Routing of a Hazmat Truck in the Presence of Weather Systems

by

Vedat Akgün
Emery Worldwide
One Emery Plaza
Vandalia, Ohio 45377, USA

Amit Parekh and Rajan Batta
Department of Industrial Engineering
University at Buffalo
State University of New York
342 Lawrence D. Bell Hall
Buffalo, NY 14260-2050, USA

Christopher M. Rump
Department of Applied Statistics and Operations Research
Bowling Green State University
Bowling Green, Ohio 43403, USA

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1To whom all correspondence should be addressed. Email: batta@eng.buffalo.edu
Abstract

This paper focuses on the effects of weather systems on hazmat routing. We start by analyzing the effects of a weather system on a vehicle traversing a single link. This helps characterize the time-dependent attributes of a link due to movement of the weather systems. This analysis is used as a building block for the problem of finding a least risk path for hazmat transportation on a network exposed to such weather systems. Several methods are offered to solve the underlying problem, and computational results are reported. We draw two conclusions from this paper. First, it is possible to determine the time dependent attributes for links on a network provided that some assumptions on the nature of the weather system are made. Second, heuristics can provide effective solutions for practical size problems while allowing for parking the vehicle to avoid weather system effects.

Scope and Purpose

Shipment of hazmat exposes the public to risk in case of an accident. Routing decisions for hazmat are dynamically affected by weather systems, since these systems dramatically change the accident probabilities, the impact radius of the hazmat, and the speed of the vehicle. The focus of this paper is on characterizing these effects under certain assumptions about weather systems, and on the development and testing of practical solution methods for this problem.
1 Introduction and Literature Review

The code of Federal Regulations [1] has a special chapter (49 CFR Parts 100-177) for hazmat listings, hazmat transportation, oil transportation and pipeline safety. Approximately 1.5 billion tons of hazmat (of which 65% are carried by truck and rail) are being transported yearly in the United States [2]. Worldwide generation of hazmat is estimated as 3-4 billion tons per year [3].

Routing is a critical factor to consider in hazmat logistics. Although the fatalities due to hazmat-related traffic accidents are almost negligible as compared to the deaths in ordinary traffic accidents, hazmat risks are considered unacceptable [4]. The main objective of hazmat routing is to determine the optimal path(s) for routing the hazmat on a network subject to certain criteria. The objective, which can be either a single or multiple criteria, is typically based on risk, equity and cost considerations. The choice of objective criteria will highly influence the selection of the “best” route in the presence of a NTMBY (Not Through My BackYard) syndrome. We consider routing of the hazmat truck with the objective to minimize the “risk” involved in the transport. Risk is represented in different ways in the literature. It has be modeled as expected consequence [5], or the population exposed to consequences due to impact [6], incident probability [7], or conditioned on the probability of the first incident [8].

Many models for hazmat routing have been proposed. The reader is referred to papers by Erkut and Verter ([4], [3] and [10]) for a complete review of such work. In what follows we provide a brief summary of some key papers. Abkowitz and Cheng [11] developed a model that incorporates risk as a cost into a framework for optimizing the routing of hazardous materials. Batta and Chiu [6] consider the problem of routing an undesirable vehicle on a network. The objective is to find a path that minimizes the weighted sum of lengths over which the vehicle is within a threshold distance from the population centers. Chin and Paul [12] lay out a bicriterion problem that minimizes the distance traveled and the population at risk within a fixed band width along the path. Gopalan et al. [13] focus on development and analysis of a model to generate an equitable set of routes for hazardous material trans-
portation. Lindner et al. [14] follow-up on the observation that risk equity between these zones is achieved only after all the shipments are done, and may be severely violated due to an accident at an intermediate stage of the shipment process. Kalelkar and Brooks [15] use decision analysis in optimizing choices regarding material transportation. Scanlon and Cantilli [16] propose a new measure for risk assessment in hazardous material transportation rather than simply risk-of-incident. Zografos and Davis [17] examine a systemwide routing of hazardous materials as a means of reducing the threat to the population residing along the links of an entire transportation network. Cox and Turnquist [18] solve a closely related problem to routing, that of scheduling of these shipments in the presence of curfews. Wi- jeratne [19] developed a stochastic multiobjective shortest path algorithm to find the set of non-dominated paths in a network that minimizes all objectives, where some or all of the attributes are uncertain. Our work is new in that it analyzes hazmat routing under the influence of weather systems.

Weather systems dynamically affect the accident probabilities, risk, equity and costs involved. There are several articles in the modelling of wind effects. Karkazis and Boffey [20] propose a model which incorporates meteorological conditions in determining the dispersion of pollutants. Patel and Horowitz [21] considered the diffusion of gases over wide areas from possible spills during transportation of hazardous material while determining the least risk path through the network. They consider specific wind direction, uniform average wind direction, maximum concentration wind direction, wind-rose averaged wind directions and speeds and multiday routing with uncertain weather conditions. However, there is a need to analyze other elements to a weather system. For example, a rain or a snow event will change both the node and link accident probabilities as well as the travel speed through the affected sections of the network. Akgiin [22] considers these factors and develops model for routing in the presence of weather systems.

Weather systems are usually modelled as a front having a certain bandwidth of effect and a direction of movement. They will effect a portion of the network for a certain amount of time. Therefore, routing and scheduling decisions of hazmat shipments based on weather
effects will bring new alternatives such as delaying the shipment, changing the route(s), re-routing, etc. In this case, a path with the minimum accident probability might become a path with a higher accident probability, or the one with minimum travel time may now be dominated by another path(s). These ramifications are, of course, due to the dynamic nature of the weather system and hence the time-varying nature of the weights on links in the transportation network. Finding a shortest path between an origin location and destination in this dynamic network gives rise to a time-dependent shortest path problem (TDSPP), which was first introduced by Cooke and Halsey [23]. Their proposed algorithm solves the TDSPP from every node to the destination using finite departure times. Thus all the link travel times also are elements of a finite set. The algorithm has theoretical computational complexity $O(V^3M)$. Dreyfus [24] suggested a label setting procedure that is a generalization of Dijkstra’s [25] static shortest path algorithm. This approach calculates the TDSP between two nodes for one departure time step in $O(V^2)$. Orda and Rom [26] proposed a procedure which unlike the Dreyfus approach is not limited to FIFO (First-in-First-out) links. They studied the problem in the context of unrestricted waiting and source waiting models. Ziliaskopoulos and Mahmassani [27] introduce an algorithm that calculates the TDSP’s from all nodes to the destination for every time step over a given time horizon.

