Simultaneous Sensor Selection and Routing of Unmanned Aerial Vehicles for Complex Mission Plans

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Abstract

Military reconnaissance missions often employ a set of unmanned aerial vehicles (UAVs) equipped with sensors to gather intelligence information from a set of known targets. UAVs are limited by the number of sensors they can hold; also attaching a sensor adds weight to the aircraft which in turn reduces the flight time available during a mission. The task of optimally assigning sensors to UAVs and routing them through a target field to maximize intelligence gain is a generalization of the team orienteering problem studied in vehicle routing literature. This work presents a mathematical programming model for simultaneous sensor selection and routing of UAVs, which solves optimally using CPLEX for very simple missions. Larger missions required the development of three heuristics, which were augmented by column generation. Results from a performance study indicated that the heuristics quickly found good solutions. Column Generation improved the solution in many instances, with minimal impact on overall solution time. The rapid nature of the solution approach allows it to be used in other mission planning tasks. A fleet sizing application is discussed as an example of its flexible usage.

Keywords: unmanned aerial vehicles, routing, sensor selection, column generation, team orienteering problem

1. Introduction

1.1. Problem Description

In unmanned aerial vehicle (UAV) mission planning, there exists a set of predetermined targets that require surveillance. The surveillance required at each target is unique and can only be satisfied with a specific sensor, or set of sensors. Surveillance benefit is obtained when UAVs visit targets with the appropriate sensors attached. The goal of mission planning is to route the UAV fleet through the target field in an effort to maximize surveillance benefit. For simplicity, mission planners often assume that the sensors attached to each UAV in the fleet are fixed. Modern UAVs have the ability to interchange sensors, providing greater flexibility in mission planning. The cost/time associated with a sensor change is usually small so swapping sensors between missions is feasible. Assuming fixed sensor attachments simplifies the complexity associated with the planning phase, but also hinders the effectiveness of the mission. The consideration of interchangeable sensors adds complexity for the following reasons:

1. **Increased Combinations:** When sensors are fixed on UAVs, each target has a unique benefit when it is visited. Consideration of interchangeable sensors adds a great deal of complexity because the benefit of visiting a target is no longer fixed. Thus, the increase in complexity is dependent on the quantity of sensors considered and the sensor capacity of the UAV.
2. **Travel Time Variability:** UAVs with predefined sensor attachments have a fixed travel range. When interchangeable sensors are considered the sensor payload weight assigned to each UAV will vary. As the payload weight increases, the travel time available for UAVs to visit targets decreases. This variation in travel time adds additional complexity to the problem.

Furthermore, each sensor may not be compatible with every type of UAV. In Section 1.2, an example case is presented to demonstrate the problem. This case will be referenced in subsequent sections to assist in the explanation of the solution approach.

1.2. Example Case

Consider six targets spread over a terrain of 100 by 80 units. Intelligence is gained when these targets are surveyed with a particular sensor. Some targets may only benefit from a single sensor, while others may benefit from a combination of sensors. The quantitative benefit of surveillance is based on priority and importance, as determined by a mission planner. Typically, this data would be derived from a prior reconnaissance mission or existing intelligence. Figure 1 details the spatial layout of targets.

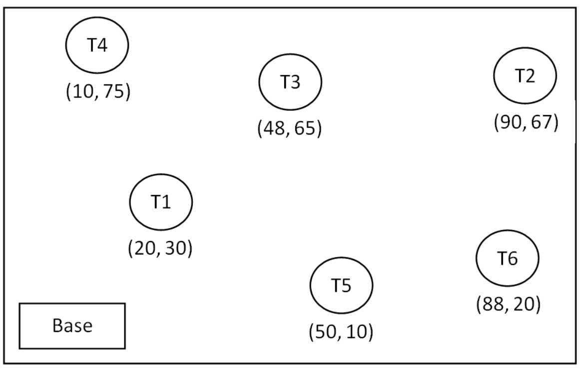


Figure 1: Spatial Arrangement of Targets and Base

The Cartesian coordinates of each target are listed in parenthesis. A single base is considered and located at the origin. All UAVs must depart from the base, survey a set of targets, and return to the base before their available travel time is depleted. The requirement for each UAV to return to the base is pertinent, as many of the UAVs and sensors are relatively expensive. However, relaxation of this requirement can be incorporated for disposable UAVs. Euclidean distance is assumed as the travel time between targets.

Next, the surveillance benefit of visiting a target will be discussed. Four sensors are considered in this example. While not explicitly defined, the sensor set could include electro-optical/infrared cameras, video recording devices, and radiation detectors. The sensor-target benefit matrix (STBM) is shown in Table 1.

Table 1: Sensor Target Benefit Matrix

	S1	**S2**	**S3**	**S4**
T1	100	0	130	135
T2	145	120	100	75
T3	80	0	0	120
T4	160	80	50	25
T5	0	0	0	300
T6	50	45	110	0

Here, if a UAV visits target T1 with sensor S1, a benefit of 100 units is obtained. Additionally, if sensor S3 was also attached, the surveillance benefit of visiting target T1 would increase to 230. Target T3 receives no benefit if it is visited by sensor S2 or S3. The characteristics of the resources available to survey the targets will now be detailed.

Two UAVs are considered, each UAV with the ability to carry two sensors. Additional attributes are detailed in Table 2.

Table 2: UAV Attributes

UAV	**Sensor Capacity**	**Unloaded Range**	**Load Limit**
1	2	300	125
2	2	350	140

The unloaded range represents the time a UAV can travel without any sensor attachments while the load limit indicates the maximum sensor payload weight. The attributes of each sensor is displayed in Table 3.

Table 3: Sensor Attributes

Sensor	**Sensor Weight**	**Travel Time Reduction**
S1	100	100
S2	75	75
S3	125	125
S4	40	40

If sensor S1 is attached to UAV 1, the travel time would reduce to 200 units. Also, sensors S1 and S2 cannot simultaneously be attached to UAV 1 because doing so would exceed the load limit. The weight of a sensor is strongly correlated to its hindrance on travel time (Scaled Composites, 2004). Thus, in this example, the travel time reduction is equivalent to the weight of a sensor. It should be noted that the mathematical formulation developed does not require this equivalence.

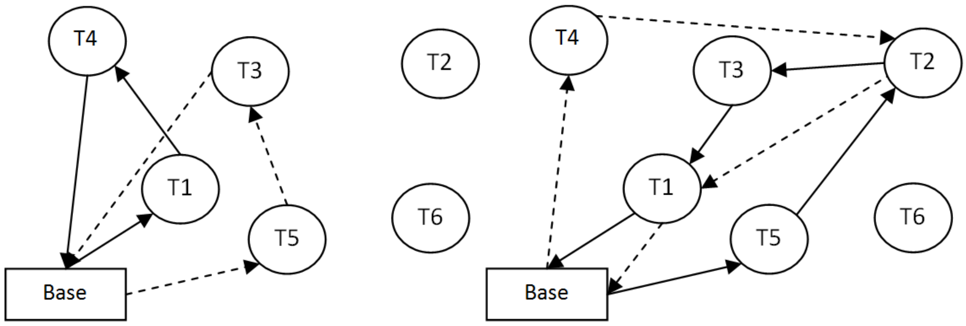
If one were to consider the sensor selection and routing aspects of the problem independently, the solution procedure may elect to assign sensors to UAVs first and then route them through the target field. Alternatively, one could create a route for each UAV first and subsequently assign sensors. Here, the former procedure will be discussed.

In a two step method that assigns sensors first and routes UAVs second, two logical approaches are suggested for the sensor assignment step.

1. **Highest Potential Benefit:** Using this approach, one would assign sensors based on their potential benefit. It is assumed that assignment precedence is based on greatest unloaded travel range. The potential benefit of a sensor is the benefit it could obtain if it was capable of visiting all targets. For the example problem, the potential benefit of sensors S1, S2, S3, and S4 is 535, 245, 390, and 655, respectively. Using this approach, one would assign sensor S3 to UAV 1 and sensors S1 and S4 to UAV 2. Note that we cannot assign both sensors S2 and S3 to UAV 2 due to payload limitations. S3 was chosen over S2 because it has a higher potential benefit.

2. **Lowest Travel Time Reduction:** The logic behind this approach is to load each UAV with sensors that minimize travel time reduction. Hence, UAVs will be able to visit more targets, even though the benefit per visit may be less than the previously stated method. Once again, assignment precedence is based on unloaded travel range, with the lightest sensor being assigned to the UAV with the lowest range. Here, sensor S4 would be assigned to UAV 1 and sensor S2 would be assigned to UAV 2.

