*} Pr(\mathcal{K}) \times E[P \mid \mathcal{K}]$$

where $\mathcal{K}^*$ is the set of all possible patterns of picks among our aisles. The enumeration of the elements of $\mathcal{K}^*$ and the calculation of their respective probabilities from $g_M(m)$ is relatively straightforward.

To calculate $E[P \mid \mathcal{K}]$, we consider the problem as a Markov reward process with three states 0, h and B, corresponding to the three cross aisles. The process will be considered to be in state

$i \in \{0, h, B\}$ when the picker is in cross aisle i (using it to travel from one storage aisle to the next). That is, when the picker enters storage aisle m via cross aisle i, makes picks in m and then departs m via cross aisle j, we consider the Markov reward process to have transitioned from state i to state j, with the reward being the expected north-south travel distance required to make all of the picks in m. We note that the initial state of the process will always be 0, and likewise once all picks have been made the system will end up in state 0. The expected total reward for a given pattern $\mathcal{K}$ will equal $E[P \mid \mathcal{K}]$.

To compute the total expected reward we will need to calculate the relevant transition probability matrices, as well as the expected reward for each possible state transition, *i.e.* the expected path length in a particular storage aisle given the cross aisles via which the picker arrived and departed. Note that the path length will also depend on both the distribution of picks in m, as determined by $f_{X_m}(x)$, and K_m, the number of picks to be made in m, as determined by the pattern $\mathcal{K}$.

We make the following additional definitions:

J_m is the 3-vector of probabilities that the picker will enter storage aisle m via the three cross aisles. Note that $J_1 = (1, 0, 0)$.

R_m^k is the 3-vector of expected rewards in aisle m given that the picker will enter m via each the three cross aisles and will make k picks in the aisle. Note that $R_m^0 = (0, 0, 0)^T$ for all m.

T_m^k is the 3 x 3 transition probability matrix for aisle m given that k picks are made in aisle m. Note that T_m^0 is the 3 by 3 identity matrix for all m.

The expected path length in storage aisle m, R_m^k, must be computed for each of the three cases:

Case 0: storage aisle m is entered at $y = 0$

Case h: storage aisle m is entered at $y = h$

Case B: storage aisle m is entered at $y = B$

The expected path lengths for the different cases are calculated as follows. Cases 0 and B are straightforward; the picker will proceed either up (if entering at $y = 0$) or down (if entering at $y = B$) the storage aisle until all picks have been made and will then exit via the closest cross aisle to the final pick location, as shown in figure 2.

Case 0: storage aisle m is entered at $y = 0$. We must travel up the storage aisle far enough to make all picks (which amounts to traveling up to X_m^+), whereupon we then leave by the closest exit point (0, h or B). Therefore the length of the optimal path depends only on the value of X_m^+. There are four possible sub-cases:

1. If $0 < X_m^+ \leq \frac{h}{2}$, the closest exit point to X_m^+ is at $y = 0$. Then the shortest possible path length is $(2 * x - 1) * w_b + w_a$, where w_a is the width of a cross aisle and w_b is the width of a pick location.

2. If $\frac{h}{2} < X_m^+ \leq h$, the closest exit point to X_m^+ is at $y = h$, and no picks are made at any locations $\geq h$. Then the shortest possible path length is $h * w_b + w_a$

3. If $h < X_m^+ \leq \frac{B+h}{2}$, the closest exit point to X_m^+ is at $y = h$, and at least one pick is made at some location $\geq h$. Then the shortest possible path length is $(h + 2 * (x - h) - 1) * w_b + 2 * w_a$

4. If $\frac{B+h}{2} < X_m^+ \leq B$, the closest exit point to X_m^+ is at $y = B$ and the shortest possible path length is $(B * w_b + 2 * w_a)$

From this we can derive an expression for the expected path length in case 0:

$$
\begin{aligned}
R_m^k(1) = E[P_m \mid \mathcal{K} \text{ and Case } 0] \quad &= \sum_{1 \leq x \leq \frac{h}{2}} f_{X_m^+}(x) \left[(2 * x - 1) * w_b + w_a\right] \\
&+ \sum_{\frac{h}{2} < x \leq h} f_{X_m^+}(x) \left[h * w_b + w_a\right] \\
&+ \sum_{h < x \leq \frac{(B+h)}{2}} f_{X_m^+}(x) \left[(h + 2 * (x - h) - 1) * w_b + 2 * w_a\right] \\
&+ \sum_{\frac{(B+h)}{2} < x \leq B} f_{X_m^+}(x) \left[B * w_b + 2 * w_a\right]
\end{aligned}
$$

Case h: storage aisle m is entered at $y = h$. In this case, the values of both X_m^+ and X_m^- are relevant, and to find the expected minimum path length we must sum over the domain of the joint pmf of X_m^+ and X_m^-. Because we may have some picks above h and some below, we must calculate the path lengths of the two possible routes (either first making all picks above h and then picks all below h, or else the reverse), and take the minimum of the two.

For all i, j such that $0 < i \leq j \leq B$ define $P^*_m(i,j)$ as the minimum path length to pick all items in aisle m given that aisle m was entered at $y = h$, $X^-_m = i$ and $X^+_m = j$. For any given i, j $P^*_m(i,j)$ is straightforwardly calculated as follows:

$$
\begin{aligned}
\text{Let } P^1_m(i,j) \quad = & \\
& (\text{the distance from the cross aisle at } y = h \text{ to } i) \\
+ & \quad (\text{the distance from } i \text{ to } j) \\
+ & \quad (\text{the distance from } j \text{ to the closest cross aisle to } j)
\end{aligned}
$$

$$
\begin{aligned}
\text{Let } P^2_m(i,j) \quad = & \\
& (\text{the distance from the cross aisle at } y = h \text{ to } j) \\
+ & \quad (\text{the distance from } j \text{ to } i) \\
+ & \quad (\text{the distance from } i \text{ to the closest cross aisle to } i)
\end{aligned}
$$

Then $P^*_m(i,j) = \min\{P^1_m(i,j), P^2_m(i,j)\}$ and

$$R^k_m(2) = E[P_m \mid \mathcal{K} \text{ and Case 3}] = \sum_{i=1}^{B} \sum_{j=i}^{B} P^*_m(i,j) f_{X^+_m, X^-_m}(i,j)$$

Note that if points i and j are either both above or both below $y = h$ then the shorter of the two paths will always be the one that makes picks in the order of increasing distance from h. This may be considered a trivial case, for which the above formula will also compute the correct path length.

