Abstract

This paper considers a bi-level hazmat transportation network design problem in which hazmat shipments have to be transported over a road network between specified origin-destination points. The bi-level framework involves a regulatory authority and hazmat carriers. The control variables for the regulatory authority are locations of hazmat response teams and which additional links to include for hazmat travel. The regulatory authority (upper level) aims to minimize the maximum transport risk incurred by a transportation zone, which is related to risk equity. Our measure of risk incorporates the average response time to the hazmat incidents. Hazmat carriers (lower level) seek to minimize their travel cost. Using optimality conditions, we reformulate the non-linear bi-level model into a single-level mixed integer linear program, which is computationally solvable for medium size problems using a commercial solver. For large size problems, we propose a greedy heuristic approach, which we empirically demonstrate to find good solutions with reasonable computational effort. We also seek a robust solution to capture stochastic characteristics of the model. Experimental results are based on popular test networks from the Sioux Falls and Albany areas.

Keywords: Hazmat emergency response team; Bi-level network design; Greedy heuristic algorithm; Equity of risk; Robust solution

1 Introduction

Although the majority of hazardous materials (hazmat) are essential for industry and human life, they are potentially harmful to people and the environment. Based on the US Department of Transportation’s statistics \(^1\), a 10-year (2004–2013) hazmat incident report indicates that highways, with 140,742 number of incidents out of 163,469 in total, have the largest portion of fatalities, injuries, and damage among all modes of transportation. This report indicates the importance of

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risk on hazmat road networks and the reason why these problems have received so much attention from OR/MS researchers.

Hazmat shipments are regulated under the Federal Hazardous Materials Transportation Act in the United States. There are two main policies available to regulatory agencies to mitigate hazmat transport risk, proactive and reactive (Marcotte et al., 2009). Proactive policies reduce the likelihood and consequences of hazmat incidents \textit{a priori} such as closure of road segments or container design. Reactive policies confine the undesirable consequences of a hazmat incident after the occurrence of the accident such as deployment of emergency response teams.

We focus on a combination of proactive and reactive policies in order to reduce the transport risk; regulating the use of road segments that can be used by hazmat carriers (proactive) and locating the Hazmat Response Teams (HRTs) (reactive). Many existing papers consider toll-setting or road closure policies to deter the hazmat carriers from using certain road segments. Consideration of network design is a relatively well-studied concept in hazmat transportation. The closest piece of work is that by Kara and Verter (2004), who propose a network design problem in which a regulator selects the road segments to be closed for hazmat shipments so as to minimize the total risk, while taking into account the hazmat carriers’ minimum-cost route choices. In contrast, we propose an alternative policy tool to regulate the use of roads for hazmat shipments which we refer to as \textit{adding roads}. In addition, we optimize placement of HRTs, recognizing that hazmat transport risk may be significantly reduced by judiciously locating emergency response teams so that they can respond to an incident in a timely manner.

This work presents three main contributions that differentiate our paper from the current literature. First, we consider simultaneous decisions on designing a road network and locating HRTs to mitigate hazmat transport risk. Second, we define a risk measure that includes the average response time to the hazmat incidents. The regulator’s objective function incorporates this definition of risk and allows us to capture the interactions between network design decisions and HRT location decisions. Regional jurisdiction of HRT guides us to consider risk equity over network zones, where each geographic zone is assigned to exactly one HRT. Third, we propose a robust solution to deal with the stochastic characteristics of hazmat accident probability and hazmat release consequences.

The remainder of this paper is organized as follows. A review of the relevant literature is provided in Section 2. Section 3 presents our mathematical formulation. Solution methodologies are proposed in Section 4, which is followed by our computational experiments in Section 5. Section 6 contains a summary and future research directions.

2 Literature Review

Three main bodies of literature are relevant to this paper: hazmat network design, equity of risk, and emergency response team locating for hazmat shipments. Each area is reviewed separately. Table 1 shows where our model sits among the available models in the literature.
2.1 Hazmat network design

There are a number of papers in HND that discuss partial or entire road closure to hazmat shipments in an existing network—see a review of network design problems proposed by Yang and Bell (1998). Kara and Verter (2004) provide a bi-level integer programming HND considering the leader-follower relationship between the government and carriers. Their model designs a network for each hazmat group based on risk impact with no interaction between these groups. The government objective is to design a minimum-total risk network, considering the minimum-travel cost route choice behavior of carriers. They reformulate their model into a single level mixed integer linear problem by replacing the follower model with its KKT conditions, and solve the linearized form of the problem with an optimization solver. Erkut and Alp (2007) formulate a bi-level tree HND as an integer programming problem to minimize the total transport risk. They developed a construction heuristic to expand the solution of the tree design problem by adding road segments to provide carriers with routing choices and help local authorities to trade off risk and cost. Later Erkut and Gzara (2008) generalize Kara and Verter’s model to the undirected road network where the solution stability of single level mixed integer linear model was guaranteed by proposing a heuristic solution method. They add cost in the upper level objective to impose a trade-off between cost and risk. Verter and Kara (2008) define a path-based hazmat transport network design problem, where the construction of a set of alternative paths makes the incorporation of carriers’ cost concern in regulator’s risk-mitigation decisions easier. The proposed model can be used for making road-closure decisions that are mutually acceptable for the regulator and the carrier. They formulate the problem as a single level integer programming assuming that the shortest path is chosen by each carrier. Bianco et al. (2009) provide a bi-level hazmat network design model where both regional and local government authorities aim to regulate hazmat shipments by imposing restrictions on the amount of hazmat flow over the network. They provide a heuristic approach to overcome the non-stability of the single-level mixed integer linearized form of the problem. Amaldi et al. (2011) propose a generalized HND where a subset of roads can be banned by the government. Their model is inspired by the fact that in some cases government regulates the hazmat transportation in tunnels, while the hazmat shipments outside tunnels remains unconstrained. They propose a bi-level integer programming, and solve the compact single-level integer programming that guarantees stability of solution. All these mathematical models seek risk minimization considering minimum-cost route choices of hazmat carriers. Recently, Sun et al. (2014) present a robust HND to model the risk uncertainty in two ways; uncertainty on each link across all shipments, and uncertainty on each link for each shipment. They extend a existing heuristic framework to solve their robust HND.

2.2 Equity of risk

Based on the definition of Keeney (1980), risk equity is the magnitude of the largest difference in the level of risk among a certain set of individuals. There are a number of papers that apply risk equity to the area of hazmat transportation. Gopalan et al. (1990) develop an integer programming formulation to find an equitable set of routes for hazmat shipments where a high degree of equity
can be achieved by nominal increase in the total risk and by imposing an average equity over each route to evenly spread the risk among zones. They apply a Lagrangian dual approach with a gap-closing procedure to optimally solve single-trip problems. Current and Ratick (1995) present a multi-objective model to analyze combined location/routing decisions involving hazmat. The model includes three objectives, related to risk, equity and cost. They impose equity of risk by defining the maximum allowable risk that any individual is exposed to at a facility site in the form of a constraint in the problem formulation, and minimize this maximum threshold as one of the objectives. Two multi-objective mixed integer programming methods are applied to generate noninferior solutions. Carotenuto et al. (2007) provide a model to generate minimal risk paths for the road transportation of hazardous materials on a given regional area. The contribution is to select paths that minimize the total risk of hazmat shipments while spreading the risk induced on the population in an equitable way. They consider an upper limit on the total hazmat transportation risk over populated links by considering a risk threshold for the links. They apply a Lagrangean relaxation to find a lower bound on the optimal solution value. Two heuristic algorithms are proposed to solve the problem. The only one paper in HND that discusses risk equity is that by Bianco et al. (2009). In their HND model, the regional authority aims to minimize total transport risk assuring risk equity, while the local authority tries to minimize the risk over the local jurisdiction. They define a maximum link total risk threshold to set an upper limit over the total risk value on each link on the network as a leader (upper level problem). They suggest a heuristic approach to find a stable feasible solution of the bi-level model. Kang et al. (2014) proposed a generalized routing problem for hazmat transport based on the hazmat value-at-risk model. They apply their model to a multi-trip multi-hazmat type problem, which determines routes that minimize the global value-at-risk value while satisfying equity constraints.