After assigning sensors using either of these methods, the UAVs can be optimally routed through the targets. Given a small problem, the optimal route can be obtained using a straightforward mathematical programming model. Figure 2 shows the optimal routes when the above approaches are used. The solid and dashed lines represent UAV 1 and UAV 2, respectively. Table 4 summarizes the results.



(a) Highest Potential Benefit (b) Lowest Travel Time Reduction

Figure 2: Route Assignments for Two-Step Heuristics

Table 4: Summary of Results for each Sensor Selection Procedure

Sensor Selection Procedure	UAV 1 Sensors	UAV 2 Sensors	Benefit
Highest Potential Benefit	S3	S1,S4	915
Lowest Travel Time Reduction	S4	S2	830

The optimal sensor and route assignment is shown in Figure 3. Clearly, the sensor selection utilized for the optimal solution is some combination of the selection procedures above. The optimal solution was acquired using the Integrated Sensor Selection and Routing Model (ISSRM). The ISSRM is a mixed integer linear programming formulation that will be detailed in Section 2. For this example, a 9% improvement is obtained due to a combined consideration of sensor assignment and routing of the aircraft.

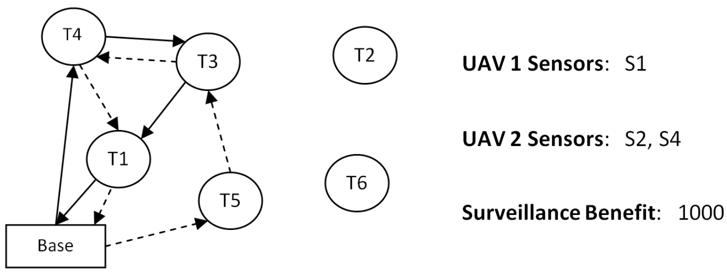


Figure 3: Optimal Solution using Integrated Sensor and Routing Model

1.3. Connection to Team Orienteering Problem

The UAV sensor selection and routing problem is a generalization of the team orienteering problem (TOP) (Butt and Cavalier, 1994). To establish this connection with the TOP, we first discuss the orienteering problem (OP) which was initially investigated by Tsiligirides (1984). In the OP, a single vehicle begins at a starting location and must reach a designated destination prior to time T_{max}. Along with the starting and ending points, a set of locations exist with an associated benefit which can be collected by the vehicle. The objective of the OP is to route the vehicle through a subset of the locations in a way which maximizes the collected benefit. Of course, the vehicle must reach the end point by time T_{max}. The problem gets its name from the sport of orienteering. In this navigation based game, players begin at a central control location and must visit other control locations where they accumulate points. The quantity of points received is typically based on how difficult it is to reach the control location. Players are disqualified if they fail to reach a predefined destination before the time limit, and the winner is the individual who accumulates the greatest number of points. Clearly, the optimal route is the solution to the OP. The TOP is identical to the OP, but multiple vehicles may be used to visit the control points.

Two papers have been published which present exact algorithms for the TOP: Boussier et al. (2007) present a branch and price algorithm while Butt and Ryan (1999) use column generation. Unfortunately, each of these approaches can only solve problems of limited size in a reasonable amount of time. Butt and Ryan find optimal solutions to problems with 100 nodes, but the vehicle travel time is limited so the average tour size contains fewer than 4 nodes. These results are not surprising, however, as the OP (a single vehicle instance of the TOP) was shown by Golden et al. (1987) to be NP-hard. For this reason, most of the literature has focused on the presentation of heuristic approaches.

Chao et al. (1996) developed a heuristic to solve the TOP. They compare their heuristic with an extension of a stochastic heuristic originally designed by Tsiligirides (1984) to solve the OP. In our paper, a variation of Tsiligirides' heuristic is also used to develop initial solutions for column generation. This heuristic will be detailed in Section 3.3.3. Tang and Miller-Hooks (2005) used a tabu search heuristic to solve the TOP. Their procedure included an adaptive memory procedure to store and update solutions. Archetti et al. (2007) developed two additional tabu search heuristics along with with a variable neighborhood search, which provided results superior to the above mentioned. An ant colony optimization approach was used by Ke et al. (2008) which produced results on par or better than Archetti et al. (2007) with quicker solution times. In a recent paper by Vansteenwegen et al. (2009) a guided local search framework was implemented to rapdily find good solutions to the TOP. For a thorough overview of the OP and TOP see Vansteenwegen et al. (2010).

In the TOP, split deliveries are not allowed and a customer may not be visited by more than one vehicle. If the sensor assignment and routing problem in this work considered a homogeneous UAV fleet with fixed sensors, it would be equivalent to the TOP. In this special case, T_{max} would be equivalent to the travel range of each UAV in the fleet. Also, the surveillance benefit gained for visiting a target would be identical for each UAV. Specifically, the benefit would correspond to the fixed sensor assignment.

Generally speaking, however, T_{max} will be different for each UAV in the fleet for two reasons. Primarily, the fleet is heterogeneous so the unloaded travel time for each UAV, T_{max}, need not be equivalent. Moreover, each sensor attachment impacts travel time differently. Since each UAV can be equipped with a unique set of sensors, impact on travel distance will not be consistent across all UAVs in the fleet. Additionally, the value associated with each target will be different for each UAV in the fleet since it also corresponds to the attached sensors. Butt and Cavalier (1994) refer to the TOP as the Multiple Tour Maximum Collection Problem (MTMCP) and assume the start and end point of each vehicle to be the same. Similarly, the work presented here assumes that each UAV takes off and lands at the same location. Table 5 compares the sensor selection and routing problem to the team orienteering problem.

Table 5: Comparison of Traditional Team Orienteering Problem (TOP) and Sensor Selection and Routing Problem (SSRP)

	Traditional TOP	**SSRP**
Allowable Travel Time	Fixed Across All Vehicles	Variable Across All Vehicles
Customer Benefit	Fixed Across All Vehicles	Variable Across All Vehicles
Start and End Nodes	Unique	Identical
Split Deliveries	Prohibited	Allowed

Additionally, the mathematical model developed here allows for the inclusion of time windows. We note that Kantor and Rosenwein (1992) initially investigated the orienteering problem with time windows and developed a tree based heuristic. The tree heuristic is compared to an insertion procedure centered on an extension of a heuristic developed by Laporte and Martello (1990). For problems with few nodes and small T_{max} values, the tree based heuristic outperforms the insertion heuristic in terms of solution quality. As the quantity of nodes, T_{max}, and time window width increase, the tree heuristic is unable to find a solution in reasonable time.

1.4. Relationship with UAV Routing and Planning Literature

In this section, we briefly review contributions that have been made in UAV routing and scheduling. There has been significant work in the area of path planning for UAVs. See, for example, the papers by Kim et al. (2008), Nikolos et al. (2003), Qu et al. (2005), and Yang and Kapila (2002). The focus of path planning is primarily on satisfying vehicle dynamic requirements as well as avoiding obstacles. Mission planning can be viewed as a complex version of path planning where the objective is to visit a sequence of targets to achieve the objectives of the mission. For example, in a recent paper by Wu et al. (2009) the mission scenario involves the delivery of a medical package to a remote location using a small UAV. The goal of our paper is related to a much more complex mission. We are dealing with targets in an area of interest that can be visited by UAVs to gain information about the targets (i.e., reconnaissance). The additional complications include the facts that the UAVs can be equipped with several alternative sensor assignments, each with its own gain values; the weight of sensors reduce the UAV range; there are time windows for the targets to be visited; and a fuel constraint exists. Thus, our contribution to the UAV routing and planning literature is in the domain of planning a complex mission, not in the domain of satisfying realistic vehicle dynamic requirements or obstacle avoidance.

1.5. Organization of Paper

Section 2 contains the formulation for the Integrated Sensor Selection and Routing Model (ISSRM), which is useful for finding a direct solution using CPLEX when the mission contains few targets and UAVs. Section 3 details a column generation heuristic, which includes a presentation of the master and subproblem formulations, a description of three heuristics used to obtain initial columns, and a heuristic for solving the subproblem. Our computational experience is detailed in Section 4 where experimental conditions are defined, small/medium sized problems and large problems are separately considered, and a fleet sizing application is discussed. The paper ends with a set of conclusions and future work directions in Section 5.

2. Integrated Sensor Selection and Routing Model

The Integrated Sensor Selection and Routing Model (ISSRM) is defined as a mixed integer linear programming formulation. The formulation is based off the single commodity problem proposed by Warrier (2001). The inputs, decision variables, and formulation are detailed below.