Case B: storage aisle m is entered at $y = B$. Then, analogously to case 0, the length of the optimal path depends only on the value of X^-_m, and is given by

$$
\begin{aligned}
R_m^k(3) = E[P_m \mid \mathcal{K} \text{ and Case B}] \quad &= \sum_{1 \leq x \leq \frac{h}{2}} f_{X_m^-}(x)\,[B * w_b + 2 * w_a] \\
&+ \sum_{\frac{h}{2} < x \leq h} f_{X_m^-}(x)\,[((B-x)+(h-x)+1) * w_b + 2 * w_a] \\
&+ \sum_{h < x \leq \frac{(B+h)}{2}} f_{X_m^-}(x)\,[(B-h) * w_b + w_a] \\
&+ \sum_{\frac{(B+h)}{2} < x \leq B} f_{X_m^-}(x)\,[(2*(B-x)+1) * w_b + w_a]
\end{aligned}
$$

The transition probability matrices T_m^k may be computed by following similar reasoning. Define

$$
\begin{aligned}
I_0 &= \{i \mid \text{ the closest cross aisle to pick point } i \text{ is at } y = 0\} \\
I_h &= \{i \mid \text{ the closest cross aisle to pick point } i \text{ is at } y = h\} \\
I_B &= \{i \mid \text{ the closest cross aisle to pick point } i \text{ is at } y = B\}
\end{aligned}
$$

Then the first row of T_m^k is computed similarly to case 0 above:

$$
\begin{aligned}
T_m^k(1,1) &= \sum_{i \in I_0} f_{X_m^+}(x) \\
T_m^k(1,2) &= \sum_{i \in I_h} f_{X_m^+}(x) \\
T_m^k(1,3) &= \sum_{i \in I_B} f_{X_m^+}(x)
\end{aligned}
$$

and the third row of T_m^k is computed similarly to case B above:

$$T_m^k(3,1) = \sum_{i \in I_0} f_{X_m^-}(x)$$
$$T_m^k(3,2) = \sum_{i \in I_h} f_{X_m^-}(x)$$
$$T_m^k(3,3) = \sum_{i \in I_B} f_{X_m^-}(x)$$

As above, the only complicated case is the middle one, because we have to consider the shorter of two paths for the picker in aisle m. Define

$$\hat{P}_m(i,j) = \begin{cases} 1 & \text{if } P_m^1(i,j) > P_m^2(i,j) \\ 0 & \text{otherwise} \end{cases}$$

Then we can calculate the middle row of T_m^k by

$$T_m^k(2,1) = \sum_{i \in I_0} \sum_{i<j \le B} f_{X_m^+, X_m^-}(i,j)\hat{P}_m(i,j) + \sum_{j \in I_0} \sum_{0<i<j} f_{X_m^+, X_m^-}(i,j)\hat{P}_m(j,i) + \sum_{i \in I_0} f_{X_m^+, X_m^-}(i,i)$$
$$T_m^k(2,2) = \sum_{i \in I_h} \sum_{i<j \le B} f_{X_m^+, X_m^-}(i,j)\hat{P}_m(i,j) + \sum_{j \in I_h} \sum_{0<i<j} f_{X_m^+, X_m^-}(i,j)\hat{P}_m(j,i) + \sum_{i \in I_h} f_{X_m^+, X_m^-}(i,i)$$
$$T_m^k(2,3) = \sum_{i \in I_B} \sum_{i<j \le B} f_{X_m^+, X_m^-}(i,j)\hat{P}_m(i,j) + \sum_{j \in I_B} \sum_{0<i<j} f_{X_m^+, X_m^-}(i,j)\hat{P}_m(j,i) + \sum_{i \in I_B} f_{X_m^+, X_m^-}(i,i)$$

Once we have computed R_m^k and T_m^k for all $m = 1, 2, \ldots, M$ and $k = 0, 1, \ldots, N$ we are ready to calculate $E[P_m \mid \mathcal{K}]$. Note that there will have to be an additional calculation of R_m^k made for the case of the final (or "easternmost") aisle with picks, because we will always exit that aisle via cross aisle 0 regardless of what picks are made there, and therefore our expected path length will be different from the usual case. The logic behind this calculation is similar enough to the foregoing that we will omit the details.

Note that $J_1 = (1, 0, 0)$ and for $m > 1$ we calculate J_m by

$$J_m = J_{m-1} T_{m-1}^{K_{m-1}}$$

Then the expected reward for aisle m is given by

$$E[P_m \mid \mathcal{K}] = J_m R_m^{K_{m-1}}$$

and $E[P \mid \mathcal{K}]$, the expected total north-south path length, is simply the sum of the aisle-by-aisle expected path lengths:

$$E[P \mid \mathcal{K}] = \sum_{m=1}^{M} E[P_m \mid \mathcal{K}] = \sum_{m=1}^{M} J_m R_m^{K_{m-1}}$$

Now as noted above we simply compute $E[P]$ by conditioning over all possible values of $\mathcal{K}$.

The algorithm will then consist of calculating R_m^k and T_m^k for all possible values of m and k, and then using R_m^k and T_m^k to compute the expected path length for each pick pattern in $\mathcal{K}^*$. Thus $E[P]$ may be computed as follows:

{ $totalPath$ is the total expected (north-south) path length }
$totalPath \leftarrow 0$
{ m is the storage aisle }
for m=1 to M **do**
  {k is the number of picks in the aisle}
  for k=0 to N **do**
    compute R_m^k
    compute T_m^k
  end for
end for
for all $\mathcal{K} \in \mathcal{K}^*$ **do**
  {$patternPath$ is the total expected path length given pattern $\mathcal{K}$}
  $patternPath \leftarrow 0$
  for m=1 to M **do**
    if $m = 1$ **then**
      $J_m \leftarrow (1, 0, 0)$
    else
      $J_m \leftarrow J_{m-1} T_{m-1}^{K_{m-1}}$

end if

$patternPath \leftarrow patternPath + J_m R_m^{K_m}$

end for

$totalPath \leftarrow totalPath + (Pr(\mathcal{K}) * patternPath)$

end for

$\{totalPath$ will now be equal to $E[P]\}$

The foregoing may be straightforwardly extended to calculating the expected path lengths if we have two or more "floating" cross aisles, at $y = h_1$, $y = h_2$, etc. If there are A cross aisles (including those at $y = 0$ and $y = B$) then the vectors J_m and R_m^k must be increased to size A and the matrices T_m^k must be increased to size A by A. The procedures for calculating J_m, R_m^k and T_m^k remain conceptually the same, although the actual calculations are more complex as we must consider a larger number of possible cross aisles via which we might exit a given storage aisle. Calculations for the fixed aisles at $y = 0$ and $y = B$ may be computed in a way quite similar to cases 0 and B above, whereas calculations involving the movable interior aisles at $y = h_1$, $y = h_2$, etc. are done as in case h. Once the J_m and R_m^k values have been computed, the procedure for calculating $E[P]$ will be identical to the case where $A = 3$.

