Conflict-Free Electric Vehicle Routing Problem with Capacitated Charging Stations and Partial Recharge

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Abstract

The adoption of electric vehicles (EVs) grows in rapid fashion in the transportation industry. However, limited battery capacity of EVs and the lack of charging facilities become challenges to obtain efficient route plans. In this paper, we propose a variant of electric vehicle routing problems considering the limited charging capacity at each station and the possibility of strategic partial recharging. A heuristic method, based on variable neighborhood search and tabu search, with simple charging time adjustment processes is presented to solve large instances of the problem. The proposed capacitated E-VRP model is tested with small size instances with a commercial optimization solver, while the heuristic method is implemented on both small and large instances. The main conclusion is that the battery capacity of the EVs and the number of chargers can significantly affect the total traveled distance.

Keywords: Electric Vehicles; Vehicle Routing Problem; Capacitated Charging Station; Partial Recharge; Heuristic

1 Background and Motivation

With the increasing interest of green logistics strategies and operations, all-electric vehicle (EV) adoption is becoming a major green logistic activity. Because of its positive effects on reducing

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greenhouse gas emission and ability of promoting urban sustainability, more and more EVs are being deployed by package delivery companies, i.e. FedEx, UPS, DHL, for last-mile deliveries (International Post Corporation, 2013). However, the big challenges of limited driving range and limited charging infrastructure of EVs hold back the widespread adoption of electric trucks in the parcel delivery service industry. A typical all-electric truck can drive 75 to 100 miles with a fully charged battery (Ingram, 2014; Daclison-Dickey, 2013) but this range can be decreased significantly by cold temperatures and by range anxiety (Botsford and Szczepanek, 2009; Tredeau and Salameh, 2009). Given that the daily driving needs of urban pickup/delivery trucks can range from 150 miles to 250 miles, the available range is not sufficient to complete the delivery route. Therefore, visits to charging stations along the routes are needed and proper planning for charging infrastructure can be a critical factor that can determine success of all-electric truck adoption.

There exist different types of charging techniques including conductive charging, inductive charging and battery swapping. Within conductive charging, where charging infrastructure is required, there are different levels of charging technologies: Level 1 charging, which is convenient but slow, needs average 20 hours to full charge the battery; Level 2 primarily charging needs 2 to 3 hours; and Level 3 fast charging intended for commercial and public applications can complete charging in 30 minutes to 1 hour (Yilmaz and Krein, 2013). A typical parcel delivery center has 100 to 200 trucks and each truck typically makes 70-90 stops daily. Given these numbers and the fact that all trucks would likely run out of battery about the same time, even using Level 3 charging technology the available chargers at a delicate charging station can be a significant problem that delays adoption of all-electric trucks in the parcel delivery service industry. Moreover, the congestion and waiting at the charging station make vehicle routing even more challenging.

In this paper, we introduce the all-electric truck routing problem while considering limited capacity at charging stations. Level 3 charging technology is assumed to be implemented for our electric-truck routing problem. Practical constraints of a typical EV routing problem are considered, including vehicle capacity restrictions and customer time window to enable time-definite deliveries. Moreover, our routing model incorporates (1) the dynamic charging time, which depends on the battery level when vehicle arriving at the station, (2) the possibility of partial recharging at the station, (3) conflict avoidance at chargers due to limited capacity, and (4) the ability to handle both pickup and delivery.

To the best of our knowledge, none of the models available in the literature of electric vehicle routing problems considers the capacitated charging stations. The limited capacity of charging stations can enforce electric vehicles to wait in the queue, which can potentially impact the feasibility of routing plans, especially when time windows at the customer locations are narrow, as well as increase the total travel time. The consideration of charging capacities naturally leads to the strategy that allows charging EVs partially. By incorporating these important issues, this paper delivers a flexible routing and operational plan.

The rest of this paper is organized as follows: In Section 2, a review of the related literature is provided. In Section 3, the E-VRP model considering the limited capacity at charging stations and partial recharging for EVs is presented in detail. In Section 4, the heuristic method used to solve our new EV routing problem is described. Section 5 conducts numeric experiments and provides corresponding results obtained from both small and large size instances with further analysis. We conclude in Section 6.

2 Literature Review

The vehicle routing problem (VRP), originally introduced by Dantzig and Ramser (1959), is one of the most widely studied problems in operations research and combinatorial optimization. The VRP aims to minimize the total transportation cost of visiting a set of customers with known demands by designing proper routes for a fleet of vehicles departing from and returning to the depot. As an important extension of VRP, the VRP with time windows (VRPTW) considers the realistic constraint that customers must be served within a specific time period (Solomon, 1987). Another remarkable extension of VRP is the capacitated VRP which considers the limited cargo capacity of vehicles (Laporte et al., 1986). Lots of existing work of VRPTW and capacitated VRP used conventional vehicles as the means of transportation. However, the increasing negative effects caused by conventional vehicles become a heavy burden on the environment. In order to reduce such burden, green logistics using a clean means of vehicles such as electric vehicles (EVs) becomes an urgent necessity (Davis and Figliozzi, 2013).

Green logistics has recently received increasing attention due to its sustainability in the long term. Electric vehicles (EVs) strongly relate to the field of green logistics, because they are powered by clean, sustainable and renewable energy sources. However, EVs have limitations of driving range and need to reach recharging facilities, which make EV routing problem (E-VRP) different from the traditional vehicle routing problem (VRP). Routing models considering alternative fuels, e.g., electricity and hydrogen fuel that are alternative to fossil fuels, have just developed in recent years; see Lin et al. (2014) for a survey. When a VRP is general enough to be used for any alternative fuel vehicles, it is called a green VRP (G-VRP) problem, while it is called an E-VRP when it focuses on EVs.

Gonçalves et al. (2011) study the VRP with pickup and delivery with a mixed fleet of EVs and conventional vehicles. They assumed that EVs can be recharged anywhere during the route and the number of recharging stops is determined by dividing the total traveled distance by the EV travel range, which is unrealistic in the actual routing cases. Conrad and Figliozzi (2011) study a recharging VRP where vehicles with limited range, wherein the vehicles are capacitated and the recharging time is assumed to be fixed. The existence of customer time windows is studied to show its influence on the objectives, including number of employed vehicles and total travel costs. Additionally, bounds are formulated to predict average tour length. A G-VRP with consideration of limited vehicle driving range and limited refueling infrastructure is proposed by Erdoğan and Miller-Hooks (2012). It assumes the possibility of visiting one or more recharging facilities during the route and the recharging time is assumed to be fixed. However, there is no capacity constraint or time window constraint in the model formulation.