2.3 Location of hazmat response teams

The HRT location problem is valued as a prominent logistic problem to mitigate hazmat transportation risk. List and Turnquist (1998) define a multi-objective problem combining a route-siting model with three main elements. They solve the problem in three separate sub-problems, i.e., routing for each origin-destination pair to find the set of routes for assignment of flows, flow assignment to routes, and locating the HRTs based on assigned flows. The objective of the latter sub-problem describes the importance of locating HRTs near links with high flow volume or large exposed population (List, 1993). Hamouda (2004) proposes a risk-based decision support model to find the optimal location of HRTs among candidate nodes to minimize the total network risk. Their model ensures that response time to any demand point does not exceed a specified threshold. Berman et al. (2007) present a methodology to determine the optimal design of a specialized team network to maximize its ability to respond to hazmat incidents in a region. The problem was represented via a maximal arc-covering model to assess the emergency response capability to transport incidents. Later, Zografos and Androutsopoulos (2008) present a decision support system to find distribution routes with respect to minimization of travel time, minimization of risk and evacuation implications,
Table 1: Our paper with respect to the HND, hazmat equity modeling, and HRT location literature

<table>
<thead>
<tr>
<th>Research Area</th>
<th>Existing Papers In Hazmat Transportation Literature</th>
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<tbody>
<tr>
<td></td>
<td>Erkut and Alp (2007)</td>
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<td>Erkut and Gzara (2008)</td>
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<td>Verter and Kara (2008)</td>
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<td>Amaldi et al. (2011)</td>
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<td>Sun et al. (2014)</td>
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<td>Bianco et al. (2009)</td>
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<td>This paper</td>
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<tr>
<td>Equity Of Risk</td>
<td>Gopalan et al. (1990)</td>
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<td></td>
<td>Current and Ratick (1995)</td>
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<td></td>
<td>Carotenuto et al. (2007)</td>
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<td>Kang et al. (2014)</td>
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<tr>
<td>Emergency Response Team Location</td>
<td>List (1993)</td>
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<tr>
<td></td>
<td>List and Turnquist (1998)</td>
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<td></td>
<td>Hamouda (2004)</td>
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<td>Berman et al. (2007)</td>
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<td>Zografos and Androutsopoulos (2008)</td>
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<td>Jiahong and Bin (2010)</td>
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<td></td>
<td>Xu et al. (2013)</td>
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</table>

while integrating the HRT location decisions with hazmat route decisions. Jiahong and Bin (2010) present an HRT location-routing problem as a maximal arc-covering model, while adding time and cost to the objectives to maximize the ability to respond to hazmat incidents in a region. Recently, Xu et al. (2013) propose a bi-level optimization model for the HND problem that considers the location of HRT.

3 Problem Definition

In our bi-level problem, the regulatory authority influences the carriers’ decisions by making additional roads available to the hazmat carriers and locating HRTs, whereas carriers can influence the leaders’ decisions by their route selections. Figure 1 shows the conceptual representation of our model. To counteract the high consequences of hazmat incidents on the network, the regulatory authority’s policy tools encourage the carriers to choose their paths such that, the selected minimum cost routes be closer to the HRTs.

3.1 Risk measurement

A popular way to estimate risk is to multiply the accident probability with estimated incident consequences to evaluate expected damage. The consequence is often measured as potential fatalities or dollar value of damage to property (Gopalan et al., 1990). In the event of an accident, the hazmat has a radius of spread that depends on the physical and chemical properties of the hazmat. Gopalan et al. (1990) showed that if \( \lambda \) represents the radius of spread, people and properties within the \( \lambda \)-neighborhood (boundary of a circle with a radius \( \lambda \) and a center at the incident location) of the accident could potentially be affected. In this paper, risk measurement is not only a multiplication of accident probability by estimated consequences, but also a function of response time to
Figure 1: The proposed bi-level model

Let $\mathcal{A}_z$ be the set of all current and potential-to-add links on zone $z$. Let $\eta^c_{ij}(m)$ be the risk associated with one hazmat shipment of type $c$ on link $(i,j)$ given that hazmat response unit $m$ responds to an incident that occurs on zone $z$. Definition of $\eta^c_{ij}(m)$ includes the accident probability and the accident consequence. Accident probability is denoted by $\rho_c l_{ij}$, where $\rho_c$ is the average per-mile accident probability of hazmat type $c$ and $l_{ij}$ is the length of link $(i,j)$, denoted by $l_{ij}$. We assume that the accident consequence is a non-decreasing function of the response time, denoted by $\xi^c_{ij}()$ for each link $(i,j)$ and hazmat type $c$. Then, we have

$$\eta^c_{ij}(m) = \rho_c l_{ij} \xi^c_{ij}(f^m_{ij}) \quad \forall (i,j) \in \mathcal{A}_z, c \in C$$

where $f^m_{ij}$ denotes the average response time (distance) from hazmat response team $m$ to the incident location happening on link $(i,j)$.

To specify the accident consequence function $\xi^c_{ij}()$, we adopt the population exposure measure $q_{ij}^c$ on the $\lambda$-neighborhood of link $(i,j)$ for hazmat type $c$ as shown in Figure 2. We suppose that the consequence increases in proportion to the response time and a linear consequence function for simplicity of the following form:

$$\xi^c_{ij}(f^m_{ij}) = q_{ij}^c \frac{f^m_{ij}}{F^c_{ij}} \quad (2)$$

where $F^c_{ij}$ is a positive scaling constant specific to link $(i,j)$ and hazmat type $c$. The consequence function (2) implicitly assumes that the impact of a hazmat incident extends beyond the $\lambda$-neighborhood, if the response time takes longer than $F^c_{ij}$. 
3.2 Bi-level model formulation

We let $\mathcal{N}$ be the set of nodes, and $\mathcal{A}$ be the set of current road links available for hazmat shipments. We define $\mathcal{A}'$ be the set of all (current and potential-to-add) road links and $\mathcal{A}' \setminus \mathcal{A}$ be the set of potential links to be available for hazmat shipments. Let $\mathcal{C}$ be a set of hazmat shipments, and for each hazmat shipment $c \in \mathcal{C}$, let $o(c)$ and $d(c)$ be, respectively, the origin and destination nodes. Let $n_c$ be the number of shipments from $o(c)$ to $d(c)$ for each $c \in \mathcal{C}$. We assume the existence of at least a path between each origin-destination pair in $G(\mathcal{N}, \mathcal{A})$. Let the binary variable $x^c_{ij}$ be equal to one if hazmat shipment $c$ traverses link $(i, j)$, and zero otherwise. Let the binary variable $y_{ij}$ be equal to one if link $(i, j) \in \mathcal{A}' \setminus \mathcal{A}$ becomes available for hazmat transport, and zero otherwise. Let the binary variable $z_m$ be equal to one if candidate site $m \in \mathcal{M}$ is opened for locating an HRT, and zero otherwise. Binary variable $v^m_z$ denotes the assignment of a HRT located at site $m \in \mathcal{M}$ to zone $z \in \mathcal{Z}$. Let $h_{ij}$ be the cost of adding link $(i, j) \in \mathcal{A}' \setminus \mathcal{A}$ and $B$ be the total available budget for adding links. Constant $p$ denotes the total number of HRT locations to be chosen. The total risk over zone $z \in \mathcal{Z}$ becomes

$$
\sum_{(i,j) \in \mathcal{A}_z} \sum_{m \in \mathcal{M}} \sum_{c \in \mathcal{C}} \rho_c l_{ij} n_c q^c_{ij} \frac{f^m_{ij}}{F_{ij}} v^m_z x^c_{ij}
$$

and we let $\theta$ be the maximum zone total risk.