To solidify the motivation for this work, we now briefly discuss a study that relates accident probabilities on roadways with weather conditions. Saccomano and Chan [7] analyzed the truck accident data for Toronto and estimated the probability of an accident on a dry urban expressway to be $2.379 \times 10^{-6}$ per mile for unrestricted visibility and $4.054 \times 10^{-6}$ for restricted visibility. This amounts to a 60% increase due to night travel or weather systems. They state that regardless of the strategy selected, random variations in the environmental influences can have significant effect on safety. Also, they conclude that minimizing risk is the best strategy to adopt. It produces incremental benefits to society which significantly exceed the incremental costs. Although transport costs increase, the savings in damages from fewer accidents and their effects is significantly greater. In the light of these conclusions we aim to minimize the expected risk for routing of hazardous material in the presence
of weather systems. This paper makes two contributions. First, it develops a method by which the effect of weather systems on link attributes and risk can be characterized. Second, it develops and tests practical solution algorithms for routing a hazmat truck in the presence of such weather systems, while allowing for the possibility of parking.

The rest of this paper is organized as follows. In Section 2 we characterize the dynamic variations of attributes of a link of the network due to movement of the weather systems. In Section 3, these time-varying link weights are utilized by our solution algorithms - which include an exact TDSPP routing algorithm (due to Ziliaskopolus and Mahmassani [27]) and a variety of heuristic methods. In Section 4, the network and other implementation details of the computational study are described. Section 5 includes a comparison of the heuristics to the optimal solution for small-scale problems and a comparison of performance among the heuristics on large-scale problems. Finally, we suggest directions for future research.

## 2 Characterizing Link Attributes

To characterize link attributes it is sufficient to focus on a single link. To maintain the analytical tractability of the approach we assume circular weather systems that move in a linear direction. In essence, we extend the model developed by Batta and Chiu [6] by incorporating the effects of a dynamic weather system e.g., modified travel speed, accident probabilities, and threshold distance of the hazmat being transported.

### 2.1 Notation

Let $G = (N, E)$ be a connected, undirected and planar transportation network with node set $N$ and link set $E$. A point $s$ on the plane has Cartesian coordinates $(x_s, y_s)$. A node $i$ has positive weight $w_i$ that typically signifies the population at this point. Similarly, a positive weight $g_{(i,j)}$ is associated with a link $(i, j)$. In addition, a population density function $f_{(i,j)}(z)$ is associated with link $(i, j)$, where $z$ ranges from 0 to $l_{(i,j)}$, the length of the link $(i, j)$. The population density function $f_{(i,j)}(z)$ is normalized such that $\int_0^{l_{(i,j)}} f_{(i,j)}(z)dz = 1$. For a single
link \((i, j)\), we assume, without loss of generality, that this link is horizontal to the Cartesian plane with the coordinates of node \(j\) being \((0, l_{i,j})\). Distances on the plane are measured by the Euclidean metric, with the distance between two points \(a\) and \(b\) on the plane being

\[
d(a, b) = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}.
\]

Let \(W\) be a weather system that is assumed to be circular with radius \(r\) and center \((x_w(0), y_w(0)) = (x_w, y_w)\) at time \(t = 0\). The weather system travels on a straight line with a \(\theta\)-degree angle from the horizontal at a constant speed of \(v_w\). The center of \(W\) at time \(t\) is denoted by \((x_w(t), y_w(t))\), where

\[
\begin{align*}
x_w(t) &= x_w + (v_w \cos \theta)t, \text{ and} \\
y_w(t) &= y_w + (v_w \sin \theta)t.
\end{align*}
\] (1)

Let \(v\) be the travel speed of the hazmat-carrying vehicle. There are finite probabilities, \(h_i\) and \(h_j\), of an accident when the vehicle travels through node \(i\) and \(j\), respectively. Along the link \((i, j)\) there is an associated function of accident rate, \(q_{i,j}\), measured in probability of an accident per unit length of movement. In general (see [6]), the values of \(h_i\) and \(h_j\) are on the order of \(10^{-7}\), and the value of \(q_{i,j}\) is on the order of \(10^{-7}/\text{mile}\).

We assume that the travel speed of the vehicle and the accident probabilities (at nodes and links) change in the presence of a weather system, and are denoted by \(v', h'_i, h'_j, q'_{i,j}\). For most situations we would expect that travel will be slower and the accident probabilities will increase inside \(W\). Therefore, we assume \(v' \leq v\), \(h_i \leq h'_i\), \(h_j \leq h'_j\), and \(q_{i,j} \leq q'_{i,j}\).

**2.2 Weather Effects on Travel Time**

Assume that the vehicle starts at node \(i\) at time \(t = 0\) and travels towards node \(j\). There are three possible cases:

1. The vehicle is never in \(W\),
2. The vehicle starts outside \(W\), and then enters \(W\) and
3. The vehicle starts in \( W \).

In the latter two cases, the vehicle either

(a) stays in \( W \) until node \( j \) is reached, or
(b) exits \( W \) before reaching node \( j \).

Note that the vehicle (while traveling from node \( i \) to node \( j \) on the straight-line link \((i, j)\)) cannot re-enter \( W \) once it exits. This follows from the fact that \( W \) is assumed to be circular in shape with a linear motion.

Let \((x(t), y(t))\) denote the location of the vehicle along the horizontal link \((i, j)\) at time \( t \). When the vehicle is traveling at speed \( v \) starting at time \( t = 0 \), these coordinates are given by \( x(t) = vt, y(t) = 0 \). The distance between the vehicle and the center of \( W \) at time \( t \), \( d(t) \), is such that

\[
    d^2(t) = [x(t) - x_{w}(t)]^2 + [y(t) - y_{w}(t)]^2 = At^2 + Bt + C,
\]

where, using (1),

\[
    A = v^2 - 2vv_{w}\cos \theta + v_{w}^2,
\]

\[
    B = 2x_{w}v_{w}\cos \theta + 2y_{w}v_{w}\sin \theta - 2vx_{w}, \text{ and}
\]

\[
    C = x_{w}^2 + y_{w}^2.
\]

The amount of time to traverse the link \((i, j)\) under normal conditions is \( \tau_{(i,j)} = l_{(i,j)}/v \).