Schneider et al. (2014) introduce E-VRP with time windows (E-VRPTW) as an extension of G-VRP, which has additional constraints on vehicle capacity and customer time window as well as consideration of dynamic recharging time based on the difference between vehicle battery level and full battery capacity. The proposed model is solved by a hybrid variable neighborhood search (VNS) approach combined with tabu search (TS) heuristic. Goeke and Schneider (2015) extend by considering a mixed fleet of EVs and conventional vehicles. Felipe et al. (2014) consider multiple charging technologies and the possibility of partial recharge to strategically reduce the travel time, but without time window consideration. Schiffer and Walther (2015) consider a location-routing problem to determine the location of charging stations, incorporating an E-VRPTW with partial recharge.

In this paper, we consider the capacity of charging stations. While there are some models that consider the charging capacity available in the literature of public charging station siting decision (Upchurch et al., 2009; Zhang et al., 2015), our model in this paper is the first of its kind in the electric or green VRP literature. We construct our model and devise an algorithm based on the work of Schneider et al. (2014). The key additional components are, as discussed in Section 1, the consideration of charging capacity and the possibility of allowing partial charging strategically. Our model utilizes partial charging to avoid potential conflicts at charging stations, as well as to reduce the total travel time as in Felipe et al. (2014).

Note that three types of capacities are considered in our model: the capacity of cargo space in each EV, the battery capacity (the size of battery cells), and the capacity of charging stations—two EVs cannot charge at a charger at the same time.

3 Electric Vehicle Routing Problem (E-VRP) considering the Capacity of Charging Stations

A typical VRPTW using conventional vehicles is known as the problem of assigning customers to vehicles and forming the routes of vehicles to visit customers in order. An extension of VRP using electric vehicles, namely E-VRP, is introduced with EV-specific challenges. Given the common constraints of VRP, E-VRP seeks the minimal total distance from the routing plans of a set of EVs which depart from the depot, visit a set of customers, and return to the depot without exceeding the EVs driving range. The driving range of each EV depends on its battery capacity which is limited. In order to let an EV to complete its route, it is necessary to have EVs charged en route.

The E-VRPTW model of Schneider et al. (2014) enforces customers to be visited within pre-specific time interval, enables recharging of EVs en route, and incorporates the limited freight capacity of EVs. However, the E-VRPTW uses the assumption that the capacity of charging stations is unlimited so that more than one EV can charge at a charger at the same time, which is an unreasonable assumption for the currently available charging facilities. The E-VRPTW also assumes that EVs are always recharged to a fixed battery level always, i.e. full battery capacity, which may not be necessary. Moreover, the E-VRPTW considers pick-up service only.

In order to consider real-word characteristics, specifically, limited number of chargers at a charging station, flexibility of partial charging for EVs, and both pick-up and delivery services, we develop an improved model from the E-VRPTW formulation of Schneider et al. (2014). Conflicts may occur at such capacitated charging stations and partial charging may help to reduce the time vehicles need to stay at a charging station. Our model resolves conflicts at capacitated charging stations and enables partial charging for EVs. Similar notations for data sets and constants from Schneider et al. (2014) are used to describe our model and a complete list of variables and parameters is summarized in Table 1.

We develop an improved model from the E-VRPTW formulation of Schneider et al. (2014) with additional consideration of important real-world characteristics, specifically, limited chargers at a charging station and flexibility of partial charging for EVs. Conflicts may occur at such capacitated charging stations and partial charging may help to reduce the time vehicles need to stay at a charging station. Our model resolves conflicts at capacitated charging stations and enables partial charging for EVs. A similar notation for sets, variables, and constants from Schneider et al. (2014) are used to describe our model. Our notation is summarized in Table 1.

3.1 Model formulation

Our E-VRP is formulated as a mixed-integer program. We let $\mathcal{V}$ denote the set of customers and K is the number of chargers, $\mathcal{F}_k$ is the set of dummy nodes for each charger $k = 1, ..., K$ and $\mathcal{F}$ is the set of dummy nodes for all chargers, i.e. $\mathcal{F}' = \bigcup_{k=1}^{K} \mathcal{F}_k$. Nodes 0 and $N + 1$ denote the same depot, and every route starts from 0 and ends at $N + 1$. Let $\mathcal{V}'_{0,N+1}$ denotes all the nodes including depot instances 0 and $N + 1$, $\mathcal{V}$ and $\mathcal{F}'$. The E-VRP is defined on a complete graph $G = (\mathcal{V}'_{0,N+1}, A)$, with the set of arcs $A = \{(i, j) : i, j \in \mathcal{V}'_{0,N+1},\ i \neq j\}$. Each arc (i, j) is associated with a distance d_{ij} and a travel time t_{ij}.

Each EV is assumed to start from depot 0 with full battery Q, with constant battery consumption rate h, and is assumed to be recharged at a charger with a constant recharging rate g. A linear recharge process is also assumed to simplify the problem. Additionally, it is assumed that each EV is not necessarily recharged to full capacity during the route, which enables partial recharging for each EV. Each customer $i \in \mathcal{V}$ is associated with a time window $[e_i, l_i]$ and a service time s_i. A set of homogeneous EVs with cargo capacity of C is located at the depot and the maximal number of available EVs is p. The objective function of this model is to minimize the total traveled distance.

