Impact of Dual-Toll Pricing in Hazmat Transportation considering Stochastic Driver Preferences

Tolou Esfandeh Masoumeh Taslimi Rajan Batta Changhyun Kwon*

Department of Industrial and Systems Engineering
University at Buffalo (SUNY), Buffalo, NY, USA
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Abstract

This paper proposes and analyzes a dual-toll setting policy for both hazmat and regular carriers to route them in a manner that controls the hazmat risk. The model minimizes total risk on the network while considering stochastic driver preferences in route selection based on toll prices and travel time. The risk measurement model computes risk by considering overall accident probability times the consequence of an accident. A computationally feasible exact solution method is developed for the case of a single toll decision variable. This exact single-toll algorithm is embedded in a cyclical algorithm to create a heuristic for the general case. A series of computational tests are performed to study the computational efficiency of the algorithm as well as the impact of various parameters on the solution. A case study is also presented. Our conclusion is that dual toll pricing offers considerable improvement versus the single toll strategy for hazmat risk mitigation in a situation where stochastic driver preferences are considered.
Keywords: Hazmat routing, dual toll pricing, value of time

1 Introduction

Based on the definition provided by the U.S. Department of Transportation Pipeline and Hazardous Material Agency, hazmat is any substance capable of posing an unreasonable risk to health, safety or property when transported in a network. Any kind of gaseous, liquid or solid which is potentially explosive, flammable, combustible or corrosive is considered as hazmat. When shipped in commerce, hazmat is generally dangerous but some much more than others. Yearly incident data acquired from the U.S. Department of Transportation Pipeline and Hazardous Materials Safety Administration reveals that in 2012, there are 5,088 accidents involving an explosive and/or flammable gas or liquid such as gasoline that resulted in 2 hospitalizations, 36 injuries, 6 fatalities and over $29 million in damages. In the same period, 322 accidents involving a nonflammable compressed gas such as sprays or perfumes caused 2 injuries and nearly $1 million clean up costs. Although the accidents

*Corresponding author: `chkwon@buffalo.edu`

involving hazmat rarely happen (low-probability incidents), if they occur, their consequences can be catastrophic (high-consequence incidents). Statistical data for the USA shows in 1998 that while there were roughly 15,000 accidents related to hazmat transportation, only 429 of them were classified as serious incidents. However, several hazmat accidents can be alarming in terms of damage and human loss. As an example, in 1993 a straightforward bus-truck collision in Quebec caused 20 deaths due to the spill and explosion of gasoline (Kara and Verter, 2004). Hazmat accidents include damage on regular vehicles and properties directly involved in the incident and/or damages on the population and environment exposed due to the hazardous materials released in the incident. In 1996 over 16,250 gallons of chlorine releasing from a derailed train in Montana resulted in 1 fatality, 787 injuries, 1000 evacuations and more than \$4.5 million in cleanup cost (U.S. Department of Transportation, 1999). More recently, in July 6th 2013, a runaway 73-car train slammed into the center of the Quebec town of Lac-Megantic in the early morning. Tank cars full of oil exploded and burned in the heart of the commercial district. Other than the \$7.7 million environmental costs, the remains of 42 people were recovered, and five people are reported missing (Pipeline and Hazardous Materials Administration (PHMSA), 2013). Due to this inherent risk, hazmat transportation continues to be a vital issue in the context of public safety and environmental preservation. A relatively unexplored risk mitigation approach in hazmat transportation is to attempt a separation of hazmat flow from regular traffic flow while guiding hazmat trucks to less populated areas.

To better comprehend the issues related to hazmat transportation, it is worthwhile considering this subject from the viewpoint of both the authorities and carriers. The main concern of the authorities is to mitigate the total risk induced by hazmat transportation. On the other hand, the main concern of carriers is to minimize transportation costs (distance, travel time) when choosing a route between a given origin-destination pair. Hazmat Transportation Network Design (HTND) policy restricts hazmat carriers from using certain road segments by imposing curfews. For instance, all parkways are restricted routes in New York State for hazmat trucks. Curfews either prevent carriers from using a road during specified time periods or totally prohibits its use. This policy does not consider the driver's preferences in route selection. In addition, curfews do not rationally adjust hazmat flows to less-risk areas.

Recently, a flexible policy, toll setting (TS), was proposed by Marcotte et al. (2009) to steer hazmat drivers to less-populated routes based on the carrier's own preferences (economic considerations) by assigning tolls to road segments. Toll pricing is widely used by traffic controllers to reduce network congestion. This reduction in congestion is achieved by assigning well-defined tolls to certain road segments so as to encourage drivers to choose less-congested routes. Further, charging toll allows network regulator to generate revenue which can be used for maintenance and network construction and also provide more flexible solutions to drivers. Therefore, toll-setting policies can yield a win-win scenario for both carriers and transportation agencies.

The idea of regulating both hazmat and regular traffic, namely dual-toll pricing (Wang et al., 2012), is quite recently proposed to encourage both drivers to the use of divergent routes. Wang et al. (2012) aim to reduce the transportation risk by dual toll pricing considering deterministic behavior of drivers in route selection. Assuming stochasticity in drivers behavior for route selection better explains their preferences in reality. Hence, our contribution is the consideration of dual toll pricing within the context of stochastic driver preferences.

The rest of this paper is organized as follows. The next section reviews the literature categorized in three main areas, namely, hazmat routing and risk measure, toll pricing, and stochastic driver preferences. This is followed by underscoring our major contribution in this paper. Section 3 develops the dual toll pricing model, encompassing the explicit definition of the problem, inclusion of the drivers' stochastic behavior, flow distribution model, risk measurement, and the mathematical model of the general problem proposed in this research. In Section 4, we propose a reformulation of the general model to a link-based model which motivates our solution algorithm. Section 5 presents a cyclic solution technique for the general problem by iteratively solving multiple single decision variable problems. Our computational experiments based upon an example problem are discussed in Section 6. Finally, Section 7 concludes the paper and offers some future research directions.

2 Literature Review

In this section, we provide separate literature reviews for risk modeling and routing methods either for regular vehicles or for hazmat trucks, proposed methods in the context of pricing, and finally, stochastic drivers preferences.

2.1 Hazmat Routing and Risk Measure

Routing of hazmat typically rely on two link attributes on the road network: accident probability and accident consequence. While the accident consequence is usually population, which may be measured by the λ-neighborhood concept (Batta and Chiu, 1988), near each link, it can also be other factors like environmental damages and property damages.

There are various measures of the risk of hazmat accidents, based on the distributional information of accident probability and consequence. Popular hazmat risk measures include expected risk (Alp, 1995; Jin and Batta, 1997), population exposure (ReVelle et al., 1991), incident probability (Saccomanno and Chan, 1985), perceived risk (Abkowitz et al., 1992), maximum risk, mean-variance, disutility (Erkut and Ingolfsson, 2000), and conditional probability (Sivakumar et al., 1993). More recently, quantile-based risk measures such as value-at-risk and conditional value-at-risk are also proposed for hazmat routing (Kang et al., 2014; Kwon, 2011; Toumazis et al., 2013). We refer readers to Erkut and Verter (1998) and Erkut and Ingolfsson (2005) for discussion of various hazmat risk measures.

In the context of dual toll pricing, Wang et al. (2012) used the duration-population-frequency risk measure to uniquely model the dependence of hazmat risk on congestion induced by non-hazmat traffic. In this paper, we define a risk measure function that extends the duration-population-frequency to consider direct damage to non-hazmat traffic so that our dual toll pricing model captures interdependence between both types of traffic.

2.2 Toll Pricing Problems

Toll pricing has been used as an efficient tool for congestion management. Generally, a congestion toll pricing problem can usually be classified as first and second best. In the first-best policy for a congested road network, transportation economists often prefer tolls that are based on marginal social cost pricing (Arnott and Small, 1994). In this policy tolls are equal to marginal external costs on each individual link and the objective is obtaining a traffic pattern which optimizes the collective use of the network. Hearn and Ramana (1998) assumes different criteria (e.g., minimizing the number of tolled streets, minimizing the total tolls collected) to define a set of tolls which optimizes the use of traffic network for the drivers. On the other hand, Johnsson-Stenman and Sterner (1998) define second-best pricing as the situation that not every single link of a transportation network can be charged by tolls. Because such kinds of tolls and other types of restrictions do not generally yield the maximum benefit possible, they are referred to as second-best.

In the context of hazmat transportation and regulation, toll policy was proposed as an alternative method to discourage hazmat carriers from using specific road segments (Marcotte et al., 2009). HTND policies aim to redirect hazmat shipments into low-risk paths. However such a policy does not consider the drivers' behavior in determining route selection. Moreover, restricting road segments does not logically distribute the hazmat flows to less risky areas. Imposing tolls on specified road segments results in hazmat carriers traveling through the less populated areas according to their own selection (due to economic and time considerations).

Bianco et al. (2012) enhance the work of Marcotte et al. (2009) by defining a new toll setting model. This new model discourages concentration of high level of risk on some links by minimizing the maximum link total risk, thus taking into account risk equity. This is achieved by linking the toll paid by a carrier on a road segment to the total risk on that road segment.

Directly relevant to this paper, Wang et al. (2012) suggest a dual-toll pricing model to control both regular and hazmat traffic, since they impact population safety and increase the accident probabilities in congested networks. We consider a dual-toll pricing policy that has the same fundamental idea, but considering stochastic drivers' route choice behavior.

2.3 Stochastic Drivers' Preferences

In the modeling of drivers' route choices, there are two main sources of uncertainty: one is the error component in the perceived travel time, and the other is uncertain distribution of value of time (VoT) among drivers, in particular, in the presence of toll charges. VoT is defined as the opportunity cost of the time that a driver spends on his or her travel from origin to destination. In nature, the former source is usually modeled as an additive stochasticity, while the latter source is modeled as a multiplicative stochasticity. Most models with additive stochasticity extend the framework of stochastic user equilibrium, proposed by Daganzo and Sheffi (1977), and they are closely related to discrete choice models (Akiva and Lerman, 1985).

To model drivers' response to toll pricing, VoT is often used to link two different route choice criteria of travel time and toll price. There are two classes of approaches to consider different VoT evaluation of different drivers. The first class, the multi-user-class approach, is based on the assumption that VoT remains constant within a user class and there are a finite number of user classes. Examples of such approaches include Cantarella and Binetti (1998) in the context of stochastic user equilibrium, Yang and Huang (2004) in congestion toll pricing, and Joksimovic et al. (2005) in dynamic toll pricing with additive stochasticity in users' travel time estimation. Also, Marcotte and Zhu (2009) prove the existence of optimal tolls for multi-user-class network equilibrium problem where the VoT parameter varies continuously among the population. In particular, they partition the population into segments where each segment is endowed with its own VoT parameter.

The second class considers continuously distributed VoT among users. Dial (1996) proposes a modeling framework, named stochastic bicriterion traffic user equilibrium, in this class, and uses a variational inequality formulation. Within this framework, Dial (1999a) considers congestion toll pricing. As in the case of the congestion toll pricing with single VoT (Hearn and Ramana, 1998), Dial (1999a) uses the marginal social cost pricing approach to derive the optimal toll, i.e., the optimal toll is computed according to the marginal social cost from the system optimal traffic flow. Dial (1999b) proposes an algorithm that computes the system optimal traffic flow with stochastic VoT. In this class, Lu et al. (2008) and Zhang et al. (2011) consider a bi-criterion dynamic user equilibrium model and a bi-criterion dynamic stochastic user equilibrium model, respectively, in the presence of toll prices.

2.4 Contributions of This Paper

In this paper, we consider a dual-toll pricing model with stochastic VoT for risk mitigation in hazmat transportation, and our model is the first of its kind. Our dual-toll pricing problem lends itself to a bi-level program where the upper level objective is to minimize the risk from hazmat transportation, and the lower level minimizes the total travel cost of regular and hazmat traffic.