Let \( t^*_{1} \) be the first time the vehicle enters \( W \) (defined to be 0 if the vehicle starts in \( W \)), \( t^*_{w} \) be the time it stays inside \( W \), and \( t^*_{2} = t^*_{1} + t^*_{w} \), be the time that the vehicle leaves \( W \).

We now present a simple method to determine the case that applies to a particular set of the parameters \((v, v', v_{w}, x_{w}, y_{w} \text{ and } \theta)\), and to compute the values of \( t^*_{1} \) and \( t^*_{2} \). To find \( t^*_{1} \) we solve the equation \( d^2(t) = r^2 \) using the vehicle location \( x(t_1) = vt, y(t_1) = 0 \) and the coordinates of the weather system \( x_{w}(t_1) = x_{w}(0) + (v_{w}\cos \theta)t_1 \) and \( y_{w}(t_1) = y_{w}(0) + (v_{w}\sin \theta)t_1 \) given by (1) at time \( t = t_1 \). Let the roots of this quadratic equation be

\[
    t^*_1 = -\frac{B \pm \sqrt{B^2 - 4AC - r^2}}{2A},
\]

where, using (1),

\[
    A = v^2 - 2vv_{w}\cos \theta + v_{w}^2,
\]

\[
    B = 2x_{w}v_{w}\cos \theta + 2y_{w}v_{w}\sin \theta - 2vx_{w}, \text{ and}
\]

\[
    C = x_{w}^2 + y_{w}^2.
\]
The smaller root, \( t_1^- \), corresponds to an entrance time. (The larger root, \( t_1^+ \), corresponds to an exit time only under the assumption that the vehicle maintains a speed \( v' = v \) inside \( \mathcal{W} \).)

Suppose the roots are real; if not, then we are in Case 1 and assign \( t_1^+ = t_2^- = -\infty \). If \( t_1^- \leq 0 \), then \( t_1^- = 0 \) and we are in Case 3; else if \( t_1^- \leq \tau_{(i,j)} \), then \( t_1^+ = t_1^- \) and we are in Case 2; else we are in Case 1 and assign \( t_1^+ = t_2^- = -\infty \).

In Cases 2 and 3, to find the time \( t_w^* \) spent inside \( \mathcal{W} \) we orient around the entrance time \( t_1^* \) and solve \( d^2(t_w) = r^2 \) using the values \( x(t_w) = vt_w^* + v't_w, \ y(t_w) = 0, \ x_w(t_w) = x_w(t_1^*) + (v_w \cos \theta)t_w \) and \( y_w(t_w) = y_w(t_1^*) + (v_w \sin \theta)t_w \). Since the vehicle is starting at time \( t_1^* \) in \( \mathcal{W} \), there exist real-valued roots \( t_w^\pm \) to this quadratic equation, where \( t_w^- \leq 0 \) (since the vehicle starts in \( \mathcal{W} \)) and \( t_w^+ \) corresponds to the exit time (starting from time \( t_1^* \)). If \( vt_w^* + v't_w^+ \geq l_{ij} \), then we are in subcase (a) with \( t_w^* = (l_{(i,j)} - vt_1^+)/v' \); else we are in subcase (b) with \( t_w^* = t_w^+ \). In either case, the vehicle leaves the weather system at time \( t_2^* = t_1^* + t_w^* \).

In the above analysis, we assume that the vehicle begins travelling at time 0. To generalize, suppose it were possible to start travel at any time \( s \) in the time window \([0, T_s]\), where \( T_s \) is the latest permissible start time. Since the weather system remains moving, whether the vehicle is or not, the above analysis can be modified to incorporate this general vehicle start time \( s \in [0, T_s] \) by replacing the coordinates of the weather system’s initial location \((x_w, y_w)\) with \( x_w^*(s) = x_w + v_w \cos \theta \cdot s \) and \( y_w^*(s) = y_w + v_w \sin \theta \cdot s \) in equation (1). In this case, the distance between the vehicle and the center of \( \mathcal{W} \) at time \( t \), \( d(t) \), is such that
\[
d^2(t) = A(t^2) + B(s) + C(s),
\]
where
\[
B(s) = B + 2v_w(v_w - v \cos \theta) \cdot s, \quad \text{and}
\]
\[
C(s) = C + v_w^2 \cdot s^2 + 2(x_wv_w \cos \theta + y_wv_w \sin \theta)s.
\]

In doing so, we discover that the time spent in the weather system while traversing the link is concave in the start time \( t_s \), as established in Theorem 1.

**Theorem 1** The time that the vehicle spends in the weather system while traversing a link is concave in the starting time \( s \in [0, T_s] \).
Proof. The time in the weather system, \( t_w^*(s) \), is now a function of \( s \). We examine the two subcases above corresponding to whether the vehicle stays in the weather system until reaching the end of the link or not.

(a) If the vehicle stays until reaching node \( j \), \( t_w^*(s) = (l_{(i,j)} - vt_1^*(s))/v' \), where \( t_1^*(s) \) is the time the vehicle enters the weather system, given that the vehicle starts at time \( s \). Thus, the concavity of \( t_w^*(s) \) can be established by showing the convexity of \( t_1^*(s) \).

From the above analysis, only Case 2 is of interest. (Otherwise, \( t_1^*(s) = 0 \), which is clearly convex in \( s \).) Here, \( t_1^*(s) = t_1^- (s) \), where

\[
t_1^- (s) = - \left( B(s) + \sqrt{B^2(s) - 4AC(s)} \right) / 2A \quad \in \quad [0, \tau_{(i,j)}].
\]

Differentiation with respect to \( s \) gives

\[
8A \cdot (t_1^-)''(s) = -4B''(s) + [B(s)^2 - 4AC(s)]^{-3/2}[2B(s)B'(s) - 4AC'(s)]^2
\]

\[
-2[B(s)^2 - 4AC(s)]^{-1/2}[2B'(s)B'(s) + 2B''(s)B(s) - 4AC''(s)],
\]

where \( B'(s) = 2v_w(v_w - v \cos \theta), B''(s) = 0, C'(s) = 2v_w^2 \cdot s + 2(x_w v_w \cos \theta + y_w v_w \sin \theta) \) and \( C''(s) = 2v_w^2 \). Also, \( A \geq 0 \) since \( v^2 - 2vv_w \cos \theta + v_w^2 \geq v^2 - 2vv_w + v_w^2 = (v - v_w)^2 \).

