Store Layout Using Location Modelling To Increase Purchases

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Abstract. This paper explores a new application for the well-known p-dispersion model from location theory to optimize the placement of items in a retail store setting. Specifically, it focuses on designing a store layout where item placement is done with the objective of maximizing the total profit earned from the sale of impulse items. It begins by presenting a simple version with a grid layout and rectilinear distance metric that is solved to optimality. Then, a two stage heuristic algorithm is developed for the general problem that employs a simulation analysis to verify the quality of the solutions generated. The performance of this two-phase algorithm is empirically tested using three different approaches: benchmarking against available results, including in the practitioner literature, empirical testing and finally, with real-world data taken from a grocery store in the western region of New York. Results attest to the effectiveness of the solutions generated by algorithm and its ability to solve larger problems than have been reported heretofore in the literature.

Keywords: Facility Layout; Store Layout, p-Dispersion; Simulation; Simulated Annealing

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1. Introduction

1.1 Motivation

especially in commonplace Purchases undertaken by customers, settings such as grocery/convenience/department stores, can be categorized as "planned" (hereafter referred to as "musthave" purchases) or "unplanned" purchase (hereafter referred to as "impulse" purchases). Per the etxtant literature in consumer behavior, while a planned purchase is characterized by deliberate, thoughtful search and evaluation that normally results in rational, accurate and better decisions (Gutierrez, 2004), impulse buying of unplanned items results from spontaneous buying stimuli (Rook and Fisher, 1995), prompted by physical proximity to desired product (Hoch and Loewenstein, 1991). In fact, as empirically validated in Hui et al. (2013), sales of impulse items increases with the exposure of customers to them. Further, studies show that almost 90 percent of people make purchases on impulse occasionally (Welles, 1986) and between 30-50 percent of all purchases are classified by the buyers themselves as impulse purchases (Bellenger et al., 1978; Cobb and Hoyer, 1986; Kollat and Willett, 1967; Shapiro, 2001; Clifford, 2006; Zhang et al. 2009, 2010; Supriya and Arora, 2010). The importance of impulse purchases is further highlighted by the fact that the sale of planned items is predetermined as is the revenue thereof and in addition to the deterministic nature of these sales, in settings such as grocery/convenience/department stores, most "musthave" items are basic commodities such as milk, bread, meats etc. that have lower profit margins. By contrast, the sale of impulse items can be influenced by the store and therefore, the marginal revenue accrued by the store depends strongly on the sales of these unplanned items. All of the above underscores the importance of strategically maximizing the sales of impulse items in a store as an effective way to increase revenue.

With regards to increasing store revenue, marketing and consumer behavior literature clearly shows that strategically placing items can have a significant positive impact on sales of both must have and impulse items (Lewison, 1996; Ghosh, 1994; Borin et. al., 1994, Levy & Weitz, 2006, Dalwadi et al. (2010), Jacobs et al. (2010)) even when the stores are virtual (Manganari et. al. (2011)). This strategic placement of items is often referred to in the literature as store layout optimization and much work has been done on this subject – see for example Borin et.al. (1994), Sharma and Baan-Hoffman (2008) and Bruzzone and Longo (2009). However, to the best of our knowledge, only two recent works (Li (2010) and Ozgormus (2015)) have

considered the use of classical model(s) from the Facilities Location literature to "optimally locate" products in the store so as to increase the likelihood of their purchase.

Using an approach similar to those of Li (2010) and Ozgormus (2015), we adapt a well-studied location model (*p*-dispersion Model) to maximize the sale of impulse items in a store and develop herein an algorithm for placement of must have items that is aimed at doing the same. Our algorithm is based on the common consumer behavior assumption in the literature cited above that when a customer visits a store, they purchase a basket of items that contains a predetermined (planned) item-list of must-have items and are maybe inclined to also buy impulse (unplanned) items but only if the customer passes by them during his/her visit to the store. As verified in Hui et. al. (2013), the more such impulse items that customers are exposed to while purchasing their must-have items, the greater will be the sales of these impulse items; thus, the total sales of impulse items depends on the total exposure of these items to all the customers visiting the store. Coupled then with our second assumption that the impulse items are located along the pathways between the must-have items, the corollary is that in order to maximize the sales of impulse items, it is necessary to "disperse" the must-have items as widely as possible within the store, necessitating customers to maximize the amount of distance traveled within the store. That leads to maximizing the exposure of the customers to the impulse items (which are assumed to be displayed between the must-have items) and hence, their sales; that, in turn, is the essence of our algorithm.

With regards to the travel patterns by customers, our algorithm assumes that each customer plans his/her route in the store using a nearest neighbor approach on his/her list of must-have items and their current location. However, we do not assume that the customer's path is deterministically known. Rather, we assume that given the current location of the customer, the likelihood that s/he selects a particular must-have item to visit next is inversely proportional to its distance from her/his current location and ties are broken arbitrarily. Finally, given that the sale of must-have items is predetermined, we define the "value" of a given store layout as the total profit earned from the sales of impulse items.

The algorithm presented is a two-stage heuristic. In the first stage, the algorithm spreads the must-have items by using the *p*-dispersion model in location theory. In doing so, we assume that the customers can be grouped into a finite number of categories such that all customers within the same category have the same list of must-have items; the algorithm then disperses all must-have items in a manner that is aimed at maximizing the overall travel by all the customers in purchasing them. In the second stage, it improves the first-stage solution by utilizing a simulated annealing based metaheuristic. Simulation analysis is used to compute the effectiveness of the solution generated in generating store revenue. We test the performance of our algorithm in different ways. First, extensive benchmarking and performance analysis is done including comparisons with data available in the academic as well as practitioner literature. Second, the running time of the algorithm is studied for different problems sizes as is the error bound of the solutions produced. Thereafter, we also perform an empirical analysis of how the algorithm's performance changes based on randomly generated input parameters followed by a case study application of the algorithm to improving the store layout of a grocery store in Western New York. Our empirical analysis reveals that the p-dispersion method develops a good initial solution and the simulated annealing metaheuristic significantly improves the initial solution. More importantly, the benchmarking and performance analysis provides evidence about the superior effectiveness of the algorithm as well as its ability to solve larger problems than have been reported heretofore in the literature.

1.2 Literature Review

Extant academic literature in marketing and consumer behavior provides ample evidence that selling floor layouts strongly influence the in-store traffic patterns, shopping behavior, shopping atmosphere and operational efficiency (Lewison, 1996; Ghosh, 1994; Levy & Weitz, 2006, Dalwadi et al. (2010), Jacobs et al. (2010)) even when the stores are virtual (Manganari et. al. (2011)). This has also attracted a lot of interest among practitioners about optimizing retail store layout design; Welles (1986), Economist (2008), Merchandising Matters (2013), Michalowicz (2015) are three well-cited instances among many from the practitioner literature. Work on this topic identifies three primary objectives in designing the layout of a store: to guide the customer around the store and entice increased purchases; creating a balance between sales and shopping space and finally, creating an effective platform for merchandise presentation. The literature also identifies three major types of store layouts. The "Grid" layout is a rectangular arrangement of displays and long aisles that generally run parallel to one another (Vrechopoulos et al., 2004). It provides customers with flexibility and speed in identifying preselected items which appear on their shopping list. (Lewison, 1996; Levy & Weitz, 2006). The second is the "Freeform Layout" that is a free flowing and asymmetric arrangement of displays and aisles, employing a variety of different sizes, shapes and styles of display. It is mainly used by large department stores (Vrechopoulos et al., 2004). The freeform layout has been shown to increase the time that customers are willing to spend in the store. (Lewison, 1996; Levy & Weitz, 2006). Finally, the "Racetrack/Boutique Layout" is one where the sales floor is organized into individual, semi-separate areas, each built around a particular shopping theme. It leads customer along the specific paths to visit as many store sections of the departments as possible, because the main aisle/corridor facilitates customer movement through the store (Vrechopoulos et al., 2004). More recently, researchers have used models from facilities design to optimize store layouts - Li (2010) and Ozgormus (2015) are two recent dissertations devoted to this subject.