Bi-level programming has a rich body of literature in transportation network regulation problems. For a recent review of applications we refer the reader to Yang and Bell (2001) and Colson et al. (2005). In terms of solution methodology, Bard (1998) provides a comprehensive overview of the state of the art bi-level optimization algorithms. Nonetheless, the solution to the dual-toll pricing problem remains challenging due to stochasticity. Even if the VoT parameter is considered as a constant, the resulting problem is a NP hard bi-level model. In addition, the regulator's objective is to minimize the risk, rather than to minimize the system travel cost as in (single) congestion toll pricing. This difference prevents us from using the marginal social cost pricing approach in this paper. While Wang et al. (2012) propose a two-stage approach, which is popular in congestion toll pricing (Hearn and Ramana, 1998), for dual-toll pricing, it is unclear how stochastic VoT can be managed within such a two-stage approach in the hazmat dual-toll pricing context. Also, our objective function is non-convex and highly nonlinear which cannot be solved to optimality to obtain

Notation	Description
$G(N,A)$	A graph of road network
N	Set of nodes
A	Set of arcs
W	Set of all O-D pairs considered for regular vehicles
P_{rs}	Set of all paths considered for regular vehicles traveling from node r to node s
Z	Set of all O-D pairs considered for hazmat trucks
Q_{rs}	Set of all paths considered for hazmat trucks traveling from node r to node s
d_{rs}	Travel demand of regular vehicles from node r to node s
e_{rs}	Travel demand of hazmat trucks from node r to node s
h_p	Regular traffic flow along path p
f_p	Hazmat traffic flow along path p
x_{ij}	Regular traffic volume in arc (i,j)
y_{ij}	Hazmat traffic volume in arc (i,j)
δ_{ij}^p	Path-arc incidence coefficient, equals 1 if arc (i,j) is contained in path p; otherwise 0.
τ_p	Total traveling time along path p
α_p	Total toll along path p for regular vehicles
θ_{ij}	Toll assigned to arc (i,j) for regular vehicles
β_p	Total toll along path p for hazmat trucks
μ_{ij}	Toll assigned to arc (i,j) for hazmat trucks
l_{ij}	The distance of arc (i,j)

Table 1: Summary of the key mathematical notation

a minimum risk flow solution. Further, Leurent (1993) develops the cost-versus-time equilibrium conditions over a network with continuously distributed VoT, variable demand and congestion effects on travel times. However, their method assumes toll prices as given parameters rather than unknown variables. In this paper, we formulate a non-convex optimization problem that is based on discrete choice modeling and provide a cyclic algorithm that solves each sub problem exactly.

3 Dual Toll Pricing Model

Let $G(N,A)$ be a network, with N being the node set and A being the arc set. Here A denotes the incidence matrix of graph G. Network links are assumed to be directed and without restriction for vehicle type. For both regular vehicles and hazmat trucks, we have separate origin destination pair sets, namely W and Z respectively. Corresponding to each O-D pair (r,s), d_{rs} and e_{rs} represent the travel demand for regular and hazmat carriers, respectively. Link travel times are flow-independent and are also assumed to be known constants. Separate tolls can be assigned for regular vehicles and hazmat trucks on each link (i,j), namely θ_{ij} and μ_{ij}. Table 1 summarizes the notation.

In the following subsections, we develop our dual-toll pricing model which aims at minimizing the risk of hazmat transportation while considering stochasticity in drivers' route selection. It is important to note that in our model, the stochasticity arises through the VoT parameter, and does not correspond to a stochastic traffic assignment problem.

3.1 Inclusion of Stochastic Driver Preferences

We now explain our definition and methodology to consider variability in driver's behavior in path selection. We define a multiplicative stochastic value of time to represent different drivers' preferences when minimizing their total travel cost.

We define the total travel cost of a path p as the sum of two components: one deterministic and the other stochastic. We let the deterministic part be defined as the total toll assigned to the path, i.e., α_p. The stochastic part, however, is the product of two components: the path p travel time, τ_p, which is assumed to be deterministic and flow-independent, and the value of time (VoT) which is stochastic with respect to the drivers traversing path p. Therefore, for the regular vehicle drivers we write

$$c_p = \gamma \tau_p + \alpha_p, \tag{1}$$

where c_p is the total traveling cost (disutility) along path p. The stochastic factor, γ, converts time to monetary unit and, as mentioned, is the VoT.

The same approach can be applied for hazmat trucks. Let k_p be the total travel cost along path p for hazmat drivers, τ_p the total travel time, and β_p the total toll assigned. Using this notation we obtain:

$$k_p = \lambda \tau_p + \beta_p, \tag{2}$$

with λ representing the VoT corresponding to hazmat carriers. It is assumed that γ and λ are both continuously distributed across the corresponding population. It is natural to consider that the non-hazmat users, which are non-atomic, have distinct VoT parameters. While for hazmat carriers, the decisions are often be enforced by the hazmat companies rather than individual drivers. Nevertheless, under the fact that hazmat carriers route choice actually reflects the decision of the companies which itself can vary across their population due to different valuation of time, a continuously distributed VoT for hazmat drivers becomes a valid assumption.

3.2 Flow Distribution Model

Network users, either regular or hazmat, have different preferences when selecting a path. We assume that a path p is chosen when its associated cost is minimum compared to all other possible paths in O-D pair (r, s). Recalling that the total travel cost along path p is $c_p = \gamma \tau_p + \alpha_p$ for regular vehicles, we define the probability of path p being selected when traveling from origin node r to destination node s as:

$$\begin{aligned}
\phi_{rs}^p(\alpha_p) &= \Pr\left[c_p = \min_{p' \in P_{rs}} c_{p'}\right] \qquad \forall (r,s) \in W \\
&= \Pr\left[c_p \leq c_{p'} \quad \forall p' \in P_{rs} \setminus \{p\}\right] \qquad \forall p \in P_{rs} \qquad \forall (r,s) \in W \\
&= \Pr\left[\gamma \tau_p + \alpha_p \leq \gamma \tau_{p'} + \alpha_{p'} \qquad \forall p' \in P_{rs} \setminus \{p\}\right] \qquad \forall p \in P_{rs} \qquad \forall (r,s) \in W \\
&= \Pr\left[\gamma(\tau_p - \tau_{p'}) \leq (\alpha_{p'} - \alpha_p) \qquad \forall p' \in P_{rs} \setminus \{p\}\right] \qquad \forall p \in P_{rs} \qquad \forall (r,s) \in W
\end{aligned} \tag{3}$$

We note that, according to (3), the probability of selecting a path p by regular vehicles is a function of the toll we assign to the path p as well as to all paths $p' \in P_{rs} \setminus \{p\}$, namely α_p and $\alpha_{p'}$. Nonetheless, we exclude $\alpha_{p'}$ for notational simplicity. One can observe that calculating $\phi^p_{rs}(\alpha_p)$ is dependent on the sign of $(\tau_p - \tau_{p'})$, which reverses the inequality in the negative case. This is discussed further in Section 3.4.

Let us assume that we know $\phi^p_{rs}(\alpha_p)$ for path p. Then, given the demand of regular vehicles of O-D pair (r, s), d_{rs}, the expected regular traffic flow along path p is

$$h_p = d_{rs}\phi^p_{rs}(\alpha_p) \qquad \forall p \in P_{rs}, \qquad (r, s) \in W, \tag{4}$$

where W is the set of all O-D pairs of regular vehicles. Consequently, the expected regular traffic volume on arc (i, j) can be written as

$$x_{ij} = \sum_{p \in P} \delta^p_{ij} h_p \qquad \forall (i, j) \in A, \tag{5}$$

where $P = \bigcup_{\forall (r,s) \in W} P_{rs}$ is the set of all paths for regular vehicles in the network. Equivalently

$$x_{ij} = \sum_{(r,s) \in W} \sum_{p \in P_{rs}} \delta^p_{ij} h_p \qquad \forall (i, j) \in A, \tag{6}$$

where

$$\delta^p_{ij} = \begin{cases} 1, & \text{if} \quad (i, j) \in p, \\ 0, & \text{otherwise.} \end{cases}$$

The same approach can be applied to hazmat trucks having total traveling cost along path p as $k_p = \lambda\tau_p + \beta_p$. The probability that a path p is chosen by a hazmat truck when traveling from origin r to destination s is

$$\begin{aligned} \psi^p_{rs}(\beta_p) &= \Pr\left[k_p = \min_{p' \in Q_{rs}} k_{p'}\right] \qquad \forall (r, s) \in Z \\ &= \Pr\left[\lambda(\tau_p - \tau'_p) \leq (\beta'_p - \beta_p) \qquad \forall p' \in Q_{rs} \setminus \{p\}\right] \qquad \forall p \in Q_{rs} \qquad \forall (r, s) \in Z. \end{aligned} \tag{7}$$

Given the travel demand of hazmat trucks from node r to node s as e_{rs}, the expected hazmat traffic flow along path p is

$$f_p = e_{rs}\psi^p_{rs}(\beta_p) \qquad \forall p \in Q_{rs}, \qquad (r, s) \in Z, \tag{8}$$

where Z is the set of all O-D pairs of hazmat trucks. In order to calculate the distributed hazmat traffic flow on each arc (i, j) we let

$$y_{ij} = \sum_{p \in Q} \delta^p_{ij} f_p = \sum_{(r,s) \in Z} \sum_{p \in Q_{rs}} \delta^p_{ij} f_p \qquad \forall (i, j) \in A, \tag{9}$$

where $Q = \bigcup_{\forall (r,s) \in Z} Q_{rs}$ is the set of all paths in the network for hazmat trucks. We now state our basic formulation of the dual toll pricing problem (P).

$$
(\mathsf{P}) \qquad \min \sum_{(i,j)\in A} R_{ij}(x_{ij}, y_{ij}), \tag{10}
$$

subject to

$$
\begin{aligned}
x_{ij} &= \sum_{p\in P} \delta_{ij}^{p} h_p \qquad \forall (i,j)\in A, && (11)\\
y_{ij} &= \sum_{p\in Q} \delta_{ij}^{p} f_p \qquad \forall (i,j)\in A, && (12)\\
h_p &= d_{rs}\phi_{rs}^{p}(\alpha_p) \qquad \forall p\in P_{rs}, (r,s)\in W, && (13)\\
f_p &= e_{rs}\psi_{rs}^{p}(\beta_p) \qquad \forall p\in Q_{rs}, (r,s)\in Z, && (14)\\
\alpha_p &\geq 0 \qquad \forall p\in P, && (15)\\
\beta_p &\geq 0 \qquad \forall p\in Q. && (16)
\end{aligned}
$$

In (P), the objective function minimizes the overall risk consequence in the network, which is the aggregated risk consequence on each arc (i, j). As we explain later in Section 3.3, the arc-based risk consequence is a function of both regular and hazmat flow on each arc (i, j) which can be derived from equations (11) through (14).

In the following subsections, we reformulate this basic model to obtain an efficient formulation for the general problem which also motivates our solution methodology. In problem (P), α_p and β_p denote the regular and hazmat tolls assigned to path p. It is important to note that due to the difficulty of collecting path based tolls, we assume that tolls are assigned with respect to arcs. For doing so, a straightforward transformation of the path tolls to the arc tolls can be applied.

Consider an arc (i, j) with θ_{ij} and μ_{ij} being the tolls assigned for regular vehicles and hazmat trucks, respectively. Thus, the path tolls can be readily computed:

$$
\alpha_p = \sum_{(i,j)\in A} \delta_{ij}^{p}\theta_{ij} \qquad \forall p\in P, \tag{17}
$$

$$
\beta_p = \sum_{(i,j)\in A} \delta_{ij}^{p}\mu_{ij} \qquad \forall p\in Q. \tag{18}
$$

In (P), we assume that tolls can have any continuous positive value. We now restrict the toll for an arc (i, j) to be within a specified lower and upper bound. We propose the following to define the bounds. Most instances of transportation networks contain three different types of links, categorized as state, interstate and local routes. Given the length of every link (mile) and the average and standard deviation for the amount of toll per mile for each type of road (measured in \$/mile), we can define a range for the toll assigned to each link for both regular and hazmat vehicles as:

$$\theta_{ij}^{\min} \leq \theta_{ij} \leq \theta_{ij}^{\max} \qquad \forall (i,j) \in A, \tag{19}$$

$$\mu_{ij}^{\min} \leq \mu_{ij} \leq \mu_{ij}^{\max} \qquad \forall (i,j) \in A. \tag{20}$$

We let $\theta_{ij}^{\min} = \mu_{ij}^{\min} = 0$ as this allows to have toll-free links; a good option from a cost perspective for network regulators.

3.3 Risk Measurement Function

If a hazmat accident happens on a road segment, it can impact regular traffic directly involved in the incident and those passing by the vicinity of the event. Other than regular vehicles, population residing in the neighborhood of the accident site and the environment can be significantly impacted in case of explosion or release of the hazmat. Hence, in the risk measurement function we compute both direct consequence and indirect consequence of a hazmat accident. Table 2 summarizes the notation used for risk measurement.