Since the root \( t_1^- (s) \) is real-valued, only the sign of the last term in brackets is uncertain. This term reduces to \( 8v^2 v_w^2 (\cos^2 \theta - 1) \leq 0 \), which establishes that \( (t_1^-)''(s) \geq 0 \). Hence, \( t_1^*(s) \) is convex, which completes the proof.

(b) If the vehicle exits the weather system before reaching node \( j \), \( t_w^* = t_w^+(s) \). Since \( t_w^+(s) \) is the larger root of the solution to a quadratic equation, a similar analysis establishes its concavity in \( s \).

This concavity result suggests that in order to minimize exposure to the weather system, a vehicle should begin travel on a link at one of the endpoints of the permissible travel window, [0, \( T_s \)].
2.3 Weather Effects on Risk

A hazmat release at point $c$ poses a threat to point $z$ if it is within a threshold distance, $\lambda$, of point $z$. However, if the point $c$ is in $W$, the threshold distance is $\lambda'$, which may be greater or smaller than $\lambda$ depending on the type of weather system and the type of hazmat. We accordingly define the indicators:

\[
\delta(z, c) = \begin{cases} 
1, & \text{if } d(z, c) \leq \lambda \\
0, & \text{otherwise}
\end{cases},
\]

and

\[
\delta'(z, c) = \begin{cases} 
1, & \text{if } d(z, c) \leq \lambda' \\
0, & \text{otherwise}
\end{cases}.
\]

We now calculate $R(i,j)$, the risk of travel on link $(i,j)$. To do this, we need to find $F(z; i, j)$, the threat that travelling on link $(i,j)$ poses on a point $z$. Once we do this, we can compute

\[
R(i,j) = \sum_{z \in N} w_z F(z; i, j) + \sum_{(l,k) \in A} \int_{l(i,k)}^{l(l,k)} g_{(l,k)} f_{(l,k)}(z) F(z; i, j) dz.
\]

An explanation of equation (2) is as follows. The first term accounts for the potential damage that the vehicle could do to nodal population centers on the network. The second term does the same for population that resides on links of the network (which is continuously distributed).

We now focus on calculating $F(z; i, j)$. To do this, we define (following the discussion in Batta and Chiu [6]) the $\lambda$-neighborhood of a link $(i,j)$ as the set of points within $\lambda$ distance from at least one point on link $(i,j)$. This $\lambda$-neighborhood is divided into four regions as shown in Figure 1.

Consider a point $z = (x_z, y_z)$ and let the intersections (if any) of a circle of radius $\lambda$ centered at $z$ with the horizontal axis be given by $x^+(z)$ and $x^-(z)$, where $x^\pm(z) = x_z \pm \sqrt{\lambda^2 - y_z^2}$, if $\lambda \geq |y_z|$. If $\lambda < |y_z|$, then there is no intersection and the point $z$ lies outside the $\lambda$-neighborhood of the link $(i,j)$. Also, if $x^-(z) < 0$ and $x^+(z) > l(i,j)$, then
the point $z$ lies outside the $\lambda$-neighborhood of the link $(i, j)$. The remaining situations are identified by the regions labelled I, II, III and IV in Figure 1.

We are now armed with the notation and concepts we need to calculate $F(z; i, j)$, the threat that travelling on link $(i, j)$ poses on a point $z$. Consider the case $x(t_1^*) \in (0, l_{(i,j)})$ and $x(t_2^*) \in (0, l_{(i,j)})$. (Other cases will be handled similarly.) Clearly, $x(t_1^*) \leq x(t_2^*)$. For this situation,

$$F(z; i, j) = h_i \delta(z, i) + \int_0^{x(t_1^*)} q_{(i,j)} \delta(z, (y, 0))dy + \int_{x(t_1^*)}^{x(t_2^*)} q'_{(i,j)} \delta'(z, (y, 0))dy + \int_{x(t_2^*)}^{l_{(i,j)}} q_{(i,j)} \delta(z, (y, 0))dy + h_j \delta(z, j).$$ (3)

Equation (3) can be explained as follows. The first and last terms account for the possibility of an accident at node $i$ and node $j$, respectively, which in this case occur with the vehicle outside the weather system $\mathcal{W}$. The second and fourth terms are to account for the possibility of an accident along the link before entering and after exiting $\mathcal{W}$, respectively. The third term is for an accident along the link while in $\mathcal{W}$.

We note that equation (3) is valid for the case where the vehicle starts at $i$ and ends at $j$. When computing the total risk of a path that had multiple links, using equation (3) would double count accident probabilities on intermediate nodes of the path. As pointed out in
Batta and Chiu [6], this can be handled by using \(h_i/2\) and \(h_j/2\) (instead of \(h_i\) and \(h_j\)) for intermediate nodes \(i\) and \(j\). Thus, equation (3) and its variants (that account for the other cases) can be used together with equation (2) to find the total risk associated with a specific path for the vehicle on the network.

3 Routing Methods

In Section 2, we analyzed the movement of a vehicle to determine the risk of travel on the link and the time taken to traverse the link in the presence of a weather system crossing the link. These time-varying link weights are now utilized to route the vehicle across a network. Suppose that the vehicle must start at an origin node, \(O\), at time \(t = 0\), and reach the destination node, \(D\), by time \(t = T\). Our objective is to find the least risk path for this vehicle from the origin to the desitnation. Since the risk and time to traverse a link are time dependent due to weather systems, this reduces to an one-to-one time-dependent shortest path problem (TDSPP).