Table 1: Variable and Parameter Definitions of the capacitated E-VRP model

Data sets	$0, N+1$	Depot instances
	$\mathcal{V}$	Set of customers $\mathcal{V} = \{1, ..., N\}$
	$\mathcal{V}_0$	Set of customers including depot instance 0, $\mathcal{V}_0 = \mathcal{V} \cup \{0\}$
	K	Number of chargers
	$\mathcal{F}_k$	Set of dummy nodes for charger k, $k = 1, ..., K$
	$\mathcal{F}'$	Set of dummy nodes for all chargers, $\mathcal{F}' = \bigcup_{k=1}^{K} \mathcal{F}_k$
	$\mathcal{F}'_0$	Set of dummy nodes for all chargers and depot instance $\{0\}$, $\mathcal{F}' = \bigcup_{k=1}^{K} \mathcal{F}_k$
	$\mathcal{V}'$	Set of customers and visits to chargers, $\mathcal{V}' = \mathcal{V} \cup \mathcal{F}'$
	$\mathcal{V}'_0$	Set of customers, visits to chargers and depot instance $\{0\}$, $\mathcal{V}'_0 = \mathcal{V}' \cup \{0\}$
	$\mathcal{V}'_{N+1}$	Set of customers, visits to chargers and depot instance $N+1$, $\mathcal{V}'_{N+1} = \mathcal{V}' \cup \{N+1\}$
	$\mathcal{V}'_{0,N+1}$	Set of customers, visits to chargers and depot instances 0 and $N+1$, $\mathcal{V}'_{0,N+1} = \mathcal{V}' \cup \{0\} \cup \{N+1\}$
Constants	$T_{\max}$	Maximal time duration of a route
	M	Big M
	p	Maximal number of available EVs at depot
	C	Vehicle demand capacity
	Q	Vehicle battery capacity
	h	Battery consumption rate
	g	Battery recharging rate
	d_{ij}	Distance between nodes i and j
	t_{ij}	Travel time between nodes i and j
	q_i	Demand at customer node i, 0 if $i \notin \mathcal{V}$
	e_i	Earliest start of service at node i
	l_i	Latest start of service at node j
	s_i	Service time at node i
Decision Variables	x_{ij}	Binary decision variable; 1 if arc (i, j) is traveled, 0 otherwise.
	τ_i	Time of arrival at node i
	u_i	Remaining cargo on arrival at node i
	y_i	Remaining battery level on arrival at node i
	z_i	Target battery level when leaving from node i
Dummy Variables	ζ_{ij}	Dummy variable for linearization, $\zeta_{ij} = z_i x_{ij}$
	ψ_{ij}	Dummy variable for linearization, $\psi_{ij} = y_i x_{ij}$
	w^1_{ij}, w^2_{ij}	Binary dummy variables for linearization

We formulate a mathematical optimization model as follows:

$$\min \sum_{i\in\mathcal{V}'_0, j\in\mathcal{V}'_{N+1},\ i\neq j} d_{ij}x_{ij} \tag{1}$$

subject to

$$\sum_{j\in\mathcal{V}'_{N+1},\ j\neq i} x_{ij} = \sum_{j\in\mathcal{V}'_0,\ j\neq i} x_{ji} \qquad \forall i \in \mathcal{V}' \tag{2}$$

$$\sum_{j\in\mathcal{V}'_{N+1},\ j\neq i} x_{ij} = 1 \qquad \forall i \in \mathcal{V} \tag{3}$$

$$\sum_{j\in\mathcal{V}'_{N+1},\ j\neq i} x_{ij} \leq 1 \qquad \forall i \in \mathcal{F}' \tag{4}$$

$$\sum_{j\in\mathcal{V}'_{N+1},\ j\neq i} x_{0j} \leq p \tag{5}$$

$$e_i \leq \tau_i \leq l_i \qquad \forall i \in \mathcal{V}'_{0,N+1} \tag{6}$$

$$\tau_j \geq \tau_i + (t_{ij} + s_{ij})x_{ij} - T_{\max}(1 - x_{ij}) \qquad \forall i \in \mathcal{V}_0, j \in \mathcal{V}'_{N+1},\ i \neq j \tag{7}$$

$$\tau_j \geq \tau_i + t_{ij}x_{ij} + g(z_i - y_i)x_{ij} - T_{\max}(1 - x_{ij}) \qquad \forall i \in \mathcal{F}', j \in \mathcal{V}'_{N+1},\ i \neq j \tag{8}$$

$$u_0 = C \tag{9}$$

$$0 \leq u_j \leq u_i - q_i x_{ij} + C(1 - x_{ij}) \qquad \forall i \in \mathcal{V}'_0, j \in \mathcal{V}'_{N+1}, i \neq j \tag{10}$$

$$y_0 = Q \tag{11}$$

$$0 \leq y_j \leq y_i - hd_{ij}x_{ij} + Q(1 - x_{ij}) \qquad \forall i \in \mathcal{V}, j \in \mathcal{V}'_{N+1}, i \neq j \tag{12}$$

$$0 \leq y_j \leq z_i - hd_{ij}x_{ij} \qquad \forall i \in \mathcal{F}'_0, j \in \mathcal{V}'_{N+1}, i \neq j \tag{13}$$

$$z_i \geq y_i \qquad \forall i \in \mathcal{F}' \tag{14}$$

$$z_i = y_i \qquad \forall i \in \mathcal{V}_0 \tag{15}$$

$$(\tau_i - \sigma_j)(\tau_j - \sigma_i) \leq 0 \qquad \forall i, j \in \mathcal{F}_k, i \neq j, k = 1, ..., K \tag{16}$$

$$\sigma_i = \tau_i + g(z_i - y_i) \qquad \forall i \in \mathcal{F}',\ j \in \mathcal{V}'_{N+1},\ i \neq j \tag{17}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in \mathcal{V}'_0, j \in \mathcal{V}'_{N+1}, i \neq j \tag{18}$$

The objective function of minimizing total traveled distance is presented in (1). Constraints (2)–(5) establish flow conservation. Constraint (2) is a flow balance equation between the numbers of incoming and outgoing arcs at each node. Constraint (3) ensures that each customer should be visited exactly once and constraint (4) enforces the connectivity from chargers to other nodes. Constraint (6) ensures that each node is visited within its time window.

Constraints (7) and (8) guarantee the time feasibility between successive nodes. Note that constraint (8) allows partial recharging. Since (8) involves nonlinear terms $(z_i - y_i)x_{ij}$, we will linearize using additional variables as explained in Section 3.2.

Constraint (9) enforces that the initial cargo capacity of EVs at the depot is full capacity C, while constraint (10) ensures a nonnegative demand load upon arrival at any node. Further, constraints (11)–(15) handle the battery levels of EVs. EVs leave the depot with full battery capacity under constraint (11) and battery level is always nonnegative when arriving at any node with constraints (12)–(13). Constraints (14)–(15) enforce the relationship between the remaining battery level upon arrival at a node and the target battery level when leaving from the node.

Finally, nonlinear constraints (16) avoid possible conflicts at a charger as described in Section 3.3, while constraint (17) establishes the leaving time of an EV from a charger to be dynamically dependent on the difference between the target battery level after recharging and the present battery level upon arriving at the charger.