5 Examples

5.1 Computing the optimal position for a single cross aisle

As we are able to compute the expected path length for each pattern, and we know the probability of each pattern, we are thus able to compute $E[P]$ for a given value of h. Once we know how to do this, the next step is to compute that value of h for which the picker's expected travel distance is minimized. We can do this by evaluating $E[P]$ for various values of h. If we wish to find the optimal location of a single floating cross-aisle using the algorithm outlined above, a single objective function evaluation remains sufficiently inexpensive that it is still feasible to use full enumeration to find the solution. As an example, we do this for an "across-aisle" storage policy, where $g_M(m)$ is the uniform distribution and for each storage aisle $f_{X_m}(x)$ is the "80-20" distribution function, so-called because 80% of picks are in the 20% of the aisle closest to the I/O point:

$$f_{X_m}(x) = \begin{cases} 0.08 & 1 \le x \le 10 \\ 0.005 & 11 \le x \le 50 \\ 0 & \text{otherwise} \end{cases}$$

Assume 50 pick locations per aisle ($B = 50$), 20 storage aisles ($M = 20$) and 5 picks per trip ($N = 5$), $w_a = 10$ and $w_b = 5$. For this case we obtain the results shown in Figure 3: the expected path length $E[P]$ as a function of h, the position of the intermediate cross aisle. The minimal value of $E[P]$ is 448.007, achieved when $h = 8$, and was found by full enumeration in 155 milliseconds.

We can also compare the expected north-south path length given optimally positioned cross aisles with the expected path length for evenly-spaced cross aisles. Table 1 shows the percentage savings achieved by moving the middle cross aisle to its optimal position.

5.2 An example with dual-command travel

Dual-command travel is the special case where the pick list size N is equal to 2. A theorem proven by Pohl et al. [2009] states that the optimal position of a single movable cross aisle with dual command-picking and a random storage policy will be between the center of the warehouse and the top cross aisle (their Proposition 1). We calculate the optimal cross-aisle position for random storage and $N = 2$ for a number of warehouse sizes. The results, as shown in table 2, agree with the theorem of Pohl et al. [2009] that the optimal cross aisle position will be beyond the midpoint of the warehouse.

5.3 The comparison of three storage policies

We now compute the optimal cross aisle positions for three different volume-based storage policies, and compare the resulting optimal expected path lengths. The storage policies considered are diagonal storage, across-aisle storage and within-aisle storage, as shown in figure 4. These storage policies were evaluated in Petersen and Schmenner [1999]. That study also considered perimeter storage, but we will disregard perimeter storage because it is clearly not suitable for the aisle-by-aisle routing policy being used here. The other three storage policies were evaluated with storage classes A B and C, occupying 20, 30 and 50 percent of storage locations respectively. The skewness of the three classes was either high, medium or low, defined as in table 3. A warehouse with 20 storage aisles was assumed, with an I/O point in at bottom center, and approximately twice as wide as deep (not including the portion of warehouse depth due to cross aisles).

The results were that across aisle storage was the most efficient in all cases. (A subset of those results, for the medium skewness level, are shown in table 4.) (The north-south distances were calculated using the algorithm of section 4, and the east-west distances were calculated using a simpler formula given in appendix B.) For the smallest pick list sizes, diagonal storage was superior to within aisle storage, for larger pick list sizes within aisle storage was more efficient than diagonal storage. It should be observed that this does not imply that across aisle storage is superior in all cases. The optimal storage policy is dependent on the routing policy used; according to Jarvis and McDowell [1991] a within-aisle storage policy is preferable when traversal routing is used, and Le-Duc and de Koster [2005] finds that an across-aisle policy is superior when return routing is used. Another factor which should not be ignored is that, as seen in table 4, within-aisle storage results in less east-west travel but more north-south travel, when compared to across-aisle storage. Thus the relative merits of the two storage policies will be sensitive to factors such as storage aisle widths. (Widening storage aisles will increase the path lengths for across-aisle storage more than it will those for within-aisle storage.)

The results for different numbers of cross aisles with across aisle storage and medium skewness are shown in table 5. One result which is apparent here (and was observed for other storage policies and skewness levels as well) is that the the benefits for additional cross aisles decrease rapidly. The first cross-aisle brings a significant benefit, especially for larger pick list sizes. The second aisle added never gives as much as a two per cent improvement, and the third either yields a very small improvement or else may even cause path lengths to increase. Also note that optimal cross aisle positions are fairly insensitive to pick list size. For three cross aisles, the optimal position for the middle aisle is always at $y = 10$. However when an increase in pick list size does cause the optimal aisle positions to change, they tend to do so in an abrupt fashion: for $A = 4$ and $2 \leq N \leq 5$ the optimal aisle positions are (0 10 39 50), and for $A = 4$ and $5 < N \leq 10$ they are (0 8 23 50).

Table 6 shows the results for different skewness levels for across aisle storage with four cross aisles. As before, we see that optimal cross aisle positions are insensitive to pick list size, and that this insensitivity is more pronounced for higher skewness levels. It is also worth noticing that in this example the percentage savings resulting from higher skewness levels increases as the pick list size increases.

6 Conclusions and Further Work

As noted by many researchers (e.g. Hausman et al. [1976]), volume-based storage policies decrease picker travel distances. Up to some point of diminishing returns, the addition of interior cross-aisles reduce travel as well. Therefore it is useful to study the use of volume-based storage policies in a warehouse with interior cross aisles. As we have seen, in the absence of random storage, the most efficient cross aisle positions will not be equally-spaced. Furthermore, as the cost of adjusting cross aisle positions is not prohibitive, practitioners will be able to benefit from knowing the maximally efficient positions for cross aisles corresponding to storage policies in use, or storage policies under consideration. We have presented a method for calculating maximally efficient cross aisle positions for a picker-to-part warehouse using arbitrary storage policies, subject to certain assumptions, most notably an assumption regarding routing policy.

This work could be developed into a tool that could be used to calculate optimal cross aisle positions, and applied to a larger number of questions. For example, warehouse shape could be varied, as so could be the number of storage aisles. Other pick list sizes could also be considered.

A more difficult task would be to relax our routing assumptions. The simulation results (see appendix A) suggest that our predicted optimal cross aisle positions will also be optimal for aisle-by-aisle routing in the absence of the naïve routing assumption. However this may not be true for all possible routing policies. Other simple routing policies exist for which path lengths could be calculated analytically (e.g. traversal routing), but such policies do not as a rule take advantage of cross aisles. Optimal routing is therefore the nut we need to crack. For optimal routing however it seems unlikely that the expected path length for a given layout could be found except by simulation. The computational effort required to find optimal positions for several cross-aisles using simulation might, however, prove prohibitively large.