Our bi-level model considers two related optimization problems. The optimal solution of the upper level model ($P_1$) is affected by the solution of the lower level model ($P_2$). The added potential links $Y = \{y_{ij}\}$, HRT assignments $V = \{v^m_z\}$ and HRT locations $Z = \{z_m\}$ are the variables controlled by the leader to assure minimization of the maximum zone total risk over the network. The followers aim to minimize the network total travel cost by controlling the variables $X = \{x^c_{ij}\}$.

$$(P_1):$$
\[
\min_{y_{ij}, z_m, v_m^z} \theta \\
\sum_{(i,j) \in A} \sum_{m \in M} \sum_{c \in C} \rho_{cij} n_{cij} \frac{f_{mij}}{v_{mij}} v_m^z x_{ij}^c \leq \theta \quad \forall z \in Z
\]
\[
\sum_{(i,j) \in A \setminus A} h_{ij} y_{ij} \leq B
\]
\[
\sum_{m \in M} z_m = p
\]
\[
\sum_{m \in M} v_m^z = 1 \quad \forall z \in Z
\]
\[
v_m^z \leq z_m \quad \forall z \in Z, m \in M
\]
\[
y_{ij} \in \{0, 1\} \quad \forall (i,j) \in A \setminus A
\]
\[
z_m \in \{0, 1\} \quad \forall m \in M
\]
\[
v_m^z \in \{0, 1\} \quad \forall z \in Z, m \in M
\]

where \( x_{ij}^c \) solves
\[
(P_2): \min \sum_{c \in C} \sum_{(i,j) \in A} n_{cij} x_{ij}^c
\]
\[
\sum_{i \in N} x_{ij}^c - \sum_{l \in N} x_{jl}^c = \begin{cases} 
1 & \text{if } j = o(c) \\
-1 & \text{if } j = d(c) \\
0 & \text{otherwise} 
\end{cases} \quad \forall j \in N, c \in C
\]
\[
x_{ij}^c \leq y_{ij} \quad \forall (i,j) \in A \setminus A, c \in C
\]
\[
x_{ij}^c \in \{0, 1\} \quad \forall (i,j) \in A, c \in C
\]

The first problem \( P_1 \) is the upper level formulation in which the leader seeks an equitable distribution of the risk over the network given a minimum total travel cost hazmat flow by the follower’s decisions. The model minimizes the maximum zone total risk (\( \theta \)) among all the zone total risk values of the existing network \( G \) by adding the appropriate available links, finding the best candidate locations for HRTs and, assigning each zone to an HRT. Constraints (4) assure equity of risk on zones, while (5) limits the total cost of links additions to the available budget. Constraint (6) indicates the total number of available HRTs and constraints (7) assign each zone on the network to exactly one hazmat response team. Constraints (8) permit assignments of opened candidate locations to zones. Problem \( P_2 \) is the lower level formulation that models the follower’s behavior of minimizing total travel cost influenced by a feasible flow assignment \( X = \{x_{ij}^c\} \), given the added links \( Y = \{y_{ij}\} \) by the leader. Constraints (13) are the flow conservation requirements, whereas (14) ensures that only the links made available by the leader can be used by the carriers. The proposed bi-level formulation has a non-linear objective in the upper level model. Since the non-linear term is a quadratic function of two binary variables, it can be linearized using common
techniques.

4 Solution Methodologies

Due to the computational difficulty of a non-linear bi-level integer programing problem (Jeroslow, 1985), we develop a single level representation of the model to small instances that are solvable using an optimization solver like CPLEX. A greedy heuristic is developed for large size problems.

4.1 Single level representation

As noted earlier, problem \( P_2 \) can be solved given a set of available added links determined by the upper problem. While the \( y_{ij} \) values are given, the constraints of problem \( P_2 \) constitute a totally unimodular matrix and its integrality requirements can be replaced by \( x_{ij}^c \geq 0 \) without loss of optimality (Kara and Verter, 2004). Based on the weak and strong duality theorems, \( P_2 \) can be replaced with the primal feasibility constraints, the dual feasibility constraints, and the reverse weak duality inequality. To obtain this we define:

- \( \pi_i^c \): dual variables associated with primal constraints (13), where \( i \in N, c \in C \)
- \( \pi_j^c \): dual variables associated with primal constraints (13), where \( i \in N, c \in C \)
- \( M \): A sufficiently large positive number

We follow the procedure in Amaldi et al. (2011) and replace the lower level model \( P_2 \) with constraints (13), (14) and the following constraints in the upper level model \( P_1 \).

\[
\pi_i^c - \pi_j^c \leq n_{c,i} \quad \forall (i, j) \in A, c \in C \quad (16)
\]

\[
\pi_i^c - \pi_j^c \leq n_{c,i} + M(1 - y_{ij}) \quad \forall (i, j) \in A' \setminus A, c \in C \quad (17)
\]

\[
\pi_{d(c)}^c - \pi_{d(c)}^c \geq \sum_{(i, j) \in A'} n_{c,i} x_{ij}^c \quad \forall c \in C \quad (18)
\]

This establishes a new single-level mixed-integer non-linear programming (MINLP) problem \( P_3 \):

\[
(P_3) : \min_{y_{ij}, z_m, v_m, x_{ij}^c} \theta
\]

\[
\sum_{(i, j) \in A} \sum_{m \in M} \sum_{c \in C} \rho_{c,i} n_{c,i} q_{ij}^c \frac{f_m}{F_{ij}} v_m x_{ij}^c \leq \theta \quad \forall z \in Z \quad (20)
\]

\[
\sum_{(i, j) \in A' \setminus A} h_{ij} y_{ij} \leq B \quad (21)
\]

\[
\sum_{m \in M} z_m = p \quad (22)
\]
\[
\sum_{m \in \mathcal{M}} v_z^m = 1 \quad \forall z \in \mathcal{Z}
\]

\[
v_z^m \leq z_m \quad \forall z \in \mathcal{Z}, m \in \mathcal{M}
\]

\[
\sum_{i \in \mathcal{N}} x_{ij}^c - \sum_{l \in \mathcal{N}} x_{jl}^c = \begin{cases} 
1 & \text{if } j = o(c) \\
1 & \text{if } j = d(c) \\
0 & \text{otherwise}
\end{cases} \quad \forall j \in \mathcal{N}, c \in \mathcal{C}
\]

\[
x_{ij}^c \leq y_{ij} \quad \forall (i, j) \in \mathcal{A}' \setminus \mathcal{A}, c \in \mathcal{C}
\]

\[
\pi_c^i - \pi_c^j \leq n_{cij} \quad \forall (i, j) \in \mathcal{A}, c \in \mathcal{C}
\]

\[
\pi_c^i - \pi_c^j \leq n_{cij} + M(1 - y_{ij}) \quad \forall (i, j) \in \mathcal{A}' \setminus \mathcal{A}, c \in \mathcal{C}
\]

\[
\pi_{o(c)}^i - \pi_{d(c)}^i \geq \sum_{(i, j) \in \mathcal{A}'} n_{cij} x_{ij}^c \quad \forall c \in \mathcal{C}
\]

\[
x_{ij}^c \in \{0, 1\} \quad \forall (i, j) \in \mathcal{A}', c \in \mathcal{C}
\]

\[
y_{ij} \in \{0, 1\} \quad \forall (i, j) \in \mathcal{A}' \setminus \mathcal{A}
\]

\[
z_m \in \{0, 1\} \quad \forall m \in \mathcal{M}
\]

\[
v_z^m \in \{0, 1\} \quad \forall z \in \mathcal{Z}, m \in \mathcal{M}
\]

\[
\pi_i^c, \pi_j^c \text{ free} \quad \forall i \in \mathcal{N}, c \in \mathcal{C}
\]