With regards to purchase of items, as stated before, impulse buying of unplanned items results from spontaneous buying stimuli (Rook and Fisher, 1995), prompted by physical proximity to desired product (Hoch and Loewenstein, 1991). Beyond spontaneity, impulse buying is also characterized as an unexpected urge to buy without regard to the consequences of the purchase decision (Rook, 1987). Stern (1962) further categorizes impulse buying behavior as pure impulse buying, reminder impulse buying, suggestion impulse buying and finally planned impulse buying. Finally, Mohan (2013) and Koo and Kim (2013) illustrate the importance of store environment, including layout, in influencing such impulse purchases.

Another strand of research relevant to our problem has studied the travel paths undertaken by customers in making their purchases. Farley and Ring (1966) developed a model to predict area-to-area transition probabilities for traffic in supermarkets and proposed a stochastic model of supermarket traffic flow that provides a framework for predicting conditional probabilities of shopper's traffic flow. Burke (1996) studied consumer grocery shopping patterns using a virtual (simulated) store. Sorensen (2003) tabulated purchase and time-of-stay statistics at different locations within an actual grocery store and Larson et al. (2005) categorized grocery paths using a clustering algorithm, and identified 14 different "canonical paths". As we have stated above, we assume in our algorithm that a customer plans his/her route in the store using a nearest neighbor approach on his/her list of must-have items and their current location and the likelihood that s/he selects a particular must-have item to visit next is inversely proportional to its distance from her/his current location with ties being broken arbitrarily.

The remaining paper is organized as follows. The next section motivates the general problem by considering a simple version and solving it optimally followed by the third section that develops the model

for the general version of the problem. Section 4 then presents a two-phase algorithm for the general problem and is followed by the next section where this algorithm is benchmarked against published results and empirically studied using randomly generated data. The sixth section then tests the algorithm in a real-world application and the seventh and final section concludes by summarizing the paper, its primary findings and suggesting future research on the subject.

2. Rectilinear Version of the Item Placement Problem

In order to motivate the general problem this section examines a simple version of the same and solves it to optimality. This simple version assumes that the store has a grid layout with identical and impenetrable parallel shelves on whom the items are placed. As common in facility layout models involving grid patterns, we also assume that the customers define distance using the rectilinear metric². While definitely a simplistic version of the general problem, it is reflective of how many stores are laid out with parallel aisles. We also assume that there is only one customer type and that the K must-have items that are common to all customers are denoted by 1, 2... k, ...,K; in other words, every customer purchases all K of these items. As explained in the introduction section, the fundamental behavioral premise of our approach is that the more a customer is exposed to impulse items while traveling inside the store to buy the must-have items, the more s/he will be exposed to these impulse items and hence, the more the likelihood of their purchase. As a result of this assumption, we can formulate the placement of must-have items as the following: find locations of the K must-have items on the parallel shelves so as to maximize the distance that the customer has to travel in purchasing them and place the impulse items along this longest path. Finally, we assume each customer will deterministically take the shortest path to travel between any pair of items, where distance is defined by the rectilinear metric in the presence of barriers represented by the shelves themselves.

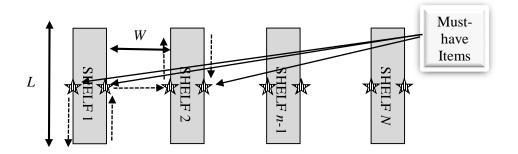


Figure 1: An Optimal Location of Must-Have Items in Grid Layout and Rectilinear Metric

Let the shelves in the store be designated by 1,2,..,N and assumed to be numbered from left to right as shown in Figure 1 with a distance of *W* between adjacent shelves and assume that the store entrance is immediately to the left of the first shelf, implying that is where all customer travel begins in the store. Further, assume that each of these impenetrable shelves is of length *L* and negligible width, as shown in the figure and that K=2N, implying that there are twice as many must-have items as shelves. All items are located on the boundary of these *N* impenetrable rectangular shelves. Given that customer travel is performed using rectilinear metric in the presence of barriers, it is easy to show that when a customer travels

² The rectilinear distance between two points $X_1 = (x_1, y_1)$ and $X_2 = (x_2, y_2)$ is $|x_1-x_2| + |y_1-y_2|$.

between adjacent must-have items on the same shelf (say first two items from left located on Shelf 1 in Figure 1), the shortest path is the travel along the boundary of the shelf itself – an example of this is illustrated by dashed lines in the figure). It also readily follows that regardless of the path taken by any customer, travelling between the *N* shelves to purchase the 2*N* must-have items necessitates any customer to travel a minimum total inter-shelf distance of W((K-2)/2). Thus, any attempt to maximize a path taken by any customer should focus solely on maximizing the distance travelled by him/her in picking adjacent items located on either side of the same shelf (e.g., the distance between the first two items located on shelf 1 in Figure 1). It is easy to show that one configuration in which this is achieved is when both items are located at the mid-point of each shelf, one on either side, as shown in Figure 1. The impulse items are then placed on each shelf between the must-have items to expose them to the customers and the total distance travelled by a customer is W((K-2)/2) + KL. This is summarized as follows and illustrated in Figure 1.

Observation 1. Given rectangular impenetrable shelves with negligible widths, twice as many must-have items as shelves and rectilinear distances, it may be assumed that an optimal placement of the must-have items that maximizes the total distance travelled by a customer to W((K-2)/2) + KL by placing them at the midpoint of each side of every shelf. Placing the impulse items between the must-have items then maximizes their exposure to the customers.

3. Model Formulation For The General Problem

This section will address the general version of the problem in this paper that seeks to optimally place musthave items so that even when customers probabilistically select nearest-neighbor paths in purchasing them, their travel inside the store is maximized. Placing the impulse items along this longest path then maximizes their exposure to them and thus, the probability of making impulse purchases. It is helpful to note here that each item is essentially what a retail store would consider a unique SKU (Stock Keeping Unit). We assume that customers are heterogeneous in that a customer's must-have item list depends on his/her individual needs and thus, there exist different categories of customers with different sets of must-have items. Specifically, we assume that there are K categories of customers, indexed by k = 1, ..., K and let $\theta_K =$ 1,..., K define the set of must-have items for customer category k. Furthermore, let $\beta_k = 1,...k$ define the set of impulse items for customer category k and l_{ik} be the number of impulse items of type i purchased by customer type k if they were to pass by this item type in their visit to the store. Finally, we let C_i denote the marginal profit earned from a unit sale of item *i*, where this profit is assumed to be given as the difference between unit sale price and the unit variable cost for this item. An inherent simplifying assumption in this is that the store has information on all marginal operational costs related to a given item, such as replenishment costs or holding costs and that these costs can be independently and accurately ascribed to each item.

Let Δ be the set of all possible layouts indexed by *j* and with elements denoted by L_j . That is L_j denotes a specific layout or arrangement of items in the store. We let $\delta_{kj} \subseteq \beta_k$ denote the set of impulse items customer type *k* passes while travelling in layout L_j and picking up his/her must-have items. Then the value of layout L_j is given by:

$$V(j) = \sum_{k=1}^{K} \sum_{i \in \delta_{kj}} C_i \ell_{i,k}$$
⁽¹⁾

Therefore our problem can be formulated as:

$$(P) \qquad \frac{Max \quad V(j)}{L_i \in \Delta}$$

$$(2)$$

Due to the combinatorially large number of elements in Δ , the problem (P) above is difficult to solve. We therefore proceed by developing a two-stage heuristic which relies on the evaluation of V(j) for a specific layout $L_j \in \Delta$. The data needed for evaluating this is as follows: the C_i be the marginal profit earned from a unit sale of item *i*; *K*, the number of customer categories; the set of must-have items for a given customer category and; set of impulse items for a given customer category. As mentioned before, we assume that (a) the customer is inclined to buy impulse items if s/he passes by the items, (b) when customers travel between must-have items, they select the next must-have item based on the distance from their current location – the smaller the distance the greater the chance of selecting that particular must-have item, with ties broken arbitrarily and (c) the distances are symmetric, i.e. the distance from location "A" to "B" is same as from "B" to "A".

In order to create the store layout and in-store travel pattern, we develop a graph representation of the store layout, with nodes corresponding to the center of item storage department areas and arcs corresponding to connections between the nodes. We do not assume that the customer's path between must-have items is deterministically known. Rather, we assume that given the current location of the customer, the likelihood that s/he selects a particular must-have item to visit next is inversely proportional to its distance from her/his current location and ties are broken arbitrarily. Given this probabilistic nature of the paths taken by the different customers, computing the value of a store layout is a nontrivial problem. In order to address the same, we developed a simulation code in VBA (Visual Basic) to evaluate the value of a layout. In that simulation, for a given store layout, the customers are programmed as independent entities that move around the store to pick up their must-have items and impulse items in accordance with the probabilistic choice rule described above in selecting the sequence of items to purchase and the paths that they take to do the same.

