

# On-line Supplement for:

## Dispatching and Loitering Policies for Unmanned Aerial Vehicles under Dynamically Arriving Multiple Priority Targets

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### 1 First Region: Supplemental Information

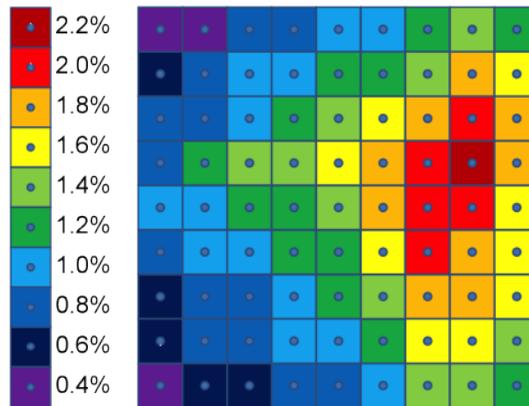


Figure 1: Mission space, with *a priori* information

The values from the percent improvement charts are included in Table 1. The greatest improvement values indicated in the paper are highlighted.

The full analysis of the which policy combination is best with full information on that mission's factor levels is included in Table 2. A Tukey test with a 95% confidence was also conducted to establish which policy combination's mean performance was not significantly different then the best.

Table 1: Percent Improvement in Policy Combinations' Performance

DP	LP	Percent Improvement		
		over DP1 and LP1	of LP under fixed DP	of DP under fixed LP
1	1	0.00 %	0.00 %	0.00 %
1	2	9.92 %	<b>9.92 %</b>	0.00 %
1	3	5.56 %	5.56 %	0.00 %
1	4	5.64 %	5.64 %	0.00 %
1	5	5.58 %	5.58 %	0.00 %
2	1	11.53 %	0.00 %	11.53 %
2	2	12.49 %	1.08 %	2.84 %
2	3	20.36 %	9.98 %	15.67 %
2	4	20.58 %	10.23 %	15.83 %
2	5	20.76 %	<b>10.44 %</b>	16.08 %
3	1	19.73 %	0.00 %	19.73 %
3	2	18.29 %	-1.80 %	9.28 %
3	3	27.13 %	9.21 %	22.83 %
3	4	27.70 %	9.92 %	23.38 %
3	5	28.12 %	<b>10.44 %</b>	23.87 %
4	1	3.29 %	0.00 %	3.29 %
4	2	13.48 %	<b>10.53 %</b>	3.95 %
4	3	9.05 %	5.95 %	3.69 %
4	4	9.11 %	6.02 %	3.68 %
4	5	9.06 %	5.97 %	3.69 %
5	1	15.05 %	0.00 %	15.05 %
5	2	16.19 %	1.34 %	6.96 %
5	3	24.19 %	10.76 %	19.72 %
5	4	24.44 %	11.06 %	19.93 %
5	5	24.67 %	<b>11.32 %</b>	20.22 %
6	1	21.84 %	0.00 %	<b>21.84 %</b>
6	2	20.52 %	-1.68 %	<b>11.77 %</b>
6	3	29.38 %	9.64 %	<b>25.22 %</b>
6	4	29.97 %	10.41 %	<b>25.78 %</b>
6	5	<b>30.35 %</b>	<b>10.89 %</b>	<b>26.23 %</b>

Table 2: Context Sensitive - Best Policy Combinations

UAV Speed	Target Arrival Rate	On-Scene Service Rate	Percent Priority 1	Best Policy		Policies not Significantly* Different then Best Policy
				DP	LP	
3	1	10	0.1	3	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5

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UAV Speed	Target Arrival Rate	On-Scene Service Rate	Percent Priority 1	Best Policy		Policies not Significantly* Different then Best Policy
				DP	LP	
3	1	10	0.35	3	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
3	1	10	0.6	3	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
3	1	20	0.1	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
3	1	20	0.35	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
3	1	20	0.6	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
3	1	30	0.1	3	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
3	1	30	0.35	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
3	1	30	0.6	3	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
3	2	10	0.1	6	5	6-2, 6-3, 6-4, 6-5
3	2	10	0.35	6	5	6-3, 6-4, 6-5
3	2	10	0.6	6	5	6-3, 6-4, 6-5
3	2	20	0.1	6	4	6-3, 6-4, 6-5
3	2	20	0.35	6	4	6-3, 6-4, 6-5
3	2	20	0.6	6	4	6-3, 6-4, 6-5
3	2	30	0.1	6	5	6-3, 6-4, 6-5
3	2	30	0.35	6	5	6-3, 6-4, 6-5
3	2	30	0.6	6	5	6-3, 6-4, 6-5
3	3	10	0.1	6	4	6-all
3	3	10	0.35	6	4	6-all
3	3	10	0.6	4	1	4-all, 5-all
3	3	20	0.1	6	4	6-all
3	3	20	0.35	6	2	6-all
3	3	20	0.6	4	3	4-all, 5-all
3	3	30	0.1	6	3	6-all
3	3	30	0.35	6	4	6-all
3	3	30	0.6	5	4	4-all, 5-all
6	1	10	0.1	3 & 6	5	2-3, 2-4, 2-5, 3-3, 3-4, 3-5, 5-3, 5-4, 5-5, 6-3, 6-4, 6-5
6	1	10	0.35	3 & 6	5	2-5, 3-3, 3-4, 3-5, 5-5, 6-3, 6-4, 6-5
6	1	10	0.6	3 & 6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	1	20	0.1	3 & 6	5	2-3, 2-4, 2-5, 3-3, 3-4, 3-5, 5-3, 5-4, 5-5, 6-3, 6-4, 6-5
6	1	20	0.35	3 & 6	5	2-5, 3-3, 3-4, 3-5, 5-5, 6-3, 6-4, 6-5
6	1	20	0.6	3 & 6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	1	30	0.1	3 & 6	5	2-3, 2-4, 2-5, 3-3, 3-4, 3-5, 5-3, 5-4, 5-5, 6-3, 6-4, 6-5
6	1	30	0.35	3 & 6	5	2-5, 3-3, 3-4, 3-5, 5-5, 6-3, 6-4, 6-5
6	1	30	0.6	3 & 6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5