In the single-level representation, \(x_{ij}^c\) and \(y_{ij}\) are determined simultaneously. This change results in the loss of the total unimodularity. Therefore, it is necessary to reimpose integrality on the \(x_{ij}^c\) variables by replacing \(x_{ij}^c \geq 0\) by \(x_{ij}^c \in \{0, 1\}\). We can linearize the quadratic term in constraints (20) by adding the following constraints:

\[
w_{ij}^{cmz} = v_z^m x_{ij}^c \quad \forall (i, j) \in \mathcal{A}', c \in \mathcal{C}, m \in \mathcal{M}, z \in \mathcal{Z}
\]

\[
v_z^m + x_{ij}^c - w_{ij}^{cmz} \leq 1 \quad \forall (i, j) \in \mathcal{A}', c \in \mathcal{C}, m \in \mathcal{M}, z \in \mathcal{Z}
\]

\[
v_z^m + x_{ij}^c \geq 2w_{ij}^{cmz} \quad \forall (i, j) \in \mathcal{A}', c \in \mathcal{C}, m \in \mathcal{M}, z \in \mathcal{Z}
\]

This results in a mixed integer linear programming (MILP) problem, for which we had success solving small size problems by CPLEX.

### 4.2 A Greedy heuristic approach

We present a greedy iterative construction algorithm for cases that CPLEX is unable to solve the model. Let \(G(\mathcal{N}, \mathcal{A})\) denote the current hazmat network and \(G'(\mathcal{N}, \mathcal{A}')\) denote the hazmat network including all the potential-to-add links and the current links. Table 2 describes the notation used for the greedy algorithm. In each iteration, the algorithm will try to add a new link and update \(G\).

**Step 0.** Given the two networks \(G\) and \(G'\), the algorithm, in each iteration, first finds the shortest paths \(P_c\) and \(P_c'\), respectively, for each OD pair \(c \in \mathcal{C}\). When there exist multiple shortest paths,
Table 2: Notation for greedy algorithm

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Objective value (minimum of maximum risk values on zones)</td>
</tr>
<tr>
<td>$\eta^c_{ij}$</td>
<td>Risk of hazmat type $c$ traveling on link $(i, j)$</td>
</tr>
<tr>
<td>$\Omega_c$</td>
<td>Vector of risk values for the links on path $P_c$</td>
</tr>
<tr>
<td>$\Omega'_c$</td>
<td>Vector of risk values for the links on path $P'_c$</td>
</tr>
</tbody>
</table>

choose any shortest path arbitrarily. We consider paths $P_c$ and $P'_c$ as sets of links so that set operations are meaningful. Construct the following subset of OD pairs:

$$W = \{ c : c \in \mathcal{C}, \text{ and } P_c \neq P'_c \}$$

If $W$ is an empty set, go to Step 5. Otherwise, construct the following set of links:

$$\mathcal{I} = \bigcup_{c \in \mathcal{C}} \{ P_c \cap P'_c \}$$

which is the set of all common links.

**Step 1.** Given the set $\mathcal{I}$, the greedy algorithm solves the following $p$-center problem to find the location of HRTs:

$$\min_{z_m, v_z^m} \theta$$

subject to:

$$\sum_{(i,j) \in A \cap \mathcal{I}} \sum_{m \in \mathcal{M}} \sum_{c \in \mathcal{C}} \rho_{c} l_{ij} n_{c} d_{ij}^c \frac{f^{m}_{ij}}{F^{c}_{ij}} v_z^m \delta_{ij}^c \leq \theta \quad \forall z \in \mathcal{Z}$$

$$\sum_{m \in \mathcal{M}} z_m = p$$

$$\sum_{m \in \mathcal{M}} v_z^m = 1 \quad \forall z \in \mathcal{Z}$$

$$v_z^m \leq z_m \quad \forall z \in \mathcal{Z}, m \in \mathcal{M}$$

$$z_m \in \{0, 1\} \quad \forall m \in \mathcal{M}$$

$$v_z^m \in \{0, 1\} \quad \forall z \in \mathcal{Z}, m \in \mathcal{M}$$

where $\delta_{ij}^c$ is a constant that equals to 1 if $(i, j) \in P_c \cap P'_c$ for each OD pair $c$ and 0 otherwise. Let the solutions be $\bar{z}_m$ and $\bar{v}_z^m$. 

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**Step 2.** From the solutions $\bar{v}_z^m$ to the $p$-center problem in Step 1, we compute

$$
\eta_{ij}^c = \rho v_{ij} n_e q_{ij} f_{ij}^m \frac{\bar{v}_z^m}{F_{ij}}
$$

when $\bar{v}_z^m = 1$ and $(i, j) \in A_z$ for all $(i, j) \in (P_c \setminus P'_c) \cup (P'_c \setminus P_c)$ and $c \in W$. Then, we find the following values:

$$
\eta_c = \max_{(i, j) \in P_c \setminus P'_c} \{\eta_{ij}^c\} \quad \forall c \in W
$$

$$
\eta'_c = \max_{(i, j) \in P'_c \setminus P_c} \{\eta_{ij}^c\} \quad \forall c \in W
$$

$$
\zeta_c = \min\{\eta_c, \eta'_c\} \quad \forall c \in W
$$

Then we find the minimum of $\{\zeta_c : c \in W\}$ and the corresponding OD pair $\bar{c}$.

**Step 3.** For the chosen OD pair $\bar{c}$, we consider the following three cases:

- **Case 1:** If $\eta_c < \eta'_c$, then update

  $$
  \mathcal{I} \leftarrow \mathcal{I} \cup (P_c \setminus P'_c)
  $$

  $$
  W \leftarrow W \setminus \{\bar{c}\}
  $$

  and go to Step 4.

- **Case 2:** If $\eta_c > \eta'_c$, then we check the remaining budget $B$.

  - If $\sum_{(i, j) \in P_c \setminus P'_c} h_{ij} \leq B$, then update

    $$
    \mathcal{I} \leftarrow \mathcal{I} \cup (P'_c \setminus P_c)
    $$

    $$
    W \leftarrow W \setminus \{\bar{c}\}
    $$

    $$
    B \leftarrow B - \sum_{(i, j) \in P_c \setminus P'_c} h_{ij}
    $$

    and go to Step 4.

  - If $\min_{(i, j) \in A \setminus A'} \{h_{ij}\} \leq B < \sum_{(i, j) \in P_c \setminus P'_c} h_{ij}$, then update

    $$
    \mathcal{I} \leftarrow \mathcal{I} \cup (P_c \setminus P'_c)
    $$

    $$
    W \leftarrow W \setminus \{\bar{c}\}
    $$

    and go to Step 4.