The above is best illustrated with an example and hence, consider the case of grocery store with the 30 item store layout shown in Figure 2. The locations and shortest distances are given by the shortest distance matrix in Table 1. The cost of items is given in Table 2. Let there be a single customer category (i.e. K = 1) and must-have and impulse items belonging to this category be $\theta_1 = \{I-4, I-9, I-15\}$ and $\beta_1 = \{I-2, I-19, I-30\}$. Based on the distance from the entrance that the first must-have item selected is: I-4 (36%), I-9 (42%), and I-15 (22%). If the customer goes to I-9 first, based on the distance measurement from I-9 that the second must-have item selected is: I-4 (35%), and I-15(65%). We present the path as Figure 3. The only impulse item visited on this path is item I-2. The value of the layout using equation (1) is therefore given by:

$$V = \sum_{k=1}^{K} \sum_{i \in \delta_k} C_i \,\ell_{i,k} = 3.59 \times 1 = \$3.59$$

4 Solution Methodology: A Two-Phase Algorithm

In this section, we present the details of our solution method to improve the impulse item revenues of the grocery store. The method we employ is carried out in two phases. In the first phase we apply a heuristic based on the *p*-dispersion model in location theory and then in the second phase we implement a simulated

annealing approach to improve upon the solution from the first phase. Moreover, we build a simulation program to verify our solution method.

The *p*-dispersion problem is known to be an NP-hard problem whose objective is to disperse the entities as far as possible in a given space. It is defined as selecting "*p*" out of "*n*" given points $(1 \le p \le n)$ in some space, where the objective is to maximize the minimum distance between any two of the selected points (Erkut, 1990). To maximally disperse the "*p*" points, maximally separated locations can be obtained by applying the greedy deletion heuristic developed by Erkut et al. (1994). This heuristic starts with a solution that contains all points and eliminates one point at each iteration, until "*p*" points are left. The point to be eliminated is one of the two closest points in the current solution. Among those two points, the one that is closest to the remaining points in the solution is eliminated. This is implemented in the first phase of our solution methodology per the following algorithm.

- 1. Find the common must-have items across all K customer categories. Let there be p1 such items. Use the p-dispersion heuristic with p = p1 to locate these items. Fix p1 must-have items at maximally dispersed locations.
- 2. Find the common must-have items across *K*-1 customer categories. Let there be p2 such items. Use the *p*-dispersion heuristic with p = p2 to locate these items. Fix p2 must-have items at maximally dispersed locations.
- 3. Repeat until common must-have" items across K/2 customer categories are located using a *p*-dispersion algorithm.

Once the "must-have" items are located, we derive the path taken by the customer to visit these must-have items on his/her list and obtain the resulting value of the layout using the simulation described before.

In the second phase of our algorithm, simulated annealing starts with the layout obtained from the p-dispersion based heuristic. The four basic ingredients required to solve simulated annealing are: a concise description of the configuration of the system; a random generator of "moves" or rearrangements of elements in a configuration; a quantitative objective function containing the trade-offs that have to be made and an annealing schedule of temperatures, and; length of time for which the system is to be evolved (Kirkpatrick et al., 1983). Given a particular layout and its associated value, as determined by the simulation, the simulated annealing algorithm requires a systematic method for choosing another neighboring solution in the next iteration. For our problem, a neighboring solution is generated by randomly choosing two items from two different candidate sets and interchanging their locations. The two sets from which items to be exchanged are selected from are: (i) must-have candidate set of m' must-have items that have not been moved from their original location and (ii) an Interchange candidate set comprising of (n-m) items other than must-have items that have not been moved from initial location.

At each iteration, a prospective solution is generated by defining a "pair-wise" exchange of must-have item with a neighbor. If the prospective solution's objective value is better than that of the best solution, then the prospective solution is saved as the best solution and becomes the current solution. If the prospective solution is not better than the best solution, then it may become the current solution with an acceptance probability determined by temperature parameter (Reeves, 1993).

The probability of acceptance of an inferior solution, denoted by P(acceptance), at each iteration of the simulated annealing algorithm is given by:

Where,

$$P (\text{acceptance}) = \exp[V(j) - V(j_0)]/t$$
(3)

V(j) = objective value of the prospective layout j $V(j_0)$ = objective value of the current layout j_0 and t = temperature.

The temperature is held fixed for each loop. At the end of each loop the temperature is dropped according to the rule:

where;

 $t = \mu[(m' - m_i)/m']$

 μ = average value of items, m' = total number of items in must-have candidate set and, m_i = iteration number.

It is best to illustrate the working of the above 2-phase algorithm with a numerical example and hence, that is what we do next. Consider the 30 item store layout shown in Figure 2. The locations and shortest distances are given by the shortest distance matrix in Table 1. The unit marginal profits of the items is given in Table 2. To test the *p*-dispersion based heuristic and simulated annealing approach we developed a simulation program and ran it for the case of K=3 (i.e. three customer categories). The set of items for the three customer categories were as follows:

Category I: Must-have items $\theta_k = \{I-2, I-11, I-12, I-13, I-23, I-25\}$ and Impulse items $\beta_k = \{I-7, I-28\}$ Category II: Must-have items $\theta_k = \{I-2, I-11, I-12, I-20, I-22, I-30\}$ and Impulse items $\beta_k = \{I-15, I-16\}$ Category III: Must-have items $\theta_k = \{I-2, I-3, I-11, I-12, I-20, I-22\}$ and Impulse items $\beta_k = \{I-24, I-29\}$

The value of the original layout based on the average obtained from ten runs of the simulation program was found to be V = 8.23.

(4)