Indcies

i, j	indices for targets($i, j = 0$ represents base location)
h	index for UAVs
s	index for sensors

Inputs

N	number of targets
O	number of UAVs
S	number of sensors
τ_h	sensor capacity for UAV h
Q_s	quantity of sensor s available at base
V_{js}	demand of sensor s at target j
R_{js}	benefit obtained when sensor s visits target j
D_{ij}	travel time from target i to target j
λ_h	unloaded travel time of UAV h
C_s	travel time reduction when sensor s is attached
δ	fuel minimization weight factor
A_{ih}	earliest time target i can be visited by UAV h
B_{ih}	latest time target i can be visited by UAV h
W_{js}	time required to deliver a single surveillance unit of sensor s to target j

Decision Variables

f_{hs}	1 if UAV h is equipped with sensor s, 0 otherwise
y_{ijh}	1 if UAV h travels from target i to target j
x^s_{ijh}	sensor visit from i to j using UAV h
z_{jsh}	delivery amount of sensor s to target j using UAV h

ISSRM Formulation

$$\text{maximize} \sum_{h=1}^{O}\sum_{j=1}^{N}\sum_{s=1}^{S} R_{js} z_{jsh} - \sum_{i=0}^{N}\sum_{j:j\neq i}^{N}\sum_{h=1}^{O} \delta D_{ijh} y_{ijh} \tag{1}$$

subject to

$$\sum_{h=1}^{O} z_{jsh} \leq V_{js} \qquad \forall j,s \tag{2}$$

$$z_{jsh} \leq V_{js} f_{hs} \qquad \forall j,s,h \tag{3}$$

$$z_{jsh} \leq V_{js} \sum_{i=0,\ i\neq j}^{N} y_{ijh} \qquad \forall j,s,h \tag{4}$$

$$\sum_{h=1}^{O} f_{hs} \leq Q_s \qquad \forall s \tag{5}$$

$$\sum_{\substack{i=0\\ i\neq j}}^{N} x_{ijh}^{s} - \sum_{\substack{i=0\\ i\neq j}}^{N} x_{jih}^{s} = z_{jsh} \qquad \forall j,h,s \tag{6}$$

$$\sum_{s=1}^{S} f_{hs} \leq \tau_h \qquad \forall h \tag{7}$$

$$\sum_{\substack{j=0\\ j\neq i}}^{N} y_{ijh} - \sum_{\substack{j=0\\ j\neq i}}^{N} y_{jih} = 0 \qquad \forall i,h \tag{8}$$

$$\sum_{j=0,\ i\neq j}^{N} y_{ijh} \leq 1 \qquad \forall i,h \tag{9}$$

$$\sum_{j=0}^{N} y_{0jh} = 1 \qquad \forall h \tag{10}$$

$$\sum_{j=0}^{N} y_{j0h} = 1 \qquad \forall h \tag{11}$$

$$t_{0h} \leq \lambda_h - \sum_{s=1}^{S} C_s f_{hs} \qquad \forall h \tag{12}$$

$$\sum_{i=0}^{N}\sum_{j=0,\ j\neq i}^{N} D_{ij} y_{ijh} + \sum_{s=1}^{S} f_{hs} \leq \lambda_h \qquad \forall h \tag{13}$$

$$x_{ijh}^{s} \leq \sum_{j=0}^{N} V_{js} f_{hs} \qquad \forall i,j,h,s \tag{14}$$

$$x_{ijh}^{s} \leq \sum_{j=0}^{N} V_{js} y_{ijh} \qquad \forall i,j,h,s \tag{15}$$

$$t_{jh} \geq D_{ij} y_{ijh} + t_{ih} - M(1 - y_{ijh}) + W_{is} z_{ish} \qquad \forall i,j,h,s \tag{16}$$

$$A_{ih} \leq t_{ih} \leq B_{ih} \qquad \forall i,h \tag{17}$$

$$f_{hs}, y_{ijh} \in \{0,1\} \tag{18}$$

The first term in the objective function (1) attempts to maximize the surveillance benefit for the entire fleet of UAVs, while the second term is included to minimize fuel cost when there are alternate optimal target sets and sensor assignments. If δ is selected properly, the optimal alternative that minimizes fuel consumption will be chosen. Selection of δ is important because if the selection is too large, fuel minimization will take precedence over maximizing surveillance benefit. The mission planner may prefer to use a larger value for δ if he or she anticipates pop-up targets to appear after the mission begins. This will allow the UAVs to have adequate fuel available to potentially visit these additional targets. Constraint (2) ensures that the total sensor surveillance at a target does not exceed the demand of the target. For example, if target 6 required three photographs from a camera sensor S2, $V_{62} = 3$. Constraint (5) makes sure that the total number of sensors assigned among all UAVs does not exceed the number of sensors available at the base. Constraints (6), (14), and (15) are included to preserve sensor delivery. In order for a UAV to provide surveillance on a target, the UAV must have the appropriate sensor equipped and the UAV must visit the target. These requirements are satisfied in constraints (3) and (4), respectively. UAV sensor capacity is represented in constraint (7). Route continuity is ensured by the inclusion of constraint (8), and constraint (9) does not allow a single UAV to visit a target more than once. Each UAV has a maximum flight time when its sensor payload is empty. This flight time is reduced when sensors are added due to the increased weight. Constraints (12) and (13) ensure that the route assignments for each UAV does not exceed the total adjusted flight time. This is performed by restricting the time at which a UAV returns to the base, t_{0h}, to be less than or equal to the adjusted flight time. Constraints (10) and (11) require that all UAVs begin and end their route at the base. Constraint (16) keeps track of the time each UAV visits a target. This constraint works in conjunction with constraint (12), but also is necessary for the inclusion of time windows which are represented in constraint (17).

The ISSRM works well for simple missions containing a relatively small number of targets, few UAVs, and minimal sensor attachments. For example, the case in the previous section with six targets, two UAVs, and four sensors was optimally solved in 15 seconds using CPLEX. However, for complex missions an optimal solution cannot be found within an acceptable time limit. Thus, a column generation heuristic was developed to quickly provide good solutions. For a survey of recent contributions in column generation, see Lubbecke and Desrosiers (2005).

3. Column Generation Heuristic

3.1. Master Problem

Column generation requires decomposition of the original problem. Here, the original problem is decomposed by UAV. Thus, the number of subproblems in the procedure is equivalent to the number of UAVs considered in the mission. The subproblems are solved at each iteration of column generation, providing new sensor and route combinations for the master problem. The inputs, decision variables, and formulation for master problem are defined as follows.

Indices

i, j	indices for targets
k	index for route/sensor combination
h	index for UAV
s	index for sensors

Inputs

(HK)	set of routes/sensor combinations for UAV h
R_{js}	benefit of delivering one unit of sensor s to target j
$F_{(hk)s}$	1 if sensor s is included in (hk), 0 otherwise
$U_{(hk)j}$	1 if target j is included in (hk), 0 otherwise
$P_{(hk)ji}$	1 if UAV h travels from j to i in (hk)
V_{js}	maximum demand for sensor s at target j
Q_s	quantity of sensor s available at base
E_{hk}	additional travel time remaining after flight path is executed for (hk)
D_{ji}	travel time from target j to target i
W_{js}	time required to deliver a single unit of sensor s to target j
A_i	earliest time UAV may arrive at location i
B_i	latest time UAV may arrive at location i

Decision Variables

$x_{(hk)}$	1 if UAV h selects route/sensor combination k, 0 otherwise
g_{hjs}	units of sensor s delivered to target j using UAV h
t_{ih}	arrival time of UAV h at location j

CG Master Problem Formulation

$$\text{maximize} \sum_j \sum_s R_{js} g_{hjs} \tag{19}$$

subject to

$$\sum_{(k)\in K} x_{(hk)} = 1 \qquad \forall h \qquad \text{Dual Cost: } \alpha \tag{20}$$

$$\sum_{(hk)\in HK} x_{(hk)} F_{(hk)s} \le Q_s \qquad \forall s \qquad \text{Dual Cost: } \beta_s \tag{21}$$

$$\sum_h g_{hjs} \le V_{js} \qquad \forall j, s \qquad \text{Dual Cost: } \nu_{js} \tag{22}$$

$$g_{hjs} \le V_{js} \sum_k F_{(hk)s} x_{(hk)} \qquad \forall j, s, (hk) \qquad \text{Dual Cost: } \psi_{js} \tag{23}$$

$$g_{hjs} \le V_{js} \sum_k U_{(hk)s} x_{(hk)} \qquad \forall j, s, (hk) \qquad \text{Dual Cost: } \xi_{js} \tag{24}$$

$$\sum_j \sum_s W_{js} g_{hjs} \le \sum_k E_{(hk)} x_{(hk)} \qquad \forall (hk) \tag{25}$$

$$t_{jh} \ge D_{ij} \sum_k P_{(hk)ij} x_{hk} + t_{ih} + W_{is} g_{mis} \qquad \forall (hk), i, j, s \qquad \text{if } P_{(hk)ij} \text{ is in } (hk) \tag{26}$$

$$A_i \le t_{ih} \le B_i \qquad \forall i, h \tag{27}$$

$$x_{(hk)} \in \{0, 1\} \tag{28}$$

The objective function of the master problem is equivalent to that of the ISSRM, with the exception of fuel minimization. Specifically, it attempts to maximize the surveillance benefit for the entire mission. Unlike the ISSRM, the master problem does not generate routes and sensor combinations. It simply selects from those that it currently has available. The selection of a single route/sensor combination for each UAV in the fleet is reflected in constraint (20). Constraint (21) states that the sensors included in the selected combinations cannot exceed the number available at the base. The cumulative sensor delivery at a target among the selected combinations cannot exceed the demand at a target. This is satisfied in constraint (22). Additionally, a combination cannot survey a target unless the combination includes the appropriate sensor and visits the correct target. These requirements are satisfied in constraints (23) and (24), respectively.