  - If $B < \min_{(i, j) \in A \setminus A'} \{h_{ij}\}$, then update

    $$
    \mathcal{A} \leftarrow \mathcal{I} \cup \left( \bigcup_{c \in W} \{P_c \setminus P'_c\} \right)
    $$
and go to Step 5.

- Case 3: If \( n_i = n'_i \), consider the vectors

\[
\Omega_c = \{n_{ij}^c : (i, j) \in P_c \setminus P'_c\}
\]
\[
\Omega'_c = \{n_{ij}^c : (i, j) \in P'_c \setminus P_c\}
\]

Since the largest elements in the two sets \( \Omega_c \) and \( \Omega'_c \) were the same, we compare the second largest elements. If \( \Omega_c \) has the greater second largest element, then we follow the steps in Case 1; if \( \Omega'_c \) does, follow Case 2. If the second largest elements are the same again, we consider the third largest elements. We repeat this process until we can break the tie. When either of the two sets has no element to compare, assume zero.

**Step 4.** If \( W \) is an empty set, update

\[ A \leftarrow I \]

and go to Step 5; otherwise go to Step 1.

**Step 5.** We have an updated set of links \( A \) that contains new links to build. Given this set \( A \), we solve the following \( p \)-center problem to find the final solution.

\[
\min_{z_m, v^m} \theta
\]

subject to:

\[
\sum_{(i, j) \in A \cap A} \sum_{m \in M} \sum_{c \in C} \rho_{cij} n_{cij} f_{ij}^m v^m z_{ij} \leq \theta \quad \forall z \in Z
\]

\[
\sum_{m \in M} z_m = p
\]

\[
\sum_{m \in M} v^m z_m = 1 \quad \forall z \in Z
\]

\[
v^m z_m \leq z_m \quad \forall z \in Z, m \in M
\]

\[
z_m \in \{0, 1\} \quad \forall m \in M
\]

\[
v^m \in \{0, 1\} \quad \forall z \in Z, m \in M
\]

where \( \delta_{ij}^c \) is a constant that equals to 1 if link \( (i, j) \in A \) for each OD pair \( c \in C \) is used and 0 otherwise. Let the final solutions be \( \theta^*, z^*_m, \) and \( v^*_m \).

To illustrate the greedy algorithm we present a simple numerical example. Figure 3 shows the networks \( G \) and \( G' \) on which there are two zones, two hazmat shipments, an available budget \( (B) \), one potential-to-add link (shown by bolded arrow), two candidate locations, and one HRT to deploy. All the parameters required to solve the model by the heuristic algorithm are shown in
Figure 3: A simple example of a network: $G$ and $G'$

Table 3: Accident probabilities and number of shipments

<table>
<thead>
<tr>
<th>OD ID</th>
<th>Origin</th>
<th>Destination</th>
<th>$n_c$</th>
<th>$\rho_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>10</td>
<td>$2 \times 10^{-7}$</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>$3 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Tables 3, and 4. In this paper, we assume the scaling constant $F_{ij}^c = 10$ for all links $(i, j) \in \mathcal{A}'$ and hazmat shipments $c \in \mathcal{C}$.

Figure 4a shows the first step. The first OD pair has different shortest paths ($P_1 \neq P'_1$), while second OD pair has the same shortest paths ($P_2 \equiv P'_2$) on networks $G$ and $G'$. Therefore, $\mathcal{W}$ contains the first OD pair and the greedy algorithm selects one of the paths $P_2$ or $P'_2$. We present the algorithm as follows:

**Steps 0 and 1:**

$P_1 = \{(1, 2), (2, 4)\}$

$P'_1 = \{(1, 2), (2, 3), (3, 4)\}$

$P_2 = \{(3, 4)\}$

$P'_2 : \{(3, 4)\}$

$\mathcal{W} = \{1\}$

$B = 1000, B' = 600$

$\mathcal{I} = \{(1, 2), (3, 4)\}$

Table 4: Length, consequences, and opening cost for the links

<table>
<thead>
<tr>
<th>Link</th>
<th>$l_{ij}$</th>
<th>$q_{ij}^1$</th>
<th>$q_{ij}^2$</th>
<th>$h_{ij}$</th>
<th>$f_{ij}^1$</th>
<th>$f_{ij}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 2)</td>
<td>2</td>
<td>$1 \times 10^6$</td>
<td>$2 \times 10^6$</td>
<td>0</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>(2, 3)</td>
<td>1</td>
<td>$1 \times 10^6$</td>
<td>$2 \times 10^6$</td>
<td>600</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>(2, 4)</td>
<td>3</td>
<td>$5 \times 10^6$</td>
<td>$2 \times 10^6$</td>
<td>0</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>(3, 4)</td>
<td>1</td>
<td>$1 \times 10^6$</td>
<td>$2 \times 10^6$</td>
<td>0</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>
After finding the location of the HRT, we can determine the shortest path that may add less risk on zone 2, if chosen by hazmat carriers traveling between the second origin-destination points. Step 3 and Step 4 show the calculations for path selection. Since $\mathcal{W}$ is empty at the end of Step 4, the algorithm implements Step 5 and finds the final solution, then it stops. Based on the results at this iteration, link $(2, 3)$ is added to network $G$ and the HRT is deployed in site 2. The cost of adding link $(2, 3)$ is 600, thus, the remaining budget is equal to 400. Also, the minimum of the maximum zone total risk belongs to zone 2 with the value of 8. Figure 4b shows the schematic representation of the final solution found by greedy heuristic algorithm.

**Steps 2 to 5:**

$$\eta_1 = \max\{\eta^1_{24}\} = 10$$
$$\eta'_1 = \max\{\eta^1_{23}, \eta^1_{34}\} = \{2, 7\} = 7$$
$$\zeta_1 = \min\{\eta_1, \eta'_1\} = \eta'_1 = 7$$
$$\bar{c} = 1$$
$$\mathcal{I} = \{(1, 2), (3, 4), (2, 3)\}$$
$$\mathcal{A} = \{(1, 2), (2, 3), (2, 4), (3, 4)\}$$
$$\mathcal{W} = \{\emptyset\}$$
$$B = 400, B' = 600$$
$$\theta^* = 8, \quad Z^* = \{z_1 = 0, z_2 = 1\}$$
$$V^* = \{v^1_1 = 0, v^1_2 = 0, v^2_1 = 1, v^2_2 = 1\}$$
4.3 Special case with unique shortest path for each OD pair

If there is a unique shortest path between each origin-destination on networks $G$ and $G'$, then the upper-level problem, called as $P_1$ in section 3, reduces to a $p$-center problem. Let $C$ be the set of all $OD$ pairs. We suppose that for all $c \in C$, OD $c$ has a unique shortest path on both networks $G$ and $G'$. In this case, we can reduce network $G'$ to $G$ with the lower level problem’s optimal solution $(\bar{x}_{ij}^c)$. All the constraints related to the set of potential-to-add links become redundant, and the original upper level problem reduces to the $p$-center optimization problem called as $P_5$:

\[
(P_5) : \\
\min_{z_m,v_m^z} \theta \\
\text{subject to:} \\
\sum_{(i,j) \in A_z} \sum_{m \in M} \sum_{c \in C} \rho_{c}d_{ij}n_{c}q_{ij}^c f_{ij}^m z_{m}^{m} x_{ij}^c \leq \theta \quad \forall z \in Z \\
\sum_{m \in M} z_m = p \\
\sum_{m \in M} v_m^z = 1 \quad \forall z \in Z \\
v_m^z \leq z_m \quad \forall z \in Z, m \in M \\
z_m \in \{0,1\} \quad \forall m \in M \\
v_m^z \in \{0,1\} \quad \forall z \in Z, m \in M \\
\bar{x}_{ij}^c \text{ is an optimal solution}
\]

Here, $\bar{x}_{ij}^c$ is the optimal solution given by the lower level problem. $P_5$ represents the general form of $p$-center problem presented by Owen and Daskin (1998). Thus for this special situation, the greedy algorithm is able to solve $P_5$ by solving only a $p$-center problem.