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A Simulation

Simulation was used to validate the results of the analytical study in two different ways. An estimate was made of the increase in average path length due to the naïve routing assumption. The path length resulting from the naïve routing assumption was compared to that resulting from the aisle by aisle routing policy used by Vaughan and Petersen [1999], where storage aisles are visited in a strict left-to-right sequence, but the shortest possible path subject to that restriction was found (using a dynamic programming algorithm). For the sample problem with the "80-20 distribution" defined in section 5.1, cross aisles at 0, 6, 29, 44 and 50 , fifty pick locations per storage aisle ($B = 50$), 20 storage aisles ($M = 20$) and a pick list size of five ($N = 5$), the results were that the average penalty of the naïve routing assumption was 8.64%, based on a sample of 100,000 picking trips. For the three-aisle case, the discrepancy for the optimal configuration (cross aisles at 0, 8 and 50) was 13.49%. It makes intuitive sense that the discrepancy should be larger in this case: when the cross aisles are fewer and therefore farther apart, the penalty for using the wrong cross aisle will be larger.

But does this larger discrepancy between the expected path lengths when using the different routing assumptions cause us to find the wrong optimal solution? Simulation was used to estimate the expected path length using the routing policy of Vaughan and Petersen [1999] for the different candidate solutions to the sample problem of the 80-20 distribution, with $N = 5$ and $M = 20$, $B = 50$ and three cross aisles, varying the position of the interior cross aisle between 1 and 49. Figure 5 plots these simulated path lengths against the values calculated analytically using the naïve routing assumption. Although the routing assumption of Vaughan and Petersen [1999] resulted in significantly shorter path lengths, the two curves are quite similar in shape and have the same minimum point (at $h = 8$).

B East-West Path Length

The expected east-west path length (total distance traveled in cross aisles) for a given storage policy and pick list size may be calculated as follows. Given the aisle-by-aisle routing policy in use, the expected east-west path length depends only on the easternmost and westernmost storage aisle being visited (the actual pick points visited does not matter). Also, if we know the easternmost and westernmost storage aisle being visited we may calculate the (exact) east-west path length. If we define

P_{ew} is the east-west path length.

W_i is the event that the westernmost storage aisle visited is aisle i

E_j is the event that the easternmost storage aisle visited is aisle j.

p_{ij} is the east-west path length given the events W_i and E_j.

w_s is the distance between storage aisles, measured midpoint-to-midpoint.

I is the location of the I/O point, expressed as a number between 1 and M. That is, $I = 10.5$ means that the I/O point is between storage aisles 10 and 11.

Then the expectation of P_{ew} may be calculated by conditioning over the values of easternmost and westernmost storage aisle visited, given pick list size N, as follows:

$$E[P_{ew} \mid N] = \sum_{i=1}^{M} \sum_{j=i}^{M} p_{ij} Pr((W_i \cap E_j) \mid N)$$

Furthermore, p_{ij} is easily calculated for each i and j:

$$p_{ij} = w_s * \{\mid I - i \mid + \mid i - j \mid + \mid I - j \mid\}$$

And $Pr((W_i \cap E_j) \mid N)$ may be calculated as follows. If $i = j$, then

$$Pr((W_i \cap E_j) \mid N) = (g_M(i))^N$$

otherwise

$$Pr((W_i \cap E_j) \mid N) = \left[\sum_{m=i}^{j} g_M(m)\right]^N \sum_{k=1}^{N-1} \sum_{l=1}^{N-k} \left[(g_M(i))^k \, (g_M(j))^l \left(\sum_{m=i+1}^{j-1} g_M(m) \right)^{N-k-l} \binom{N}{k} \binom{N-k}{l} \right]$$

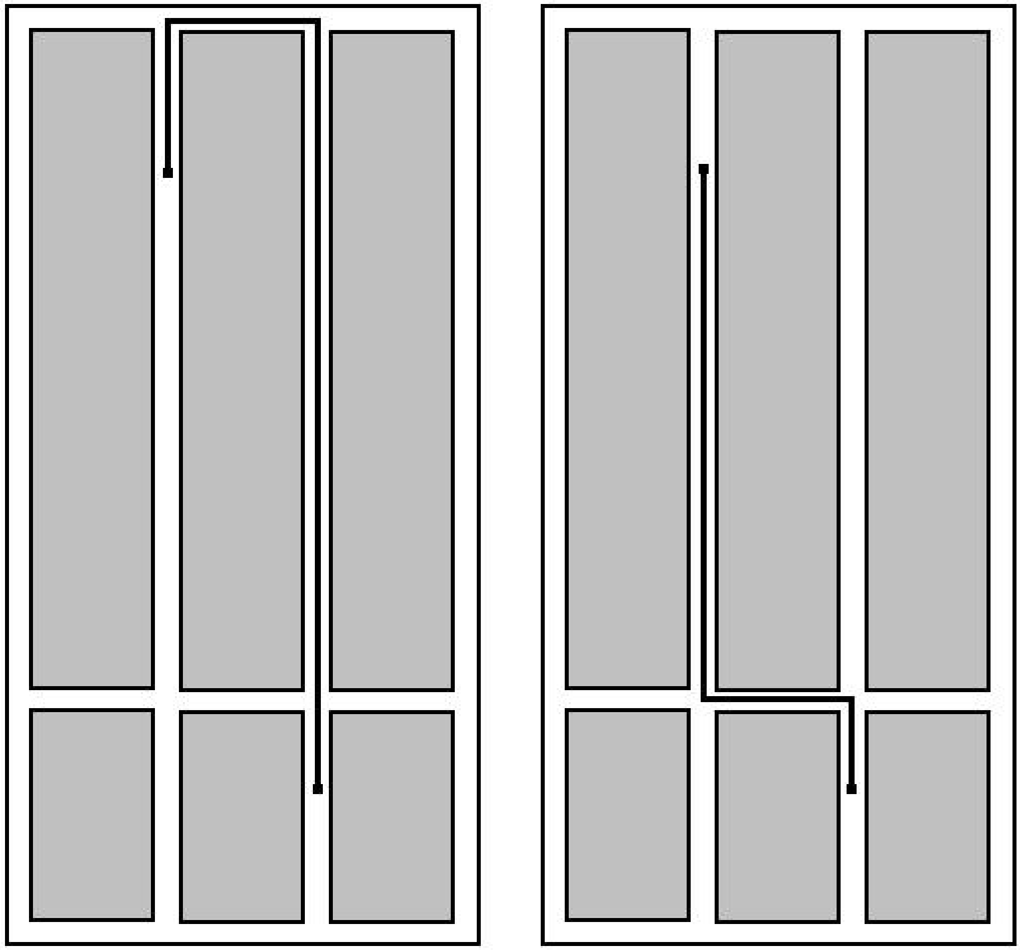

Figure 1: In the above example, the “naïve” routing method (shown on the left) results in a longer path length than the optimal routing (shown on the right).