	Table 1. Shortest Distance Matrix																															
ITEM		0	I-1	1-2	I-3	I-4	I-5	I-6	I-7	I-8	I-9	۱- 10	۱- 11	۱- 12	۱- 13	۱- 14	l- 15	۱- 16	I- 17	۱- 18	۱- 19	I- 20	- 21	- 22	۱- 23	I- 24	l- 25	۱- 26	- 27	۱- 28	l- 29	I- 30
	LOC	ENT/EXIT	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	L20	L21	L22	L23	L24	L25	L26	L27	L28	L29	L30
0	ENT/EXIT	0	10	20	30	40	50	65	55	45	35	25	25	35	45	55	65	80	70	60	50	40	40	50	60	70	80	95	85	75	65	55
I-1	L1	10	0	10	20	30	40	55	45	35	25	15	15	25	35	45	55	70	60	50	40	30	30	40	50	60	70	85	75	65	55	45
I-2	L2	20	10	0	10	20	30	45	55	45	35	25	25	35	45	55	45	60	70	60	50	40	40	50	60	70	60	75	85	75	65	55
I-3	L3	30	20	10	0	10	20	35	45	55	45	35	35	45	55	45	35	50	60	70	60	50	50	60	70	60	50	65	75	85	75	65
I-4	L4	40	30	20	10	0	10	25	35	45	55	45	45	55	45	35	25	40	50	60	70	60	60	70	60	50	40	55	65	75	85	75
I-5	L5	50	40	30	20	10	0	15	25	35	45	55	55	45	35	25	15	30	40	50	60	70	70	60	50	40	30	45	55	65	75	85
I-6	L6	65	55	45	35	25	15	0	30	20	30	40	40	30	20	10	20	15	25	35	45	55	55	45	35	25	15	30	40	50	60	70
I-7	L7	55	45	55	45	35	25	10	0	10	20	30	30	20	10	20	10	25	35	45	55	45	45	55	45	35	25	40	50	60	70	60
I-8	L8	45	35	45	55	45	35	20	10	0	10	20	20	10	20	10	20	35	45	55	45	35	35	45	55	45	35	50	60	70	60	50
I-9	L9	35	25	35	45	55	45	30	20	10	0	10	10	20	10	20	30	45	55	45	35	25	25	35	45	55	45	60	70	60	50	40
I-10	L10	25	15	25	35	45	55	40	30	20	10	0	20	10	20	30	40	55	45	35	25	15	15	25	35	45	55	70	60	50	40	30
I-11	L11	25	15	25	35	45	55	40	30	20	10	20	0	10	20	30	40	55	45	35	25	15	15	25	35	45	55	70	60	50	40	30
I-12	L12	35	25	35	45	55	45	30	20	10	20	10	10	0	10	20	30	45	55	45	35	25	25	35	45	55	45	60	70	60	50	40
I-13	L13	45	35	45	55	45	35	20	10	20	10	20	20	10	0	10	20	35	45	55	45	35	35	45	55	45	35	50	60	70	60	50
I-14	L14	55	45	55	45	35	25	10	20	10	20	30	30	20	10	0	10	25	35	45	55	45	45	55	45	35	25	40	50	60	70	60
I-15	L15	65	55	45	35	25	15	20	10	20	30	40	40	30	20	10	0	15	25	35	45	55	55	45	35	25	15	30	40	50	60	70
I-16	L16	80	70	60	50	40	30	15	25	35	45	55	55	45	35	25	15	0	10	20	30	40	40	30	20	10	20	15	25	35	45	55
I-17	L17	70	60	70	60	50	40	25	35	45	55	45	45	55	45	35	25	10	0	10	20	30	30	20	10	20	10	25	35	45	55	45
I-18	L18	60	50	60	70	60	50	35	45	55	45	35	35	45	55	45	35	20	10	0	10	20	20	10	20	10	20	35	45	55	45	35
I-19	L19	50	40	50	60	70	60	45	55	45	35	25	25	35	45	55	45	30	20	10	0	10	10	20	10	20	30	45	55	45	35	25
I-20	L20	40	30	40	50	60	70	55	45	35	25	15	15	25	35	45	55	40	30	20	10	0	20	10	20	30	40	55	45	35	25	15
I-21	L21	40	30	40	50	60	70	55	45	35	25	15	15	25	35	45	55	40	30	20	10	20	0	10	20	30	40	55	45	35	25	15
I-22	L22	50	40	50	60	70	60	45	55	45	35	25	25	35	45	55	45	30	20	10	20	10	10	0	10	20	30	45	55	45	35	25
I-23	L23	60	50	60	70	60	50	35	45	55	45	35	35	45	55	45	35	20	10	20	10	20	20	10	0	10	20	35	45	55	45	35
I-24	L24	70	60	70	60	50	40	25	35	45	55	45	45	55	45	35	25	10	20	10	20	30	30	20	10	0	10	25	35	45	55	45
I-25	L25	80	70	60	50	40	30	15	25	35	45	55	55	45	35	25	15	20	10	20	30	40	40	30	20	10	0	15	25	35	45	55
I-26	L26	95	85	75	65	55	45	30	40	50	60	70	70	60	50	40	30	15	25	35	45	55	55	45	35	25	15	0	10	20	30	40
I-27	L27	85	75	85	75	65	55	40	50	60	70	60	60	70	60	50	40	25	35	45	55	45	45	55	45	35	25	10	0	10	20	30
I-28	L28	75	65	75	85	75	65	50	60	70	60	50	50	60	70	60	50	35	45	55	45	35	35	45	55	45	35	20	10	0	10	20
I-29	L29	65	55	65	75	85	75	60	70	60	50	40	40	50	60	70	60	45	55	45	35	25	25	35	45	55	45	30	20	10	0	10
I-30	L30	55	45	55	65	75	85	70	60	50	40	30	30	40	50	60	70	55	45	35	25	15	15	25	35	45	55	40	30	20	10	0

Table 1. Shortest Distance Matrix

ITEM NO.	I-1	I-2	I-3	I-4	I-5	I-6	I-7	I-8	I-9	I-10	I-11	I-12	I-13	I-14	I-15
MARGINAL	1.24	3.59	0.69	1.29	3.09	2.49	1.79	0.59	3.69	5.99	2.79	2.99	1.79	2.79	2.49
PROFIT(\$)															
ITEM NO.	I-16	I-17	I-18	I-19	I-20	I-21	I-22	I-23	I-24	I-25	I-26	I-27	I-28	I-29	I-30
MARGINAL	2.59	2.29	1.85	2.99	6.99	4.39	2.29	3.25	3.79	4.49	1.69	2.59	0.35	3.33	2.59
PROFIT (\$)															

Table 2. Price Index

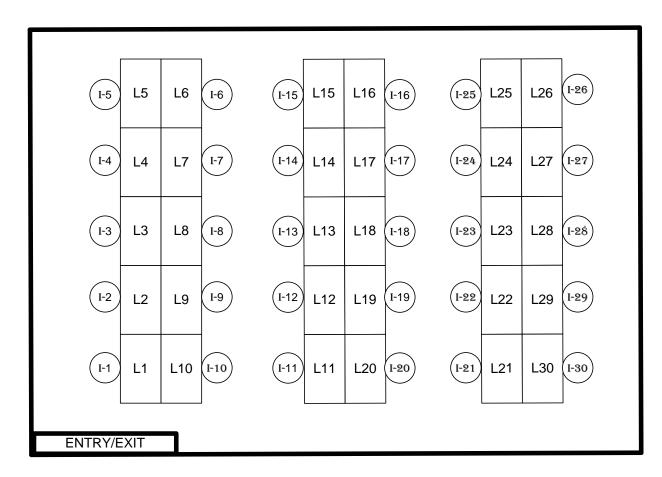


Figure 2. Grid Layout

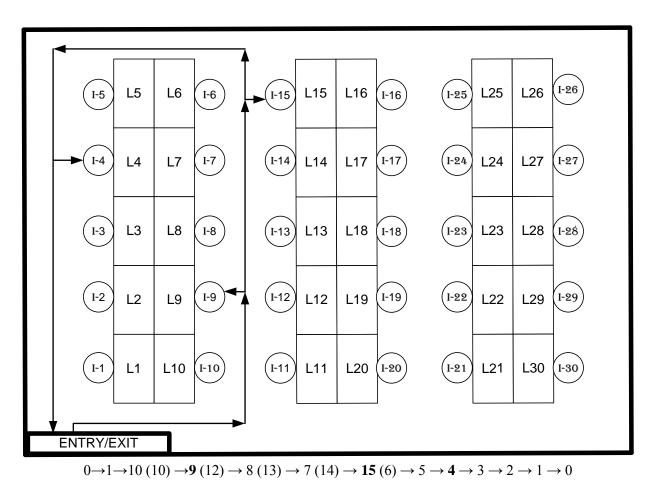


Figure 3. Path of Travel

Let us now improve this current layout above by using the 1st phase of our algorithm that uses the *p*-dispersion heuristic. We start by applying this heuristic to all K=3 categories of customers and derive the value *p* (*p* common points) based on the example. Three must-have items I-2, I-11, and I-12 are common to all customer categories; hence the initial value of *p* is equal to p1=3. The three maximally dispersed locations obtained from the greedy deletion heuristic are L8, L18 and L28. We then exchange the locations of I-2, I-11, and I-12 with I-8, I-18, and I-28 respectively and fix these must-have items at these locations.

We again apply the *p*-dispersion based heuristic to find the common must-have items in *K*-1 customer categories. Two items I-20 and I-22 are common to Category II and Category III; hence the second value of *p* is p2 = 2. We then obtain L5 and L30 as the two maximally dispersed locations from the greedy deletion heuristic. Items I-20 and I-22 are exchanged with I-5 and I-30 respectively and the must-have items are fixed at these locations. The value of this new layout (again obtained by taking the average of ten runs of the simulation program) was found to be V = 13.25.

Next, we apply the second phase of the algorithm and further improve the layout using simulated annealing, which leads us to exchange locations I-4 and I-25 with I-23 and I-26, respectively. The value of this revised layout as obtained by taking the average of ten runs of the simulation program was found to be V = 14.04. Therefore in this example it can be seen that the *p*-dispersion phase yields a significant increase in value of the layout, whereas the simulated annealing phase yields a marginal further improvement.

5 Benchmarking and Performance Analysis

This section focuses on analyzing the performance of our all using two different approaches. The first approach is to perform an extensive empirical study to determine the effectiveness of the algorithm wherein we test the behavior of the algorithm when the factors that affect the computational complexity of the input parameters are varied. The second part of the analysis benchmarks the results from the algorithm against available results in the academic as well as practitioner literature. Thereafter, we determine the growth in the time taken by the algorithm to solve increasingly larger problems and empirically estimate the error bounds of the solutions produced by this algorithm.