Note that the key differences from the model of Schneider et al. (2014) are constraints (8) and (16) for allowing partial recharge and avoiding conflicts at chargers, respectively. We explain these constraints in more detail in the subsequent sections.

3.2 Partial charging at a charger

In the EVRPTW of Schneider et al. (2014), EVs are assumed to be always recharged to full battery capacity Q at the charging station. Decision variable y_i specifies the battery level for an EV arriving at node i and the charging time for an EV at a charger depends on the charging amount: the difference between Q and y_i.

In our model, partial charging for EVs at the chargers is incorporated. In addition to y_i, a new decision variable z_i , indicating target battery level when leaving from node i, is introduced to enable partial charging. If node i is a customer node, the target battery level z_i remains the same as y_i. If node i is a charger node, this indicates that at a charging station, an EV is not necessarily charged to full battery capacity Q and only the difference between z_i and y_i needs to be recharged.

This also affects the charging time for an EV at a charger.

Suppose an EV travels from charger i to any other possible node j (and $\neq i$). When τ_i and σ_i denote EV's time of arrival at charger i and time to leave from node i, respectively, the relationship between τ_i and σ_i is as follows:

$$\sigma_i = \tau_i + g(z_i - y_i) \qquad \forall i \in \mathcal{F}',\ j \in \mathcal{V}'_{N+1},\ i \neq j$$

where $(z_i - y_i)$ is the amount of battery level to be recharged at charger i, and $g(z_i - y_i)$ is the time that takes to recharge.

Similarly, τ_j is EV's time of arrival at node j and time to leave from node j, respectively. The constraint of arrival times between two nodes i and j is formulated as follows:

$$\tau_j \geq \tau_i + t_{ij}x_{ij} + g(z_i - y_i)x_{ij} - T_{\max}(1 - x_{ij}) \qquad \forall i \in \mathcal{F}',\ j \in \mathcal{V}'_{N+1},\ i \neq j \tag{19}$$

In order to linearize the quadratic constraint (19), we first let $\zeta_{ij} = z_i x_{ij}$ and introduce the following additional constraints:

$$\zeta_{ij} \geq 0 \tag{20}$$

$$\zeta_{ij} \leq z_i \tag{21}$$

$$\zeta_{ij} \leq M x_{ij} \tag{22}$$

$$\zeta_{ij} \geq z_i + M(x_{ij} - 1) \tag{23}$$

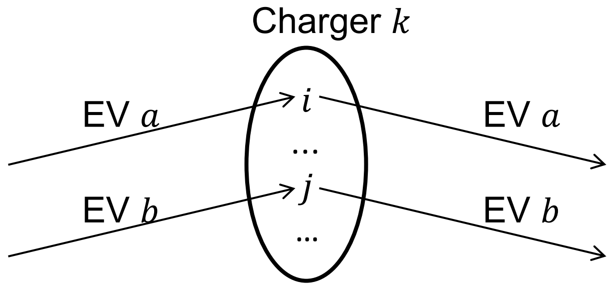
which will ensure $\zeta_{ij} = z_i$ when $x_{ij} = 1$, and $\zeta_{ij} = 0$ when $x_{ij} = 0$. Similarly, we let $\psi_{ij} = y_i x_{ij}$ and

$$\psi_{ij} \geq 0 \tag{24}$$

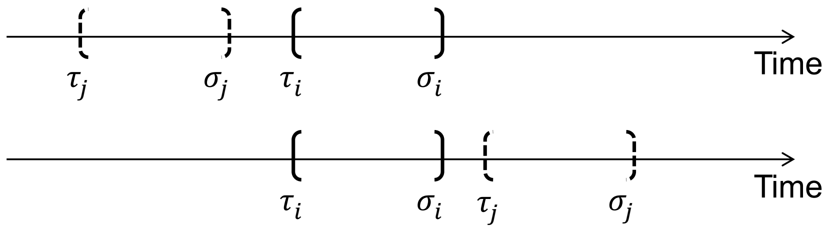
$$\psi_{ij} \leq y_i \tag{25}$$

$$\psi_{ij} \leq M x_{ij} \tag{26}$$

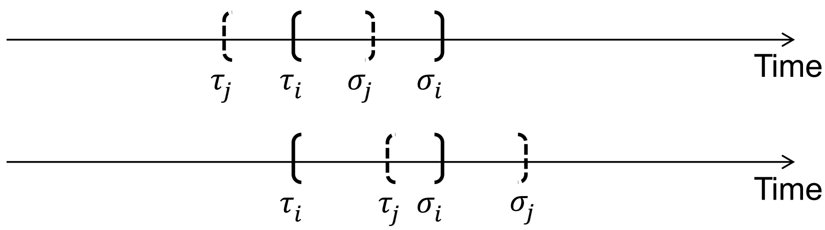
$$\psi_{ij} \geq y_i + M(x_{ij} - 1) \tag{27}$$



(a) Representation of a charger k and its dummy nodes i and j



(b) Cases when there is no conflict at a charger



(c) Cases when there are conflicts at the charger

Figure 1: Two vehicles at a charger

Finally, with above additional constraints, constraint (19) is replaced by:

$$\tau_j \geq \tau_i + t_{ij}x_{ij} + g(\zeta_{ij} - \psi_{ij}) - T_{\max}(1 - x_{ij}) \qquad \forall i \in \mathcal{F}',\ j \in \mathcal{V}'_{N+1},\ i \neq j \tag{28}$$

3.3 Conflict-free

This section explains constraint (16) and introduces a linearization method. Suppose both EV a and EV b need to be recharged at the charger k in order to complete the routes. With reference to Figure 1a, $i, j \in \mathcal{F}_k$ are dummy nodes for the same charger k, and EV a is recharged at dummy node i while EV b is recharged at dummy node j. τ_i is the arrival time of EV a at node i and σ_i is the time EV a leaves node i after finishing recharging. Similarly, τ_j is the arrival time of EV b at node j and σ_j is the time EV b leaves node j after finishing recharging. As shown in Figure 1b, no conflict exists if there is no time overlap of EVs a and b being recharged at the charger k. However, if time overlap exists, there will be conflicts at the charger as presented in Figure 1c. In order to avoid the conflicts at the same charger k, we present the following constraint:

$$(\tau_i - \sigma_j)(\tau_j - \sigma_i) \leq 0 \tag{29}$$

for all $i, j \in \mathcal{F}_k, i \neq j, k = 1, ..., K$.