When the distributed flow of regular vehicles and hazmat trucks on arc (i,j) is x_{ij} and y_{ij}, respectively, the risk associated with arc (i,j) is calculated by:

$$R_{ij}(x_{ij}, y_{ij}) = (\omega_1 x_{ij}^2 y_{ij} t_{ij} + \omega_2 y_{ij} \rho_{ij} s_{ij}) \pi_{ij}^r \tag{21}$$

In (21), x_{ij} and y_{ij} are computed by the flow distribution methodology explained in Section 3.2. Thus, the risk function is indirectly dependent on the decision variables θ_{ij} and μ_{ij}. In (21), the first term with weight factor ω_1 is the measure of direct consequence of an accident. We assume that the direct consequence of the risk is influenced linearly by hazmat flow and the arc travel time, and nonlinearly by regular traffic flow. We adapt this novel function for the calculus of the direct consequences to account for the sever impact of a hazmat accident when the network is congested. In this sense, any nonlinear function can be a good estimate. A quadratic function is being considered in our model for the sake of its simplicity. The second term with ω_2 measures the indirect consequence of a hazmat accident. Motivated by the risk assessment model proposed by Abkowitz and Cheng (1988), once a hazmat accident occurs in link (i,j) and it is followed by a release or an explosion, a certain fraction of the population, $\rho_{ij} s_{ij}$, in the vicinity of the accident site is exposed. To compute s_{ij}, we use the λ-neighborhood method presented by Gopalan et al. (1990), where λ represents the radius of spread for the hazmat.

We note that the arc travel time, t_{ij}, is flow-independent and π_{ij}^r, the probability of hazmat accident release, is usually in the range of 10^{-7} to 10^{-6} per mile traveled (Abkowitz and Cheng, 1988). The weight factors ω_1 and ω_2 are conversion factors making the two terms of the same unit (usually the consequences are reported by the \$ damage amount) and the same order of magnitude.

Notation	Description
x_{ij}	Regular traffic volume in arc (i,j)
y_{ij}	Hazmat traffic volume in arc (i,j)
t_{ij}	Link (i,j) travel time
ρ_{ij}	Population density on arc (i,j) per square mile
s_{ij}	The impact area (square mile) on arc (i,j)
π^r_{ij}	Probability of a hazmat accident resulting in release incident on arc (i,j)

Table 2: Summary of notation used for the risk measurement

3.4 Dual Toll Pricing Problem

This subsection motivates our procedure to derive a formula for a path p selection probability. Once equations (3) and (7) are calculated, we can transform our basic model (P) to the general model that we aim to solve mathematically.

From (21), the objective of the dual toll pricing problem is to minimize the following function:

$$\sum_{(i,j)\in A} R_{ij}(x_{ij}, y_{ij}) + \epsilon \sum_{(i,j)\in A} (x_{ij}\theta_{ij} + y_{ij}\mu_{ij})$$

In the above equation, the first term measures the overall risk in the network, while the second term represents the total toll collected, multiplied by a very small constant $\epsilon > 0$, and added to the objective function in order to break any ties between set of tolls with the same risk value. That is, for any two given solutions of toll set with equal risk, we choose the one with the least total toll collected. Hence, the objective function is written as

$$\sum_{(i,j)\in A} \left[\Big\{ \omega_1 (x_{ij})^2 (y_{ij}) t_{ij} + \omega_2 (y_{ij}) \rho_{ij} s_{ij} \Big\} \pi^r_{ij} + \epsilon (x_{ij}\theta_{ij} + y_{ij}\mu_{ij}) \right] \tag{22}$$

In (P), we substitute equation (13) in (11) and equation (14) in (12) to introduce x_{ij} and y_{ij} in terms of path selection probability. Replacing the new definitions of x_{ij} and y_{ij} in equation (22), we develop an objective function containing probabilistic terms:

$$\begin{aligned} \min \sum_{(i,j)\in A} \Bigg[\Bigg\{ \omega_1 \Bigg(\sum_{(r,s)\in W} \sum_{p\in P_{rs}} \delta^p_{ij} d_{rs} \phi^p_{rs}(\alpha_p) \Bigg)^2 \Bigg(\sum_{(r,s)\in Z} \sum_{p\in Q_{rs}} \delta^p_{ij} e_{rs} \psi^p_{rs}(\beta_p) \Bigg) t_{ij} \\ + \omega_2 \Bigg(\sum_{(r,s)\in Z} \sum_{p\in Q_{rs}} \delta^p_{ij} e_{rs} \psi^p_{rs}(\beta_p) \Bigg) \rho_{ij} s_{ij} \Bigg\} \pi^r_{ij} \\ + \epsilon \Bigg(\sum_{(r,s)\in W} \sum_{p\in P_{rs}} \delta^p_{ij} d_{rs} \phi^p_{rs}(\alpha_p) \Bigg) \theta_{ij} + \Bigg(\sum_{(r,s)\in Z} \sum_{p\in Q_{rs}} \delta^p_{ij} e_{rs} \psi^p_{rs}(\beta_p) \Bigg) \mu_{ij} \Bigg] \end{aligned} \tag{23}$$

We also consider the two side constraints defined in Section 3.2 for problem (P).

Obtaining the probabilistic terms in (23) requires the information of the distribution of the VoT parameter. The literature on the distribution of the VoT is very limited. Small et al. (2005) use econometric techniques to study the distribution of commuters' preferences for speedy and reliable highway travel. Fosgerau (2006) applies various nonparametric and semiparametric methods to the estimation of the distribution of the VoT according to an empirical dataset. Nonetheless, there is not a widely used distribution for the VoT. In the present work, we assume that γ and λ are uniformly distributed throughout the population of regular and hazmat trip makers, respectively. This assumption is not only simplifying but also helps better demonstrate our solution methodology proposed in Section 5. We now can derive expressions for the probabilities, i.e., $\phi^p_{rs}(\alpha_p)$ and $\psi^p_{rs}(\beta_p)$ to replace in equation (23).

For regular vehicles, we assume $\gamma \in \text{Uniform}[a_\gamma, b_\gamma]$. Hence, in order to find the probability of selecting a path p, i.e., $\phi^p_{rs}(\alpha_p)$, we need to define a distance in the interval $[a_\gamma, b_\gamma]$ in which the path p travel cost, c_p, is smaller than that of any other path p' for O-D pair (r, s). More precisely, for each path p in an O-D pair there may be a certain subset of interval $[a_\gamma, b_\gamma]$ which minimizes c_p. Therefore, our major task becomes finding such an interval for each path. A similar discussion is valid for hazmat carriers.

Note that equation (23) states the probabilities as functions of the path toll decision variable, i.e., α_p and β_p, whereas we are interested in calculating the link tolls. Hence, in the following subsection, we suggest an alternative approach to replace $\phi^p_{rs}(\alpha_p)$, and $\psi^p_{rs}(\beta_p)$ in (23), with functions of link toll decision variable, namely θ and μ.

4 Reformulation Based on Link Tolls

In this section, our focus is to transform problem (P) with the objective defined as equation (23) to a link-based problem. This requires replacing $\phi^p_{rs}(\alpha_p)$ and $\psi^p_{rs}(\beta_p)$ in equation (23) with their equivalent definitions. Let us first describe our approach for the regular vehicles. The same approach can be applied for hazmat trucks.

4.1 Path Choice Probability for Regular Vehicles

Referring back to equation (3), the path p choice probability for regular vehicles is

$$\begin{aligned}
\phi^p_{rs}(\alpha_p) &= \Pr[c_p = \min_{p' \in P_{rs}} c_{p'}] \\
&= \Pr[\gamma(\tau_p - \tau_{p'}) \leq (\alpha_{p'} - \alpha_p) \qquad \forall p' \in P_{rs} \setminus \{p\}] \\
&= \Pr\Bigg[\gamma \leq (\alpha_{p'} - \alpha_p)/(\tau_p - \tau_{p'}) \quad \forall p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} > 0 \\
&\qquad\qquad \text{and } \gamma \geq (\alpha_{p'} - \alpha_p)/(\tau_p - \tau_{p'}) \quad \forall p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} < 0\Bigg] \qquad (24) \\
&= \Pr\Bigg[\max_{p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} < 0} (\alpha_{p'} - \alpha_p)/(\tau_p - \tau_{p'}) \leq \gamma
\end{aligned}$$

$$\leq \min_{p' \in P_{rs} \setminus \{p\}: \tau_p - \tau_{p'} > 0} (\alpha_{p'} - \alpha_p)/(\tau_p - \tau_{p'}) \Bigg] \tag{25}$$

for all $p \in P_{rs}, (r,s) \in W$. Note that we assume $\tau_p - \tau_{p'} \neq 0$ for all p' in (24). Later in this section, we explain how we can obtain $\phi^p_{rs}(\alpha_p)$ when $\tau_p - \tau_{p'} = 0$.

For every path p', equation (3) provides an interval in $[a_\gamma, b_\gamma]$ in which path p is dominant over p'. In order to find the overall probability of path p selection, a joint interval of $[a_\gamma, b_\gamma]$ over all paths p' must be derived. In equation (24), the inequality depends on the sign of coefficient $(\tau_p - \tau'_p)$. Thus we separate those p' paths with positive coefficient $(\tau_p - \tau'_p)$ from negative ones. Using probabilistic rules, the joint interval for those p' paths with positive $(\tau_p - \tau'_p)$ coefficient is the "min" of the intervals defined by equation (3) as $\frac{\alpha_{p'} - \alpha_p}{\tau_p - \tau_{p'}}$. On the other hand, for those p' with negative $(\tau_p - \tau'_p)$, the inequality reverses and the joint interval is achieved by "max" of $\frac{\alpha_{p'} - \alpha_p}{\tau_p - \tau_{p'}}$. Consequently, the distance in the interval $[a_\gamma, b_\gamma]$ in which path p has the smallest travel cost among all other paths in an O-D pair is the shared distance between the two "min" and "max" operators. This completes the explanation of equation (25).

We now express (25) as a function of link tolls θ, using the uniform distribution properties:

$$\phi^p_{rs}(\theta) = \left[\frac{u^p_{rs}(\theta) - v^p_{rs}(\theta)}{b_\gamma - a_\gamma} \right]_0^1, \tag{26}$$

where $[x]_a^b = \max(a, \min(b, x))$, and u and v are two new sets of variables for every path p in OD pair (r,s), defined as follows:

$$\begin{aligned} u^p_{rs}(\theta) &= \left[\min_{p' \in P_{rs} \setminus \{p\}: \tau_p - \tau_{p'} > 0} \left\{ \frac{\alpha_{p'} - \alpha_p}{\tau_p - \tau_{p'}} \right\} \right]_{a_\gamma}^{b_\gamma} \\ &= \left[\min_{p' \in P_{rs} \setminus \{p\}: \tau_p - \tau_{p'} > 0} \left\{ \frac{\sum_{(i,j)\in A} \delta^{p'}_{ij} \theta_{ij} - \sum_{(i,j)\in A} \delta^{p}_{ij} \theta_{ij}}{\tau_p - \tau_{p'}} \right\} \right]_{a_\gamma}^{b_\gamma} \end{aligned} \tag{27}$$

and

$$\begin{aligned} v^p_{rs}(\theta) &= \left[\max_{p' \in P_{rs} \setminus \{p\}: \tau_p - \tau_{p'} < 0} \left\{ \frac{\alpha_{p'} - \alpha_p}{\tau_p - \tau_{p'}} \right\} \right]_{a_\gamma}^{b_\gamma} \\ &= \left[\max_{p' \in P_{rs} \setminus \{p\}: \tau_p - \tau_{p'} < 0} \left\{ \frac{\sum_{(i,j)\in A} \delta^{p'}_{ij} \theta_{ij} - \sum_{(i,j)\in A} \delta^{p}_{ij} \theta_{ij}}{\tau_p - \tau_{p'}} \right\} \right]_{a_\gamma}^{b_\gamma} \end{aligned} \tag{28}$$

In obtaining link-based expressions, we use equation (17) to calculate path tolls in terms of link tolls.

From a geometric perspective, equation (27) first finds the intersections of linear functions of the form $\frac{\alpha_{p'} - \alpha_p}{\tau_p - \tau_{p'}}$ in the range of $\theta^{\min}$ and $\theta^{\max}$. Then, in each interval between two intersections, the min operator finds the line with the smallest value. Finally, the selected line is projected on two constant lines a_γ and b_γ to ensure that u^p_{rs} falls in the specified range. The term in the inner bracket of equation (28) represents a series of linear functions as well. However, here we find the line with the largest value in every interval between two intersections in the range of $\theta^{\min}$ and $\theta^{\max}$. Similarly the selected line is projected between a_γ and b_γ. Due to the fact that we select a linear function in every interval between two intersection points, both u^p_{rs} and v^p_{rs} represent piecewise-linear functions of θ projected onto the interval $[a_\gamma, b_\gamma]$. More explanation on how to calculate u^p_{rs} and v^p_{rs} is provided in Section 5.