5.1 Empirical Analysis

Paired T-test

Here, we wanted to empirically test whether or not our algorithm improved the value of a store's layout and if so, what the average improvement would be. To that end, we selected 16 different grid patterns similar to the one shown in Figure 2 with the total number of items (must-have and impulse) ranging from 30-45. For each such test case, we randomly picked must-have items and impulse items and assigned them randomly to the placement spots available (similar to Figure 2, the number of placement spots in the grid pattern was equal to the number of items to be placed in each test case) to determine the original layout. Then, we ran each phase of our two phase algorithm. For every layout, the initial one as well as the ones produced at the end of each phase of our algorithm, the VBA simulation code was used 10 times and the average of these 10 simulations was denoted as the value of that layout. We then tested the results from these 16 experiments for impulse value for the original layout and the same for the improved layouts produced by the *p*-dispersion heuristic (1st phase of our algorithm) and simulated annealing (2nd phase of our algorithm). The results of the paired t-test for *p*-dispersed impulse value minus initial impulse value are displayed in Table 3.

	N	Mean	StDev	SE Mean
<i>p</i> -dispersed impulse value	16	15.18	3.17	0.79
Original impulse value	16	13.66	3.46	0.87
Difference	16	1.53	0.29	0.08

Table 3. Paired T-Test p-dispersed impulse value minus Original impulse value

95% CI for mean difference: (-2.164, -0.873)

T-Test of mean difference = 0 (vs not = 0): T-Value = -5.01 P-Value = 0.000

From Table 3, we see that the P-Value of the test is 0. This suggests that there is a significant difference in the impulse values of the *p*-dispersed layout and the original layout. Specifically, the *p*-dispersed impulse value (mean = 14.47) is higher than the original impulse value (mean = 12.13). Therefore, there is a significant increase in impulse value through implementation of the *p*-dispersion based heuristic.

The results of the paired t-test for simulated annealing impulse value minus *p*-dispersed impulse value are displayed in Table 4. From Table 4, we see that the P-Value of the test is 0.00. This suggests that there is significant difference in the impulse values of the *p*-dispersed layout and the simulated annealing layout. Specifically, the simulated annealing impulse value (mean = 15.712) is slightly higher than the p-dispersed impulse value (mean = 15.712).

Thus our empirical results showed that the average improvement in the value of a layout due to the application of our algorithm was (15.712-12.13)/12.13 = 29.53%.

	Ν	Mean	StDev	SE Mean
SA impulse value	16	15.712	3.453	0.863
p-dispersion impulse value	16	15.175	3.462	0.866
Difference	16	0.573	0.009	0.003

 Table 4. Paired T-Test SA impulse value minus p- dispersed impulse value

95% CI for mean difference: (-0.6590, -0.4139)

T-Test of mean difference = 0 (vs not = 0): T-Value = -9.33 P-Value = 0.000

Factorial Design

For this second set of empirical tests, testing was carried out for a store layout in which the 30 items were arranged in a grid format. In order to capture the variety of the unit marginal profit earned per impulse item, they were randomly differentiated into three categories and given weights 1, 2 and 3, where one implies a low marginal unit profit and 3 implies a high marginal unit profit. Three different sets of must-have and impulse items were tested to ensure robustness of our solution. Finally, all layout values reported are the result of the average value obtained from ten runs of the simulation model using that layout.

A 2 factorial design allows the simultaneous study of effects that several factors may have on the response. A full factorial design measures the response at all combinations of the experimental factor levels. The effect of a factor is defined by the change in response produced by change in level of a factor. This is called as a main effect and it refers to primary factors of interest in the experiment. The number of runs needed

to exhaust all combinations for our 2 factorial design is 8. We took 3 replications at each run for a total of 24 data points. The algorithms were tested for all possible combinations and output parameters recorded. Graphs were plotted to determine the significance of each factor.

Since the essential objective of the algorithm is to maximally disperse the must-have items to increase the number of impulse buys in the store, the output parameter of interest is the impulse value (aggregate marginal profit) of the layout. Considering the influences of those factors significant to the improvement of value, we also need to analyze the nuances between the original layout and the one that applies our algorithm. To that end, we divided the factors into two groups: Customer segmentation factors (number of customer categories) and Algorithmic factors (Number of common must-have items and total number of must-have items). Additionally, these factors have to be varied to determine the response at all combinations of the experimental factor levels since the effect of a factor is defined by the change in response produced by change in level of a factor. In this case, we varied the factors at two levels: LOW and HIGH. The resulting values of different factors levels are found in Table 5.

Table 5. Factor Levels											
Factor Type	Factor	LOW	HIGH								
Customer Factor	No. of customer categories	3	5								
Algorithmic Factor	Common must-have items	2	4								
Algorithmic Factor	Number of must-have items	4	6								

Table 5. Factor Levels

We performed ANOVA of two different groups of customers to understand the significance of different factors. The ANOVA Table for main effects is shown in Table 6 and Table 7.

Table 0. ANOVA Table (Impulse value group 1)									
Source	DF	Seq SS	Adj SS	Adj MS	F	р			
No. of Common must have items	1	0.344	0.344	0.344	0.45	0.511			
No. of must have item	1	11.363	11.363	11.363	14.79	0.001			
No. of Customer categories	1	371.633	371.633	371.633	483.83	0.000			

 Table 6. ANOVA Table (Impulse value group 1)

Tuble // The of the Tuble (Impulse funde group =)									
Source	DF	Seq SS	Adj SS	Adj MS	F	р			
No. of Common must have	1	1.118	1.118	1.118	1.330	0.263			
items									
No. of must have item	1	11.833	11.833	11.833	14.040	0.001			
No. of Customer categories	1	363.712	363.712	363.712	431.440	0.000			

It can be seen from the analysis in Table 6 and Table 7, that the main effects 'No. of Customer Categories' and 'No. of must-have items' are significant. The R^2 value of 95.57 % and 95.07% (refer to Appendix B

and Appendix C) shows that the main effects explain the variance in impulse value extremely well. The regression equations of two different groups we derive from our analysis are:

and

Impulse Value = 17.14 - 0.1197 (No. of Common must have item) + 0.6881 (No. of must-have items) + 3.94 (No. of Customer Categories)

Impulse Value = 17.17 - 0.2159 (No. of Common must have item) + 0.7 (No. of must-have items) + 3.89 (No. of Customer Categories)

Turning to a graphical analysis of the results, refer to figures 4 and 5. As these two figures reveal, the impulse value of a layout increases with increase in the number of must-have items. This can be attributed to the fact that when the number of items on the customers' must-have list increases, he/she travels more in the store increasing the likelihood of passing by an impulse item and thereby increasing the value of the layout. We can also see that as the number of customer categories increases there is a significant increase in the impulse value of the layout. The increase in number of customer categories leads to more different impulse items, in other words, there are more impulse buying, thereby increasing the value of the layout. From Figures 4 and 5, we observe a small decrease in impulse revenue as we increase the number of common must-have items. However the decrease is insignificant and does not change the value of the layout drastically in our case. A possible explanation is that our algorithm is efficient regardless of how many common must-have items overlap among customers.

Next we performed ANOVA of two different groups of customers to understand the significance of different factors of impulse values improvement after applying our algorithm. The ANOVA Table for main effects is shown in Table 8 and Table 9.

The R^2 value of 15.14 % and 42.27% (refer to Appendix D and Appendix E) shows that the main effects do not explain the variance in impulse value very well. For group 1, it can be seen from the analysis in Table 8 that only the main effect 'No. of Customer Categories' is significant. However, for group2, that the main effects 'No. of Customer Categories' and 'No. of must-have items' are both significant. The regression equations of two different groups we derive from our analysis are:

Impulse Value = 0.64 - 0.015 (No. of Common must have item) -0.026 (No. of must-have items) + -0.15 (No. of Customer Categories)

and

Impulse Value = 0.55 - 0.023 (No. of Common must have item) -0.122 (No. of must-have items) -0.139 (No. of Customer Categories)

The graphs in figures 6 and 7 (pertaining to group 1 and group 2), reveal that the improvement will decrease when the numbers of customer categories increase to the HIGH level. This can be attributed to the fact that when customer categories increase, the variety of the impulse items would increase. Therefore, although we are able to disperse the must-have items to some better locations, some impulse items would still remain out of the customer's reach. This can explain that the marginal improvement will decrease when the impulse value increases.