3.1 Time Dependent Shortest Path Problem (TDSPP)

We can solve the TDSPP using an “exact” algorithm – this method is exact under the premise that time has been discretized finely enough. To do this we can use a \(O(|N|^3M^2)\) label correcting algorithm that is described in Ziliaskopolus and Mahamassani [27], here \(M\) is a large integer such that \(t_0\) to \(t_0 + M\delta\) is time interval of interest, \(t_0\) is the starting time, and \(\delta\) is a small time interval during which weather conditions remain unchanged. We refer the reader to [27] for details about the procedure. The only modification in our implementation of this procedure is that we use the modified link lengths calculated as described in Section 2.3.

Memory and computational requirements can be reduced by finding an upper bound, \(U_i\), and lower bound, \(L_i\) for the possible arrival times at node \(i\). In this way there are fewer labels for each node. This can be achieved as follows:

1. Find for each link the minimum possible travel time \(t_{ij}^* = \min \{t_{ij}(t)\} \quad t = 1, 2, \ldots, T.\)
2. Use Dijkstra’s labeling algorithm to find the minimum travel time (using $t_{ij}^*$) between the origin and all other nodes (including the destination). Let the minimum arrival time to a node $j$ be $L_j$. Therefore the minimum time label for node $j$ is $t = L_j$.

3. Use Dijkstra’s labeling algorithm to find the minimum travel time (using $t_{ij}^*$) between a node and the destination. Let the minimum arrival time from node $j$ to destination be $U_j$. Then $T - U_j$ is the latest possible departure time from node $j$ so that the destination is not reached later than $T$. Therefore this is the maximum time label for node $j$.

The TDSPP algorithm essentially permits parking after every $\delta$ units of time. Thus an optimal path may have numerous instances of parking, i.e. the vehicle may be scheduled to stop several times enroute to the destination. In practice there are likely to be limitations on the number of times the hazmat carrying truck can park and also the minimum/maximum amount of time that it should park. The algorithm doesn’t take this into consideration. Hence the algorithm will give much lower risk values than are practically possible. Also, due to memory and computation time limitations, this algorithm can be used realistically only for small-sized networks. The computational runs showed that the algorithm could solve a path between O-D pairs whose path length varied from 122 to 845 miles with run times varying from 3 to 51 minutes. In these runs, time was discretized in intervals of 5 minutes. As the time intervals are made smaller and/or the network size increases, the computational time and memory requirements become highly intensive. For this reason, in the rest of this section we describe several heuristic methods (based on similar algorithms available in the literature) which can be used for solving practical problems. These methods are designed to work in reasonable time and also limit the number of parking opportunities to just one.

3.2 $k$-Shortest Path Heuristic (KPATH)

A logical approach is to search through a set of paths and select the best one. In this heuristic, a set of $k$-shortest paths is found for the network with time-invariant weights and
these paths are then evaluated using time-variant weights. The paths are evaluated for improvement due to parking at a node, and the path with the least weight is selected. The steps involved in the KPATH heuristic are as follows:

1. Yen’s $k$-shortest path algorithm [29] is used with static weights i.e. the expected risk calculated ignoring the weather system effects. The expected risk ignoring the weather system effects is calculated using the procedure laid out in [6]. This is used as link weights to generate the $k$ shortest paths.

2. After this set of shortest paths is found, the weight of each path is evaluated with the dynamic weights.

3. The slack time for each path is calculated. The initial value of slack time is the difference between the travel time of the path and $T$, the maximum time allowed by which the shipment has to reach destination node $D$.

4. Each path is evaluated for improvement by assigning all the slack time to a node on the path.

5. If the destination is not reached by time $T$ due to addition of this slack, i.e. parking on the node, then the slack is reduced by one unit and the procedure is repeated iteratively till the time constraint is met.

6. Steps 4 and 5 are repeated for each node on the vehicle’s path and the best solution is reported.

Thus, each path is optimized by considering parking at every node on the path such that the $T$ time limit is met. The path with the least consequence is selected. For the problems we considered, the travel time is less than ten hours, so in a practical sense the driver would stop at most once, rather than making several small stops as suggested by the TDSPP algorithm. The fact that we choose to do an “all or nothing” parking assignment is justified by the result of Theorem 1.
A potential drawback of this heuristic is the fact that most of the $k$ shortest paths are small variations of each other. Therefore, it will likely be difficult to find spatially dissimilar and feasible paths to avoid the weather systems unless $k$ is very large – which increases the computational time and memory requirements substantially.

### 3.3 Dissimilar Path Heuristic (DISSIM)

Another heuristic is based on the dissimilar path method [30]. In this heuristic, a set of paths is first found by using either the KPATH or IPM [31]. The dissimilar path method [30] is then applied to obtain a subset of paths of maximum disparity. A stricter time limit of $T^*$ is set, which accounts for the travel time increase due to weather systems. Paths obtained from the dissimilar path method with travel time greater than $T^*$ are discarded. Then the remaining paths are evaluated with the dynamic weights, and for benefits due to parking using the same procedure as in the KPATH Heuristic. The advantage of this method is that it has the potential of identifying spatially dissimilar paths that may avoid the weather systems.

### 3.4 Iterative Weather System Heuristic (IWS)

This heuristic is based on the “Iterative Penalty Method” [31], where the procedure first finds the shortest path, and then penalizes the arcs (or nodes) on this path using a preset penalty factor. The two-step procedure continues until the desired number of alternate paths are found. In the IWS the penalties are based on the dynamic weights induced by the weather system(s). Steps of the heuristic are as follows:

1. Find the optimal path with static weights and save it as an incumbent solution.
2. Calculate the link weights by incorporating the weather system effects.
   (a) If there is no change in the link weights, then the current path is optimal.
   (b) Else update the weights for the affected links.
3. Repeat until
   (a) there is no change in the link weights, or
(b) the current path is identical to the incumbent path.

The advantages of this heuristic over the IPM are (i) the penalty mechanism is simple and fixed, and (ii) only the links that are affected by the weather system are penalized. However, there are drawbacks: these penalties are permanent. When a link is penalized, it is based on the time when the vehicle enters the link. However, the same weight is used in the subsequent iterations even though the entry time of the link may be different under different paths that utilize that link.

The stopping criteria could be changed so that even if the current path is identical to the incumbent, the links affected could be penalized again, and a new path could be found either until no weight change occurs or a threshold number of iterations are performed with no change in the incumbent path. The path obtained using the IWS is then evaluated for improvement using the available slack time as described in Section 3.2.