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				DP	LP	
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6	2	10	0.35	3 & 6	5	2-5, 3-3, 3-4, 3-5, 5-5, 6-3, 6-4, 6-5
6	2	10	0.6	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
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6	2	20	0.6	3	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	2	30	0.1	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	2	30	0.35	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	2	30	0.6	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	3	10	0.1	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	3	10	0.35	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	3	10	0.6	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	3	20	0.1	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	3	20	0.35	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	3	20	0.6	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	3	30	0.1	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	3	30	0.35	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
6	3	30	0.6	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	1	10	0.1	3 & 6	5	2-3, 2-4, 2-5, 3-3, 3-4, 3-5, 5-3, 5-4, 5-5, 6-3, 6-4, 6-5
9	1	10	0.35	3 & 6	5	2-3, 2-4, 2-5, 3-3, 3-4, 3-5, 5-3, 5-4, 5-5, 6-3, 6-4, 6-5
9	1	10	0.6	3 & 6	5	2-5, 3-4, 3-5, 5-5, 6-3, 6-4, 6-5
9	1	20	0.1	3 & 6	5	2-3, 2-4, 2-5, 3-3, 3-4, 3-5, 5-3, 5-4, 5-5, 6-3, 6-4, 6-5
9	1	20	0.35	3 & 6	5	2-3, 2-4, 2-5, 3-3, 3-4, 3-5, 5-3, 5-4, 5-5, 6-3, 6-4, 6-5
9	1	20	0.6	3 & 6	5	2-5, 3-3, 3-4, 3-5, 5-5, 6-3, 6-4, 6-5
9	1	30	0.1	3 & 6	5	2-3, 2-4, 2-5, 3-3, 3-4, 3-5, 5-3, 5-4, 5-5, 6-3, 6-4, 6-5
9	1	30	0.35	3 & 6	5	2-3, 2-4, 2-5, 3-3, 3-4, 3-5, 5-3, 5-4, 5-5, 6-3, 6-4, 6-5
9	1	30	0.6	3 & 6	5	2-5, 3-3, 3-4, 3-5, 5-5, 6-3, 6-4, 6-5
9	2	10	0.1	3 & 6	5	2-5, 3-3, 3-4, 3-5, 5-4, 5-5, 6-3, 6-4, 6-5
9	2	10	0.35	3 & 6	5	2-3, 2-4, 2-5, 3-3, 3-4, 3-5, 5-3, 5-4, 5-5, 6-3, 6-4, 6-5

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UAV Speed	Target Arrival Rate	On-Scene Service Rate	Percent Priority 1	Best Policy		Policies not Significantly* Different then Best Policy
				DP	LP	
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9	2	20	0.6	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	2	30	0.1	3 & 6	5	2-5, 3-3, 3-4, 3-5, 5-4, 5-5, 6-3, 6-4, 6-5
9	2	30	0.35	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	2	30	0.6	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	3	10	0.1	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	3	10	0.35	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	3	10	0.6	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	3	20	0.1	3	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	3	20	0.35	3	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	3	20	0.6	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	3	30	0.1	6	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	3	30	0.35	3	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5
9	3	30	0.6	3	5	3-3, 3-4, 3-5, 6-3, 6-4, 6-5

\* Tukey 95% confidence interval for pairwise comparison of policy combinations' mean performance

## 2 Second Region: Simulation Study

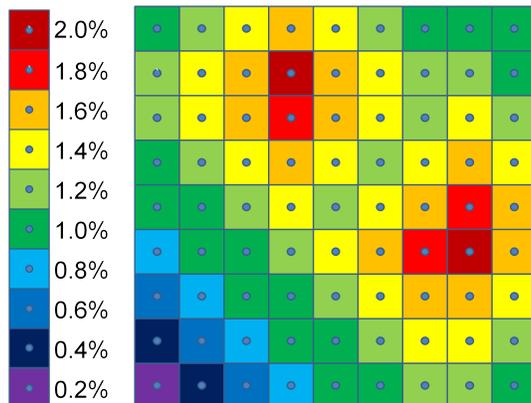


Figure 2: Second Mission space, with *a priori* information

The study and analysis of results for this second mission space will be included in this online supplement. Although interesting, the second mission space produced similar results. Two points of consideration must be considered when comparing these two mission environments: (1) The utilization of UAVs will be higher as total travel is greater in the second mission space, due to distribution of target arrival being spread out and (2) UAV initiated policy will dominate the dispatching decision and loitering will rarely occur under some of the environmental factor levels, discussed in Section 2.3.