The constraints mentioned thus far tie in closely to those included in the ISSRM. Since the subproblem is generating solutions for each UAV independently, there is a chance that the selected master problem could select combinations that collectively deliver more of a sensor than demanded at a target. Constraint (22) prevents excess delivery, but at least one UAV will have additional unused flight time. Constraints (25) and (26) allow the UAVs to utilize unused travel time surveying other targets in their route. It is important to reiterate that the extra travel time can only be used at targets in the route generated in the master problem. A UAV may not use this extra time to visit additional targets outside of the specified route.

Lastly, constraint (27) specifies the time windows for each target. Next, the subproblem will be discussed.

3.2. Subproblem

The LP relaxation of the master problem is solved to obtain dual costs which are passed to the objective function of the subproblem. The subproblem is as follows.

Indices

i, j	indices for targets
h	index for UAV
s	index for sensors

Inputs

C_{sh}	travel time reduction when sensor s is attached to UAV h
τ_h	quantity of sensors UAV h can carry
λ_h	unloaded range of UAV h
W_{js}	time required to deliver one unit of sensor s to target j
D_{ji}	travel time from target i to target j
R_{js}	benefit of delivering sensor s to target j
V_{js}	demand of sensor s at target j
A_i	earliest arrival time for target i
B_i	latest arrival time for target i

Decision Variables

f_s	1 if sensor s is selected, 0 otherwise
y_{ij}	1 if UAV travels from target i to target j, 0 otherwise
t_i	arrival time of UAV at target i
z_{js}	delivery amount of sensor s to target j

CG Subproblem Formulation

maximize

$$\sum_{j,s} R_{js} z_{js} - \alpha - \sum_{s} \beta_s f_s - \sum_{j,s} \nu_{js} z_{js} - \sum_{j,s} \psi_{js}(z_{js} - V_{js} f_s) - \sum_{j,s} \xi_{js}(z_{js} - V_{js} \sum_{\substack{i \\ i \neq j}} y_{ij}) \quad (29)$$

subject to

$$z_{js} \leq V_{js} f_s \qquad \forall j, s \quad (30)$$

$$z_{js} \leq V_{js} \sum_{i} y_{ij} \qquad \forall j, s \quad (31)$$

$$\sum_{s} f_s \leq \tau_h \quad (32)$$

$$\sum_{\substack{j \\ j \neq i}} y_{ij} - \sum_{\substack{j \\ j \neq i}} y_{ji} = 0 \qquad \forall i \quad (33)$$

$$\sum_{\substack{j \\ i \neq j}} y_{ij} \leq 1 \qquad \forall i \quad (34)$$

$$\sum_{j} y_{0j} = 1 \quad (35)$$

$$\sum_{j} y_{j0} = 1 \quad (36)$$

$$t_0 \leq \lambda_h - \sum_{s} f_s C_{sh} \quad (37)$$

$$\sum_{i=0} \sum_{\substack{j=0 \\ j \neq i}} D_{ij} y_{ij} + \sum_{s=1} f_s \leq \lambda_h \quad (38)$$

$$t_j \geq D_{ij} y_{ij} + t_i - M(1 - y_{ij}) + W_{is} z_{is} \qquad \forall i, j, s \quad (39)$$

$$A_i \leq t_i \leq B_i \qquad \forall i \quad (40)$$

$$f_s, y_{ij} \quad \in \{0, 1\} \quad (41)$$

As mentioned earlier, the goal of the subproblem is to generate a beneficial route/sensor combination to pass back into the master problem. A separate subproblem is solved for each UAV, and thus, the number of subproblems is equivalent to the number of UAVs. A beneficial solution is one with a positive reduced cost. Here, the objective function is the reduced cost. Thus, if the objective function assumes a positive value, then adding the corresponding route/sensor combination into the master problem will improve the master problem's objective provided that this column can be brought into the current basis at a non-zero level. Similar to those found in the ISSRM, constraints (30) and (31) ensure that a UAV cannot deliver a sensor to a target unless the target is visited and the sensor is attached to the UAV. The maximum sensor attachments allowed per UAV are modeled in constraint (32). Route continuity is preserved in constraint (33), while constraint (34) does not allow the same target to be visited multiple times. Constraints (37) and (38) guarantee that the UAV's return time to the base does not exceed its total travel time adjusted for sensor attachments. Constraints (35) and (36) force the UAV to start and end at the base location. Constraint (39) is used to preserve the cumulative travel time during the course of the route. This is used in conjunction with constraint (37) and also with constraint (40) which specifies the time windows.

As the number of sensors and targets increases, the problem becomes difficult to solve optimally. However, if a quick solution can be found that corresponds to a positive reduced cost, the procedure can terminate and this solution can be added back to the master problem. Extensive testing revealed that the solution time to obtain a beneficial reduced cost for mid to large size problems was exorbitant. The goal of the subproblem is to determine the sensor/route combination for a UAV that yields the maximum benefit in reduced cost. However, any sensor/route combination with a a positive reduced cost suffices in terms of its potential to improve the objective function of the master problem. With this realization we focus on an efficient heuristic to solve the subproblem in Section 3.4. Section 3.3 describes the means for generating initial columns for the master problem.

3.3. Initial Columns

Three heuristic are presented to generate initial columns for the master problem. The first two heuristics, which will be referred to as Heuristic I and Heuristic II, employ a generative approach that assigns sensors to the UAV fleet first and subsequently routes them through the targets. They share the approach for sensor selection, but execute routing decisions differently. Heuristic I makes use of deterministic routing decisions while Heuristic II utilizes a stochastic approach. The sensor selection technique used in Heuristics I and II will be discussed in Section 3.3.1. Sections 3.3.2 and 3.3.3 detail the routing approach used for Heuristic I and Heuristic II, respectively. The third heuristic is based on a local search and is the focus of Section 3.3.4.

3.3.1. Sensor Selection

Sensor selection is based on the premise that the most favorable sensors will be those which have the greatest surveillance potential and the lowest travel distance degradation. The sensor potential to weight ratio (SP/W) shown in Equation (42) gives insight on which sensors to select.

$$\mathrm{SP/W} = \frac{\text{Potential Sensor Benefit}}{\text{Travel Distance Reduction}} \tag{42}$$

The sensor with the highest SP/W ratio is assigned to the UAV with the longest travel distance and is removed from the assignable sensor pool. Sensors are assigned to the remaining UAVs in the same manner. Tables 6 and 7 show the SP/W calculations for the example case presented earlier.

Table 6: Potential Sensor Benefit (PSB) For Sensors S1, S2, S3, and S4

Target	**S1**	**S2**	**S3**	**S4**
1	100	0	130	135
2	145	120	100	75
3	80	0	0	120
4	160	80	50	25
5	0	0	0	300
6	50	45	110	0
PSB	535	245	390	655

Table 7: SP/W Ratios

Sensor	**SP/W Ratio**
S1	5.35
S2	3.27
S3	3.12
S4	16.375

Sensor S4 has the highest SP/W Ratio so it is assigned to UAV 2, which has the greatest unloaded travel range. Next, Sensor S1 is assigned to UAV 1. The travel ranges of UAV 1 and UAV 2 after they are loaded with sensors are 200 and 310, respectively.