From a management standpoint, we observed that an increase in number of customer categories has a decreasing effect on the impulse revenue improvement.

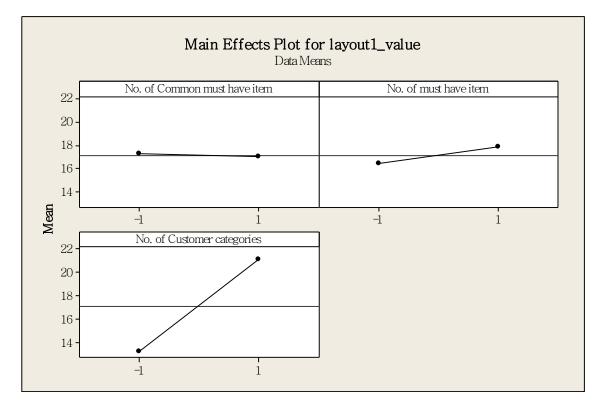


Figure 4. Effect of Main Effects on Impulse Value of group 1

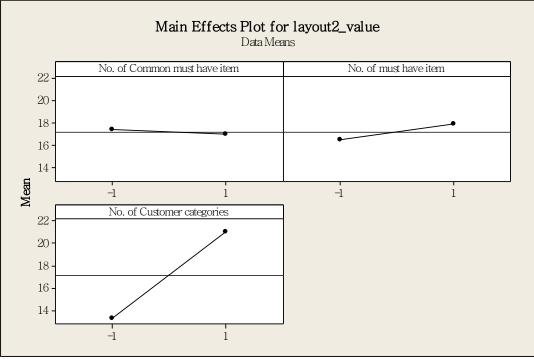


Figure 5. Effect of Main Effects on Impulse Value of group 2

Tuble of Hite v					8	
Source	DF	Seq SS	Adj SS	Adj MS	F	р
No. of Common must have	1	0.00526	0.00526	0.005256	0.07	0.800
items						
No. of must have item	1	0.01592	0.01592	0.015918	0.200	0.66
No. of Customer categories	1	0.54531	0.54531	0.545313	6.840	0.017

 Table 8. ANOVA Table (Impulse Value Improvement of group 1)

 Table 9. ANOVA Table (Impulse Value Improvement group 2)

Source	DF	Seq SS	Adj SS	Adj MS	F	р
No. of Common must have items	1	0.01271	0.01271	0.01271	0.30	0.589
No. of must have item	1	0.35956	0.35956	0.35956	8.54	0.008
No. of Customer categories	1	0.46273	0.46273	0.46273	11	0.003

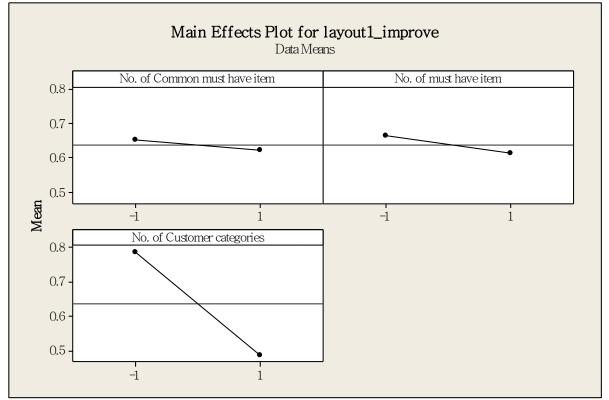


Figure 6. Effect of Main Effects on Impulse Value Improvement of group 1

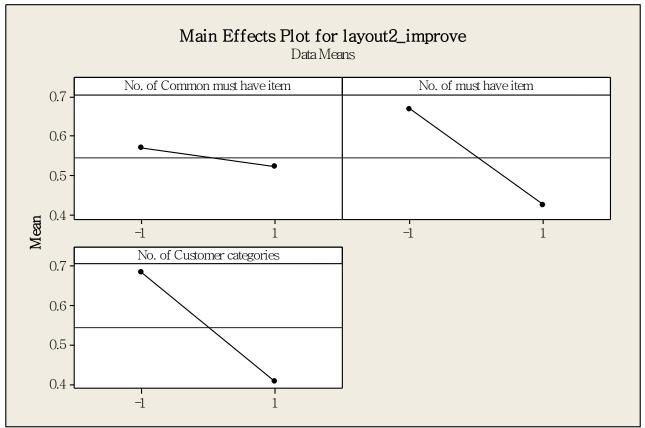


Figure 7. Effect of Main Effects on Impulse Value Improvement of group 2

5.2 Benchmarking Analysis

Since the ultimate goal of our algorithm is to improve overall sales at the store through optimal product placement, as a first step in benchmarking the quality of the solutions produced by the algorithm, we examined the practitioner literature for "normal" year-over-year improvement in same-store sales reported by various retail companies in USA. Table 10 summarizes the same-store sales percentage retailers reported in 2015 by some major American retailers.

	Tuble 100 mpr		Store Sales. Sciect Results From OS Retain	
	Reference	Store	Technique Used To Improve Same- store sales	Improvement Reported
1.	[52]	Publix	Improved Customer Service	4.5%
2	[53]	Costco	Restructuring of membership fees	7%
3	[54]	Sprouts	Restructuring of product pricing	5.8%
4	[55]	Kroger	Exploring new markets	5.4%
5	[56]	Weis	Restructuring of product pricing, Improved Customer Service	4.7%
6	[57]	Dollar Tree	Increase in average item price	2.1%

Table 10. Improvement In	Same-Store Sales:	Select Results From	n US Retailers (2015)
1 abic 10, mpi 0, cmcm m	anne-store sales.	bullet Results FION	100 Ketanets (2013)

			Increase in customers	
7	[58]	Target	Restructuring of product promotion	2.4%
8	[59]	Dollar General	Increase in average item price Increase in customers	2.8%
9	[60]	Five Below	Restructuring of product pricing Restructuring of product assortment	3.4%
10	[61]	Walgreens	Increase in market share	6.4%

As is evident from Table 10, in an effort to improve store sales, companies have adopted strategies ranging from restructuring of product pricing, promotion or product assortment (Sprouts, Weis, Target) to exploration of new markets (Walgreens, Kroger), improving customer service (Publix, Weis) and even changing membership fee structure (Costco). Both Dollar Tree and Dollar General reported improvements based on an increase in the average item price and the number of customers served. However, none reported using a product placement based layout optimization strategy espoused by our algorithm - a support for the novelty of our approach. Notwithstanding the strategies used by the different retailers in Table 10, the average improvement in same-store sales is 4%. As shown in Section 5.1 before, empirical testing of our algorithm on randomly generated data indicates a much bigger average improvement of 29.35% and Section 6 will demonstrate an even greater improvement of 66.38% for data obtained from a grocery store in Western New York.

Next, we benchmarked our algorithm against published improvement results in the academic literature on store layout optimization algorithms – the three most relevant to the problem we are studying are reported in Table 11 below. The model proposed by Coskun (2012), was a comprehensive model for shelf space allocation for a product category, and the model that was able to improve a retailer's profit by 7%. Another shelf management problem was developed by Russell and Urban (2010), where the authors categorized individual products as part of a product family. The maximum improvement that was reported in that model was 2.61%. As is the case with the improvements reported in the practitioner literature above, the performance of our algorithm reported in Sections 5.1 and 6 is superior overall. Furthermore, as we show next, our algorithm is capable of handling larger problem sizes than the ones reported in these papers.