5 Experimental Results

We organize our experimental results into four parts. The first part analyzes the benefits of jointly deciding on network design and locations of the HRTs. The second shows sensitivity analysis with respect to the number of available HRTs and the available budget. For parts one and two, the single-level form of the proposed model is optimally solved by means of an on-the-shelf branch and bound algorithm embedded in the CPLEX 12.6 solver. The third part examines the efficiency of the greedy algorithm for large-size network instances using a network derived from Albany, New York. The last part seeks a robust solution to the problem using the greedy algorithm. The heuristic algorithm is implemented in Java. All algorithms ran on a PC with 2 GB of RAM. We report the main features of the model as follows:

- $|C|$: number of OD pairs
5.1 Benefits of joint network design and location of HRTs

The data set we used for the material in Sections 5.1 and 5.2 corresponds to the Sioux Falls Road Network (Figure 5) with 24 nodes and 76 links. We consider 8 zones, 10 candidate sites for locating HRTs, and 14 potential-to-add links for this network. We define zones in such a way that each link belongs to only one zone. If a link belongs to two or more zones, that link can be broken into sub-links such that each sub-link belongs to only one zone and represents as a link itself. We indicate the candidate sites with black rectangles and the potential-to-add links with bolded arrows.
Table 5: Comparison between the joint and separate decision making to locate HRTs.

<table>
<thead>
<tr>
<th>NS</th>
<th>Travel Cost</th>
<th>$\theta^*$</th>
<th>Opened Sites</th>
<th>Opened Links</th>
<th>Travel Cost</th>
<th>$\theta^*$</th>
<th>Opened Sites</th>
<th>Opened Links</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6529</td>
<td>198.1</td>
<td>1, 9</td>
<td>4</td>
<td>8127</td>
<td>173.8</td>
<td>1, 3</td>
<td>10, 32</td>
</tr>
<tr>
<td>10</td>
<td>13139</td>
<td>317.5</td>
<td>1, 2</td>
<td>4</td>
<td>15853</td>
<td>290.2</td>
<td>1, 2</td>
<td>10, 32</td>
</tr>
<tr>
<td>15</td>
<td>20059</td>
<td>472.1</td>
<td>1, 2</td>
<td>4, 10</td>
<td>20466</td>
<td>461.2</td>
<td>1, 2</td>
<td>4, 7</td>
</tr>
<tr>
<td>20</td>
<td>29254</td>
<td>565.2</td>
<td>1, 10</td>
<td>4, 10, 32</td>
<td>29254</td>
<td>565.2</td>
<td>1, 10</td>
<td>4, 10, 32</td>
</tr>
<tr>
<td>25</td>
<td>34564</td>
<td>702.3</td>
<td>1, 7</td>
<td>4, 10, 29, 32</td>
<td>34605</td>
<td>612.3</td>
<td>1, 10</td>
<td>4, 10, 32</td>
</tr>
</tbody>
</table>

The results in Table 5 are presented for two different ways of calculating risk measurement. First, designing the road network for hazmat carriers based on the network design model proposed by Kara and Verter (2004), and then, locating hazmat response teams on the designated network by solving a $p$-center problem with the objective of minimization of maximum zone total risk. Second, a joint decision making process of designing a hazmat transportation network and locating HRTs by solving our proposed bi-level model in section 3. All problem instances in Table 5 are solved for the fixed values of parameters $B=1000$ and $p=4$.

As it is evident in Table 5, our proposed model gives a smaller maximum zone total risk value, $\theta^*$, for all instances except for the instance having 20 number of OD pairs. The objective value $\theta^*$ is the same for both methods in Table 5 for only one instance problem ($|C|=20$), and this happens when adding links does not help reducing the maximum risk on zones. For the instance with $|C|=25$, we have remarkable reduction in risk ($\theta^*$) by a nominal increase in the travel cost. Overall, our bi-level model shows a 12% improvement in the objective function, which is substantial.

5.2 Sensitivity analysis

There are two types of cost: cost of link addition and cost of HRT deployment. In order to evaluate the effect of cost on risk of hazmat shipments, a sensitivity analysis is performed based on the number of available HRTs and the total available budget for link addition.

Tables 6 and 7 show the results for different sizes of test problems on the Sioux Falls network. The optimal risk values ($\theta^*$) in Table 6 indicate that by increasing the number of available HRTs, a larger number of candidate sites can be opened. Consequently, the maximum zone total risk decreases, but increasing the number of HRTs to more than a specific level may not help to reduce the risk. That level introduces the optimal number of needed HRTs. As an example, for instances 5 and 6 in Table 6, increasing in number of HRTs from 3 to 4 does not show any reduction in $\theta^*$. Moreover, Table 7 represents the impact of budget level from 0 to 1000 on $\theta^*$. It is clearly shown that by increasing the budget from 0 to 400, we achieve a remarkable reduction in risk values. There is no more change in $\theta^*$ for a budget level larger than 400. Run time spent to find the optimal solutions ranges from 3 seconds to 48 hours in tables 6 and 7, implying that run time for different test problems is highly dependent on the network’s topology, number of hazmat shipments, and the model’s parameters.
Table 6: Results for the single-level MIP formulation of the bi-level model for # of HRT=1, 2, 3, and 4.

<table>
<thead>
<tr>
<th>Inst.</th>
<th></th>
<th>C</th>
<th></th>
<th># HRT</th>
<th>(^*\theta)</th>
<th>Travel Cost</th>
<th>Opened Links</th>
<th>Opened Sites</th>
<th>Run Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
<td>176.1</td>
<td>6529</td>
<td>4</td>
<td>1, 2, 3, 4</td>
<td>3 sec</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>3</td>
<td>176.1</td>
<td>6529</td>
<td>4</td>
<td>1, 2, 4</td>
<td>9 sec</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>2</td>
<td>183.7</td>
<td>6529</td>
<td>4</td>
<td>1, 4</td>
<td>3 sec</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1</td>
<td>210.5</td>
<td>6529</td>
<td>4</td>
<td>1</td>
<td>3 sec</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>4</td>
<td>294.9</td>
<td>13139</td>
<td>4</td>
<td>1, 2, 4, 10</td>
<td>18 sec</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>3</td>
<td>294.9</td>
<td>13139</td>
<td>4</td>
<td>1, 2, 4</td>
<td>20 sec</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>2</td>
<td>307.3</td>
<td>13139</td>
<td>4</td>
<td>1, 2</td>
<td>17 sec</td>
<td></td>
<td></td>
</tr>
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<td>8</td>
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<td>1</td>
<td>358.0</td>
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<td>4</td>
<td>1</td>
<td>7 sec</td>
<td></td>
<td></td>
</tr>
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<td>9</td>
<td>15</td>
<td>4</td>
<td>443.7</td>
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<td>1, 2, 4, 10</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>3</td>
<td>447.8</td>
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<td>1</td>
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<td>12 sec</td>
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<td></td>
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<td>13</td>
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<td>4</td>
<td>531.1</td>
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<td>10 min, 11 sec</td>
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<td></td>
</tr>
<tr>
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<td>20</td>
<td>2</td>
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<td>7 min, 34 sec</td>
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<td>17 min, 35 sec</td>
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<td></td>
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<tr>
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<td>25</td>
<td>1</td>
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<td>77 sec</td>
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<td>45295</td>
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<td>74 min, 20 sec</td>
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<td></td>
</tr>
<tr>
<td>22</td>
<td>35</td>
<td>3</td>
<td>692.1</td>
<td>45295</td>
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<td>56 min, 12 sec</td>
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<td>38 min, 30 sec</td>
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<td>55168</td>
<td>4, 10, 21, 29, 32</td>
<td>1</td>
<td>2 min</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7: Results for the single-level MIP formulation of the bi-level model for $B=0, 200, 400, 600, 800,$ and 1000.