3.3.2. Heuristic I Route Selection

The principle behind the routing portion of both heuristics is to visit targets that supply the greatest surveillance benefit and require the least amount of fuel. The process is done iteratively among each UAV in the mission and continuously accesses and updates three matrices. The remaining benefit matrix (RBM) stores the surveillance benefit for each sensor remaining at all targets. As UAVs visit targets with the appropriate sensor(s), the values in this table are updated. The distance benefit matrix (DBM) combines the information from the RBM with the current UAV location to create a numerical value which defines the benefit of traveling to a target. Finally, the remaining travel time for each UAV is stored in the remaining travel distance matrix (RTDM).

The routing algorithm for Heuristic I is defined in Table 8.

Table 8: Heuristic I Routing Algorithm

Step 0	Initialize RBM, DBM, and RTDM Set current UAV target to 0 (location of base) Set current UAV $n = 0$
Step 1	Set current UAV to $n + 1$ and proceed to **Step2**
Step 2	Scan DBM and select target with highest value, then proceed to **Step 3** If no remaining targets exist, proceed to **Step 5**
Step 3	Check feasibility of target selection If feasible, proceed to **Step 4** If infeasible, remove target as an option and return to **Step 2**
Step 4	Update RBM, DBM, and RTDM
Step 5	Return current UAV to base If all UAVs have returned to base, **end** procedure Otherwise, proceed to **Step 1**

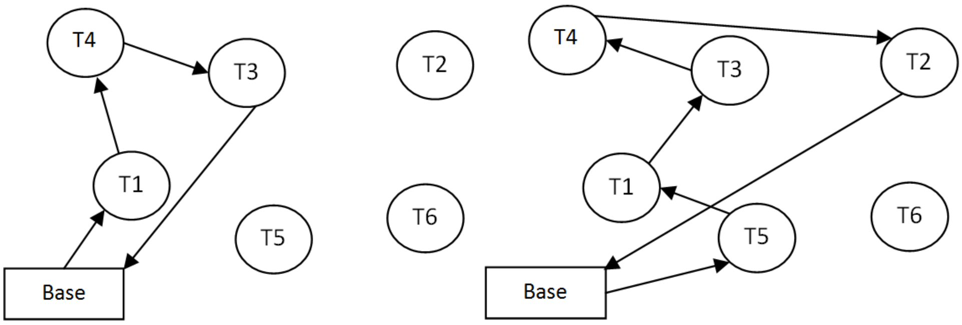
Continuing with the example case presented in Section 1.2, this algorithm can be applied to determine routes for both of the UAVs in the fleet. The DBM for UAV 1 and UAV 2 are shown in Table 9 and are used to determine the first target to visit.

Table 9: Distance Benefit Matrix For UAV 1 and UAV 2

UAV 1		**UAV 2**	
Target	**Values**	**Target**	**Value**
1	**2.17**	1	2.93
2	2.13	2	1.10
3	1.04	3	1.57
4	1.89	4	0.30
5	0	**5**	**4.96**
6	0.57	6	0

The value of visiting a target is the result when the remaining benefit for a target is divided by the travel distance to the target from the current location. For example, the value of target 1 with UAV 1 is obtained by dividing 100 (remaining benefit of visiting target 1 with UAV 1) by 46.06 (travel distance from the base to target 1). The target with the highest value is selected as it provides the highest surveillance benefit per unit of travel. In this case, targets 1 and 5 are selected for UAV 1 and UAV 2, respectively. Next, a feasibility check is performed. This check ensures that a UAV will have enough fuel remaining to return to the base after it visits the selected target.

For UAV 1, the sum of the distance to target 1 and the distance from target 1 back to the base is less than the remaining travel distance indicating that this destination is feasible. The decision to visit target 1 is finalized and the RBM and RTDM are updated. The process is repeated for UAV 2. If the target with the highest DBM value was an infeasible option, the next best target would be selected and checked for feasibility. The routing procedure ends when all of the targets are either infeasible or have a value of zero. At this point, the UAV is routed back to the base and the heuristic concludes. If the procedure were run to completion, the routes in Figure 4 would be obtained. Table 10 summarizes the results.



(a) Final Route For UAV 1 (b) Lowest Travel Time Reduction

Figure 4: Route Assignments For Two Step Heuristics

Table 10: Summary of Results For Heuristic I

UAV 1 Sensors	UAV 2 Sensors	Surveillance Benefit	Optimality Gap
S1	S4	995	%0.5

The heuristic provides excellent results for the example case with an optimality gap of only 0.5%. While this is certainly not the case for all problems, the heuristic obtains good solutions to use for initial columns in the column generation procedure.

3.3.3. Heuristic II Route Selection

The routing procedure developed for Heuristic II is based on the construction procedure used for Heuristic I. The key difference is the inclusion of a stochastic decision for each iteration of the procedure. In Heuristic I, the target with most attractive DBM value was selected. Here, the top four feasible targets ($j = 1 - 4$) are considered and one is stochastically selected. When fewer than four feasible targets with positive DBM values remain, the procedure is adjusted to select among them. Assuming that four targets are among those to be selected, a weight is assigned to them as shown in Equation (43).

$$W_i = \frac{B_i}{\sum_{j=1}^{4} B_j} \tag{43}$$

W_i is the weight assigned to target i, while B_i and B_j are the benefits of visiting targets i and j, respectively. Each target i is assigned a section of a 0 - 1 scale as shown in Figure 5.

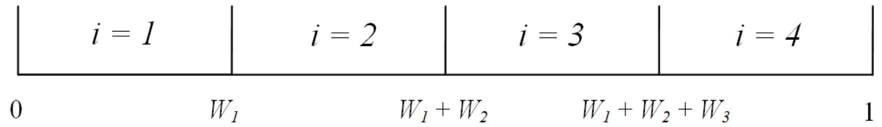


Figure 5: Graphical Depiction of Target Selection For Heuristic II

Next, a number is randomly generated between 0 and 1. A target is selected based on the random number's location on the scale. After selection, the matrices are updated as in Heuristic I and the process continues until all feasible targets are exhausted for every UAV.

Heuristic II has two advantages over Heuristic I. Primarily, due to its stochastic nature, it allows for the solution to avoid getting stuck at a local minimum. Furthermore, it allows for multiple initial columns to be generated for column generation. The computational results shown in Section 4 indicate a significant advantage of using Heuristic II over Heuristic I.

3.3.4. Local Search Heuristic

A local search heuristic was developed to solve the simultaneous sensor selection and routing problem and is outlined in Table 11. The basis of the heuristic relies on sensor benefit similarities between targets. Intuitively, targets that have similar sensor benefit requirements should be included on the same route for several reasons. Primarily, a sensor or small subset of sensors can provide similar surveillance for every target that is visited. It would not be advantageous to develop routes comprised of targets with completely disjoint sensor benefits because benefit will not be achieved at each target unless an equivalent number of sensors is attached. Attaching many sensors hinders travel range which in turn reduces the quantity of targets that may be visited on a given route. Equation (44) is used to determine the similarity, χ_{ab}, between a pair of targets.

$$\chi_{ab} = \sum_{s} |R_{as} - R_{bs}| \tag{44}$$

Here, R_{as} and R_{bs} indicate the benefit obtained when the first and second members of the target pair are visited by sensor s, respectively. A value of zero indicates that the targets have identical benefit requirements. As the value increases, the sensor benefits of the target pair become dissimilar.

Table 11: Local Search Heuristic

Step 0	Set iteration counter to 0 Set no improvement iteration counter to 0 Set iteration limit Set no improvement iteration limit Create initial routes for each vehicle
Step 1	Remove targets from routes until sensors can feasibly be added for all UAVs and solve the sensor assignment problem
Step 2	Add most beneficial targets to UAV routes until no additional targets can feasibly be added Compare mission effectiveness with current best and update if necessary If an improved solution was not found, increase no improvement iteration counter by 1
Step 3	Increase iteration counter If iteration counter or no improvement counter reach their respective limits, stop. Otherwise, go to **Step 1**

Along with setting iteration information, **Step 1** establishes routes for each UAV considered in the problem. To establish these initial routes, the similarity rating is computed for each target pair. The pair of targets A, B that are most dissimilar (have the highest similarity rating) are selected, as indicated in Equation (45).

$$A, B = \max \sum_{s} |R_{as} - R_{bs}| \tag{45}$$

The process continues by selecting targets that are most dissimilar to previously selected targets, until the number of targets selected is equivalent to the number of UAVs in the mission. These targets will be the first added to each of the UAV routes, with the target farthest away from the base being assigned to the UAV with the greatest range. After the first target is assigned, additional targets are assigned which are most similar in terms of sensor requirements to those existing on the route. The idea is to diversify the type of surveillance performed by each UAV route, while maximizing the surveillance each UAV performs with a subset of sensors. The latter is accomplished by assigning targets to routes that have low similarity scores $\chi_{a,b}$ with each other. The addition of targets to each route ceases when no targets can be added without exceeding the travel time of the UAV.