Note that (29) is nonlinear. To linearize, we introduce new binary variables and a big M parameter. For each pair of dummy nodes for the same charger, i.e. (i, j) for charger k, new binary variables w_{ij}^1 and w_{ij}^2 are introduced. The non-linear constraint is transformed as follows:

$$\begin{aligned} \tau_i - \sigma_j &\geq -M(1 - w_{ij}^1) \\ \tau_j - \sigma_i &\geq -M(1 - w_{ij}^2) \\ w_{ij}^1 + w_{ij}^2 &\geq 1 \\ w_{ij}^1, w_{ij}^2 &\in \{0, 1\} \end{aligned}$$

for all $i, j \in \mathcal{F}_k, i \neq j, k = 1, ..., K$.

4 A Heuristic Method

Our solution method for the capacitated E-VRP (1)–(18) has the objective of minimizing total traveled distance and is developed from the hybrid method proposed by Schneider et al. (2014), wherein a variable neighborhood search (VNS) heuristic is combined with a tabu search (TS) method for the intensification phase of VNS and simulated annealing (SA) for acceptance criterion. Table 2 provides an overview of the VNS/TS hybrid heuristic proposed by Schneider et al. (2014).

A typical VNS algorithm has three components including shaking phase, local search and stopping criterion. In this hybrid VNS/TS algorithm, given a current best solution S, VNS randomly generates a neighboring solution S' in predefined neighborhood $N(S)$ of S. Next, TS is applied as local search on S' to determine the local optimal S''. If S'' satisfies the acceptance criterion, the VNS accepts solution S'' and restarts with S'' and its neighborhood. Instead of accepting only the improving solutions, the SA acceptance criterion allows acceptance of deteriorating solutions to further diversify

Table 2: An overview of the VNS/TS method of Schneider et al. (2014)

Initialization.
Initial solution S.
Generate a set of neighborhoods of S: N_k for $k = 1, ..., k_{\text{max}}$.
Choose a stopping condition.

Iterations.
Repeat the following steps until the stopping condition is satisfied.

1. Set $k = 1$.
2. Repeat the following steps until $k = k_{\text{max}}$.
   (a) *Shaking:* Generate a random solution S' from the k-th neighborhood of S_k.
   (b) *TS local search:* Apply TS with S' as initial solution and S'' is the obtained local optimum.
   (c) *SA acceptance criteria:* Check the acceptance condition, if satisfied, accept and make move to S'', i.e. set $S = S''$, continue and restart search from Step 1; otherwise set $k = k + 1$ and go to Step 2(a).

the search.

To solve the capacitated E-VRP (1)–(18), we first apply the VNS/TS algorithm of Schneider et al. (2014). The solution obtained may have conflicts at charging stations and will recharge EVs always to the full battery capacity. In our heuristic method, we try to adjust charging plans in the obtained solution to resolve the conflict. While the VNS/TS algorithm minimizes the total traveled distance, our heuristic method also tries to minimize the makespan of the entire pickup/delivery plan.

Figure 2 presents the flow chart of our solution method. In Step 1, the VNS/TS algorithm is applied to E-VRP to find the best feasible solution. In Step 2, we compute time durations of charging stations for the current best feasible solution and check any conflicts observed at the charging stations. In Step 3, we tweak the time of visiting and leaving charging stations route by route to remove conflicts found at the charging stations, without violating any of the customer time windows.

In Step 3, as illustrated in Figure 2, suppose we have a conflict between route r and route k. There are two cases: (i) the vehicle in route r arrives first, and (ii) the vehicle in route k arrives first. In Case (i), determine the minimum charging time in routes r and k required for completing the

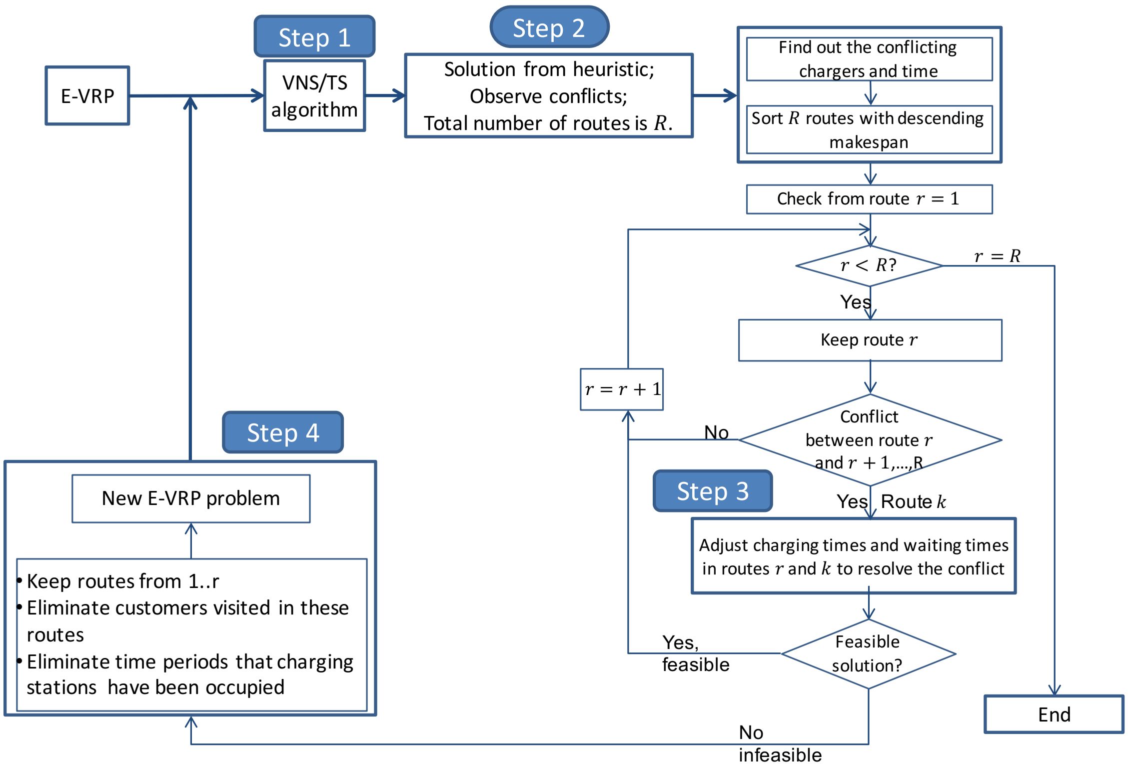


Figure 2: Flowchart of the revised heuristic method

route without running out of battery, and determine the latest departure time from the charging station in route k that does not violate all time window constraints at all the subsequent customer nodes. To avoid conflict, try the following adjustments in order:

1. In route k, keep the charging time at the minimum required level, and increase the waiting time as much as possible.

2. In route r, keep the charging time at the minimum required level.

Perform the second adjustment for route r only if the first adjustment cannot resolve the conflict. If both adjustments fail to resolve the conflict, we proceed to Step 4.