### 2.1 Design of Experiment

A full factorial experiment was performed for the second mission space. Only two factors are considered for effect of system performance: size of region and mean arrival rate of targets; each factor is tested at three levels. The previously considered factors mean on-scene service rate for targets and distribution of targets in priority classes were found, through ANOVA, to minimal explain variation. Therefore to reduce the size of the supplemental study these were set to fixed values; On-Scene Service Rate was set at 10, and Percentage of Targets at Priority Level 1, 2, and 3 were set at .35, .3, and .35 respectively.

The different levels for the speed of UAVs (region size) and mean arrival rate of targets can be described as follows: the medium and high levels are two and three times as large as the low level, respectively. Low factor level for speed, arrival rate, and on-scene service rate were chosen such that  $s = 3$ ,  $\lambda = 1$ , and  $\mu = 10$ , respectively. The resulting values computed are shown in Table 3.

Table 3: Factor Level Values

<b>Factors</b>	<b>Factor Levels</b>		
	Low	Medium	High
UAV Speed	3	6	9
Target Arrival Rate	1	2	3

The simulation is run for 20 replications of 600 time units with 100 time units of warm-up. The number of replications was chosen such that the confidence interval, for unknown mean and unknown standard deviation, is at least at a level of 90%.

## 2.2 Results Analysis

The results of a full factorial analysis indicate that all main effects and two way interactions are significant. The two environmental factors (speed and arrival rate) effect performance as intuitively expected: as the size of the region decreases (i.e. the speed of the UAVs increase), performance of the system increases; as arrival rate increases, performance decreases. The results of the region size are intuitive as the measure used is based on the time until target is serviced.

Table 4: Mean Weighted Reward for TiQ based on each factor’s level

<b>Factor</b>	Low	Medium	High
UAV Speed	42.1471	22.0049	12.7838
Target Arrival Rate	30.2424	25.4479	19.7006

The main effect of the decision policy and loiter policy choice indicated that the sixth decision policy, STTD and CtbAU, and the fifth loitering policy, dynamic recalculation of p-median w/CoBU, should be chosen respectively if knowledge of the other choice and system condition is only known to fall within the tested factor levels. The trimmed mean performance of choosing a decision policy or a loitering policy can be seen in Table 5. Through an analysis of variance test, the speed of UAVs has the most significant effect explaining 67.57% of variation of in results. Arrival rate of targets and the decision policy implemented explain 8.08% and 10.69% of variation, respectively. The context sensitive analysis in Section 2.3 will separate the effect of the environmental factors, including UAV speed, from the affect of the decision policy and loitering policy choice on performance.

Table 5: Trimmed Mean Weighted Reward for TiQ for each policy selection

		1	2	3	4	5	6
Decision Policy	Mean	17.9371	21.6450	27.7995	24.3843	28.6367	33.3536
	SE	0.4180	0.5679	0.5641	0.3083	0.4401	0.4485
Loiter Policy	Mean	23.4244	24.7171	26.5129	26.5300	26.5975	
	SE	0.3966	0.4015	0.4772	0.4794	0.4801	

### 2.3 Decision Policy and Loitering Policy Performance in Context Sensitive Environments

Following the results of the policy combination analysis over all factor levels, the same results were observed for most of the specific factor combination findings; the policy combination of DP6 and LP5 has the best performance. The policy combination of DP6 and LP4 has the best performance in two of context sensitive situation but this combination is never significantly different then the combination of DP6 and LP5.

Table 6: Context Sensitive - Best Policy Combinations

UAV Speed	Target Arrival Rate	Best Policy		Policies not Significantly* Different then Best Policy
		DP	LP	
3	1	6	5	3-3, 6-3, 3-4, 6-4, 3-5
3	2	6	5	3-3, 6-3, 3-4, 6-4, 3-5
3	3	6	5	3-3, 6-3, 3-4, 6-4, 3-5
6	1	6	4	6-3, 6-5
6	2	6	5	6-3, 6-4
6	3	6	5	6-3, 6-4
9	1	6	4	6-1, 6-2, 6-3, 6-5
9	2	6	4	6-1, 6-2, 6-3, 6-5
9	3	6	4	6-3, 6-5

\* Tukey 95% confidence interval for pairwise comparison of policy combinations' mean performance

## Acknowledgments

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