| Inst | $|C|$ | $B$ | $\theta^*$ | Travel Cost | Opened Links | Opened Sites | Run Time |
|------|-----|-----|---------|------------|-------------|-------------|----------|
| 1    | 25  | 1000| 577.0   | 36564      | 4, 10, 29, 32 | 1, 3, 4, 10 | 5 min, 39 sec |
| 2    | 25  | 800 | 577.0   | 36564      | 4, 10, 29, 32 | 1, 3, 4, 10 | 8 min, 42 sec |
| 3    | 25  | 600 | 577.0   | 36564      | 4, 10, 29, 32 | 1, 3, 4, 10 | 43 min, 32 sec |
| 4    | 25  | 400 | 577.0   | 34605      | 4, 10, 32    | 1, 3, 4, 10 | 59 min, 40 sec |
| 5    | 25  | 200 | 686.0   | 42760      | 10           | 1, 3, 4, 10 | 22 min, 35 sec |
| 6    | 25  | 0   | 842.2   | 46204      | -            | 1, 3, 4, 10 | 12 sec |
| 7    | 30  | 1000| 684.1   | 39739      | 4, 10, 32    | 1, 2, 4, 10 | 8 min, 48 sec |
| 8    | 30  | 800 | 684.1   | 39739      | 4, 10, 32    | 1, 2, 4, 10 | 12 min, 42 sec |
| 9    | 30  | 600 | 684.1   | 39739      | 4, 10, 32    | 1, 2, 4, 10 | 16 min, 18 sec |
| 10   | 30  | 400 | 684.1   | 39739      | 4, 10, 32    | 1, 2, 4, 10 | 1 hr, 6 min |
| 11   | 30  | 200 | 793.1   | 47894      | 10           | 1, 2, 4, 10 | 21 min, 18 sec |
| 12   | 30  | 0   | 949.4   | 51338      | -            | 1, 2, 4, 10 | 12 sec |
| 13   | 35  | 1000| 684.1   | 45295      | 4, 10, 21, 29, 32 | 1, 2, 4, 10 | 74 min, 20 sec |
| 14   | 35  | 800 | 684.1   | 45295      | 4, 10, 21, 29, 32 | 1, 2, 4, 10 | 2 hr, 54 min |
| 15   | 35  | 600 | 684.1   | 45366      | 4, 10, 32    | 1, 2, 4, 10 | 3 hr, 37 min |
| 16   | 35  | 400 | 684.1   | 45366      | 4, 10, 32    | 1, 2, 4, 10 | 6 hr, 1 min |
| 17   | 35  | 200 | 793.1   | 53521      | 10           | 1, 2, 4, 10 | 3 hr, 5 min |
| 18   | 35  | 0   | 949.4   | 57452      | -            | 1, 3, 4, 10 | 7 sec |
| 19   | 40  | 1000| 684.1   | 50827      | 4, 10, 32    | 1, 2, 4, 10 | 5 hr, 37 min |
| 20   | 40  | 800 | 684.1   | 50827      | 4, 10, 32    | 1, 2, 4, 10 | 44 min, 50 sec |
| 21   | 40  | 600 | 684.1   | 50827      | 4, 10, 32    | 1, 2, 4, 10 | 2 hr, 2 min |
| 22   | 40  | 400 | 684.1   | 50827      | 4, 10, 32    | 1, 2, 4, 10 | 3 hr, 32 min |
| 23   | 40  | 200 | 793.1   | 58982      | 10           | 1, 3, 4, 10 | 47 min, 30 sec |
| 24   | 40  | 0   | 949.4   | 62290      | -            | 1, 3, 4, 10 | 20 sec |
| 25   | 45  | 1000| 684.1   | 55168      | 4, 10, 21, 29, 32 | 1, 2, 4, 10 | 36 min, 40 sec |
| 26   | 45  | 800 | 684.1   | 55168      | 4, 10, 21, 29, 32 | 1, 2, 4, 10 | 8 hr, 45 min |
| 27   | 45  | 600 | 684.1   | 55168      | 4, 10, 29, 32 | 1, 2, 4, 10 | 1 day, 20 hrs |
| 28   | 45  | 400 | 684.1   | 55315      | 4, 10, 32    | 1, 2, 4, 10 | 13 hrs, 1 min |
| 29   | 45  | 200 | 793.1   | 63470      | 10           | 1, 2, 4, 10 | 10 hr, 52 min |
| 30   | 45  | 0   | 949.4   | 66914      | -            | 1, 2, 4, 10 | 25 sec |
5.3 Efficiency of the greedy heuristic

We consider Albany’s road network with 149 links, 90 nodes, 6 zones, 7 potential-to-add links, and 21 candidate sites as a larger network to investigate efficiency of the greedy algorithm. Table 8 shows the results for 50 test problems (10 instances for each \(|C|=5, 10, 20, 30, 40\)). The optimal objective values found by CPLEX (Opt. Obj.) are very close to the heuristic objective values (Heu. Obj.) for all instances, even for some of them the Opt. Obj. values are equal to their corresponding Heu. Obj. values. By comparing run times in both approaches, we notice that greedy heuristic is computationally efficient and able to solve all sizes of test problems presented in Table 8 in less than 3 minutes. On the other hand, run time spent to find the optimal solutions ranges from 4 seconds to 10 hours and 30 minutes. The average gap for every 10 instances of the same size \(|C|\), ranges from 0.06% to 1.54%. We tried to find the optimal solution for a test problem with \(|C|=60\), but CPLEX was unable to solve the single-level model. For such cases, the greedy algorithm appears to provide a viable alternative.

5.4 Robust solution development

For hazmat problems, precise estimates of the data and their distribution information are almost impossible. In addition to this property of hazmat accidents, the accident consequences depend on the nature of accidents, hazmat types, population and properties surrounding the release points. As per a report prepared for Federal Motor Carrier Safety Administration in year 2001, which is available on the US Department of Transportation Pipeline and Hazardous Materials Safety Administration website (http://phmsa.dot.gov), a total likelihood of accidents for the portrait year was developed for all of the hazmat categories based on the historical record. All hazmats are separated into nine classes according to the Code of Federal Regulations. Table 9 shows these nine hazmat classes and their corresponding accident probabilities (\(\rho_c\)). All statistics and information presented in this section are taken from this report.

There are two kinds of enroute hazmat accidents: release and non-release. Since we consider HRT’s response time to enroute hazmat incidents in our model, we confine the data presented for the impact of hazmat accidents to the release type of accidents. The estimated occurrence rates are presented in Table 10 for three release types: release only, fire but no explosion, and explosion. We calculate the hazmat accident rates by dividing the number of hazmat release type by the total number of release incidents (for each type of accident release).