In Case (ii), we determine the minimum required charging times in routes r and k. Adjust charging plans as follows, again one by one in the specified order:

1. In route k, keep the charging time at the minimum required level.

2. In route r, keep the charging time at the minimum required level.

3. In route r, increase the waiting time at the charging station, but without leaving the charging station later than the original departure time.

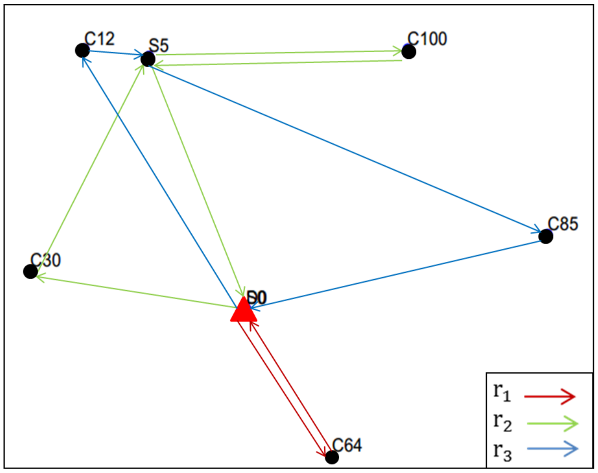
In the last adjustment, we want to keep the same departure time of the vehicle in route r from the charging station, since route r is the critical route that determines the makespan of the entire routing plan. If all adjustment attempts fail to resolve the conflict, go to Step 4.

Since the simple adjustment process in Step 3 failed to create a feasible solution, we generate a new E-VRP in Step 4 eliminating occupied time durations of charging stations and already visited customers and the apply VNS/TS heuristic to solve the new problem.

A simple example with 5 customers to illustrate our method is shown in Figure 3. The solution for this example using VNS/TS is as presented in the figure and three routes are generated. Conflicts are observed at charging station S_5 in route r_2 and r_3. The makespans of r_1, r_2, and r_3 are 374, 907, and 857, respectively. In order to resolve conflicts, at first, routes are ranked by makespan in descending order that is r_2, r_3 and r_1. Secondly, keep r_2, we check charging station S_5 in r_2, and find the conflicts with r_3. Then, recognizing that S_5 is occupied by r_2 from time 476 to 491 we postpone the available time of S_5 in r_3 to start from time 491. Therefore, S_5 is occupied by r_3 from time 491 to 504. Lastly we update serving time of the nodes in r_3 which are visited after S_5 and check if their time windows are satisfied. In this example, after tweaking the time at charging station, the current solution is still feasible and it is then accepted to be the final solution without conflicts at the charging station. Repeat this process until all routes are checked and all conflicts are resolved. In the end, both the total traveled distance and the minimal makespan remain the same as the solution before resolving the conflicts at the charging stations. Therefore, this example also shows that resolving conflicts at charging stations does not necessarily increase the total distance or makespan.

5 Numerical Experiments

In this section, we conduct numerical experiments to test and evaluate the performance of the proposed model. In order to be able to evaluate the solution quality, instances are newly designed to be solved by the commercial solver CPLEX and the heuristic. Our instances are created from instances used for E-VRPTW by Schneider et al. (2014), which are first generated based on



- Depot: D_0
- Customers: $C_{12}, C_{30}, C_{64}, C_{85}, C_{100}$
- Charging stations: S_5
- Solution: 3 vehicles with routing
  - r_1: $D_0 \to C_{64} \to S_0$
  - r_2: $D_0 \to C_{30} \to S_5 \to C_{100} \to S_5 \to S_0$
  - r_3: $D_0 \to C_{12} \to S_5 \to C_{85} \to S_0$
- Distance cost: 295.92
- Minimal makespan: 907
- Conflicts at the charging station exist:
  - r_2 visits S_5 during $[476, 491]$.
  - r_3 visits S_5 during $[472, 485]$.

Figure 3: A simple example with conflict at the charger

benchmark instances for the VRPTW proposed by Solomon (1987). These instances are with 5, 10 and 15 customers. Given an instance of Schneider et al. (2014), we first reduce the available chargers to create possible conflicts at charging stations. Such reduction of charging stations may make it impossible to satisfy the original customer time windows. In such a case we create new time windows to make the problem instance feasible. Moreover, each charging station is capacitated with limited number of chargers. In our case the capacity of each charging station is assumed to be 1.

Instances are labeled as in the first column of Table 3: for example, m101c5, wherein ‘m101’ is a unique instance index and ‘c5’ means there are 5 customers. All numerical experiments are performed on a desktop computer with an Intel Xeon E5-2630 2.40 GHz Processor with 32 GB RAM, running Windows 7 Enterprise.

5.1 Performance of the heuristic method on small size instances

Small size instances with up to 15 customers are solved by both the CPLEX solver using the formulation proposed in Section 3 and the heuristic approach presented in Section 4. Results of those instances are obtained and compared in Table 3. Solutions presented in the table are the optimal solution and results are presented in multiple criterions, including number of vehicles denoted by n, total distance by d and time to solve by t in seconds. Instances are solved to optimality in a few seconds to at most 3 minutes in the CPLEX solver. For the heuristic, the solution provided for each instance is the best found solution in 10 runs. The results show the ability of both the

Table 3: Comparison of Results obtained from CPLEX Solver and Heuristic for Small-size Instances. Note that n denotes the number of vehicles, d the total traveled distance, t the time to solve the instance, and Δd the difference between results from CPLEX solver and heuristic.