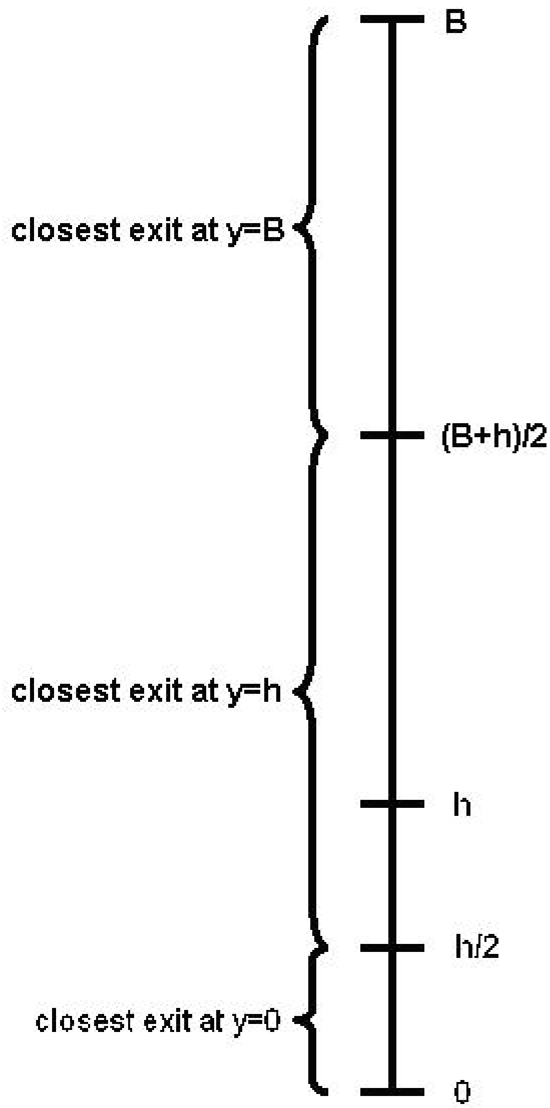

Figure 2: In the case where we enter a storage aisle at $y = 0$ or $y = B$, we make all picks and then exit via the closest cross aisle to our last pick location.

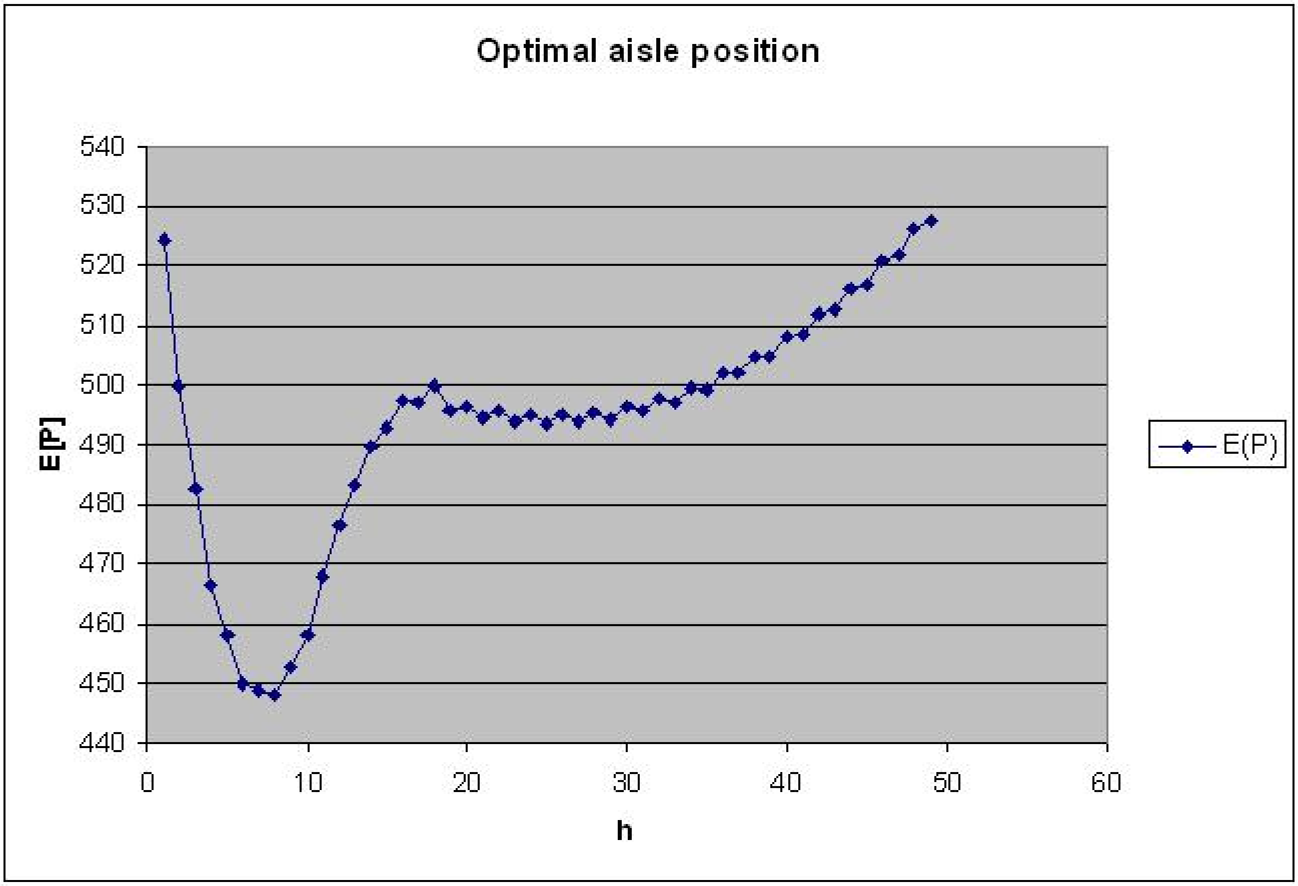

Figure 3: Expected north-south path length $E[P]$ as a function of cross-aisle location (h) for the "80-20" distribution, with 50 pick locations per aisle, 20 storage aisles and 5 picks per trip, w_a=10 and w_b=5.