Next, sensors are assigned to each UAV in the fleet by solving the Sensor Selection Model.

Sensor Selection Model

$$\text{maximize}$$
$$\sum_{j,s,h} R_{js} z_{jsh} \tag{46}$$
$$\text{subject to}$$
$$\sum_{h} f_{hs} \leq Q_s \qquad \forall s \tag{47}$$
$$\sum_{h} z_{jsh} \leq V_{js} \qquad \forall j, s \tag{48}$$
$$z_{jsh} \leq f_{sh} \qquad \forall j, s, h \tag{49}$$
$$z_{jsh} \leq Y_{jh} \qquad \forall j, s, h \tag{50}$$
$$\sum_{s} f_{hs} \leq \tau_h \qquad \forall h \tag{51}$$
$$\sum_{s} C_s f_{hs} \leq \Psi_h \qquad \forall h \tag{52}$$
$$\sum_{s} f_{hs} \geq 1 \qquad \forall h \tag{53}$$
$$f_{hs} \in \{0, 1\} \qquad \forall h, s \tag{54}$$

The formulation follows that of the ISSRM, with the objective of maximizing surveillance benefit. The difference is that this formulation is only concerned with sensor placement and utilizes existing routes determined from other steps in the heuristic. The routes are represented in the binary variable Y_{jh}, where a value of 1 indicates that that UAV h visited target j and a value of 0 indicates no visitation took place. Additionally, Constraint (52) ensures that the sensor attachments do not violate the remaining travel time of a UAV.

Clearly, if the remaining travel time of the UAV is not great enough to assign the lightest sensor, the model return with infeasibility due to Constraint (53). Prior to solving the model, a set number of targets is randomly removed from each route to accommodate sensor assignments. The number of targets removed will determine the quantity and type of sensors that can be added when the model is solved. Thus, prior to removing targets, a set of sensors is randomly selected. Targets are randomly removed in a sequential manner until the selected sensor set can feasibly be assigned. It should be noted that the sensor set may be any size up to that permitted by the UAV.

After the model is solved and sensors are assigned to the UAVs, there is a reasonable possibility that additional travel time will remain and thus, opportunity for additional target insertions. Equation (55) is the basis for target insertions.

$$\frac{\sum_{s \in S} R_{js} + \sum_{s \notin S} \frac{R_{js}}{C_s}}{(Cost_{Insertion})(1 + \sum_h I_{jh}) + \sum_{b \in r_{current}} \chi_{jb}} \tag{55}$$

Step 2 evaluates Equation (55) for each target not currently included in a given route. Targets included in other routes are not omitted from consideration. The first term of the numerator evaluates how well target j benefits the mission with the current set of sensors, while the second term evaluates the targets benefit potential if included with additional sensors. The second term also considers the travel time hindrance incurred when a sensor is attached. The first term of the denominator considers the product of insertion cost and the number of times the considered target appears in other routes. The similarity, as discussed in Equation (44) of the entering target with existing targets in the current route, $r_{current}$, is evaluated in the second term. Looking at the numerator and denominator independently, it is clear that a desirable target would have a large numerator and small denominator. Therefore, targets with the highest overall value are considered first for entry.

Targets are ranked for each route and sequentially added based on feasibility, until all feasible target entries are exhausted. After this step is completed for all routes, the mission effectiveness is evaluated, and compared to the best discovered thus far, which is updated accordingly. The heuristic updates the iteration counter and concludes if the maximum number of iterations has been reached, or a predefined number of iterations are subsequently performed with no solution improvement. Otherwise, the procedure returns to **Step 1**.

3.4. Two-Phase Sensor Selection and Routing Heuristic

As mentioned in Section 3.2, a heuristic was necessary to solve the subproblem. The Two Phase Sensor Selection and Routing Heuristic (TPSSRH) assigns sensors in the first phase and selects routes in the second phase. All of the constraints found in the subproblem are satisfied, and the heuristic makes decisions using derivations of the objective function. Phase I begins by calculating the potential sensor benefit of each sensor included in the mission as shown in Equation (56). This value is calculated assuming that each sensor can visit all of the targets.

$$\text{Potential Benefit}_s = \sum_j R_{js} + \sum_j \psi_{js} V_{js} - \beta_s \tag{56}$$

Next, the sensors are ranked with respect to their potential benefit, and the top three sensors are selected. The final step of phase one creates a list indicating the benefit of visiting all targets with each sensor independently, and all combinations of these sensors. For simplicity, single sensors will be referred to as a combination. Hence, seven lists will be developed. The value of each target in the list is calculated by Equation (57).

$$\text{Benefit of Visiting Target}_j = \sum_{s \in S} R_{js} - \sum_{s \in S} \nu_{js} - \sum_{s \in S} \psi_{js} \tag{57}$$

S represents the set of sensors considered in each of the seven combinations. Clearly, if a UAV was only capable of holding two sensors, only six of the seven combinations should be used.

The procedure can also be adjusted to accommodate UAVs capable of holding more than three sensors by increasing the number of individual sensors considered. This concludes the first phase of the procedure.

One of the generated sensor combinations and associated target lists is selected at random and passed to the second phase of procedure which is outlined in Table 12. This phase utilizes a technique presented by Bodin et al. (1983).

Table 12: Two-Phase Sensor Selection and Routing Heuristic

Step 0	Set l, P, R_0, and $T = 0$ Choose α, set τ = adjusted travel time
Step 1	Assume UAV begins at base Compute δt and $\Delta P - R_0 \Delta t$ for all targets i in the target list passed in from Phase I Select i that maximizes Δt and $\Delta P - R_0 \Delta t$ and satisfies time constraints Insert target i in the current route, remove i from target list and proceed to **Step 2**
Step 2	Update l, R_l, P, τ, and T, then proceed to **Step 3**
Step 3	Compute Δt and ΔP for all possible insertions of remaining targets in the target list Select target from list that maximizes $\Delta P - R_0 \Delta t$ and satisfies time constraints If such a target exists, insert it in the route, remove target from list, and proceed to **Step 2** If such a target does not exist, go to **Step 5**
Step 4	End procedure and check reduced cost of the solution

The iteration number is defined as l, while P and T correspond to the cumulative benefit and cumulative travel time, respectively. R_l is defined as the worth of a time unit at iteration l. This value is a ratio of the cumulative sensor benefit to the cumulative travel time. Thus, the value of R_l will be high when the cumulative sensor benefit is large relative to the cumulative travel time. α is used to prevent rapid fluctuations in R_l and introduce randomness to the procedure. This will be discussed shortly. In **Step 0**, P, T, and R_0 are set to 0 and a value for α is selected. τ is set to the travel time adjusted for the attached sensors passed in from the first phase of the algorithm.

Step 1 assumes that each UAV begins at the base location. The change in time Δt and change in benefit ΔP is computed for each target i in the target list, which is passed in from the first phase. Δt and ΔP are computed by Equations (58) and (59), respectively.

$$\Delta t = t(0, i) + t(i, 0) \tag{58}$$

$$\Delta P = P(i) \tag{59}$$

The target i which maximizes ΔP, does not violate $\Delta t \leq \tau$, and satisfies the appropriate time window is selected. Once selected, target i is removed from the target list. This concludes the first step of phase two.

Step 2 updates all of the variables. The iteration, cumulative benefit, cumulative travel time, and remaining travel time are updated according to Equations (60) through (63).

$$l = l + 1 \tag{60}$$

$$P = P_{l-1} + \Delta P \tag{61}$$

$$T = T_{l-1} + \Delta T \tag{62}$$

$$\tau = \tau_{l-1} - \Delta T \tag{63}$$

To update the worth of a time unit, Equation (64) is evaluated.

$$R_l = \frac{\alpha P}{T} + (1 - \alpha)(R_{l-1}) \tag{64}$$

As mentioned earlier, α is used to prevent extreme changes in R_l. It is similar to the way in which a smoothing constant is used in forecasting models. Furthermore, if the procedure stalls and continuously generates identical columns, the value of α can be altered to introduce randomization.

Step 3 adds subsequent targets to the route. For each target k in the target list, Δt and ΔP - $R_l(\Delta t)$ are computed for insertion between all existing targets i and j currently in the route. The calculations are performed using Equations (65) and (66).

$$\Delta t = t(i,k) + t(k,j) - t(i,j) \tag{65}$$

$$\Delta t - R_{l-1}\Delta t = P(k) - R_l \Delta t \tag{66}$$

The triplet that maximizes ΔP - $R_l(\Delta t)$ and satisfies $T + \Delta t \leq \tau$ as well as time window constraints for k and all existing targets in the route is selected. If such a triplet exists, target k is inserted into the current route and removed from the target list. The process then moves to **Step 2**. If a triplet does not exist that satisfies these requirements, no additional targets can feasibly be added to the route and the process advances to **Step 4**.