	Capacitated E-VRP			Heuristic			
Instance	n	d	t (sec)	n	d	t (sec)	Δd (%)
m101c5	3	274.81	3.34	3	274.81	0.53	0.00
m104c5	2	209.74	3.08	2	209.74	0.47	0.00
m205c5	2	140.70	2.97	2	140.70	0.61	0.00
m206c5	2	270.83	2.96	2	270.83	0.89	0.00
m101c10	3	369.69	9.22	3	369.69	1.35	0.00
m104c10	2	273.93	12.34	2	273.93	2.49	0.00
m202c10	2	243.20	4.72	2	243.20	1.78	0.00
m205c10	2	242.88	2.14	2	242.88	1.54	0.00
m103c15	3	348.54	1779.84	3	348.54	85.32	0.00
m106c15	3	253.62	8.28	3	253.62	27.01	0.00
m202c15	3	374.89	131.78	3	374.89	29.52	0.00
m208c15	2	301.83	6.60	2	301.83	20.31	0.00

proposed capacitated E-VRP model and heuristic to solve small size instances. The last column in the table, Δd, denotes the difference between the total traveled distance given by Heuristic and CPLEX solver for each instance. It is shown in the table that there is no difference for the total traveled distance obtained from CPLEX solver and heuristic for all instances. It is clearly seen from the table that the numbers of EVs employed in both approaches are the same. The results also shows the proposed capacitated E-VRP model is able to avoid possible conflicts at chargers and enable the partial recharging for EVs at charger.

5.2 Performance of the heuristic method on large size instances

To generate large size instances, we start with the small size instances used in previous Section 5.1. We first fix the location of the depot. Then we randomly select and merge small size instances to have 100 customers and re-scale them with reference to the depot. Later, we allocate 20 chargers in the manner that every customer is reachable from the depot with no greater than two chargers. At last, we apply different time windows to customers to create a set of large size instances. Large size instances with 100 customers and 20 chargers are solved by the heuristic and results are presented in Table 4. These instances have the same distribution of customers and allocation of chargers but different time windows for customers. For the solutions obtained from the heuristic, the number of

Table 4: Results of Large Size Instances Using Heuristic Method. Note that n_1 and n_2 denote the number of vehicles for two cases, respectively; d_1 and d_2 the total traveled distance for two cases, respectively; and Δd_2 the increase percentage when comparing d_2 to d_1.

	Higher Battery Capacity		Lower Battery Capacity		
Instance	n_1	d_1	n_2	d_2	Δd_2 (%)
m101c100	8	897.23	8	904.21	0.78
m102c100	8	908.62	8	913.33	0.52
m103c100	8	946.80	8	957.63	1.14
m104c100	8	908.75	8	909.51	0.08
m105c100	8	944.32	9	950.23	0.63
m106c100	9	885.70	9	885.87	0.02
m107c100	9	946.42	9	965.62	2.03
m108c100	10	977.24	9	985.87	0.88
m109c100	8	897.23	8	906.21	1.00
m110c100	10	904.81	9	913.33	0.94
m111c100	8	941.04	10	948.41	0.78
m112c100	10	977.30	10	987.68	1.06

employed EVs and the total traveled distance are given as columns n and d in Table 4. Instances are solved under both higher battery capacity of 62 and lower battery capacity of 50.

Results show that heuristic method is able to handle larger size E-VRP with capacitated charging stations. It is seen from the results that, for these instances with the same distribution of nodes, time windows of customers will affect the number of employed EVs and total traveled distance. In general, the tighter the time windows for customers, the higher the total traveled distance. It is also observed that for the same instance, different levels of battery capacity has impact on the solution that, lower battery capacity will result in higher total traveled distance and possibly larger number of employed EVs. The maximal increase of total traveled distance with lower battery capacity is 2.03% while the minimal increase is 0.02%, compared to that with higher battery capacity. The average increase of total traveled distance is 0.82%.

5.3 Impact of the number of chargers

Further analysis is conducted on the instances to show the impact of number of chargers on the solution. Instance m101c100 is selected to use as illustration. Initially, this instance has 100 customers and 20 chargers and it is solved without using all chargers as present in Section 5.2. Then we classify chargers into two subsets. One subset including the chargers used in the solution is named U_1, and the other including the rest of chargers is named U_2. Next step is to delete a charger each time and re-solve the instance. Chargers subset from U_2 is first selected to be deleted one by one. Once chargers from U_2 are deleted, we begin to delete chargers from subset U_1. Instance is re-solved by heuristic method and corresponding solutions for this instance with different number of available chargers are obtained by heuristic method and are as shown in Figure 4.

In Figure 4, number of chargers is the horizontal axis and obtained total traveled distance is the vertical axis. It is observed in the figure that, in the beginning, total traveled distance remains the same as 897.23 for the instance with number of chargers being 20 and decreasing to 15. Starting from the case that the number of chargers becoming 14, it is shown in the figure that the total traveled distance increases as number of chargers decreases. When the number of chargers is 8, the total traveled distance is 1035.95, which is 15.46% increased from the case with 20 chargers. Once number of chargers decreases to 7 or less, the problem becomes infeasible. This result shows that number of charger has a significant influence on the solution of total traveled distance. The more chargers, the smaller total traveled distance possibly. It is also seen from the figure that there exists a lower bound of number of chargers below which the problem will become infeasible; similarly, an upper bound exists that above that point, the total traveled distance may not be improved any more.

5.4 Impact of charging rate

Instance m101c100 is selected to test for the impact of charging rate on the solution. It is set up with 8 available chargers and full battery capacity $Q = 62$. Charging rate, as a parameter, varies in different experiments. Initially, the charging rate is $g = 2.07$ so that the time of recharging to full battery capacity is $\frac{Q}{g} = 30$ minutes. Gradually, it decreases to 1.38, 1.03, 0.83, 0.69, 0.59, 0.52, in such a way the time of charging to full battery capacity time increase from 30 to 45, 60, 75, 90, 105, and 120 minutes, respectively. We use the proposed heuristic method to solve these instances with different charging rate.