N	north-south Path length, h=8	north-south Path length, h=25	difference	percent savings
2	189.24	201.33	12.09	6.01
3	259.21	287.48	28.27	9.83
4	328.42	371.83	43.41	11.67
5	397.31	454.89	57.58	12.66
6	465.11	536.10	70.99	13.24
7	531.75	615.42	83.67	13.60
8	597.70	693.45	95.75	13.81
9	662.80	769.70	106.90	13.89
10	726.66	843.99	117.33	13.90

Table 1: Savings of optimal cross-aisle position compared to centered cross-aisle for an across-aisle storage policy with two storage classes (the "80-20" distribution), with B=50, M=25, and N=2 thru 10, based on 1,000,000 simulation runs per value of N. The optimal aisle position of h=8 saved between 6 and 13.9 percent as compared to the centered cross aisle at h=25.

N	M	P^*	h^*	N	M	P^*	h^*
2	2	382.00	30	2	16	404.02	27
2	3	390.45	28	2	17	404.20	27
2	4	394.63	28	2	18	404.37	27
2	5	397.14	28	2	19	404.51	27
2	6	398.81	28	2	20	404.64	27
2	7	400.00	28	2	21	404.76	27
2	8	400.90	28	2	22	404.87	27
2	9	401.59	28	2	23	404.97	27
2	10	402.15	28	2	24	405.06	27
2	11	402.61	28	2	25	405.14	27
2	12	402.99	28	2	26	405.22	27
2	13	403.31	27	2	27	405.29	27
2	14	403.58	27	2	28	405.35	27
2	15	403.82	27	2	29	405.41	27
				2	30	405.47	27

Table 2: Optimal cross-aisle position h^* and minimal expected path length P^* for dual-command operation for the uniform distribution, with B=50 and M between 2 and 30. The value of h^* is always greater than 25, which agrees with proposition 1 of Pohl et al. [2009]

Skewness	percent in class A	percent in class B	percent in class C
Low	40	40	20
Medium	60	30	10
High	80	15	5

Table 3: Three different skewness levels used.

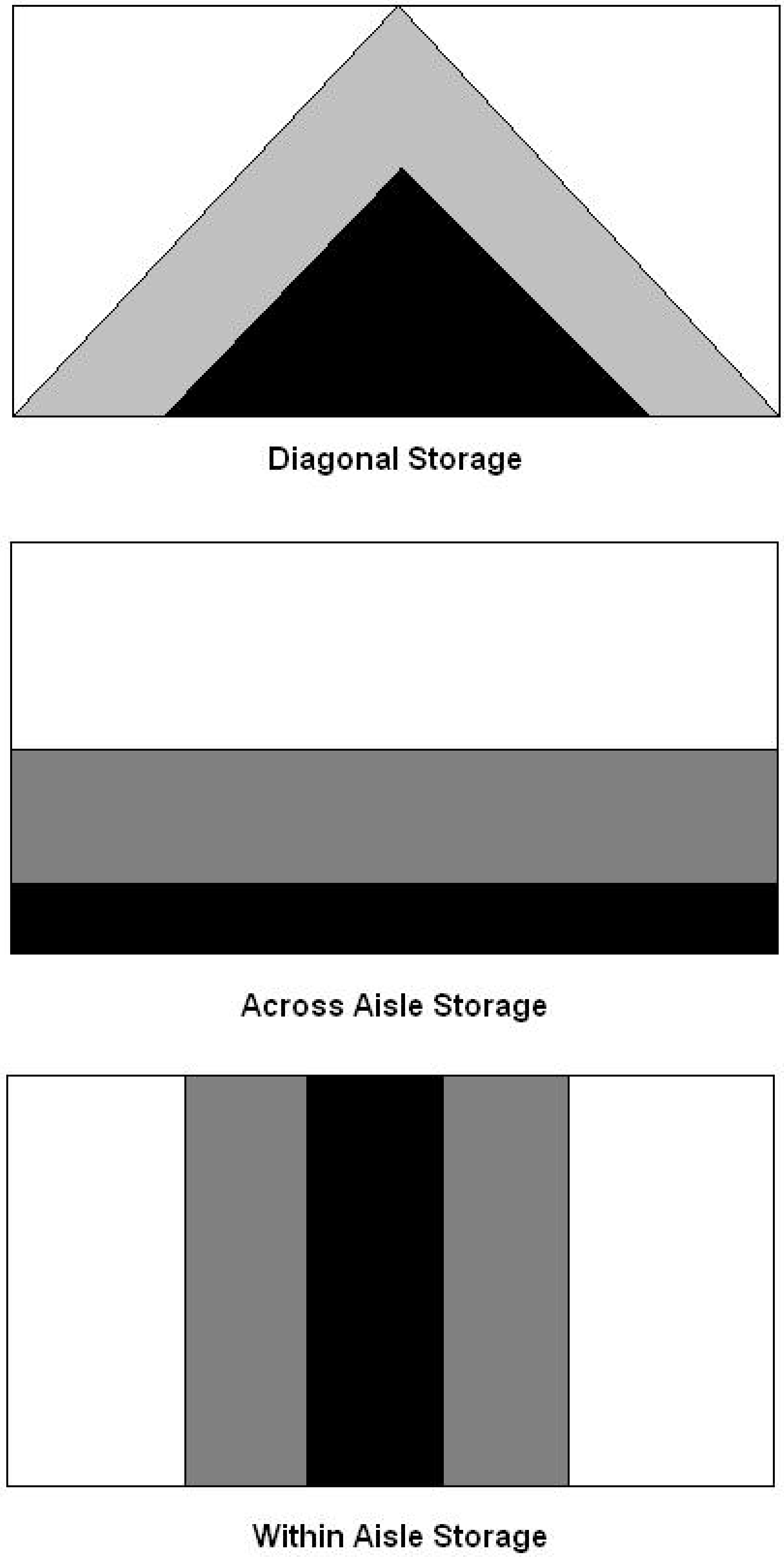

Figure 4: Diagonal, across aisle and within aisle storage with three storage classes. The I/O point is at bottom center. After Petersen and Schmenner [1999].