One can note that, for an O-D pair (r, s), there are numerous paths, among which only a few may be assigned positive flows. Leurent (1993) refers to such alternatives as efficient paths which are those with no alternative that would be both quicker and cheaper. That is, for an efficient path, there always exists some positive VoT for which the path ensures the minimum total traveling cost. In this case, equation (26) is positive with u^p_{rs} being greater than v^p_{rs}. For other cases, drivers never use path p to travel from r to s due to there is at least one path p' which is both quicker and cheaper.

It is also worth mentioning that, for an O-D pair (r, s), if there are two efficient paths p_1 and p_2 with similar attributes, i.e., $\alpha_1 = \alpha_2$, and $\tau_1 = \tau_2$, the subintervals $[v^{p_1}_{rs}(\theta), u^{p_1}_{rs}(\theta)]$ and $[v^{p_2}_{rs}(\theta), u^{p_2}_{rs}(\theta)]$ overlap. In this case, we assume that the O-D demand, d_{rs}, is split equally between the two paths. In general, if multiple of such paths exist for an O-D pair $(r, s) \in W$ which are denoted by set Ω_{rs}, the corresponding O-D flow is distributed according to the following:

$$h_{p_i} = \frac{d_{rs}}{|\Omega_{rs}|}\phi^{p_i}_{rs}(\alpha_{p_i}) \qquad , \forall p_i \in \Omega_{rs}, \tag{29}$$

where $|\Omega_{rs}|$ denotes the cardinality of set Ω_{rs}, and $\phi^{p_i}_{rs}(\alpha_{p_i})$ is equivalent among all the elements $p_i \in \Omega_{rs}$. This procedure has been incorporated throughout the implementation of the solution algorithm provided in Section 5. However, these spacial cases have been excluded from the general model representation for the sake of notational simplicity.

4.2 Path Choice Probability for Hazmat Trucks

For hazmat trucks, a similar approach can be applied by defining two new variables m^p_{rs} and n^p_{rs} for every path p with the stochastic factor λ following the uniform distribution $[a_\lambda, b_\lambda]$. To find the probability of selection for a path p in O-D pair $(r, s) \in Z$ with toll β_p, i.e., $\psi^p_{rs}(\beta_p)$, we determine an interval in the range $[a_\lambda, b_\lambda]$ in which path p is preferred compared to other paths p' in O-D pair (r, s). To find such an interval, we represent equation (7) as follows:

$$\begin{aligned}
\psi^p_{rs}(\beta_p) &= \Pr\left[k_p = \min_{p' \in Q_{rs}} k_{p'}\right] \\
&= \Pr\left[\lambda(\tau_p - \tau_{p'}) \leq (\beta_{p'} - \beta_p) \qquad \forall p' \in Q_{rs} \setminus \{p\}\right] \\
&= \Pr\left[\lambda \leq (\beta_{p'} - \beta_p)/(\tau_p - \tau_{p'}) \quad \forall p' \in Q_{rs} \setminus \{p\} : \tau_p - \tau_{p'} > 0\right.
\end{aligned}$$

$$\text{and } \lambda \geq (\beta_{p'} - \beta_p)/(\tau_p - \tau_{p'}) \quad \forall p' \in Q_{rs} \setminus \{p\} : \tau_p - \tau_{p'} < 0\Big] \tag{30}$$

$$= \Pr\Bigg[\max_{p' \in Q_{rs} \setminus \{p\}: \tau_p - \tau_{p'} < 0} (\beta_{p'} - \beta_p)/(\tau_p - \tau_{p'}) \leq \lambda \leq \min_{p' \in Q_{rs} \setminus \{p\}: \tau_p - \tau_{p'} > 0} (\beta_{p'} - \beta_p)/(\tau_p - \tau_{p'}) \Bigg] \tag{31}$$

for all path $p \in Q_{rs}, (r, s) \in Z$. For every path p', equation (7) yields an interval in $[a_\lambda, b_\lambda]$ in which path p is dominant over p'. In equation (30), we separate the p' paths depending on the sign of the coefficient $(\tau_p - \tau_{p'})$. The intersection of the intervals obtained from p' paths with a positive $(\tau_p - \tau_{p'})$ coefficient is the "min" of the intervals which are defined by equation (7) as $\frac{\beta_{p'} - \beta_p}{\tau_p - \tau_{p'}}$. While for those p' paths with negative $(\tau_p - \tau_{p'})$, the intersection of the intervals is equal to the "max" of $\frac{\beta_{p'} - \beta_p}{\tau_p - \tau_{p'}}$. Now, combining the two intervals achieved by "min" and "max" operators, we can find the overall shared interval in the range $[a_\lambda, b_\lambda]$ in which path p is chosen over any other path p'. This is shown in equation (31). Consequently, the selection probability of a path p in an O-D pair $(r, s) \in Z$, i.e., equation (31), can be defined as a function of link tolls μ:

$$\psi^p_{rs}(\mu) = \left[\frac{m^p_{rs}(\mu) - n^p_{rs}(\mu)}{b_\lambda - a_\lambda} \right]_0^1, \tag{32}$$

where m and n, the two new sets of variables introduced for every path p in O-D pair (r, s), are defined as the following:

$$\begin{aligned} m^p_{rs}(\mu) &= \left[\min_{p' \in Q_{rs} \setminus \{p\}: \tau_p - \tau_{p'} > 0} \left\{ \frac{\beta_{p'} - \beta_p}{\tau_p - \tau_{p'}} \right\} \right]_{a_\lambda}^{b_\lambda} \\ &= \left[\min_{p' \in Q_{rs} \setminus \{p\}: \tau_p - \tau_{p'} > 0} \left\{ \frac{\sum_{(i,j) \in A} \delta^{p'}_{ij} \mu_{ij} - \sum_{(i,j) \in A} \delta^p_{ij} \mu_{ij}}{\tau_p - \tau_{p'}} \right\} \right]_{a_\lambda}^{b_\lambda} \end{aligned} \tag{33}$$

and

$$\begin{aligned} n^p_{rs}(\mu) &= \left[\max_{p' \in Q_{rs} \setminus \{p\}: \tau_p - \tau_{p'} < 0} \left\{ \frac{\beta_{p'} - \beta_p}{\tau_p - \tau_{p'}} \right\} \right]_{a_\lambda}^{b_\lambda} \\ &= \left[\max_{p' \in Q_{rs} \setminus \{p\}: \tau_p - \tau_{p'} < 0} \left\{ \frac{\sum_{(i,j) \in A} \delta^{p'}_{ij} \mu_{ij} - \sum_{(i,j) \in A} \delta^p_{ij} \mu_{ij}}{\tau_p - \tau_{p'}} \right\} \right]_{a_\lambda}^{b_\lambda} \end{aligned} \tag{34}$$

where μ is a vector of hazmat tolls, i.e., (μ_{ij}). Note that we use equation (18) in the above definitions to represent path tolls by link tolls.

Equation (33) finds the intersections of linear functions of the form $\frac{\beta_{p'} - \beta_p}{\tau_p - \tau_{p'}}$ in the range of $\mu^{\min}$ and $\mu^{\max}$, and then in each interval between two intersections selects the line with the smallest value. Finally, the selected line is projected on a_λ and b_λ. On the other hand, equation (34) finds the line with the largest value in every interval between two intersections of linear functions. The selected line is projected between a_λ and b_λ. As in the case for regular vehicles, we select a linear function in every interval between two intersection points, namely m^p_{rs} and n^p_{rs}. This gives us piecewise-linear functions of μ projected onto interval $[a_\lambda, b_\lambda]$ in the range of $\mu^{\min}$ and $\mu^{\max}$.

Consequently, if there exists such an interval in which path p outperforms the other paths $p' \in Q_{rs}$, i.e., path p is efficient, then m^p_{rs} becomes greater than n^p_{rs}. That is, equation (32) represents a positive value as the path p selection probability.

Similar to regular vehicles, for hazmat carriers case, there might be multiple efficient paths in an O-D pair $(r,s) \in Z$ whose the corresponding subintervals $[n^p_{rs}(\mu), m^p_{rs}(\mu)]$ overlap. Our assumption is that the demand e_{rs} is equally distributed among these paths. That is

$$f_{p_j} = \frac{e_{rs}}{|\Upsilon_{rs}|}\psi^{p_j}_{rs}(\beta_{p_j}) \qquad \forall p_j \in \Upsilon_{rs}, \tag{35}$$

where Υ_{rs} denotes the set of all paths in O-D pair (r,s), with overlapping subintervals, whose cardinality is $|\Upsilon_{rs}|$. One can also note that the selection probability $\psi^{p_j}_{rs}(\beta_{p_j})$ is equivalent for all paths $p_j \in \Upsilon_{rs}$. Although our model representation in the current section disregards these special cases for notational simplicity, they have been fully implemented in the solution procedure described in Section 5.

4.3 Pairwise comparison of paths with equivalent travel times

In calculating path p choice probability, there is another case that might occur when comparing paths p and p'. In particular, it is possible that when using equation (3), the coefficient $\tau_p - \tau_{p'} = 0$ for some path $p' \in P_{rs} \setminus \{p\}$. The same situation may also happen when calculating $\psi^p_{rs}(\mu)$ in equation (7) for some $p' \in Q_{rs} \setminus \{p\}$. We explain such a case for regular vehicles; the same calculation is valid for hazmat trucks. The explanation below is valid for the case of having only one decision variable (e.g., θ_{lk}) in the problem.

Suppose that, for a path $p \in P_{rs}, (r,s) \in W$, there exists at least one path $p' \in P_{rs} \setminus \{p\}$ where the coefficient $\tau_p - \tau_{p'} = 0$. In this case, the path p choice probability for regular vehicles, i.e., equation (3), can be reconstructed as follows:

$$\begin{aligned}
\phi^p_{rs}(\alpha_p) &= \Pr[c_p = \min_{p' \in P_{rs}} c_{p'}] \\
&= \Pr[c_p \le c_{p'} \quad \forall p' \in P_{rs} \setminus \{p\}] \\
&= \Pr[(c_p \le c_{p'} \quad \forall p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} \neq 0) \\
&\qquad\qquad \text{and} \quad (c_p \le c_{p'} \quad \forall p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} = 0)]
\end{aligned} \tag{36}$$

In the above equation, obtaining an interval wherein path p dominates all $p' : \tau_p - \tau_{p'} \neq 0$, i.e., the term $(c_p \le c_{p'} \quad \forall p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} \neq 0)$, has been already explained using equations (25)-(28). This divides the range of $[\theta^{\min}_{lk}, \theta^{\max}_{lk}]$ to sub-intervals each corresponding with a distinct pair of $u^p_{rs}(\theta_{lk})$ and $v^p_{rs}(\theta_{lk})$. We note that in the second term, the interval does not depend on γ

due to the following calculation

$$\forall p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} = 0 : \qquad \begin{aligned} (c_p \le c_{p'}) &\Leftrightarrow (\alpha_p + \gamma\tau_p \le \alpha_{p'} + \gamma\tau_{p'}) \\ &\Leftrightarrow (\alpha_p + \gamma(\tau_p - \tau_{p'}) - \alpha_{p'} \le 0) \\ &\Leftrightarrow (\alpha_p - \alpha_{p'} \le 0). \end{aligned}$$

Two cases are possible when obtaining an interval wherein path p outperforms path p' with $\tau_p - \tau_{p'} = 0$. Let us denote such an interval by $[c_p \le c_{p'}]$ in the following calculation:

- Case 1: $\alpha_p - \alpha_{p'}$ does not contain the decision variable θ_{lk}. In this case α_p and $\alpha_{p'}$ are two known constants. Thus,

$$[c_p \le c_{p'}] = \begin{cases} [\theta_{lk}^{\min}, \theta_{lk}^{\max}] & \text{(for all } \theta_{lk} \in [\theta_{lk}^{\min}, \theta_{lk}^{\max}] \text{ path } p \text{ outperforms } p') \quad \text{if } \alpha_p \le \alpha_{p'} \\ \emptyset & \text{(path } p \text{ is always dominated by path } p') \quad \text{otherwise} \end{cases}$$

- Case 2: $\alpha_p - \alpha_{p'}$ contains the decision variable θ_{lk}. In this case $[\alpha_p - \alpha_{p'} \le 0]$ takes on the form $[\theta_{lk} \le C]$ where C is a constant. Thus,

$$[c_p \le c_{p'}] = \begin{cases} \emptyset & \text{(path } p \text{ is always dominated by path } p') \quad \text{if} \quad C < \theta_{lk}^{\min} \\ [\theta_{lk}^{\min}, \theta_{lk}^{\max}] & \text{(for all } \theta_{lk} \in [\theta_{lk}^{\min}, \theta_{lk}^{\max}] \text{ path } p \text{ outperforms } p') \quad \text{if} \quad C \ge \theta_{lk}^{\max} \\ [\theta_{lk}^{\min}, C] & \text{if} \quad C \in [\theta_{lk}^{\min}, \theta_{lk}^{\max}] \end{cases}$$

Thus, for each $p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} = 0$, the above calculation provides a set of sub-intervals of θ_{lk} in the range of $[\theta_{lk}^{\min}, \theta_{lk}^{\max}]$. In order to complete the calculation of $\phi_{rs}^{p}(\alpha_p)$, one needs to intersect the interval of θ_{lk} defined by $(c_p \le c_{p'} \quad \forall p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} \ne 0)$ with the ones obtained from the second term, i.e., $(c_p \le c_{p'} \quad \forall p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} = 0)$.