Step 4 checks the reduced cost of the discovered solution. If it is favorable, the route and sensor combination is added to the master problem as a new column. If the reduced cost is not favorable the current sensor combination is removed from consideration. A new combination is randomly selected from phase one, and phase two is restarted. If all sensor combinations are exhausted and a favorable solution is not found, no column is added for the UAV at the current CG iteration. However, a favorable column may still be discovered for the UAV in a future CG iteration. Column generation ends when an iteration is unable to find a favorable column for all UAVs, or a predefined maximum number of columns are generated.

4. Computational Experience

4.1. Experimental Conditions

The computational experiments were carried out using software developed in C++ and Java. CPLEX 12.1 was used to solve the ISSRM, the sensor selection phase of the local search heuristic, and the master problem for the column generation heuristic. All testing was carried out on a PC with a 3.00GHz Intel Core 2 Duo processor and 4GB of RAM. Since the authors were unaware of any existing instances of this problem, fourteen scenarios were developed. Ten replications of each scenario were run, for a total of one hundred and forty test cases. Table 13 summarizes the test cases.

Table 13: Summary of Test Cases

Number of Targets	**Fleet Size**	**Dimensions of Target Field**
15	2	100 x 100
30	2,3,4	100 x 100
50	3,4,5,6	150 x 150
100	3,4,5,6,7,8	200 x 200

Target locations were uniformly distributed within the target field and three sensors were considered for all test cases. The benefit assigned for each sensor/target pair was generated as follows. The probability that a target had benefit from the first sensor was 1, while the probabilities for sensor 2 and sensor 3 were 0.8 and 0.64, respectively. Given that a target benefited from a sensor, the actual benefit value was assigned using the distribution in Table 14.

Table 14: Distribution of Sensor/Target Benefit

Benefit Range	**Probability**
1 - 2	0.1
3 - 5	0.1
6 - 8	0.3
9 - 12	0.5

The unloaded range of each UAV in the fleet is dependent on the number considered in the problem. The first UAV had an unloaded range of 250 units and each subsequent UAV inclusion added an additional 50 units. Hence, if an eighth UAV was considered, its unloaded range was 600 units. The distance reduction imposed for sensor 1, sensor 2, and sensor 3 was 125, 40, and

100, respectively. Column Generation was run executed for a maximum of 1000 iterations in all test cases. While we generated and tested problems with time windows, they tend to make the problem easier to solve. For this reason, the numerical tests do not include time windows.

4.2. Small and Medium Sized Problems

CPLEX was given two hours to solve the test cases presented in Table 15. These results are reported in the ISSRM column.

Table 15: Results For 15 Targets and 2 UAVs

Run #	ISSRM	Heuristic I	Heuristic II	Local Search	CG	Percent Increase
1	113	69	74	113	113	0.0%
2	97	65	74	83	103	24.1%
3	148	118	132	138	138	0.0%
4	116	95	104	104	104	0.0%
5	130	105	127	101	127	0.0%
6	120	92	131	51	131	0.0%
7	93	58	88	84	95	7.9%
8	74	74	82	82	84	2.4%
9	114	90	103	93	103	0.0%
10	104	105	110	115	115	0.0%

The solution times for Heuristic I and Heuristic II were essentially instantaneous and Column Generation solutions were obtained within 10 seconds for all cases. The maximum number of iterations for Local Search was set to 50,000 and terminated when no improvement was found in 1,000 subsequent iterations. Local Search and Heuristic II independently found the best solution in 40% of the cases. In the remaining 20% of the cases, Local Search and Heuristic II both found the best solution. Heuristic I was outperformed by either Heuristic II or Local Search in each test case. Column Generation was started with the heuristic solution that provided the best result. In the event of a tie, both solutions were used as initial columns. In 30% of the cases, Column Generation improved the solution. Of these cases, the average improvement in solution quality was 11.5%. Based on these results, Heuristic II and Local Search provide acceptable results for problems of relatively small size. Heuristic I was outperformed by Heuristic II in every instance and does not provide any significant saving in computation time. The use of Column Generation is justified as it significantly improved the solution in several instances with minimal increase in solution time.

The results for moderately sized test cases containing 30 targets are shown in Tables 16 - 18. A comparison is made between the ISSRM's progress after predefined time intervals, each of the three heuristics, and Column Generation. Column Generation results are independently provided when initiated using initial columns from Heuristic I, II and Local Search. For many instances, CPLEX was not able to find either a feasible solution in the desired time interval which is reported in the table as NS. Column Generation solutions were obtained within 20 seconds for all runs. With the exception of run # 12, Local Search was the best initial heuristic solution and was improved by Column Generation in many of the test cases.

Table 16: Results For 30 Targets and 2 UAVs

Run #	**ISSRM 1 min**	**ISSRM 3 min**	**ISSRM 5 min**	**Heuristic I**	**Heuristic II**	**CG HI,II**	**Local Search**	**CG LS**
11	148	148	148	127	161	168	215	215
12	106	106	106	124	142	148	139	139
13	94	94	121	129	153	153	188	188
14	121	159	159	149	157	159	199	199
15	69	90	101	100	103	107	151	151
16	39	80	80	113	124	124	145	145
17	160	170	170	173	174	185	225	225
18	89	89	89	86	91	101	144	144
19	115	130	130	102	109	121	184	184
20	85	85	99	123	136	139	181	181

Table 17: Results For 30 Targets and 3 UAVs

Run #	**ISSRM 1 min**	**ISSRM 3 min**	**ISSRM 5 min**	**Heuristic I**	**Heuristic II**	**CG HI,II**	**Local Search**	**CG LS**
21	NS	NS	125	127	160	174	273	273
22	NS	NS	NS	183	219	219	258	258
23	NS	NS	NS	140	163	181	236	236
24	NS	97	97	151	176	190	242	242
25	NS	NS	NS	186	206	224	285	285
26	NS	115	115	140	163	181	233	240
27	NS	0	0	164	177	181	279	287
28	NS	NS	NS	143	164	178	205	230
29	NS	NS	NS	194	213	222	267	267
30	NS	NS	NS	175	192	199	295	295

Table 18: Results For 30 Targets and 4 UAVs

Run #	ISSRM 1 min	ISSRM 3 min	ISSRM 5 min	Heuristic I	Heuristic II	CG HI,II	Local Search	CG LS
31	NS	NS	NS	229	250	250	301	301
32	NS	NS	NS	250	294	304	384	384
33	NS	NS	NS	211	251	256	341	350
34	NS	NS	NS	263	305	319	388	388
35	NS	NS	NS	231	271	281	343	348
36	NS	NS	NS	273	275	283	409	409
37	NS	NS	NS	252	262	283	370	377
38	NS	NS	NS	212	252	359	260	270
39	NS	NS	NS	283	317	319	432	432
40	NS	NS	NS	252	273	278	350	361

The results presented in Tables 16 - 18 represent moderately sized problems. CPLEX was given 5 minutes to solve the ISSRM and was outperformed by heuristic solutions in all thirty runs. Local Search solved quickly with solution times ranging from 2.74 - 10.88 seconds. The average solution time for Local Search was 5.21 seconds. When a solution is needed quickly for problems of this size, it is evident that the three heuristics augmented by CG provide results that are more favorable than those produced by directly solving the ISSRM.

4.3. Large Size Problems

The results for larger problems are displayed in Table 19. For simplicity, average results for each scenario are provided. The final integer master problem was unable to converge in some of the test cases. Thus, a time limit of 5 minutes was imposed on the solution procedure.

Table 19: Results For Large Problems

Target Quantity	Fleet Size	Heuristic I	Heuristic II	Local Search	CG	% Improvement Min	Avg	Max
50	3	125	140.3	208.1	211.5	0.0	1.9	5.9
50	4	198.7	231.4	293.6	296.1	0.0	0.8	4.0
50	5	253.6	297.2	394.5	402.1	0.0	1.9	5.0
50	6	305.6	366.1	478.9	488.5	0.0	2.1	6.5
100	3	152.7	173.8	253.2	254.9	0.0	0.7	5.0
100	4	236.1	266.5	341.1	344.6	0.0	1.0	6.0
100	5	279.4	311.7	426	431.4	0.0	1.4	5.2
100	6	395.6	434.8	565.9	572.2	0.0	1.1	3.4
100	7	436.5	481.4	653.4	664.3	0.0	1.6	2.9
100	8	548.1	583.8	738.6	752.4	0.8	1.9	3.9

Local Search was the superior algorithm for each test case, and Heuristic II once again outperformed Heuristic I in every scenario. Column generation was able to improve the initial solution in 60 out of the 100 test cases. The right most column in Table 19 represents the average overall percent improvement and includes the 40 cases where CG did not improve the solution. For the 60 cases where column generation did improve the solution, the average percent improvement was 2.4%, with a maximum improvement of 6.5% and a minimum improvement of 0.2%. Once again, the use of Column Generation is justified as it provides noticeable improvement while adding relatively little to the total computation time.