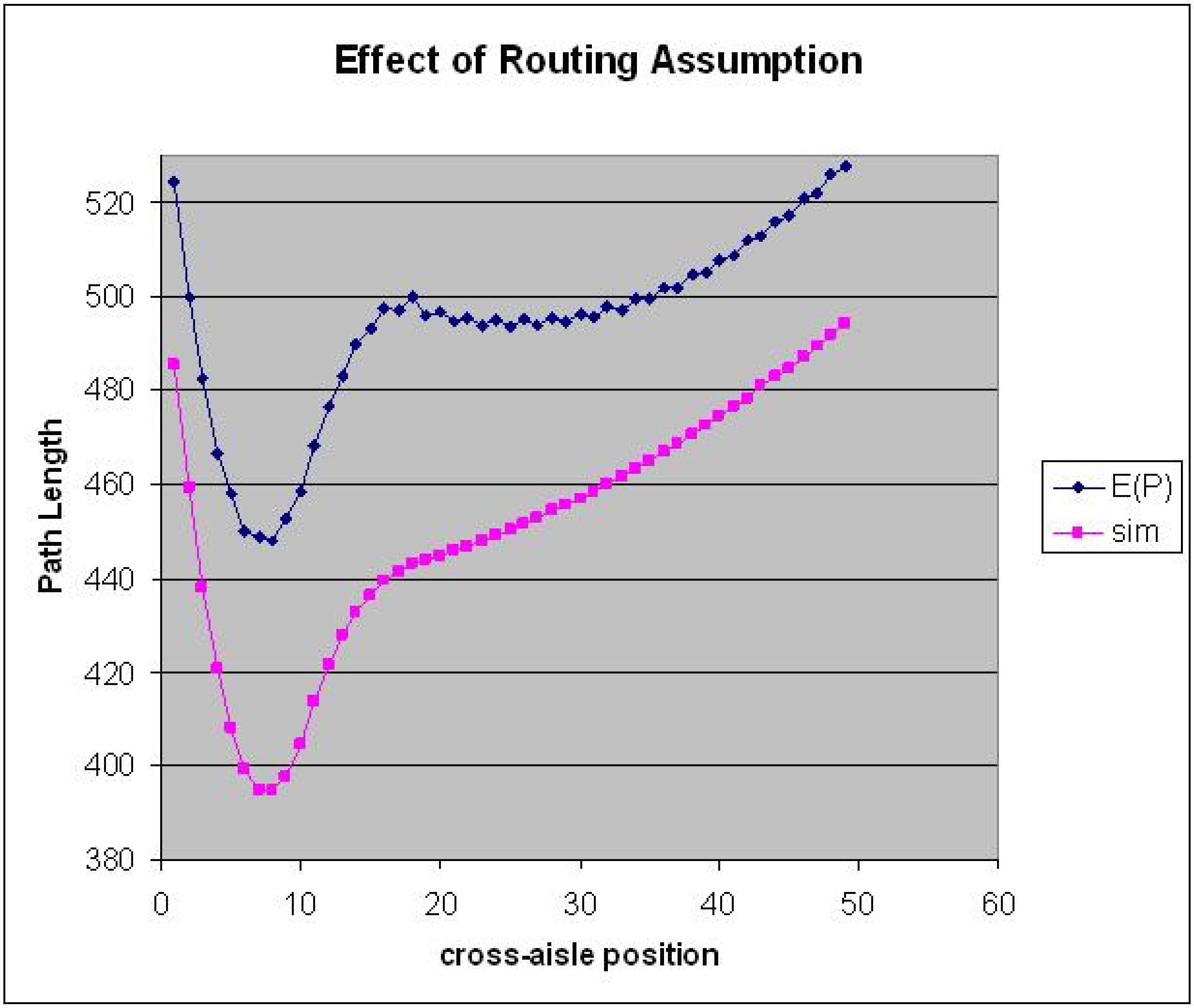

Figure 5: Simulation results for the 80-20 distribution, with $N = 5$ and $M = 20$, 50 pick locations per aisle and 3 cross-aisles, with the position of the middle aisle varying between 1 and 49. The higher of the two curves shows the analytical result for $E[P]$ given the naïve routing assumption, and lower curve shows the simulation result for the same aisle configurations, given the routing assumption used by Vaughan and Petersen [1999]. Although the naïve routing assumption results in a significant penalty (longer trip lengths), the optimal aisle position is the same in both cases ($h = 8$).

		Diagonal Storage Path Length			Across Aisle Storage Path Length			Within Aisle Storage Path Length		
A	N	north-south	east-west	total	north-south	east-west	total	north-south	east-west	total
2	2	199.06	301.62	500.68	134.25	320.63	454.88	228.38	281.23	509.60
	3	287.18	333.67	620.85	198.38	362.19	560.57	312.22	303.09	615.30
	4	369.83	354.16	724.00	260.53	387.08	647.61	390.37	318.05	708.42
	5	448.29	368.87	817.16	320.78	403.65	724.42	463.13	329.56	792.69
	6	523.15	380.14	903.28	379.05	415.45	794.49	530.58	338.95	869.54
	7	594.70	389.15	983.84	435.84	424.27	860.12	593.09	346.90	939.99
	8	663.12	396.56	1059.68	490.80	431.11	921.91	651.18	353.78	1004.96
	9	728.57	402.81	1131.38	544.12	436.57	980.68	705.32	359.84	1065.17
	10	791.20	408.16	1199.35	595.86	441.01	1036.86	755.94	365.25	1121.19
3	2	161.40	301.62	463.01	124.07	320.63	444.70	210.04	281.23	491.27
	3	229.61	333.67	563.28	171.56	362.19	533.75	270.67	303.09	573.76
	4	292.50	354.16	646.66	218.60	387.08	605.68	330.45	318.05	648.50
	5	351.84	368.87	720.71	264.75	403.65	668.39	388.16	329.56	717.72
	6	409.02	380.14	789.15	309.81	415.45	725.25	443.03	338.95	781.98
	7	464.54	389.15	853.69	354.07	424.27	778.34	495.09	346.90	841.99
	8	518.58	396.56	915.14	397.32	431.11	828.43	544.54	353.78	898.32
	9	571.09	402.81	973.90	439.67	436.57	876.23	591.60	359.84	951.44
	10	622.11	408.16	1024.91	481.14	441.01	922.15	636.44	365.25	1001.70
4	2	157.08	301.62	458.70	123.26	320.63	443.89	209.57	281.23	490.80
	3	221.06	333.67	554.73	169.17	362.19	531.36	267.00	303.09	570.08
	4	279.87	354.16	634.03	214.66	387.08	601.75	322.95	318.05	641.00
	5	335.56	368.87	704.42	259.28	403.65	662.93	376.92	329.56	706.48
	6	389.10	380.14	769.23	302.20	415.45	717.64	428.47	338.95	767.42
	7	440.99	389.15	830.13	344.15	424.27	768.42	477.44	346.90	824.34
	8	491.48	396.56	888.04	385.12	431.11	816.24	524.08	353.78	877.86
	9	540.63	402.81	943.43	425.22	436.57	861.79	568.54	359.84	928.38
	10	588.45	408.16	996.61	464.46	441.01	905.47	611.01	365.25	976.27
5	2	157.35	301.62	458.96	123.38	320.63	444.01	210.19	281.23	491.42
	3	221.31	333.67	554.98	169.35	362.19	531.54	267.47	303.09	570.56
	4	280.05	354.16	634.22	214.90	387.08	601.98	323.22	318.05	641.26
	5	335.29	368.87	704.16	258.55	403.65	662.20	377.14	329.56	706.70
	6	388.22	380.14	768.36	300.89	415.45	716.33	428.46	338.95	767.41
	7	439.49	389.15	828.63	342.47	424.27	766.75	477.29	346.90	824.19
	8	489.44	396.56	886.00	383.12	431.11	814.23	523.77	353.78	877.55
	9	538.13	402.81	940.94	422.93	436.57	859.49	568.19	359.84	928.03
	10	585.59	408.16	993.75	461.92	441.01	902.92	610.71	365.25	975.96