It is observed from the results that the lower the charging rate, the higher the total distance. This can be explained by the fact that the increasing of recharging time may lead to the violation of customer time windows. If any violation occurs, the vehicles will be re-routed or additional vehicles

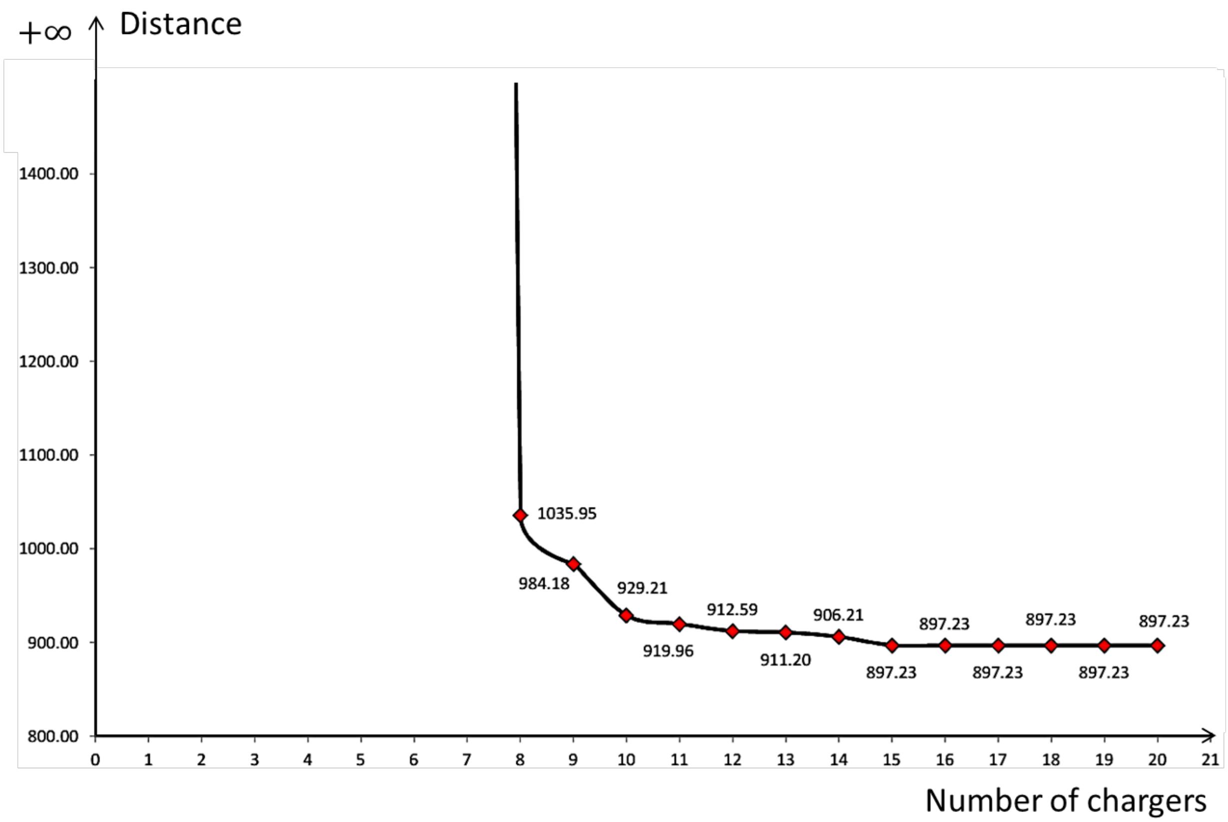


Figure 4: Impact of number of chargers on the total traveled distance

Table 5: Results of instances with different charging rate, when the battery capacity is fixed at 62.

Recharging rate, g	Q/g	Number of vehicles	total traveled distance	Makespan
2.07	30	8	1035.95	2819.30
1.38	45	8	1082.77	2853.95
1.03	60	10	1168.53	2818.62
0.83	75	10	1168.53	2818.62
0.69	90	10	1168.53	2818.62
0.59	105	10	1168.53	2818.62
0.52	120	10	1168.53	2818.62

Table 6: Results of instances with different battery capacities , when the recharging rate is fixed at $g = 2.07$.

Battery Capacity, Q	Q/g	Number of vehicles	total traveled distance	Makespan
62	30	8	1035.95	2819.30
93	45	9	1044.91	2776.54
124	60	8	1043.52	2772.67
155	75	8	1038.81	2764.35
186	90	7	1009.43	2759.35
217	105	7	1009.43	2759.35
248	120	7	1009.43	2759.35

will be deployed.

5.5 Impact of battery capacity

Instance m101c100 with 8 available chargers, and a fixed recharging rate $g = 2.07$ is selected to test the impact of battery capacity on the solution. Battery capacity Q increases from 62 to 248 in such a way the recharging time varies from 30 to 120 minutes. The results are shown in Table 6. As the battery capacity increases, the total traveled distance first increases, and then decreases monotonically, reaching to a stationary level at the end. The first unexpected increase is related to the increase in the number of vehicles used from 8 to 9. By deploying an additional vehicle when the battery capacity is changed from 62 to 93, the algorithm was able to decrease the makespan. The trend afterward is intuitive.

6 Conclusions

In this paper, we propose an improved E-VRP model developed from the EVRPTW model of Schneider et al. (2014) to minimize the total traveled distance. Customer time windows and vehicle capacity constraints are incorporated in the capacitated model. Furthermore, our capacitated E-VRP model considers the limited capacity at the charging stations and enables partial recharging for EVs at chargers. Also, EVs are possibly recharged to a necessary battery level other than full battery capacity at the chargers during the routes.

We develop a modified heuristic method from VNS/TS heuristic to obtain the solution. In the numerical experiments, both small and large size instances are tested and presented. Small size instances are solved by both the CPLEX solver using the formulation of capacitated E-VRP model and heuristic method based on VNS/TS. The study shows the capability of capacitated E-VRP model to handle conflicts at charging stations and dynamic recharging for EVs. Also for small size instances, it is shown that heuristic method has much better performance with regard to computational time when compared to using CPLEX solve with capacitated E-VRP model. Heuristic method is also tested to be able to solve large size instances. The results from large size instances show that the total traveled distances will decrease when the battery capacity is higher. Further analysis of number of chargers shows that it has influence on the total traveled distance. Proper number of chargers is needed to reduce both the total traveled distance and the cost of establishing the charging facilities.

Future work could explore the influence of locations and capacities of charging stations and allocate the charging stations to proper locations with optimized capacity, so that the total costs including cost of total traveled distance and cost of establishing such charging facilities can be reduced. Also, the possible alternative way to solve a large size problem is to split the service area into sections that have density of customers, so that in each cluster the EV may have sufficient battery to serve without recharging en route.

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