	Reference	Size of Problems Tested	Technique Used To Improve Store Layout	Improvement Reported
1	[62]	N/A	Category Correlation Matrix	N/A
2	[63]	10	Continuous and Discrete Optimization Model	2.61%
3	[64]	20	Shelf Allocation Optimization	7%

 Table 11. Store Layout Improvement: Results From the Academic Literature

The third step in our benchmarking analysis tested the running time of the algorithm for increasing sizes of the problem. For that purpose, we constructed data sets with three customer categories (i.e., K=3) with p1 being the number of common "must-have" items for all 3 customer categories, p2 as the same for all 2 categories and n representing the total number of items (must-have and impulse) for sale at a store. Further, in order to standardize the results, we assumed that p1 and p2 are 10% and 6% (approx., rounded up) respectively of n (the total number of items for sale). The hardware used was an Intel(R) Xeon(R) CPU

E5-2630 v3 @ 2.40GHz computer with 64-bit operating system and installed memory of 32MB, which is commonly available in USA. The results, as shown in Figure 8, represent the average running time of 10 simulations for each data set. As evident from same, our algorithm is able to solve problems with up to 700 items, with p1=70 and p2=42 in about 33 minutes with this hardware. To the best of our knowledge, this is significantly larger than the size of problems solved in the published academic literature.

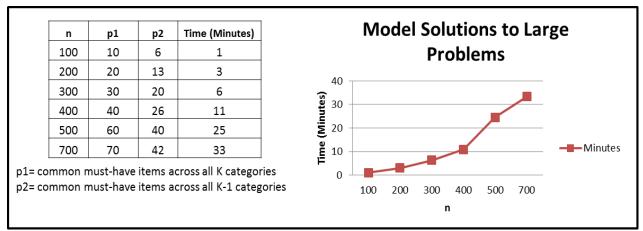


Figure 8. Running Time vs Problem Size

As a heuristic, our algorithm is not guaranteed to produce an optimal solution. Therefore, the fourth and final part of our performance benchmarking analysis involved an estimation of the error bound of the solution produced by our algorithm. For that purpose, instances of problems were randomly generated for each case of n = 25,...55, p1 = 10% (approx., rounded up) and p2 = 6% (approx., rounded up) and each instance was first solved to optimality using complete enumeration³. Thereafter, the same instance was solved 10 times with our two-phase algorithm and the average solution noted. The results are shown in Figure 9. As evident therein, the largest gap between the optimal solution and that obtained by our two-phase algorithm was 3.6%, indicating that (at least for problems sizes studied) the error bound of our algorithm is fairly low.

³ We were unable to solve problems larger than 55 to optimality using complete enumeration because of memory errors in the computer.

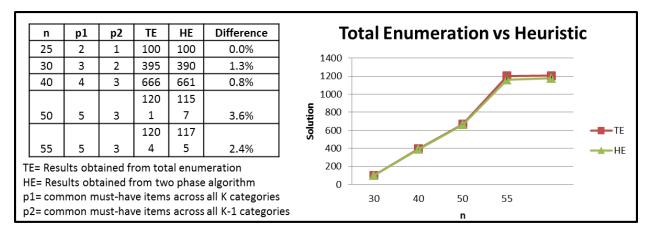


Figure 9. Error Bound

In concluding this section on analyzing the performance of our algorithm, the benchmarking done and the subsequent empirical analysis based on randomly generated data allow us to make the following observations.

(1) The statistical results suggests that the two-phase algorithm proposed with p-dispersion based heuristic in the first phase followed by simulated annealing metaheuristic in the second, are effective in increasing the impulse value of the layout. Per the experiments, the average improvement in the value of a layout reported in our empirical study is 29.53%.

(2) The *p*-dispersion based heuristic gives a good starting solution with a significantly improved layout in terms of impulse revenue and thereafter, the simulated annealing metaheuristic improves the initial solution provided by the *p*-dispersion based heuristic method.

(3) The benchmarking analysis performed shows that our two-phase algorithm produces improvements that are superior to those reported in the academic as well as practitioner literature. Further, the algorithm is capable of solving larger problems that those reported heretofore in the academic literature.

(4) The error bound of the solutions produced by our two-phase algorithm is small and below 5% for all the cases we tested.

6. Case Study

In an effort to investigate the efficacy of our proposed algorithm in a real-world setting, we tested its practical for the layout of a grocery store in Western New York, whose identity is withheld upon the request of store management. The targeted grocery store stocks fresh vegetables every Thursday. Therefore, buying fresh vegetables is the main reason for almost every customer who visits the store on Thursdays. However, store management felt that the display of vegetables was not spatially dispersed enough to make customers travel around the store, leading to poor impulse buying. That, in turn, why this store was selected for the case study. Data collection has been described in Appendix A. As, mentioned therein, 50 samples were

collected, and based on the list of identified must-have items, we classified the customers into three categories. For each of these three categories, we also identified the list of impulse items by analyzing the survey data.

Based on these categories, we ran the simulation program for ten runs and took the average value. For the original layout, the result was V = 10.65. For the layout obtained after *p*-dispersion phase of our algorithm the result improved to V = 16.38 and finally, after the completion of the second phase involving the simulated annealing metaheuristic, the impulse value of the layout reached V = 17.72. Thus, the value of the new layout had a significant improvement compared to the original layout with an overall improvement of 66.38%. In other words, the results from our case study based on an actual grocery store environment matched those from our computational analysis in that significant improvements were achieved by applying our two-phase algorithm to optimize the layout by dispersing the must-have items to maximize the sales of the impulse items.

7. Summary and Future Research

This work provides a location theory-based algorithmic framework for placement of common must-have items in a grocery/convenience store so as to generate higher impulse revenue. The solution methodology proposed is a two-phase algorithm where the first phase produces a good solution by solving the *p*-dispersion problem that maximally separates the common must-have items, resulting in a greater likelihood of the customer getting exposed to the impulse items while on their path of travel. Then the simulated annealing procedure improves the solution obtained from the *p*-dispersion phase by ensuring that it does not get stuck in a local optima. Finally, a simulation program was implemented for calculating the impulse revenue. The performance of the algorithm was evaluated using benchmarking, empirical analysis and real-world data taken from a New York grocery store. All attest to the effectiveness of the algorithm in improving store revenue. Experiments conducted on randomly generated data point to a 29.35% improvement in impulse buys by using our algorithm and testing on real-world data from a grocery store in Western New York resulted in a 66.38% improvement of the same. Benchmarking these performances against available results from practitioner and academic literature show that they are better overall. Finally, the algorithm can solve larger problems than what has been reported heretofore in the literature and produced low error bounds for the problems tested.

As with any attempt to model a real-world problem, the framework presented in this paper has limitations and in turn, these provide immediate arenas for future work. One is that we do not make any assumptions about relatedness of the items and overlaps in product segmentation data. In real-world problems, such overlaps may influence impulse purchases since a customer purchasing must-have items A and B may end up purchasing different impulse items than another one purchasing must-have items A and C. The influence of this relationship should be explored further in a further enhancement of our paper. A second limitation arises from the fact that common occurrences in retail environments such as stockouts, substitutions, cannibalization by substitute products are not considered in our model. Third and finally, we have implicitly assumed a permanently increasing relationship between the exposure of a customer to impulse items and his/her impulse purchases. This is reflected in our assumption that maximizing the distance that a customer walks to purchase the must-have items will always result in greater exposure and therefore, greater sales of impulse items. In reality, this may result in excessive walking by customers and thus, reneging by some on impulse purchases. In other words, a more realistic modeling should assume a nonlinear, perhaps concave, relationship between the distance walked by a customer and the number of impulse purchases made by him/her. This also should be investigated in a future enhancement of our model. Finally, it must be borne in mind that the improvements reported in the empirical tests as well as the case study conducted depend on how good or bad the initial layout is. It should be expected that for mature retail chains, initial layouts would already be effective in promoting impulse buys and hence, percentage improvements reported by our algorithm would be less than what has been observed in this paper. This therefore calls for future work that is dedicated entirely to the testing of our algorithm (and other similar ones) on multiple real-world case studies to assess their relative performance.

Other enhancements and extensions are also possible. In this paper, the *p*-dispersion based and simulated annealing heuristic considered the exchanges between items if their respective storage areas had a similar physical size. Constraints like temperature conditions required for a particular item can be considered to ensure more precise placement of items. The store network considered was assumed to have a single compartment shelf or storage area for item placement, so only one item is placed in one storage area. A more complex and realistic scenario can be tested using multiple levels for shelves that store multiple items. The algorithms developed in this work were tested on a small network of items (between 30-45) arranged in a grid format with single entry/exit location. But in reality, a store network might contain hundreds of item nodes with multiple entry/exit locations. Also there are other formats of a store layout like racetrack and freeform. Part of the future work might be to test the performance of algorithms over large problems with multiple entry/exit locations and different layout formats.