Table 4: Comparison of diagonal, across-aisle and within aisle storage policies with medium skewness level. N is the pick list size and A is the number of cross aisles. Here $M = 20$, $B = 50$, $w_a = 8$, $w_b = 2.5$ and the center-to-center distance between adjacent storage aisles is 12.5.

A	N	CPU	optimal aisle positions	north-south distance	east-west distance	total distance	percent savings
2	2	< 1	0 50	134.25	320.63	454.88	-
	3	< 1	0 50	198.38	362.19	560.57	-
	4	< 1	0 50	260.53	387.08	647.61	-
	5	< 1	0 50	320.78	403.65	724.42	-
	6	< 1	0 50	379.05	415.45	794.49	-
	7	< 1	0 50	435.84	424.27	860.12	-
	8	< 1	0 50	490.80	431.11	921.91	-
	9	< 1	0 50	544.12	436.57	980.68	-
	10	< 1	0 50	595.86	441.01	1036.86	-
3	2	< 1	0 10 50	124.07	320.63	444.70	2.24
	3	< 1	0 10 50	171.56	362.19	533.75	4.78
	4	< 1	0 10 50	218.60	387.08	605.68	6.47
	5	< 1	0 10 50	264.75	403.65	668.39	7.73
	6	< 1	0 10 50	309.81	415.45	725.25	8.72
	7	< 1	0 10 50	354.07	424.27	778.34	9.51
	8	< 1	0 10 50	397.32	431.11	828.43	10.14
	9	< 1	0 10 50	439.67	436.57	876.23	10.65
	10	< 1	0 10 50	481.14	441.01	922.15	11.06
4	2	5	0 10 39 50	123.26	320.63	443.89	0.18
	3	4	0 10 39 50	169.17	362.19	531.36	0.45
	4	6	0 10 39 50	214.66	387.08	601.75	0.65
	5	6	0 10 39 50	259.28	403.65	662.93	0.82
	6	6	0 8 23 50	302.20	415.45	717.64	1.05
	7	7	0 8 23 50	344.15	424.27	768.42	1.27
	8	7	0 8 23 50	385.12	431.11	816.24	1.47
	9	9	0 8 23 50	425.22	436.57	861.79	1.65
	10	9	0 8 23 50	464.46	441.01	905.47	1.81
5	2	102	0 10 39 49 50	123.38	320.63	444.01	-0.03
	3	111	0 10 39 49 50	169.35	362.19	531.54	-0.03
	4	117	0 10 39 49 50	214.90	387.08	601.98	-0.04
	5	128	0 8 21 42 50	258.55	403.65	662.20	0.11
	6	138	0 8 21 42 50	300.89	415.45	716.33	0.18
	7	146	0 8 21 42 50	342.47	424.27	766.75	0.22
	8	156	0 8 21 42 50	383.12	431.11	814.23	0.25
	9	166	0 8 21 42 50	422.93	436.57	859.49	0.27
	10	186	0 8 21 42 50	461.92	441.01	902.92	0.28

Table 5: Optimal cross aisle positions for an "across-aisle" storage policy with medium skewness level. N is the pick list size and A is the number of cross aisles. The savings is the percentage reduction in expected travel distance resulting in the addition of a cross aisle. Here $M = 20$, $B = 50$, $w_a = 8$, $w_b = 2.5$ and the center-to-center distance between adjacent storage aisles is 12.5. CPU is the number of seconds required to calculate the optimal aisle positions.

Skew	N	optimal aisle positions	Path Length north-south	east-west	total	percent savings
Low	2	0 18 43 50	156.94	320.63	477.56	
	3	0 16 39 50	212.22	362.19	574.41	-
	4	0 14 37 50	266.64	387.08	653.73	-
	5	0 14 37 50	319.72	403.65	723.37	-
	6	0 14 35 50	371.42	415.45	786.87	-
	7	0 14 35 50	422.09	424.27	846.36	-
	8	0 14 35 50	471.49	431.11	902.60	-
	9	0 8 23 50	519.75	436.57	956.31	-
	10	0 8 23 50	566.45	441.01	1007.46	-
Medium	2	0 10 39 50	123.26	320.63	443.89	7.05
	3	0 10 39 50	169.17	362.19	531.36	7.49
	4	0 10 39 50	214.66	387.08	601.75	7.95
	5	0 10 39 50	259.28	403.65	662.93	8.36
	6	0 8 23 50	302.20	415.45	717.64	8.80
	7	0 8 23 50	344.15	424.27	768.42	9.21
	8	0 8 23 50	385.12	431.11	816.24	9.57
	9	0 8 23 50	425.22	436.57	861.79	9.88
	10	0 8 23 50	464.46	441.01	905.47	10.12
High	2	0 10 41 50	92.45	320.63	413.08	6.94
	3	0 8 41 50	127.94	362.19	490.13	7.76
	4	0 8 41 50	162.53	387.08	549.61	8.66
	5	0 8 41 50	196.48	403.65	600.13	9.47
	6	0 8 41 50	229.65	415.45	645.09	10.11
	7	0 8 41 50	262.24	424.27	686.52	10.66
	8	0 8 41 50	294.11	431.11	725.23	11.15
	9	0 8 41 50	325.34	436.57	761.91	11.59
	10	0 8 41 50	355.94	441.01	796.95	11.98

Table 6: Comparison of the across-aisle storage policy with four cross aisles and different skewness levels. N is the pick list size. Here $M = 20$, $B = 50$, $w_a = 8$, $w_b = 2.5$ and the center-to-center distance between adjacent storage aisles is 12.5. For medium skewness, the percent savings is how much was saved in comparison with low skewness (for the same pick list size), and for high skewness, the percent savings is how much was saved in comparison with medium skewness (for the same pick list size).