3.5 **Myopic Shortest Path Heuristic (MYOPIC)**

This heuristic uses time-dependent link weights in a greedy manner to find a solution to the time-dependent shortest path problem. The only difference between this method and traditional shortest path method is that the link weights are calculated dynamically due to the time-dependent weights. First the minimum temporary label is selected and then it is made permanent after the adjacent nodes with temporary labels are checked for updates. The final path got from the heuristic is evaluated for improvement due to parking using the method described in Section 3.2.

The drawback of this heuristic is that it keeps just one label for each node regardless of the entrance time to the node. For example, consider two partial paths $P_1$ and $P_2$ from the origin to node $i$ with total weights of $w_1$ and $w_2$ and with arrival times $t_1$ and $t_2$, respectively. Assume that $w_1 < w_2$ and $t_1 \neq t_2$. According to the heuristic, $P_1$ will dominate $P_2$ even though $t_1$ and $t_2$ are not equal. It is very well possible that the path $P_2$ could end with a shorter path since the weight of the further links are time-dependent. The only way to overcome this drawback is to keep multiple labels for each node, which turns out to be the
exact algorithm for this problem.

### 4 Computational Framework

In this section we describe the computational framework that we developed to test the TDSPP algorithm and the heuristics developed in Section 3. The analysis was conducted under a variety of conditions. All methods were coded using C++ and Java and implemented on a Pentium III, 800 Mhz personal computer with 392 MB of memory. The TDSPP algorithm was implemented by dynamically increasing the number of labels for nodes instead of allocating the memory at the beginning. Dynamic memory usage was also employed with the heuristics whenever applicable.

#### 4.1 Network Development

The network of the State of Texas is used to implement the exact algorithm and heuristics. We used the road data from the National Transportation Atlas Databases (1997), NTAD97, from the Bureau of Transportation Statistics. We included interstate, US and State highways in the network. County highways, local and rural roads have been excluded. There are 5,521 nodes and 6,756 links in our network, which is depicted in Figure 2.

Average speeds for the interstate, US and State highways were considered to be 65, 55 and 50 miles per hour, respectively. Certain link attributes such as accident rates and population at risk are not included in NTAD97. Calculation of these attributes as well as incorporating weather systems are discussed below.

#### 4.2 Calculation of Link Attributes

##### 4.2.1 Accident Rates

For illustrative purposes in this research, we used the accident rates from a Highway Safety Information System report [28] that used truck accident data from Utah for 1985-1987 to determine truck accident rates as a function of road type. We assumed that the adverse
weather conditions would increase those rates by 60% [7]. Table 1 gives the accident rates used in the experiments.

<table>
<thead>
<tr>
<th>Road Type</th>
<th>Accident Probability (Normal) (per mile x 10^{-6})</th>
<th>Accident Probability (In Weather System) (per mile x 10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interstate</td>
<td>0.89</td>
<td>1.42</td>
</tr>
<tr>
<td>US Highways</td>
<td>1.87</td>
<td>2.99</td>
</tr>
<tr>
<td>State Highways</td>
<td>1.45</td>
<td>2.32</td>
</tr>
</tbody>
</table>

Table 1: Truck Accident Rates for Numerical Experiment.

4.2.2 Population at Risk

There are two important parameters used in the calculation of population at risk for each link. These are the average population density around the link and the impact radius.
Population at risk is calculated by multiplying the population density within the area of a circle of the given radius. The impact radius is entered by the user and was taken as two miles for the initial analysis. Note that the radius may depend on the type of hazardous materials and weather conditions.

Link population densities were calculated by using census data from the 1990 TIGER database. The entire US is divided into polygons by the census bureau. Population within each polygon and the area of each polygon are known. Population density is assumed to be uniform within a polygon. Each link is defined by shape points so that the location of the link could be portrayed graphically. For example, a link with 14.68 miles length in our network has 73 shape points, each of which is defined by longitude and latitude. In order to determine the average population density of a link, we overlayed the shape points of the link with the census polygons by using Geographic Information System (GIS) software, recording the population density for each shape point. Then, we took the average of these values to find the approximate population density of the link.

Using approximate population densities, we were able to approximate the expected population at risk for a given radius. There are different ways of calculating the population at risk such as calculating the total population at risk for a given bandwidth around the link. Since our main objective is to focus on the effects of weather systems, we chose one of these methods for illustrative purposes.

4.2.3 Weather Systems

We assume the weather systems to be circular with a constant radius, speed and direction. We also assume that the initial location of a weather system is known. The number of weather systems varied between three and five. As defaults, we set the impact radius of a weather system as two miles and assumed the vehicle travel speed is 25% less in a weather system. The method explained in Section 2 was used to calculate the link travel time and accident probabilities dynamically.

If there are multiple weather systems affecting the same link, entrance and departure
times are calculated for each weather system. Then, the union of these intervals is found. For example, consider a link that is affected by three weather systems, which have entrance and departure time intervals of $[1, 3]$, $[2, 4]$ and $[5, 7]$. The union of these intervals is $\{[1, 4], [5, 7]\}$.

The underlying assumption in this calculation is that the slower travel due to the first weather system does not affect the critical times for the second weather system and so on. Given the fact that the link lengths are very small in the NTAD97 database, it would be unlikely for a short link to be affected by more than one weather system at different time intervals, so this drawback is not significant.

5 Computational Experiments

We focused on three goals for the numerical experiments. Firstly, we test the behavior of the TDSPP algorithm for different values of $\delta$ since its performance depends on how finely time is discretized. We then empirically determine the size of problems that can be solved to optimality in less than one hour. Our main goal was to determine how well the heuristics perform for these problems. Finally, we implemented the heuristics on larger-scale problems to compare them in terms of solution quality and solution time.

We limited the number of paths for the KPATH heuristic to ten paths. Also we implemented the DISSIM heuristic by using the ten paths generated by the KPATH heuristic, selecting five dissimilar ones. Expected consequence is the sum-product of the accident probabilities and the population for the links on the selected path.

In the tables that follow, algorithm runtime is given in seconds, travel distance is given in miles, and travel time in minutes. The ‘number of nodes’ column represents the number of nodes on the solution path.