Later in Section 5, we will explain that our proposed solution methodology decomposes the multiple toll decision variable problem into single decision variable subproblems whose solutions are employed in constructing the solution to the general problem. Consequently, our solution algorithm relies only on calculating path selection probabilities in the single toll decision variable case which obviates the need to explain the calculus of $[c_p \le c_{p'} \quad \forall p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} = 0]$ in the multiple decision variable case.

For hazmat trucks with a single toll decision variable as μ_{lk}, when there exists a path $p' \in Q_{rs} \setminus \{p\}$ with $\tau_p - \tau_{p'} = 0$, a similar discussion to the regular vehicles case can be applied after reconstructing equation (7). That is, intersecting the interval defined by equations (31)–(34) with $[k_p \le k_{p'}]$, obtained from $p' : \tau_p - \tau_{p'} = 0$, completes the calculation of $\psi_{rs}^{p}(\mu)$.

4.4 Link-Based Dual Toll Pricing Model

Consequently, substituting $\phi_{rs}^p(\theta)$ from (26) and $\psi_{rs}^p(\mu)$ from (32) into equation (23) and incorporating constraints (19) and (20), the model presented in (P) is now transformed to (Q):

$$
\begin{aligned}
(\mathsf{Q}) \quad \min_{\theta,\mu} \sum_{(i,j)\in A} \Bigg[\Bigg\{ \omega_1 \Bigg(\sum_{(r,s)\in W} \sum_{p\in P_{rs}} \delta_{ij}^p d_{rs} \frac{u_{rs}^p(\theta) - v_{rs}^p(\theta)}{b_\gamma - a_\gamma} \Bigg)^2 \Bigg(\sum_{(r,s)\in Z} \sum_{p\in Q_{rs}} \delta_{ij}^p e_{rs} \frac{m_{rs}^p(\mu) - n_{rs}^p(\mu)}{b_\lambda - a_\lambda} \Bigg) t_{ij} \\
+ \omega_2 \Bigg(\sum_{(r,s)\in Z} \sum_{p\in Q_{rs}} \delta_{ij}^p e_{rs} \frac{m_{rs}^p(\mu) - n_{rs}^p(\mu)}{b_\lambda - a_\lambda} \Bigg) \rho_{ij} s_{ij} \Bigg\} \pi_{ij}^r \\
+ \epsilon \Bigg(\sum_{(r,s)\in W} \sum_{p\in P_{rs}} \delta_{ij}^p d_{rs} \frac{u_{rs}^p(\theta) - v_{rs}^p(\theta)}{b_\gamma - a_\gamma} \Bigg) \theta_{ij} \\
+ \Bigg(\sum_{(r,s)\in Z} \sum_{p\in Q_{rs}} \delta_{ij}^p e_{rs} \frac{m_{rs}^p(\mu) - n_{rs}^p(\mu)}{b_\lambda - a_\lambda} \Bigg) \mu_{ij} \Bigg] \quad (37)
\end{aligned}
$$

subject to

$$\theta_{ij}^{\min} \le \theta_{ij} \le \theta_{ij}^{\max} \qquad \forall (i,j) \in A \qquad (38)$$

$$\mu_{ij}^{\min} \le \mu_{ij} \le \mu_{ij}^{\max} \qquad \forall (i,j) \in A \qquad (39)$$

where $\theta = (\theta_{ij})$ and $\mu = (\mu_{ij})$.

In (Q), the new decision variables are $u_{rs}^p, v_{rs}^p, m_{rs}^p$ and n_{rs}^p, which themselves are based on arc tolls θ_{ij} and μ_{ij}. The two constraints, equations (38) and (39) are the lower and upper bounds for the single tolls assigned to each arc (i,j) for regular vehicles and hazmat trucks, respectively.

5 Solution Algorithm

The principle difficulty with formulation (Q) is the highly nonlinear nature of the objective function. Even for the simple case where there is only one link (l,k) with an unknown toll for regular vehicles and an unknown toll for hazmat trucks, namely θ_{lk} and μ_{lk}, the objective function is nonlinear of order three. The order of nonlinearity sharply increases as the number of links with toll variable increases. A standard approach to solve a constrained nonlinear optimization problem is to convert it to an unconstrained nonlinear problem and apply an iterative search process. We take a different approach in this paper. An optimal procedure for the single toll decision variable case is first developed (which is possible since the sub-problems for this case are either linear or quadratic). After this, a heuristic procedure is developed for the general case (with multiple toll decision variables) that sequentially uses solutions of appropriately defined single toll problems.

5.1 Single-Toll Variable Case

Suppose that there is only one link (l, k) in the network having an unspecified toll for either regular vehicles or hazmat trucks. All other link tolls are known constants. Thus, the single decision variable is either θ_{lk} or μ_{lk}. The reason we propose this approach is that finding the solution to u_{rs}^{p}, v_{rs}^{p}, m_{rs}^{p} and n_{rs}^{p} defined in Section 3.4 is complicated in the multiple toll decision variable case.

When a regular toll, θ_{lk}, is the only decision variable and all other tolls are known constants, the objective function in equation (37) is quadratic. Recall that from equations (27) and (28), to find u_{rs}^{p} and v_{rs}^{p} for every path p, the interval $[\theta_{lk}^{\min}, \theta_{lk}^{\max}]$ is divided into sub-intervals. Hence, a comparison of the local optimal solution of each sub-interval gives the global optimum.

In the case that a hazmat toll, μ_{lk}, is the only decision variable and all other link tolls are known constants, equation (37) becomes a linear function of μ_{lk}. With the same reasoning as for the regular vehicles' case, the interval of $[\mu_{lk}^{\min}, \mu_{lk}^{\max}]$ is divided into sub-intervals. The only difference with the regular vehicles' case is that the objective function is at most a linear function in each sub-interval. Hence, the global optimal solution of a piece-wise linear function can be easily found by comparing only the extreme points of the sub-intervals.

We now formalize this presentation. First, we discuss the algorithm for an unknown regular toll θ_{lk} and represent the corresponding single-toll variable algorithm as $(\mathcal{R})$, and next a similar algorithm $(\mathcal{H})$ is represented for an unknown hazmat toll μ_{lk}.

Consider the case where a regular toll, θ_{lk} is unknown. Hence, in (27) and (28) for $u_{rs}^{p}(\theta)$ and $v_{rs}^{p}(\theta)$, there is at most one variable θ_{lk} in the numerator while other parameters are given. Therefore

$$\frac{\sum_{(i,j)\in A} \delta_{ij}^{p'}\theta_{ij} - \sum_{(i,j)\in A} \delta_{ij}^{p}\theta_{ij}}{\tau_p - \tau_{p'}} \qquad \forall p' \in P_{rs} \setminus \{p\},$$

represents either a linear function of θ_{lk} or a constant value. To simplify functions $u_{rs}^{p}(\theta_{lk})$ and $v_{rs}^{p}(\theta_{lk})$ we define the following:

$$u_{rs}^{pp'}(\theta_{lk}) = \frac{\sum_{(i,j)\in A} \delta_{ij}^{p'}\theta_{ij} - \sum_{(i,j)\in A} \delta_{ij}^{p}\theta_{ij}}{\tau_p - \tau_{p'}} \qquad \forall p' \in P_{rs} \setminus \{p\}, \tau_p - \tau_{p'} > 0. \tag{40}$$

$$v_{rs}^{pp'}(\theta_{lk}) = \frac{\sum_{(i,j)\in A} \delta_{ij}^{p'}\theta_{ij} - \sum_{(i,j)\in A} \delta_{ij}^{p}\theta_{ij}}{\tau_p - \tau_{p'}} \qquad \forall p' \in P_{rs} \setminus \{p\}, \tau_p - \tau_{p'} < 0. \tag{41}$$

Equations (40) and (41) represent two sets of linear functions of a single variable θ_{lk} for which we are interested in finding the minimum and maximum, respectively.

The approach we apply can itself satisfy all constraints of (Q) in the single decision variable case. From (38) and (39) in (Q), only one constraint, namely $\theta_{lk}^{\min} \leq \theta_{lk} \leq \theta_{lk}^{\max}$, remains active. To find $u_{rs}^{p}(\theta_{lk})$, we find the intersections of linear functions $u_{rs}^{pp'}(\theta_{lk})$ in the range $\theta_{lk}^{\min}$ and $\theta_{lk}^{\max}$; this yields the intervals of the form

$$[\theta_{lk}^{\min}, \theta_{lk,1}^{(1)}), \quad [\theta_{lk,1}^{(1)}, \theta_{lk,2}^{(1)}), \quad \ldots, \quad [\theta_{lk,x-2}^{(1)}, \theta_{lk,x-1}^{(1)}), \quad [\theta_{lk,x-1}^{(1)}, \theta_{lk}^{\max}],$$

for path p. In each interval, we pick the line with the minimum value among all $u_{rs}^{pp'}(\theta_{lk})$ lines while keeping that value between a_γ and b_γ, the upper and lower limit of the uniform distribution for the VoT factor, γ. On the other hand, for $v_{rs}^{p}(\theta_{lk})$, finding the intersections of linear functions $v_{rs}^{pp'}(\theta_{lk})$ divides the range $\theta_{lk}^{\min}$ and $\theta_{lk}^{\max}$ to

$$[\theta_{lk}^{\min}, \theta_{lk,1}^{(2)}), \quad [\theta_{lk,1}^{(2)}, \theta_{lk,2}^{(2)}), \quad \ldots, \quad [\theta_{lk,y-2}^{(2)}, \theta_{lk,y-1}^{(2)}), \quad [\theta_{lk,y-1}^{(2)}, \theta_{lk}^{\max}],$$

for path p. Then in each of these sub-intervals a $v_{rs}^{pp'}(\theta_{lk})$ with the maximum value denotes $v_{rs}^{p}(\theta_{lk})$. In order to obtain sub-intervals in which neither $u_{rs}^{p}(\theta_{lk})$ nor $v_{rs}^{p}(\theta_{lk})$ changes, we mix the two sets of intersections and reorder to obtain the following

$$[\theta_{lk}^{\min}, \theta_{lk,1}^{(3)}), \quad [\theta_{lk,1}^{(3)}, \theta_{lk,2}^{(3)}), \quad \ldots, \quad [\theta_{lk,z-2}^{(3)}, \theta_{lk,z-1}^{(3)}), \quad [\theta_{lk,z-1}^{(3)}, \theta_{lk}^{\max}]$$

Note that the set of intersection points defined by $\theta_{lk,z}^{(3)}$ denotes the intervals wherein path p outperforms $p' \in P_{rs} \setminus \{p\}$ with $\tau_p - \tau_{p'} \neq 0$. Hence, the remaining part is to combine this set with the intervals obtained for p dominating $p' \in P_{rs} \setminus \{p\}$ with $\tau_p - \tau_{p'} = 0$. Let us assume the new set of intersections for path p is defined as below after reordering

$$[\theta_{lk}^{\min}, \theta_{lk,1}), \quad [\theta_{lk,1}, \theta_{lk,2}), \quad \ldots, \quad [\theta_{lk,h-2}, \theta_{lk,h-1}), \quad [\theta_{lk,h-1}, \theta_{lk}^{\max}]$$

To account for cases with $p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} = 0$ in calculating the path p selection probability, i.e., $\phi_{rs}^{p}(\theta_{lk})$, we need to modify u_{rs}^{p} and v_{rs}^{p} in each of the sub-intervals. That is, if a sub-interval, defined by $\theta_{lk,z}^{(3)}$, intersects with the ones obtained from all $p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} = 0$, the u_{rs}^{p} and v_{rs}^{p} remain unchanged within the sub-interval. Otherwise, if there is no intersection for least one $p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} = 0$, the u_{rs}^{p} and v_{rs}^{p} should be set to zero for the corresponding sub-interval.