Acknowledgment. The authors wish to acknowledge the helpful comments of two anonymous referees. Incorporating them has substantially improved the paper.

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Appendix

Appendix A: Data Collection and Analysis for The Case Study

For data collection and analysis, we divided the whole store floor into eight sections based on the fact that within each section the products are similar. For impulse value calculations we took the average price of a product in each section (since data on unit marginal profit was considered classified by the store management). To classify customer categories we developed a survey shown below was administered to 50 customers.

1. What are the must buy items for you at the store (primary items that you plan to purchase today?

2. If you pass by the items which are listed below, what is the probability you will buy it? (Please, assume you have no stock at home and give your most intuitive answer)

(i lease, assume jou nave no stock at nome and give j	our most mea			
	Most likely	Likely	Unlikely	Most unlikely
Cookies				
Cooking Ingredients				
Canned Food				
Packaged Noodle Items				
Processed Breads (Buns, Cakes, Pancakes etc.)				
Cooking Sauces				
Instant food items				
Processed Hot Beverages (teas, coffees, cocoa drinks)				
Sodas/caffeinated beverages				
Other:(Please specify)				

As evident, the first question in the survey is about must-have items for different customers. The second question is asking about impulse items that may be purchased by customers. After data collection, we determined the relationship between must-have items and impulse items in the following manner: the term "Most likely" was given a score as 4 and the score decreased by one at each level of likelihood of purchase. According to the data collected, the most popular product is vegetables so that we assume vegetable is the common must-have items among customers. Besides vegetables, the customers also listed other different must-have items. Based on the data collected, we classified the customers into three categories as follows:

Category I: must-have items $\theta_k = \{I-1, I-10, I-15, I-16, I-21\}$

Category II: must-have items $\theta_k = \{I-1, I-10, I-15, I-16, I-30\}$

Category III: must-have items $\theta_k = \{I-1, I-10, I-15, I-16, I-26\}$

In order to know the impulse items of each category, we took the average score of question 2 as the referent. Accordingly, we choose the highest two as the impulse items in each category. Here is the final result:

Category I: must-have items $\theta_k = \{I-1, I-10, I-15, I-16, I-21\}$ and

impulse items
$$\beta_k = \{I-27, I-28\}$$

Category II: must-have items $\theta_k = I-1$, I-10, I-15, I-16, I-30} and

impulse items $\beta_k = \{I-26, I-20\}$

Category III: must-have items $\theta_k = I-1$, I-10, I-15, I-16, I-26} and impulse items $\beta_k = \{I-19, I-29\}$

Appendix B: Factorial Design Table

Order	No. of Common	No. of must	No. of Customer	Original	P-dispersion	Simulated annealing
	must have item	have item	categories	layout value	value	value
1	-1	-1	-1	7.858393026	12.7379251	13.75850427
2	1	-1	-1	8.095413728	13.25179212	13.25179212
3	-1	1	-1	10.86464141	13.46237572	14.96006057
4	1	1	-1	10.74405212	13.68121291	14.64866774
5	-1	-1	1	15.09678	19.89000624	20.95180821
6	1	-1	1	15.2335583	20.38037366	20.38037366
7	-1	1	1	18.42532207	20.55060811	22.20310841
8	1	1	1	18.24589037	20.51638362	21.7539911
9	-1	-1	-1	6.004403361	12.16470795	13.37699877
10	1	-1	-1	5.775112742	12.04285403	12.04285403
11	-1	1	-1	9.346074399	13.09702886	13.75154105
12	1	1	-1	9.390475695	12.99408866	13.30055973
13	-1	-1	1	10.79731313	19.16656682	19.95920438
14	1	-1	1	10.53346102	18.19850416	18.19850416
15	-1	1	1	14.97698015	19.99143632	20.68673789
16	1	1	1	14.61743685	18.99419955	20.83679062
17	-1	-1	-1	6.968725835	10.70626217	12.16132857
18	1	-1	-1	7.243096293	11.51917545	11.51917545
19	-1	1	-1	7.534395382	12.09782749	13.26678582
20	1	1	-1	7.401208033	12.52540583	13.20429611
21	-1	-1	1	15.60475303	20.03961963	20.62580735
22	1	-1	1	15.75946952	21.30508826	21.30508826
23	-1	1	1	17.02334569	20.97462341	22.84578755
24	1	1	1	16.91457791	21.93595426	22.92497507

Appendix C: General Linear Model (Value of Layout 1)

Source	DF	Seq SS	Adj SS	Adj MS	F	Р
Main Effects	3	383.34	383.34	127.78	166.36	0.00
No. of Common must have item	1	0.34	0.344	0.34	0.45	0.51
No. of must have item	1	11.36	11.36	11.363	14.79	0.00
No. of Customer categories	1	371.63	371.63	371.63	483.830	0.00
Residual Error	20	15.36	15.36	0.77		
Lack of Fit	4	0.54	0.54	0.13	0.14	0.963
Pure Error	16	14.83	14.83	0.93		
Total	23	398.70				

R-Sq = 96.15% R-Sq(pred) = 94.45% R-Sq(adj) = 95.57%

Source	DF	Seq SS	Adj SS	Adj MS	F	Р
Main Effects	3	376.663	376.663	125.554	148.930	0.000
No. of Common must have item	1	1.118	1.118	1.118	1.330	0.263
No. of must have item	1	11.833	11.833	11.833	14.040	0.001
No. of Customer categories	1	363.712	363.712	363.712	431.440	0.000
Residual Error	20	16.861	16.861	0.843		
Lack of Fit	4	0.815	0.815	0.204	0.200	0.933
Pure Error	16	16.046	16.046	1.003		
Total	23	393.524				

Appendix D: General Linear Model (Value of Layout 2)

R-Sq = 95.72% R-Sq(pred) = 93.83% R-Sq(adj) = 95.07%

Appendix E: General Linear Model (Improvement of Layout 1)

Source	DF	Seq SS	Adj SS	Adj MS	F	Р
Main Effects	3	0.56649	0.56649	0.188829	2.37	0.101
No. of Common must have item	1	0.00526	0.00526	0.005256	0.07	0.8
No. of must have item	1	0.01592	0.01592	0.015918	0.2	0.66
No. of Customer categories	1	0.54531	0.54531	0.545313	6.84	0.017
Residual Error	20	1.5947	1.5947	0.079735		
Lack of Fit	4	0.0272	0.0272	0.0068	0.07	0.99
Pure Error	16	1.5675	1.5675	0.097969		
Total	23	2.16118				

R-Sq = 26.21% R-Sq(pred) = 0.00% R-Sq(adj) = 15.14%

Appendix F: General Linear Model (Improvement of Layout 2)

				Adj		
Source	DF	Seq SS	Adj SS	MS	F	Р
Main Effects	3	0.835	0.835	0.27833	6.61	0.003
No. of Common must have items	1	0.01271	0.01271	0.01271	0.3	0.589
No. of must have item	1	0.35956	0.35956	0.35956	8.54	0.008
No. of Customer categories	1	0.46273	0.46273	0.46273	11	0.003
Residual Error	20	0.84157	0.84157	0.04208		
Lack of Fit	4	0.04539	0.04539	0.01135	0.23	0.919
Pure Error	16	0.79618	0.79618	0.04976		
Total	23	1.67657				

 $R\text{-}Sq = 49.80\% \quad R\text{-}Sq(pred) = 27.72\% \quad R\text{-}Sq(adj) = 42.27\%$

Appendix 0: 1 area 1- rest								
Original impulse Value (\$)	p-disp impulse value (\$)	SA impulse value (\$)						
12.96791	13.63794	14.09078						
10.93634	11.07142	11.8137						
9.661359	11.42419	12.21212						
10.89489	14.10279	14.22584						
12.44378	12.58273	12.95807						
14.02544	14.36654	15.12402						
15.53402	16.17129	16.7819						
13.89049	17.27324	17.77378						
10.89121	12.80375	13.12486						
13.13141	13.42528	14.46712						
14.00723	17.24424	17.78377						
10.29376	12.07126	12.37399						
22.11665	24.80997	25.27898						
14.0312	15.64507	15.96247						
17.9396	18.01694	18.69031						
15.74383	18.15752	18.72562						

Appendix G: Paired T- Test