4.4. Fleet Sizing Application

As evident from Sections 4.2 and 4.3, column generation provides a good solution to the sensor selection and routing problem and solves relatively fast. This behavior allows the solution approach to be used as a tool for fleet sizing. A mission planner may be unsure as to how many UAVs he or she should allocate to a mission. Alternative solutions for a mission could be obtained quickly for a varying number of UAVs. The mission planner could then evaluate the surveillance benefit for each of the alternatives and make a decision. A fifteen target case with three sensors is presented for sake of illustration. The case was generated using the approach described in Section 4.1. The unloaded range of each UAV is 350 units.

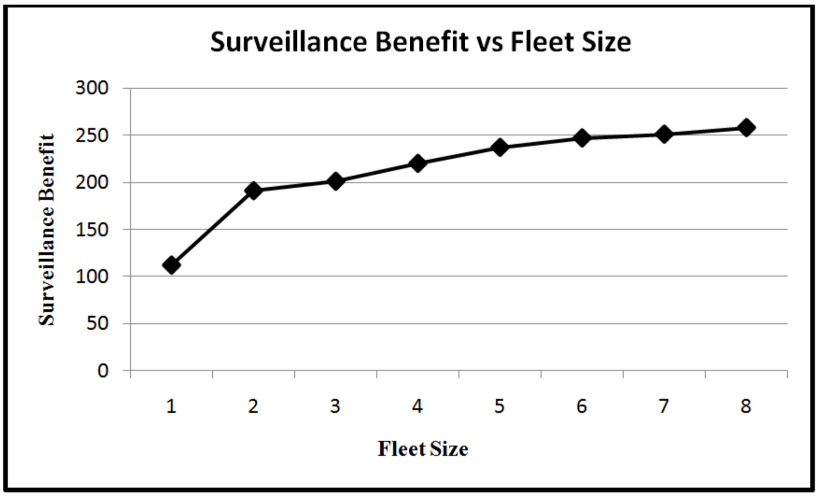


Figure 6: Fleet Sizing Example

Here, the surveillance benefit jumps significantly when the fleet size increases from one to two UAVs. As the fleet size rises from 2 to 6, the surveillance benefit experiences a linear increase. The addition of 7 or 8 UAVs has a marginal impact on surveillance benefit.

While fleet size is ultimately determined by the mission planner, the algorithms presented in this work can certainly aid in the decision. For example, an analysis of Figure 6 may lead a mission planner to question the inclusion of more than 6 UAVs. The cost per UAV and cost per unit of surveillance benefit may also be factored into the decision.

5. Conclusions and Future Work

The sensor selection and routing of unmanned aerial vehicles can be modeled as a generalization of the team orienteering problem. While the ISSRM developed in this work solves well using CPLEX for small problems, larger problems required the use of heuristics augmented with column generation. All three heuristics solved quickly, with local search and Heuristic II consistently outperforming Heuristic I. For small problems, Local Search and Heuristic II shared the ability to provide the best solution. As the problem size grows, Local Search consistently provides superior results. Column Generation improved the solution in many of the test cases, but also did not significantly increase the overall solution time. Thus, it should be included in the solution procedure. Also, since the solution approach finds a good solution very quickly, it provides a foundation for applications beyond sensor and route selection. This work demonstrated the solution approach functioning as a fleet sizing tool for a single mission. This idea can be extended to multiple missions and will be briefly discussed.

Consider the role of a mission planner with a set of target clusters requiring surveillance. The target clusters are not in close proximity, so each will require a separate mission plan. If the clusters were close to one another, only one mission plan would be required which could be obtained using the procedures presented in this work. Also, it is assumed that the surveillance of each target cluster is simultaneous, so UAVs and sensors may not be shared among mission plans. Thus, the resources allocated to each target cluster will play a significant role in the success across all missions. Since the solution procedure presented solves quickly, multiple allocation scenarios could be evaluated in a short amount of time. Furthermore, the solution procedure could be implemented within a systematic approach designed to optimize UAV and sensor allocations across multiple simultaneous missions.

Lastly, the ISSRM could not find an optimal solution for relatively small test cases. The authors are currently investigating a branch and cut approach as a more effective method to prove optimality for simultaneous sensor selection and routing problems.

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References

Archetti, C., A. Hertz, M. G. Speranza. 2007. Metaheuristics for the team orienteering problem. *Journal of Heuristics* **13** 49–76.

Bodin, L., B. Golden, A. Assad, M. Ball. 1983. Special issue - routing and scheduling of vehicles and crews - the state of the art. *Computers & Operations Research* **10**.

Boussier, S., D. Feillet, M. Gendreau. 2007. An exact algorithm for team orienteering problems. *4OR: A Quarterly Journal of Operations Research* **5** 211–230.

Butt, S. E., T. M. Cavalier. 1994. A heuristic for the multiple tour maximum collection problem. *Computers & Operations Research* **21** 101–111.

Butt, S. E., D. M. Ryan. 1999. An optimal solution procedure for the multiple tour maximum collection problem using column generation. *Computers & Operations Research* **26** 427–441.

Chao, I. M., B. L. Golden, E. A. Wasil. 1996. The team orienteering problem. *European Journal of Operational Research* **88** 464–474.

Golden, B., L. Levy, R. Vohra. 1987. The orienteering problem. *Naval Research Logistics* **34** 307–318.

Kantor, M. G., M. B. Rosenwein. 1992. The orienteering problem with time windows. *Journal of the Operational Research Society* **43** 629–635.

Ke, L., C. Archetti, Z. Feng. 2008. Ants can solve the team orienteering problem. *Computers & Industrial Engineering* **54** 648–665.

Kim, Y., D. W. Gu, I. Postlethwaite. 2008. Real-time path planning with limited information for autonomous unmanned air vehicles. *Automatica* **44** 696–712.

Laporte, G., S. Martello. 1990. The selective traveling salesman problem. *Discrete Applied Mathematics* **26** 193–207.

Lubbecke, M. E., J. Desrosiers. 2005. Selected topics in column generation. *Operations Research* **53** 1007–1023.

Nikolos, I. K., K. P. Valavanis, N. C. Tsourveloudis, A. N. Kostaras. 2003. Evolutionary algorithm based offline/online path planner for uav navigation. *IEEE Transactions on Systems Man and Cybernetics Part B-Cybernetics* **33** 898–912.

Qu, Y., Q. Pan, J. Yan. 2005. Flight path planning of UAV based on heuristically search and genetic algorithms. *31st Annual Conference of the IEEE Industrial Electronics Society*. Raleigh, North Carolina, USA.

Scaled Composites. 2004. Proteus payload user's guide. `http://www.scaled.com/images/uploads/projects/proteus_payload_users_guide.pdf`.

Tang, H., E. Miller-Hooks. 2005. A tabu search heuristic for the team orienteering problem. *Computers & Operations Research* **32** 1379–1407.

Tsiligirides, T. 1984. Heuristic methods applied to orienteering. *Journal of the Operational Research Society* **35** 797–809.

Vansteenwegen, P., W. Souffriau, G.V. Berghe, D. Oudheusden. 2009. A guided local search metaheuristic for the team orienteering problem. *European Journal of Operational Research* **196** 118–127.

Vansteenwegen, P., W. Souffriau, D. Oudheusden. 2010. The orienteering problem: A survey. *European Journal of Operational Research* **In Press, Corrected Proof**.

Warrier, A. 2001. A decision support tool for the inventory allocation and vehicle routing problem. Thesis, Department of Industrial and Systems Engineering, State University of New York at Buffalo.

Wu, P., D. Campbell, T. Merz. 2009. On-board multi-objective mission planning for unmanned aerial vehicles. *IEEE Aerospace Conference*. Big Sky, Montana, USA, 1–10.

Yang, G., V. Kapila. 2002. Optimal path planning for unmanned air vehicles with kinematic and tactical constraints. *Proceedings of the 41st IEEE Conference on Decision and Control*. Las Vegas, Nevada, USA, 1301–1306.