After this modification, u_{rs}^{p} and v_{rs}^{p} are still either a constant or a linear function of θ_{lk} in each of the sub-intervals which replacing them in equation (37) yields at most a quadratic function in each corresponding sub-interval. The global solution of this piece-wise quadratic function is achieved by finding the local optimum of every sub-interval and comparing all local optima with extreme points. We obtain the following proposition:

Proposition 1. Suppose that a *regular* toll in arc $(l,k) \in A$, $\theta_{lk} \in \mathbb{R}$, is the single variable of interest. Let $f(\theta_{lk})$ represent the objective function of problem (Q) as a function of the single regular toll θ_{lk}. Consider the subintervals

$$[\theta_{lk}^{\min}, \theta_{lk,1}), \quad [\theta_{lk,1}, \theta_{lk,2}), \quad \ldots, \quad [\theta_{lk,h-2}, \theta_{lk,h-1}), \quad [\theta_{lk,h-1}, \theta_{lk}^{\max}]$$

and denote the objective function in the i-th interval by $f_i(\cdot)$. Then an optimal toll is found by

$$\theta_{lk}^{*} = \arg \min_{\theta_{lk} \in \Theta_{lk}} f(\theta_{lk}),$$

where $\Theta_{lk} = \{\hat{\theta}_{lk,1}, \hat{\theta}_{lk,2}, \hat{\theta}_{lk,3}, \ldots, \hat{\theta}_{lk,h}\}$, and $\hat{\theta}_{lk,i}$ is the local minimum in the i^{th} interval, i.e.

$\hat{\theta}_{lk,i} = [\bar{\theta}_{lk,i}]_{\theta_{lk,i-1}}^{\theta_{lk,i}}$, $f_i(\bar{\theta}_{lk,i}) = \min_{\theta_{lk,i-1} \leq \theta_{lk} \leq \theta_{lk,i}} f_i(\theta_{lk})$, and $f'_i(\bar{\theta}_{lk,i}) = 0$.

Consequently, we propose algorithm ($\mathcal{R}$) for obtaining the exact solution of the single regular toll, θ_{lk}, problem as below:

Algorithm ($\mathcal{R}$)

Step 1: For each path $p \in P_{rs}$ and $(r,s) \in W$, use equation (40) and find the $u_{rs}^{pp'}(\theta_{lk})$ as linear functions of θ_{lk}.

Step 2: Find the intersections of the lines, $u_{rs}^{pp'}(\theta_{lk})$, to obtain the sub-intervals in the range $\theta_{lk}^{\min}$ and $\theta_{lk}^{\max}$. In each sub-interval, pick the function of the line with the minimum value as $u_{rs}^{p}(\theta_{lk})$. This step obtains a piece-wise linear concave function as $u_{rs}^{p}(\theta_{lk})$ in the specified range for θ_{lk}.

Step 3: For each path $p \in P_{rs}$ and $(r,s) \in W$, use equation (41) and find the $v_{rs}^{pp'}(\theta_{lk})$ as linear functions of θ_{lk}.

Step 4: Find the intersections of the lines, $v_{rs}^{pp'}(\theta_{lk})$, to obtain the sub-intervals in the range $\theta_{lk}^{\min}$ and $\theta_{lk}^{\max}$. In each sub-interval, select the function of the line with the maximum value as $v_{rs}^{p}(\theta_{lk})$. This step yields a piece-wise linear convex function as $v_{rs}^{p}(\theta_{lk})$ in the specified range for θ_{lk}.

Step 5: Combine and sort all the sub-intervals. For every path p, $\forall p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} = 0$, find the interval in the range $\theta_{lk}^{\min}$ and $\theta_{lk}^{\max}$ wherein path p dominates p'. If the intervals obtained for all such p' intersect with a sub-interval, do not change $u_{rs}^{p}(\theta_{lk})$ and $v_{rs}^{p}(\theta_{lk})$ within the sub-interval. Otherwise, set $u_{rs}^{p}(\theta_{lk})$ and $v_{rs}^{p}(\theta_{lk})$ to zero.

Step 6: Combine and sort all the sub-intervals, and in each sub-interval find the objective function based on θ_{lk} by substituting u_{rs}^{p} and v_{rs}^{p} in (37). (The objective function is at most a quadratic function)

Step 7: Find the minimum of the objective function in each sub-interval as the local minima.

Step 8: Compare the local minima and extreme points to obtain global optimum.

Figure 1 illustrates the exact algorithm ($\mathcal{R}$) when a regular toll θ_{lk} is unknown.

For hazmat trucks, when μ_{lk} is the only unknown toll in the network, a similar approach can be used to obtain $m_{rs}^{p}(\mu_{lk})$ and $n_{rs}^{p}(\mu_{lk})$ in (33) and (34) since they are similarly either a minimum or a maximum of linear functions—$m_{rs}^{pp'}(\mu_{lk})$ and $n_{rs}^{pp'}(\mu_{lk})$—of single decision variable μ_{lk} where

$$m_{rs}^{pp'}(\mu_{lk}) = \frac{\sum_{(i,j)\in A} \delta_{ij}^{p'} \mu_{ij} - \sum_{(i,j)\in A} \delta_{ij}^{p} \mu_{ij}}{\tau_p - \tau_{p'}} \qquad \forall p' \in Q_{rs} \setminus \{p\}, \tau_p - \tau_{p'} > 0, \tag{42}$$

$$n_{rs}^{pp'}(\mu_{lk}) = \frac{\sum_{(i,j)\in A} \delta_{ij}^{p'} \mu_{ij} - \sum_{(i,j)\in A} \delta_{ij}^{p} \mu_{ij}}{\tau_p - \tau_{p'}} \qquad \forall p' \in Q_{rs} \setminus \{p\}, \tau_p - \tau_{p'} < 0. \tag{43}$$

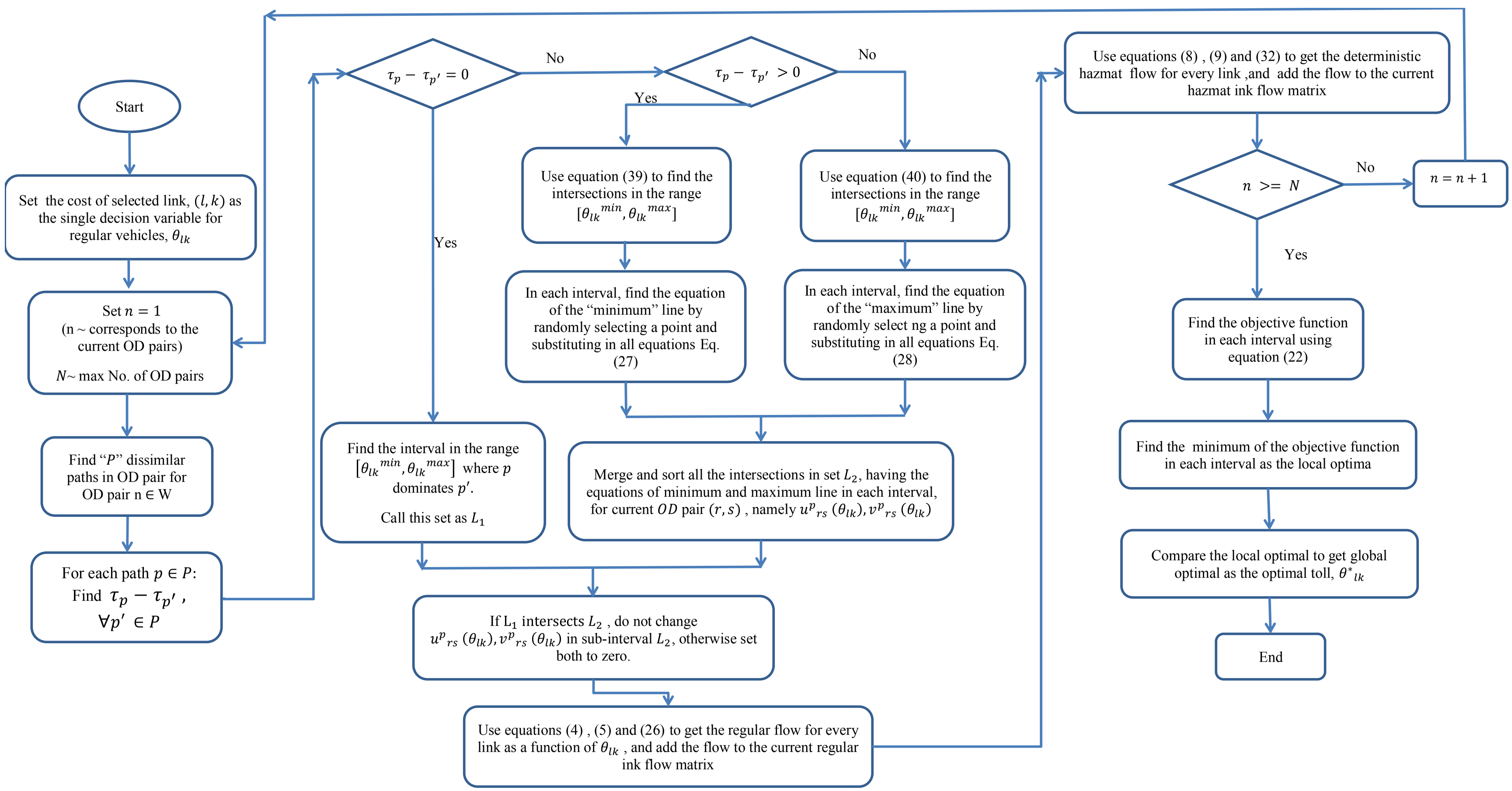

Figure 1: Flowchart of the exact algorithm for the single regular toll decision variable case

Equations (42) and (43) represent two sets of linear functions of a single variable μ_{lk}. Here, the only active constraint of the problem (Q) is the bounding constraint $\mu_{lk}^{\min} \leq \mu_{lk} \leq \mu_{lk}^{\max}$. To obtain $m_{rs}^{p}(\mu_{lk})$, finding the intersections of linear functions of the form equation (42) divides the $[\mu_{lk}^{\min}, \mu_{lk}^{\max}]$ to sub-intervals

$$[\mu_{lk}^{\min}, \mu_{lk,1}^{(1)}), \quad [\mu_{lk,1}^{(1)}, \mu_{lk,2}^{(1)}), \quad \ldots, \quad [\mu_{lk,q-2}^{(1)}, \mu_{lk,q-1}^{(1)}), \quad [\mu_{lk,q-1}^{(1)}, \mu_{lk}^{\max}].$$

On the other hand, for obtaining $n_{rs}^{p}(\mu_{lk})$ in (34), the interval $[\mu_{lk}^{\min}, \mu_{lk}^{\max}]$ is again divided into sub-intervals by finding the intersections of the lines of the form (43) as following:

$$[\mu_{lk}^{\min}, \mu_{lk,1}^{(2)}), \quad [\mu_{lk,1}^{(2)}, \mu_{lk,2}^{(2)}), \quad \ldots, \quad [\mu_{lk,r-2}^{(2)}, \mu_{lk,r-1}^{(2)}), \quad [\mu_{lk,r-1}^{(2)}, \mu_{lk}^{\max}].$$

The set of sub-intervals in which neither $m_{rs}^{p}(\mu_{lk})$ nor $n_{rs}^{p}(\mu_{lk})$ changes is consequently achieved by merging all the intersection points as below:

$$[\mu_{lk}^{\min}, \mu_{lk,1}^{(3)}), \quad [\mu_{lk,1}^{(3)}, \mu_{lk,2}^{(3)}), \quad \ldots, \quad [\mu_{lk,s-2}^{(3)}, \mu_{lk,s-1}^{(3)}), \quad [\mu_{s-1}^{(3)}, \mu_{lk}^{\max}]$$

Note that we constantly keep the minimum and maximum in the range $[a_\lambda, b_\lambda]$ since the VoT of hazmat drivers, λ, follows a uniform distribution in this range. One last thing is to combine and order the intervals obtained for all path $p' \in P_{rs} \setminus \{p\} : \tau_p - \tau_{p'} = 0$ with the sub-intervals and modify $m_{rs}^{p}(\mu_{lk})$ and $n_{rs}^{p}(\mu_{lk})$ similar to the approach described for the regular case. We let the final set of sub-intervals be denoted by

$$[\mu_{lk}^{\min}, \mu_{lk,1}), \quad [\mu_{lk,1}, \mu_{lk,2}), \quad \ldots, \quad [\mu_{lk,t-2}, \mu_{lk,t-1}), \quad [\mu_{t-1}, \mu_{lk}^{\max}]$$

A rather similar procedure with different notation can be applied in the case of an unknown hazmat toll which we call Algorithm $(\mathcal{H})$. The main difference between the two algorithms $(\mathcal{H})$ and $(\mathcal{R})$ is in Step 6. Indeed, in $(\mathcal{H})$, replacing $m_{rs}^{p}(\mu_{lk})$ and $n_{rs}^{p}(\mu_{lk})$ in equation (37) yields at most a linear function of μ_{lk} in every sub-interval. Hence, comparing to single toll case for regular vehicles the problem is even easier in that we need to find the global optimum of a piece-wise linear function. Since, the optimal point of every linear function can be achieved by simply comparing its extreme points, in Algorithm $(\mathcal{H})$ Step 6 is changed to compare all the extreme points of the sub-intervals to obtain the global optimum. This motivates the following proposition that is relevant to Algorithm $(\mathcal{H})$ for a single toll for hazmat trucks:

Proposition 2. Suppose that a *hazmat* toll, $\mu_{lk} \in \mathbb{R}$, is the single variable of interest. Let $g(\mu_{lk})$ represent the objective function of problem (Q) as a function of the single hazmat toll μ_{lk}. Consider the subintervals

$$[\mu_{lk}^{\min}, \mu_{lk,1}), \ [\mu_{lk,1}, \mu_{lk,2}), \ \ldots, \ [\mu_{lk,t-2}, \mu_{lk,t-1}), \ [\mu_{lk,t-1}, \mu_{lk}^{\max}]$$

and denote the objective function in the i-th interval by $g_i(\cdot)$. Then an optimal toll is found by

$$\mu_{lk}^* = \arg \min_{\mu_{lk} \in M_{lk}} g(\mu_{lk}),$$

where $M_{lk} = \{\mu_{lk}^{\min}, \mu_{lk,1}, \mu_{lk,2}, \ldots, \mu_{lk,t-1}, \mu_{lk}^{\max}\}$.