Table 11 shows the breakdown of estimated annual release accident impact costs into the three mentioned types. The total impact cost includes all costs of clean up, product loss, carrier damage, property damage, environmental damage, injury, fatality, evacuation and delay. Having estimated annual total cost and estimated annual total number of incidents, estimated total impact cost per incident can be easily found. Although we obtain the estimated annual impact costs or consequences for each incident release type \(r\) (\(q_r\)), estimated annual release rates for hazmat type \(c\) (\(\phi_c\)), and accident probabilities for hazmat type \(c\) (\(\rho_c\)), we still need to consider stochasticity and randomness to deal with the insufficiency of available data. That is, we may be able to obtain the minimum
Table 8: Comparison between the optimal and the heuristic solutions of the bi-level model

<table>
<thead>
<tr>
<th></th>
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<td>94106</td>
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<tr>
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<td>1.19</td>
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<td>49368.92</td>
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<td>71374.22</td>
<td>754049</td>
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<td>2.11</td>
<td>1.54</td>
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</table>

22
### Table 9: Hazmat accident rate per mile

<table>
<thead>
<tr>
<th>Hazmat Category</th>
<th>$\rho_c \times 10^{-7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1 — Explosives</td>
<td>6.58170</td>
</tr>
<tr>
<td>Class 2 — Gases</td>
<td>2.37209</td>
</tr>
<tr>
<td>Class 3 — Flammable liquids</td>
<td>4.96414</td>
</tr>
<tr>
<td>Class 4 — Flammable solids</td>
<td>6.85756</td>
</tr>
<tr>
<td>Class 5 — Oxidizers and organic peroxides</td>
<td>3.03833</td>
</tr>
<tr>
<td>Class 6 — Toxic (poison) materials and infectious substances</td>
<td>2.29576</td>
</tr>
<tr>
<td>Class 7 — Radioactive materials</td>
<td>3.94605</td>
</tr>
<tr>
<td>Class 8 — Corrosive materials</td>
<td>1.32109</td>
</tr>
<tr>
<td>Class 9 — Miscellaneous dangerous goods</td>
<td>7.16646</td>
</tr>
</tbody>
</table>

### Table 10: Estimated annual rate for all release accident types

<table>
<thead>
<tr>
<th>Hazmat Category</th>
<th>Estimated annual number of release accidents</th>
<th>Estimated annual release accident rate ($\phi_r %$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fire</td>
<td>Explosion</td>
</tr>
<tr>
<td>1</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>22.02</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.30</td>
</tr>
<tr>
<td>All Categories</td>
<td>65.20</td>
<td>24.42</td>
</tr>
</tbody>
</table>
Table 11: Probability distributions for estimated annual total impact costs for all incident types

<table>
<thead>
<tr>
<th>Incident Type</th>
<th>Total cost</th>
<th>Total number</th>
<th>Total cost per incident</th>
<th>Distribution of estimated impact cost (consequences)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enroute accident release only</td>
<td>276392494</td>
<td>678.02</td>
<td>407646.5</td>
<td>Uniform[400000, 500000]</td>
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<tr>
<td>Enroute accident fire</td>
<td>77160758</td>
<td>65.02</td>
<td>1186723.4</td>
<td>Uniform[1150000, 1250000]</td>
</tr>
<tr>
<td>Enroute accident explosion</td>
<td>62208606</td>
<td>24.42</td>
<td>2547236.3</td>
<td>Uniform[2500000, 2600000]</td>
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</tbody>
</table>

possible and maximum possible values, without specific distributional information. We define the interval data (uniform distribution) for \( q^r \) as shown in Table 11. In fact, the accident probability for each hazmat incident release type \( (\pi_c^r) \) can be calculated as \( \pi_c^r = \rho_c \times \phi_c^r \).

We consider interval \( [\pi_c^r - 0.5 \times 10^{-5}, \pi_c^r + 0.5 \times 10^{-5}] \) as the range in which each hazmat accident probability can change. Then, we randomly generate the model’s parameters in their interval data for 9 hazmat types and 3 enroute release incident types (3 \( \times \) 9 scenarios). All scenarios are defined for a test problem with 60 number of OD pairs on Albany’s road network. In order to find the best policy for all these 27 scenarios, we can take advantage of the computational efficiency of our greedy heuristic algorithm and solve the proposed model for all scenarios. The optimal policy does not depend on any single scenario but on all the 27 scenarios under consideration. Worst-case optimal policies provide guaranteed optimal performance for designed networks within the specified scenario range indicating the uncertainty. In such cases, finding the robust solution is crucial in risk assessment and the main tool used is minimax method, which suggests robust policies with guaranteed optimal performance (Rustem and Howe, 2009). Figure 6 presents the heuristic results for all scenarios, where the numbers on z-axis indicate the objective values \( (\theta^\psi) \) associated with policy \( \tau \), found by heuristic approach for scenario \( \psi = \tau \), and applied for scenarios \( \psi \neq \tau \), \( \psi = 1 \) to 27. Clearly, the \( \theta^\psi \) values where \( \psi = \tau \) have to be the minimum values for \( \psi = 1 \) to 27. It is noted that the heuristic algorithm does not necessarily find the optimal objective values. We obtained 6 different policies (solutions) for all 27 scenarios. As it is evident from Figure 6, policies 2 and 4 have smaller risk values in their worst-case scenarios (scenario 22). To choose the best policy, the minmax method has been applied as follow:

\[
\min_{\text{Policy}} \max_{\text{Scenario}_\psi} \{\theta^\psi\} \quad (45)
\]

Using data in shown Figure 6, we have: \( \min\{\theta_2^{22}, \theta_4^{22}\} = \theta_4^{22} = 534.3 \). Therefore, policy 4 is selected as the robust solution for all the 27 scenarios under consideration.

6 Summary and Future Research

In this paper, we have proposed a bi-level network design model for hazmat transportation. The model aims to minimize the maximum zone total risk and guarantees risk equity. There is a leader-follower relationship between the regulatory authorities and hazmat carriers. The authority tries to find the best locations among all candidate sites to deploy HRTs and to make additional
road segments available to hazmat carriers. On the other hand, the hazmat carriers select their minimum-cost routes on the designated network. Authority’s decisions about opening road segments and locating HRTs expand the possible route choices for hazmat carriers and help to reduce the average response time to a hazmat incident. The presented non-linear bi-level model is reformulated into a single-level mixed integer linear problem. The single-level model is solved using CPLEX 12.6 for a small size network. The greedy heuristic approach is able to find very good (near optimal or sometimes optimal) solutions in a short time period for large size test problems. Experimental results show that joint decision of network design and deployment of emergency response team results in significantly better risk reduction. Increasing the total number of available HRTs for deployment and the total available budget for link addition has a remarkable impact on risk mitigation.

We also conclude that the greedy algorithm is computationally efficient and delivered high quality solutions. Finally, a robust solution is obtained for 27 scenarios under consideration by applying the proposed heuristic approach for a large size test problem ($|C|=60$).

In practice, other emergency response units may be dispatched to the incident site, such as, Emergency Medical Service, Fire, and Police. This suggests a future work dedicated to joint deployment of all emergency units. Furthermore, the average response time is highly dependent on traffic congestion and incident location, thus, a robust solution should be investigated considering uncertainty in determining the response time. Another research opportunity is to consider the problem of adding additional HRTs to a situation where a certain number of HRTs already exist.
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References


