Recursion

Lecture Question

Restriction: No state is allowed in this question. Specifically, the keyword "var" is banned. (ie. You are expected to use a recursive solution)

Method: In a package named "functions" create an object named "Algebra" with a method named "factor" that takes an Int as a parameters and returns the prime factorization of that parameter as a List of Ints.

The following apply to this method:

- If the input is negative or 0, return an empty list
- If the input is 1, return an empty list (Note: This is different than the original question)
- Do not include 1 in the output for any inputs
- The order of the factors in the output List is undefined

Example: functions.Algebra.factor(12) can return List(2,2,3) -or- List(2,3,2) -or- List(3,2,2)

Unit Testing: Testing will be checked by AutoLab, though you are encouraged to use your tests from the last time we had this question (Be sure to update your test for 1 as the input)

Recursion Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Computes the geometric sum of the input
  - ex: if n == 3, geometric sum is 3+2+1 == 6

Recursion Example

```
def computeGeometricSum(n: Int): Int ={
  if(n <= 0){
    0
  }else{
    n + computeGeometricSum(n - 1)
  }
}


def main(args: Array[String]): Unit = {
  val result: Int = computeGeometricSum(3)
  println(result)
}
```

- Base Case:
  - An input with a trivial output
  - Geometric sum of 0 is defined as 0
  - We could also add 1 -> 1 as a base case

Recursion Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Recursive Step:
  - Any input that is not a base case will put another recursive call on the stack
  - Write the recursive step with the assumption that the recursive call will return the correct value

Recursion Example

```
def computeGeometricSum(n: Int): Int ={
  if(n <= 0){
    0
  }else{
    n + computeGeometricSum(n - 1)
  }
}


def main(args: Array[String]): Unit = {
  val result: Int = computeGeometricSum(3)
  println(result)
}
```

- Recursive calls must get closer to the base case
  - All calls must eventually reach a base case or we'll go infinite
  - n-1 is closer to n<=0 than n
  - Regardless of the original value of n, it will eventually be decremented until the base case condition is true

Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	name:n, value:1
	pointer -> line 5

- Each recursive calls creates a new stack frame
- Each frame remembers where it will resume running when it's on the top of the stack

Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	name:n, value:1
	pointer -> line 5
Method Frame	name:n, value:0
	pointer -> line 1

- New frames start at the first line of the method
- Top frame on the stack executes the method one line at a time

Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	name:n, value:1
	pointer -> line 5
Method Frame	name:n, value:0
	pointer -> line 3

- Recursive calls are added to the stack until a base case is reached

Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- When a method call returns, its frame is destroyed
- The calling frame resumes and uses the returned value

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	name:n, value:1
	pointer -> line 5
Method Frame	returning 0

Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	name:n, value:1
	pointer -> line 5
	gets return value of 0

- Method continues after the recursive call
- Sums n + return value and returns this value

Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	returning 1

- This frame reaches the end of the method
- Repeat the process

Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
	gets return value of 1

- Return value
- Pop off the stack
- Resume execution of the top frame

Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	returning 6

- Process continues until all recursive calls resolve
- Last frame returns to main

Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Main continues with the result from the recursive calls

Program Stack	
Main Frame	args
	pointer -> line 11
	gets return value of 6

Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Main continues with the result from the recursive calls

Program Stack	
Main Frame	args
	name:result, value:6
	pointer -> line 12

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Recall anagrams
  - Rewritten to use functional programming and no vars
  - The syntax may not fully make sense until Monday's lecture

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Base Case
  - A String of length 1 is itself its only anagram
  - If the length is 1, return a new list containing only that String

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Base Case Note
  - We will eventually return a list containing all anagrams from the top level call
  - The base case is the only time we create a new List

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Recursive Step
  - For each character in the input String
    - Remove that character and make recursive call with the remaining characters
    - Append the removed character to all the returned anagrams

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Recursive Step
  - We write this code with the assumption that our recursive calls will return all the anagrams of the new Strings
  - If our logic is sound, this assumption will be true through the power of recursion!

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Always reach a base case
  - We always make recursive calls on the input String with 1 character removed
    - newString.length == input.length -1
  - This always gets us closer to the base case

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Always reach a base case
  - When the base case is reached and returned, our logic starts working for us
  - If this code does append the removed character each returned anagram, output is generated starting at the base case

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Example:
  - input == "at"
  - Makes 2 recursive calls to the base case
    - "a" and "t" are returned
  - Append "t" to "a" and "a" to "t" (The removed characters)
  - Return ["at", "ta"] to the next recursive call with an input of length 3

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Functional Programming notes (More detail Monday)
  - yield: Creates a data structure containing the last expression that was evaluated on each iteration of the loop

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Functional Programming notes (More detail Monday)
  - map: Creates a new data structure by applying a function to each element (Similar to Monday's lecture question)
    - The _ is shorthand syntax we can use when a function only has 1 input and its type can be inferred

Anagrams Example

```scala
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Functional Programming notes (More detail Monday)
  - Scala data structures come with many helpful FP style methods
  - Flatten: Creates a single List from a List of Lists containing all the elements from each List
  - Distinct: Creates a new List with all duplicate values removed

Lecture Question

Restriction: No state is allowed in this question. Specifically, the keyword "var" is banned. (ie. You are expected to use a recursive solution)

Method: In a package named "functions" create an object named "Algebra" with a method named "factor" that takes an Int as a parameters and returns the prime factorization of that parameter as a List of Ints.

The following apply to this method:

- If the input is negative or 0, return an empty list
- If the input is 1, return an empty list (Note: This is different than the original question)
- Do not include 1 in the output for any inputs
- The order of the factors in the output List is undefined

Example: functions.Algebra.factor(12) can return List(2,2,3) -or- List(2,3,2) -or- List(3,2,2)

Unit Testing: Testing will be checked by AutoLab, though you are encouraged to use your tests from the last time we had this question (Be sure to update your test for 1 as the input)