5.2 Cyclic Algorithm and Dissimilar Paths

This subsection addresses a heuristic algorithm for solving the dual toll pricing model with stochastic drivers' preferences as expressed in (37) through (39) for the case of multiple regular or hazmat tolls. A cyclic algorithm, which aims to sequentially solve single toll problems, is proposed.

Before explaining the algorithm, it is worth mentioning that the set of all paths for each O-D pair, i.e., P_{rs} for regular vehicles and Q_{rs} for hazmat trucks, is extremely large for a real-scale network. Considering all of the possible paths is most likely not efficient due to the complexity that they add to the single-decision variable case algorithm. Hence in this situation, instead of considering all paths of an O-D pair, we suggest generating a set of dissimilar paths for regular vehicles and hazmat trucks as P_{rs} and Q_{rs}.

The concept of dissimilar paths is presented to reduce the search space of the path-based models, and prevent generating infeasible or undesirable paths in large capacitated network flow problems. In hazmat transportation problems, generation of dissimilar paths is widely applied to ensure spreading the risk in an equitable way when several hazmat carriers move from a given origin to a given destination (Kang et al., 2014). Moreover, in some cases, the best path choice is not available due to environmental problems such as inclement weather conditions or permits and restrictions for hazmat transport. In this situation, providing appropriate dissimilar alternative paths would be helpful. To obtain spatially different routes, Akgün et al. (2000) generate a large set of candidate paths, and select a subset using a dispersion model which maximizes the minimum dissimilarity. In another approach, Dell'Olmo et al. (2005) find a set of pareto-optimal paths for an O-D pair using a multi-criteria shortest path algorithm. Then, by approximating the impact area of a hazmat accident, they define a dissimilarity index between every pair of paths to derive the most spatially different ones. Dadkar et al. (2008) develop a k-shortest path algorithm to identify a set of routes for the hazmat transportation which trade-off between geographic diversity and performance.

In general, the k-shortest path algorithm finds path choices based on minimum distance/travel time which results in spatially similar paths. This set of paths is undesirable in hazmat transportation since the geographically similar road segments increase the risk induced on the population. On the other hand, restricting the set of available paths to a subset of dissimilar paths possibly forbids a priori to the drivers the selection of their minimum cost paths. Whereas, the objective of the dual toll setting problem is to influence drivers path choices with tolls in order to reduce the risk while leaving them the chance of selecting their desired paths. Nonetheless, in our approach we try to obviate this issue by considering both types of perspectives.

For dissimilar path generation, Kuby et al. (1997) propose the Minimax method which provides a set of dissimilar relatively short paths. The method starts with providing k-shortest paths for an O-D pair using a proper k-shortest path algorithm, e.g., Yen's algorithm (Yen, 1971). In the next step of the method, the k paths are processed and a dissimilar subset (DS) is constructed. For doing so, an index is defined to measure the desirability of a candidate path in DS. If we select the shortest path as the first path in DS, then the second path is selected among all other $k-1$-shortest paths according to two criteria, minimizing the length of the path, and minimizing the similarity with the existing path in DS. This process continues until a desired number of dissimilar paths are selected. We believe that the Minimax method is promising to the objective of the dual toll pricing problem as it obtains the dissimilar paths from the k first desired paths of the drivers.

The index being used is:

$$M(p_i, p_j) = \frac{(1+\beta)d^s(p_i, p_j) + d^n(p_i, p_j)}{d(p_1)},$$

where β is a weight factor used to emphasize the dissimilarity term. For a pair of paths i and j, i.e., p_i and p_j, $d^s(\cdot,\cdot)$ denotes the shared length of the two paths and $d^n(\cdot,\cdot)$ represents the non-shared length. Note that $d(p_1)$ is the length of the shortest path for the O-D pair which we enter to DS initially. The model is equivalent to

$$\min_{j \in \text{DS}} [\max_{i \in \text{DS}} \{M(p_i, p_j)\}].$$

In general, the number of k-shortest paths required to find a good solution with the minimax method depends on the type of the network and the location of the OD pair.

Now, let us introduce the cyclic algorithm to obtain a solution to the dual toll pricing problem with multiple toll decision variables for regular and hazmat commuters.

Cyclic Algorithm (Algorithm ($\mathcal{C}$))

Step 0: Initialization. Let $\overline{A}$ denote the set of tollable links in the network. Order the toll decision variables such that a combined decision vector $\xi = [\mu_1, \theta_1, \mu_2, \theta_2, \ldots, \mu_{|\overline{A}|}, \theta_{|\overline{A}|}]^\top$ is constructed. Choose an initial feasible toll vector, e.g., $\xi^0 = 0$. Set the cycle counter to $k = 1$.

Step 1: Reset. Set the iteration index; $n = 1$.

Step 2: Solve a single-variable problem. Set ξ_n^k as the decision variable, and solve the corresponding single-decision variable sub problem using either Algorithm ($\mathcal{R}$) or ($\mathcal{H}$) to find the optimal value for ξ_n^k, given all other toll values as $\xi_m^k = \xi_m^k$ for all $m < n$ and $\xi_m^k = \xi_m^{k-1}$ for all $m > n$.

Step 3: Check if the cycle k is complete. If $n = 2|A|$, go to Step 4; otherwise set $n = n + 1$ and go to Step 2.

Step 4: Convergence check. If $||\xi^k - \xi^{k-1}|| < \varepsilon$ for a preset tolerance $\varepsilon > 0$, stop; otherwise, set $k = k + 1$, and go to Step 1 in order to run more cycles.

Every cycle of Algorithm ($\mathcal{C}$) consists of $2|\overline{A}|$ iterations with $|\overline{A}|$ denoting the number of tollable links. In each iteration either a regular toll or a hazmat toll is being optimized while the other tolls are set to their most updated values. Since, the decision variables are ordered by link, in every two iterations of a cycle, the optimal values of the regular and hazmat tolls associated with a link are being identified, which is followed by another two iterations for the next link. Also, as we aim to consider the impact of both regular and hazmat flow distributions in the network, it is reasonable to set one regular toll followed by one hazmat toll. This approach helps the model sense the presence of hazmat trucks and capture their impact on the objective function value.

6 Computational Experiments

By performing experiments we seek to explore computational efficiency and convergence of the algorithm, impact of the dual toll pricing versus single toll pricing, and the impact of the number of the tollable links on risk mitigation.

There are many established test problems in the transportation literature. We have selected the Sioux Falls network shown in Figure 2 for our testing. Sioux Falls data including start nodes, end nodes, trip table (i.e. Origin-Destination pairs and O-D demands) and link free flow travel time are available on the Transportation Network Test Problems website (Bar-Gera, 2013). Using the Census site information, we estimated link population and link distance data. In addition, we randomly generated lower and upper limits for link toll for both regular vehicles and hazmat trucks. The Sioux Falls network contains 24 nodes and 76 links assuming that all nodes can be either origin or destination. Matlab version 2012a was used for coding the algorithms. We performed all the experiments on an Intel (R) Processor 2.99 Ghz computer.

In order to study the computational efficiency and convergence of the proposed heuristic algorithm, we ran Algorithm ($\mathcal{C}$) for the Sioux Falls network for different combinations of network characteristics, namely number of O-D pairs, number of the k-shortest paths, number of the p-dissimilar paths and number of tollable links for regular vehicles and hazmat trucks.

Table 3 summarizes the results. For all the experimentations, we assume $\pi^r_{ij} = 10^{-6}$, equivalent for all links (i, j). Also, we let $\omega_1 = \omega_2 = 0.5$ to equally account for direct and indirect consequences of an accident, and set $\epsilon = 0.01$ in the tie breaking term. As the stopping criteria we set the maximum percentage of change of all toll values to be no more than 5%, i.e., $\varepsilon = 0.05$, where

$$\frac{\xi^k_m - \xi^{k-1}_m}{\xi^{k-1}_m} \times 100 < 0.05$$

denotes the percent change in toll value of link m from cycle $k-1$ to k.

The run time values in Table 3 illustrate the time required until the convergence of the cyclic algorithm in an increasing order. One observes that the computational effort is highly affected by the number of O-D pairs, number of the k-shortest paths, number of the p-dissimilar paths, as well as the number of tollable links being considered. The first row in Table 3 shows our initial

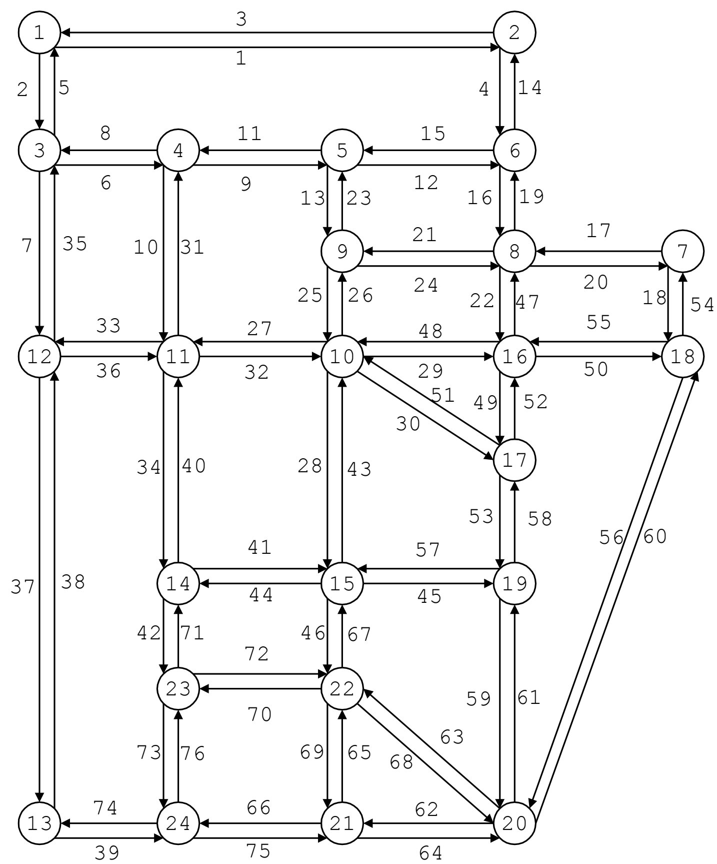

Figure 2: The network of Sioux Falls, SD used for the experiments

setting for the single O-D pair problem. From rows 1 through 6 we can see that the computational time grows with the number of O-D pairs. More specifically, rows 4 and 5 clearly show the effect of considering more O-D pairs on the computational time for fixed number of k-shortest paths, p dissimilar paths, and tollable links considered. This behavior is obviously due to increasing number of paths being compared in steps 2 and 4 of the single-decision variable algorithm, i.e., $(\mathcal{R})$ or $(\mathcal{H})$. By comparing rows 2 and 3 we can conclude that adding the number of tollable links increases the computational time as more single-decision variable algorithms are performed in each cycle in order to identify the unknown toll values. Also, rows 3 and 4 illustrate that the run time increases when enlarging the set of k-shortest paths and p-dissimilar paths with the same reasoning as for the number of O-D pairs. One primary and solvable reason behind large computation times is coding in Matlab, which is selected for the sake of its convenience and graphical representation. We believe that redoing the algorithms in another programming language such as C++ or Java and applying code optimization techniques would significantly improve its performance.