5.1 Performance Analysis of TDSPP Algorithm

The performance of the exact algorithm is studied for ten origin-destination (O-D) pairs, for different values of $\delta$. The values of $\delta$ considered are 1, 5, 10, 15, 20, and 25. For these ten problems the computational runtime decreases as the $\delta$ is increased. The number of weather
systems was varied between one to five. Since the results show similar characteristics across the O-D pairs, we only present the results for one such O-D pair in Table 2.

<table>
<thead>
<tr>
<th>Time Interval $\delta$</th>
<th>Travel Distance</th>
<th>Travel Time</th>
<th>No. of Nodes</th>
<th>Expected Effect $\times 10^{-4}$</th>
<th>Algorithm Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>115.5</td>
<td>163</td>
<td>14</td>
<td>1.29</td>
<td>1819</td>
</tr>
<tr>
<td>5</td>
<td>115.5</td>
<td>165</td>
<td>14</td>
<td>1.29</td>
<td>1714</td>
</tr>
<tr>
<td>10</td>
<td>15.5</td>
<td>160</td>
<td>3</td>
<td>2.41</td>
<td>467</td>
</tr>
<tr>
<td>15</td>
<td>15.5</td>
<td>150</td>
<td>3</td>
<td>2.41</td>
<td>202</td>
</tr>
<tr>
<td>20</td>
<td>15.5</td>
<td>140</td>
<td>3</td>
<td>2.41</td>
<td>119</td>
</tr>
<tr>
<td>25</td>
<td>15.5</td>
<td>125</td>
<td>3</td>
<td>2.41</td>
<td>65</td>
</tr>
</tbody>
</table>

Table 2: Performance of TDSPP Algorithm for O-D Pair 1.

Here, the large difference in travel distances is because the first two solution paths (for $\delta = 1, 5$), minimize the risk of the path by taking a longer route with little parking. The path taken avoids the weather system by selecting links that are not affected by the weather system. The remaining paths (for $\delta = 10, 15, 20$ and 25) follow a much shorter route in terms of travel distance (this is also reflected in the number of nodes visited by the paths) and exhibit extensive parking times. The results prove the importance of selecting an appropriate value for $\delta$. It can be observed that as the value of $\delta$ is decreased the computational runtime increases dramatically. It should be also stated that as $\delta$ decreases the memory requirements increases as we are storing labels for each time interval. Furthermore, it must be noted that as the value of $\delta$ is increased the solution obtained from the algorithm varies. It can be seen in Table 2 that the path obtained from the algorithm changes for a value of $\delta > 5$ minutes. Thus the value of $\delta$ affects solution quality. The question we are faced with is: How finely should the time be discretized to assure the optimality of the algorithm? The answer to this depends on the network, the time limit for reaching the destination, the weather systems, acceptable runtime and memory constraints.
5.2 Comparison of Heuristic Methods with TDSPP

We determined ten origin-destination (O-D) pairs, for which the length of the minimum consequence path varied between 122 and 845 miles. For these ten problems, the solution time of the TDSPP algorithm varied between 3 seconds and 51 minutes. The solution time for the TDSPP algorithm strongly depends on the location of the origin and the destination if the road network is dense, then the solution time tends to be higher.

Since the results for the different O-D pairs are similar, we present the results for one of the time consuming O-D pair in Table 3. We normalized the expected consequence value along the optimal route between this O-D pair to a baseline of 1.0 in order to make the comparison easier.

<table>
<thead>
<tr>
<th>Algorithmic Method</th>
<th>Travel Distance</th>
<th>Travel Time</th>
<th>No. of Nodes</th>
<th>Normalized Expected Effect</th>
<th>Algorithm Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>KPATH</td>
<td>701.1</td>
<td>765</td>
<td>42</td>
<td>1.16</td>
<td>425</td>
</tr>
<tr>
<td>DISSIM</td>
<td>701.1</td>
<td>765</td>
<td>42</td>
<td>1.16</td>
<td>589</td>
</tr>
<tr>
<td>IWS</td>
<td>571.4</td>
<td>803</td>
<td>48</td>
<td>1.14</td>
<td>405</td>
</tr>
<tr>
<td>MYOPIC</td>
<td>652.5</td>
<td>1014</td>
<td>56</td>
<td>1.13</td>
<td>388</td>
</tr>
<tr>
<td>TDSPP</td>
<td>652.5</td>
<td>934</td>
<td>56</td>
<td>1.00</td>
<td>1853</td>
</tr>
</tbody>
</table>

Table 3: Comparison of Heuristic Methods with TDSPP.

For this O-D Pair, none of the heuristics was able to find the minimum consequence path of TDSPP, yet all, found a path within 16% of optimality. Also, all the heuristics required much less computation time than the optimal algorithm. The main reason due to which there is such a significant difference in the solution quality between the TDSPP algorithm and the heuristics is parking. The TDSPP method allows for a very large number of parking opportunities, whereas the heuristics only allow for at most one such opportunity. Since, in practice, there are significant limitations on the number of times parking can be done to avoid weather systems, the heuristic results are likely to be much more reasonable than would be otherwise indicated by Table 3.
Comparing the heuristics we find that KPATH and DISSIM perform the worst. Of the other heuristics, MYOPIC found the best path. Its best path (near 13% of optimality) was not found by the other heuristics. The second-best path (at 14%) was found by IWS and the third-best path (at 16%) was found by KPATH and DISSIM. The path for the KPATH and DISSIM heuristics are identical, which is not surprising given the fact that the DISSIM heuristic selected from the KPATH set of paths.

The effect of the weather systems on the consequence is apparent if we examine the paths that are generated. The paths try to avoid links that are affected by the weather systems. The main reason for this is the increase in accident probabilities and hence consequence due to the weather systems. Details of these paths are not shown for the sake of brevity. Interested readers can view these details in the recent thesis by Parekh [32].

5.3 Comparison of Heuristics for Large Problems

As the size of the problem increases, the TDSPP algorithm becomes very slow and computationally demanding. For larger-scale problems, we suggest that the heuristic methods be used. We therefore, test the heuristics for larger problems.

We again selected ten O-D pairs, for which the length is about three times larger than the routes considered in Section 5.2. The computational results for three representative O-D Pairs 1, 3 and 6 are presented in Tables 4, 5 and 6, respectively.