We now study the impact of dual toll pricing versus single toll pricing on the risk mitigation. A single toll pricing policy considers only hazmat toll pricing. In this case, equation (3) for regular

No.	# of OD pairs	# of k-shortest paths	# of p-dissimilar paths	# of tollable links -regular vehicles	# of tollable links -hazmat trucks	Run time
1	1	10	8	10	7	7min 50sec
2	5	10	10	10	7	42min 39sec
3	5	10	10	13	9	55min 20sec
4	5	20	15	10	7	1hr 31min 42sec
5	10	20	15	10	7	2hr 36min 12sec
6	24	10	8	10	7	3hr 7min 4sec

Table 3: Cyclic algorithm performance under different test problems from the Sioux Falls network

No.	# of tollable links-hazmat	Initial toll	Initial risk	Optimal toll	Optimal risk	Run time	# of cycles	Avg.# of sub-intervals
1	1	μ=(0)	8.740E4	μ^*=(10)	5.738E4	19min 12sec	2	41
2	2	μ=(10,0)	5.738E4	μ^*=(12,12)	5.683E4	1hr 8min 28sec	3	79
3	4	μ=(12,12,0,0)	5.683E4	μ^*=(12,10,9.6,0)	4.626E4	4hr 24min 23sec	3	124
4	8	μ=(12,10,9.6,0,0,0,0,0)	4.626E4	μ^*=(8,0,8,13,8,0,0,0)	4.519E4	8hr 7min 15sec	3	541

Table 4: Risk value for single toll pricing under different number of tollable links

drivers reduces to the following

$$\begin{aligned}\phi^p_{rs}(\alpha_p) &= \Pr[\gamma\tau_p + \alpha_p \le \gamma\tau_{p'} + \alpha_{p'} \qquad \forall p' \in P_{rs} \setminus \{p\}] \qquad \forall p \in P_{rs} \qquad \forall (r,s) \in W \\ &= \Pr[\tau_p \le \tau_{p'} \qquad \forall p' \in P_{rs} \setminus \{p\}] \qquad \forall p \in P_{rs} \qquad \forall (r,s) \in W,\end{aligned}$$

which implies that regular drivers simply take the shortest path towards their destination. Note that the above equation considers a toll-free network for the regular vehicles, i.e. $\alpha_p = 0, \forall p \in P$. However, when regular tolls are given constants in the network, a similar calculation can be applied towards minimizing the total travel cost of regular drivers. We perform our experiments on the Sioux Falls network with 150 O-D pairs for regular vehicles, and 10 O-D pairs for hazmat. For each O-D pair, 20 shortest paths and 15 dissimilar paths have been generated. Every test problem is different than others in the number of tollable links being identified which varies between 1, 2, 4, and 8. The tollable links are $(7,18)$, $(21,20)$, $(19,20)$, $(2,6)$, $(6,8)$, $(7,8)$, $(10,15)$, and $(10,16)$. The performance criterion is the total risk value associated with the network.

We start with solving the single toll pricing problem having only one tollable link for hazmat. We consider the initial solution for this toll to be zero. The solution of the current problem is an exact optimal solution based on algorithm $(\mathcal{H})$. This solution which is the most updated toll value for the first tollable link is then used as the initial toll setting in algorithm $(\mathcal{C})$ for solving the next problem having two tollable links. Note that we set the initial toll value of the second unknown link to zero. We repeat a similar procedure for the test problems having 4 and 8 tollable links. Table 4 shows the results.

One can realize that, with more number of tollable links, the regulator has more control over the network to influence the drivers path choices towards minimizing the overall risk. This has been shown in Table 4 by constantly decreasing risk values as we increase the number of tollable links. Although such reduction in the risk most likely requires redirecting the drivers from their

No.	# of tollable links -hazmat & regular	Initial toll	Initial risk	Optimal toll	Optimal risk	Run time	# of cycles	Avg. # of sub-intervals
1	1	μ=(10) θ=(0)	5.738E4	μ^*=(12) θ^*=(4.5112)	5.478E4	36min 2sec	2	40
2	2	μ=(12,0) θ=(4.5112,0)	5.478E4	μ^*=(12,0) θ^*=(8,0)	5.088E4	1hr 5min 16sec	2	77
3	4	μ=(12,0,0,0) θ=(8,0,0,0)	5.088E4	μ^*=(10,0,10,12.8571) θ^*=(8,0,4,8)	4.485E4	5hr 12min 3sec	2	323
4	8	μ=(10,0,10,12.8571,0,0,0,0) θ=(8,0,4,8,0,0,0,0)	4.485E4	μ^*=(10,0,10,10,10,0,0,0) θ^*=(8,0,4,8,4.036,0,6,0)	3.942E4	19hr 27min 38sec	3	683

Table 5: Risk value for dual toll pricing under different number of tollable links

original shortest paths, it retains flexibility in hazmat drivers' route selection according to their own preferences. In conducting the experiments, we note that to ensure the regulator can effectively deviate the drivers, the tollable links must exist on the paths of the dissimilar path set. Otherwise, such toll setting policy obviously cannot impact the risk mitigation target. Indeed, in such cases, the drivers simply choose their shortest path which in contrast might lead to higher congestion and risk in certain segments. Whereas properly selecting the tollable links, such as the populated and congested links, can help achieving higher improvements in risk as the drivers route themselves to less populated and less congested areas towards minimizing their travel cost. In Figure 3, dark shaded bars represent the risk associated with the single toll pricing which displays the decreasing trend in risk value when increasing the number of tollable links.

In order to demonstrate the complexity of the calculations, we also report the number of cycles required until the converge of algorithm ($\mathcal{C}$), as well as the average number of the sub-intervals generated during the single-decision variable algorithm for obtaining each toll. We observe that another reason behind the excessive run times of the algorithm is probably due to the relatively high number of such sub-intervals. Indeed, the most time consuming part is obtaining the sub-intervals by pairwise comparison of all the alternative paths. To increase the efficiency, we suggest to reduce the set of paths by considering only the efficient ones. That is, to solely compare pairs of efficient paths with no alternative being both quicker and cheaper (Leurent, 1993). This will not only eliminate the unnecessary calculation for obtaining the critical sub-intervals but also significantly improve the computation time required in steps 2, 4 and 5 of the single decision variable algorithm.

To explore the impact of dual toll pricing, we perform our proposed cyclic algorithm on the Sioux Falls network with the same number of O-D pairs, k-shortest paths and p-dissimilar paths. In the first problem, we are interested in setting dual tolls for one link. As the initial value of the hazmat toll, we consider the solution of the counterpart problem in the single toll pricing case, i.e., Table 4, while we set the initial regular toll as zero. Based upon the same set of tollable links in the network, we summarize the results in Table 5. The familiar non-increasing behavior of the risk value is observed when we increase the number of tollable links. This trend is shown in Figure 3 by light shaded bars.

Another major conclusion of the experiments is the effect of dual toll pricing in comparison with single toll pricing on the overall risk value of the network. As it is shown in Figure 3, although both

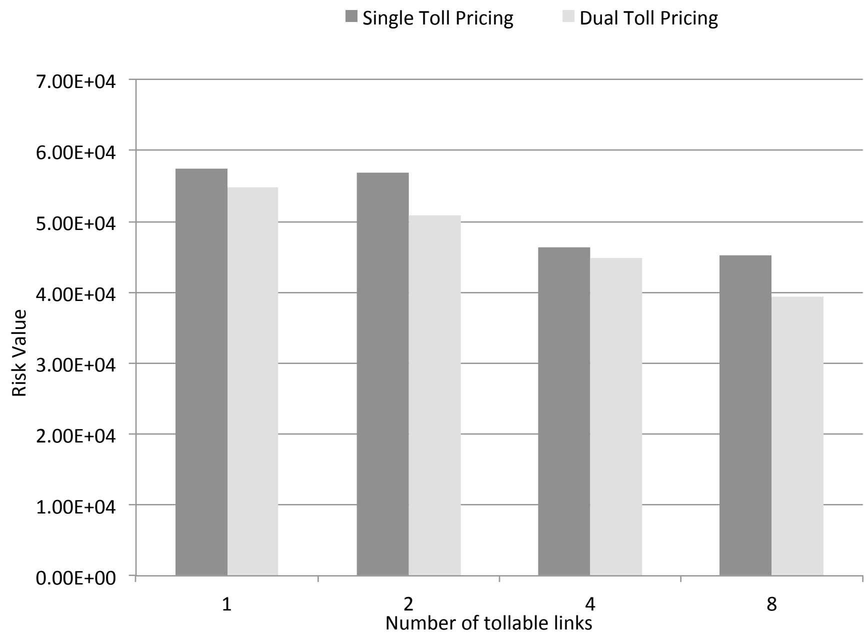

Figure 3: Risk mitigation performance of single toll pricing versus dual toll pricing under different number of tollable links

single and dual toll setting can efficiently mitigate the risk, for every point of the two graphs the risk of network in the dual toll case is less than that of the single toll case. This illustrates the benefit of dual toll pricing in the context of hazmat transportation. Indeed, with dual toll pricing hazmat drivers and regular vehicles more effectively route themselves to less risky areas of the network while they minimize their travel cost. Since this selection also includes less congested segments of the network, both regular and hazmat drivers choose their respective routes in a manner that tends to spatially separate their traffic.

7 Summary, Conclusions and Future Research

This paper studies dual toll pricing as a tool to route hazmat trucks and regular vehicles, so as to control risk while offering routing flexibility to both drivers of hazmat trucks and regular vehicles. The route's perceived cost by the drivers has two parts, namely the travel time and the cost. With the stochastic value of time as the coefficient of the first part, we model the drivers' preferences as a probabilistic function. This framework allows us to consider different patterns in drivers' behavior toward route selection. We formulate the problem as a mathematical optimization model. Then, a change in the decision variables is introduced to solve the proposed model. In terms of solution methodology, we first develop an exact method for the single toll case, where we are allowed to change the toll price in only one link. A cyclic algorithm, which iteratively solves single toll problems until convergence, is applied as a heuristic solution technique for the case with multiple tolls. Finally, we test our algorithm on a well-known Sioux Falls transportation network to study the efficiency of the model and the impact of the dual toll pricing.

Our computational experiments on various test problems with different numbers of O-D pairs, k-shortest paths, p-dissimilar paths and tollable links demonstrate that all these features have an increasing impact on the run time. Although we postulate that the time constraint can be improved if we apply other programming languages such as C++ with code optimization techniques, the time frame proposed by our algorithm for solving the problem is still reasonable, since we are concerned with strategic decision making (i.e. toll setting) that usually requires only a few problem solving for long-term planning.

From the second part of our experiments, we conclude that as we impose tolls on more road segments the total risk value decreases. Increasing the number of tollable links amplifies the ability of the regulator to influence the drivers' path choice. Without considering the cost associated with toll booths, toll pricing can obviously ensure safer hazmat transportation.

We also studied the effect of dual toll pricing versus single toll pricing. Indeed, while both policies show a non-increasing trend in the risk value when we impose tolls on more road segments, the risk value associated with dual toll pricing is less in all problem instances when compared to single toll pricing. This implies that setting tolls for both regular and hazmat vehicles is more effective than just hazmat toll pricing. Dual toll pricing not only provides flexible solutions for network regulators but also suggests acceptable choices to both types of carriers such that they are motivated to use divergent routes. In addition, the excess revenue of such policy prepares a good financial resource, which can be applied in construction and maintenance of the network.

Although toll pricing has been a popular traffic control tool to offer flexible alternatives to network users, the recently introduced concept of dual toll pricing is a relatively unexplored area in hazmat routing. There exists plenty of room to expand the problem in terms of the efficiency of the solution algorithm and applicability of the proposed model in real world problems. Our current model does not consider congestion and flow dependent link travel times. Thus, an important direction for future work is to consider the impact of regular traffic flow on the path's travel cost function while they distribute in the network. Indeed, we can ignore the effect of hazmat flow due to its low volume. In addition, while this paper assumes uniformly distributed value of time, from the flow distribution model, we believe that our approach could be useful in addressing other distributions as well. This can be achieved by employing their corresponding cumulative distribution functions. Hence, another future research opportunity is to expand our solution methodology for a general distribution of the value of time.

Despite the fact that imposing dual toll for a fixed period of time ensures safer hazmat transportation, another extension of the problem is to study dynamic dual toll pricing which allows the regulator to set dual tolls in a time-sensitive basis. It is recommended that time dependent dual tolls are studied to better capture network conditions and provide both network users and regulators with more acceptable routing alternatives.

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