<table>
<thead>
<tr>
<th>Algorithmic Method</th>
<th>Travel Distance</th>
<th>Travel Time</th>
<th>No. of Nodes</th>
<th>Expected Effect x 10^{-3}</th>
<th>Algorithm Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>KPATH</td>
<td>2369.8</td>
<td>2567</td>
<td>127</td>
<td>9.1538</td>
<td>1236</td>
</tr>
<tr>
<td>DISSIM</td>
<td>2369.8</td>
<td>2567</td>
<td>127</td>
<td>9.1538</td>
<td>536</td>
</tr>
<tr>
<td>IWS</td>
<td>2193.2</td>
<td>2393</td>
<td>108</td>
<td>5.7831</td>
<td>344</td>
</tr>
<tr>
<td>MYOPIC</td>
<td>2193.2</td>
<td>2393</td>
<td>108</td>
<td>5.3015</td>
<td>294</td>
</tr>
</tbody>
</table>

Table 4: Comparison of Heuristic Methods for O-D Pair 1.
Table 5: Comparison of Heuristic Methods for O-D Pair 3.

<table>
<thead>
<tr>
<th>Algorithmic Method</th>
<th>Travel Distance</th>
<th>Travel Time</th>
<th>No. of Nodes</th>
<th>Expected Effect x 10^{-3}</th>
<th>Algorithm Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>KPATH</td>
<td>97.5</td>
<td>107</td>
<td>25</td>
<td>2.0276</td>
<td>116</td>
</tr>
<tr>
<td>DISSIM</td>
<td>37.8</td>
<td>42</td>
<td>20</td>
<td>3.5384</td>
<td>38</td>
</tr>
<tr>
<td>IWS</td>
<td>87.6</td>
<td>98</td>
<td>23</td>
<td>1.1573</td>
<td>276</td>
</tr>
<tr>
<td>MYOPIC</td>
<td>81.1</td>
<td>119</td>
<td>19</td>
<td>1.0211</td>
<td>63</td>
</tr>
</tbody>
</table>

Table 6: Comparison of Heuristic Methods for O-D Pair 6.

<table>
<thead>
<tr>
<th>Algorithmic Method</th>
<th>Travel Distance</th>
<th>Travel Time</th>
<th>No. of Nodes</th>
<th>Expected Effect x 10^{-3}</th>
<th>Algorithm Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>KPATH</td>
<td>668.2</td>
<td>734</td>
<td>78</td>
<td>13.3776</td>
<td>376</td>
</tr>
<tr>
<td>DISSIM</td>
<td>670.1</td>
<td>745</td>
<td>56</td>
<td>13.6308</td>
<td>453</td>
</tr>
<tr>
<td>IWS</td>
<td>745.8</td>
<td>803</td>
<td>70</td>
<td>7.4578</td>
<td>283</td>
</tr>
<tr>
<td>MYOPIC</td>
<td>728.1</td>
<td>1227</td>
<td>70</td>
<td>7.0395</td>
<td>274</td>
</tr>
</tbody>
</table>

Since they each generate many paths, the KPATH and DISSIM heuristics are fairly consistent in providing a good set of paths. The results of these two very similar due to the fact that the DISSIM method is based on the KPATH heuristic. The runtimes for DISSIM are more than that for KPATH when the network is dense (as seen in O-D Pair 6), as the 10 shortest path are easily generated by the Yen’s $k$-Shortest path heuristic and also these paths aren’t too long (in terms of distance and number of nodes visited), hence the second part of the algorithm that explores improvements due to parking is also performed quickly. However when the network is sparse DISSIM displays better runtime than KPATH (as seen in O-D Pair 1 and O-D Pair 3). Another important observation is that paths obtained from the KPATH and DISSIM methods do not have any parking time. This is because these
heuristics use paths that avoid the weather system.

Among the heuristics, the IWS and MYOPIC heuristics typically find the best solution. Also, the IWS and MYOPIC heuristics usually have shorter runtimes than the other heuristics. It should be noted though that when we have most of the links of the path affected by weather systems, as in Table 4 for O-D Pair 3, the runtimes for IWS are significantly higher.

5.4 Conclusions

We can conclude that the exact method may be computationally demanding for large networks. We analyzed the $O-D$ pairs that were solved by the TDSPP algorithm in order to illustrate the large variance in the solution time for TDSPP – which varied between 3 seconds and 55 minutes. Table 7 shows the solution time by using TDSPP for those pairs and the number of links in the solution.

<table>
<thead>
<tr>
<th>$O-D$ Pair</th>
<th>Solution Time (in seconds)</th>
<th>Number of Links</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1714</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>636</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>2356</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>1856</td>
<td>28</td>
</tr>
<tr>
<td>5</td>
<td>2582</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>28</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 7: Solution Time Versus Number of Links in the Optimal Solution for TDSPP.

For most O-D pairs a larger number of links implies longer solution time. However, this is not a general pattern – e.g. Pair 3. We conclude that the problem size one can expect to solve optimally depends on several parameters, e.g. network density, O-D pair selected, number of nodes and links in the network, and the number of time intervals.

We developed heuristic methods to solve larger problems. From our computational tests, we observe that the MYOPIC and IWS methods display the best results. The MYOPIC and IWS also have significantly better runtimes than the other heuristics. All the heuristics
evaluate for improvement by allowing parking at a single node on the path. The heuristics either avoid the weather system effects by parking at a strategic location and time (MYOPIC and IWS) or by using alternate links that are not influenced by the weather system (KPATH and DISSIM). The latter strategy, however, results in paths that have higher travel distances.

6 Future Research

We end this paper by stating two directions for future work in this area.

A possible extension of this work is for an environment in which there is uncertainty associated with the size and movement of a weather system. The level of complexity in such a situation can be decreased by using Geographical Information Systems (GIS) to capture, store, manipulate, analyze and display the spatially referenced and associated tabular data needed for these algorithms [33].

Another fruitful direction for future research is to incorporate the risk due to parking in the model. We have assumed that there is zero risk when the vehicle carrying hazmat is parked. This is not realistic since accidents are possible (though admittedly less likely) when a vehicle is parked.

References


